

PROCEEDINGS

Nonlinear Dynamics Modeling and Stability–Controllability Analysis of Two-Spacecraft Radial Coulomb Formations

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ABSTRACT

With the rapid development of on-orbit servicing, space debris removal, and space situational awareness missions, spacecraft on-orbit operations are evolving toward higher precision, longer mission duration, and lower energy consumption. Conventional formation keeping and hovering typically rely on continuous thrust or frequent impulsive maneuvers, which not only increase system complexity but also lead to substantial propellant consumption, thereby limiting mission lifetime and overall cost-efficiency. In this context, Coulomb formation flying, which utilizes electrostatic interactions between charged spacecraft to generate controllable inter-satellite forces, has emerged as a promising propellant-free or low-consumption alternative. By regulating onboard charges, sustained and adjustable forces can be produced without conventional propulsion systems, enabling long-duration hovering and formation maintenance. Furthermore, two-spacecraft formations can be extended to distributed space-based sensing systems, such as spaceborne synthetic aperture radar (SAR), for on-orbit detection and early warning of space debris and other non-cooperative targets, demonstrating significant engineering potential.

Despite these advantages, the dynamic modeling and analysis of Coulomb formations remain challenging. Existing studies are predominantly based on relative motion frameworks, where the deputy spacecraft is described with respect to a reference spacecraft using the Clohessy–Wiltshire (CW) equations. While such approaches offer analytical simplicity under local linear approximations, their state variables—typically relative position and velocity components—lack explicit geometric interpretation, making it difficult to characterize formation scale and spatial orientation. Consequently, their applicability to complex formation configuration design and control is limited. Alternatively, some studies adopt geometric parameters such as inter-satellite distance, pitch angle, and yaw angle as state variables to construct formation dynamics models, thereby enhancing geometric interpretability. However, these approaches often rely on simplifying assumptions, such as neglecting higher-order coupling terms or approximating nonlinear effects, which compromise model accuracy. More importantly, a two-spacecraft formation inherently exhibits six degrees of freedom, whereas these reduced-order representations describe only three degrees of freedom using formation geometry parameters. As a result, the system state space is incompletely represented, particularly due to the omission of the formation center-of-mass orbital dynamics. This deficiency prevents a consistent description of the intrinsic coupling between overall orbital motion and internal configuration evolution.

To address these limitations, this paper develops a complete and coupled nonlinear dynamic modeling framework with explicit geometric representation for two-spacecraft radial Coulomb formations. By introducing a unified state vector consisting of the inter-satellite distance, formation attitude angles, and center-of-mass orbital states, a systematic nonlinear dynamic model incorporating Coulomb interactions is derived under a hybrid coordinate framework combining the inertial frame, the local-vertical local-horizontal (LVLH) frame, and a formation-fixed frame. The proposed framework enables, for the first time, a consistent and unified description of the coupling



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between overall orbital motion and internal configuration evolution, thereby overcoming the inherent limitations of existing reduced-order or decoupled models. For control-oriented analysis, the nonlinear model is further linearized around equilibrium configurations, yielding a structured linear dynamic model with explicit parameter dependence.

Based on the nonlinear model, the dynamic evolution of the system under typical orbital perturbations and parameter uncertainties is investigated. The results reveal the intrinsic coupling mechanisms between Coulomb-induced nonlinear effects and orbital dynamics, and clarify how these interactions govern the stability evolution of the formation, providing essential insights for robust control design. Furthermore, using the linearized model, the stability and controllability characteristics of the system are systematically analyzed through eigenvalue analysis and controllability criteria under varying charge configurations, formation scales, and attitude parameters. It is shown that system stability exhibits strong sensitivity to charge distribution and formation geometry, with distinct stability regimes emerging under different parameter configurations, while improving stability margins generally requires increased charge regulation capability or higher control sensitivity. These findings explicitly demonstrate an inherent trade-off among stability margin, control effort, and configuration constraints.

In summary, this work establishes a unified nonlinear dynamic modeling framework for Coulomb-based two-spacecraft formations and systematically elucidates their fundamental dynamic characteristics and control mechanisms from the perspectives of stability and controllability. The main contributions of this work can be summarized as follows: 1) A complete and coupled nonlinear dynamic modeling framework with explicit geometric representation is developed, enabling a consistent description of the coupled evolution of orbital motion and formation configuration. 2) The nonlinear dynamic behaviors under typical perturbations are systematically analyzed, revealing the intrinsic coupling mechanisms between Coulomb-induced nonlinear effects and orbital dynamics, and providing insights into the stability evolution of the formation. 3) The stability and controllability properties are systematically characterized, demonstrating the sensitivity of system stability to charge distribution and formation geometry, as well as the existence of distinct stability regimes and inherent trade-offs among stability margin, control effort, and configuration constraints. These results provide theoretical support for the design of Coulomb-based formation control strategies and offer a scalable foundation for extending the proposed framework to multi-spacecraft Coulomb formations and complex on-orbit operations.

KEYWORDS

Coulomb formation; nonlinear dynamics; stability analysis; controllability analysis

Acknowledgement: The authors would like to thank the reviewers and editors for their valuable comments and constructive suggestions, which have helped improve the quality of this paper.

Funding Statement: This research was supported by the National Key R&D Program of China: Gravitational Wave Detection Project (No.2024YFC2207900), the National Natural Science Foundation of China (No.12472046), the Laboratory Foundation of the National Key Laboratory on Vacuum Technology and Physics (No.2025-JCJQ-LB-071), and the Innovation Foundation for Doctoral Dissertations of Northwestern Polytechnical University (No.CX2026007).

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have influenced the research and outcomes presented in this paper, ensuring the integrity and impartiality of the work.