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Liveness and boundedness preservations of sharing synthesis of Petri net based representation for embedded systems

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Petri net based Representation for Embedded Systems (PRES+) is an outstanding methodology for analysis, modeling and verification of embedded systems. State space explosion is an awful problem for PRES+ to model and analyze large complex embedded systems. In order to solve this problem, we concern with a method for expending PRES+ model by using synthesis approach. A kind of sharing synthesis operation for PRES+ is proposed in this paper. Under some conditions liveness and boundedness will be preserved by using this sharing synthesis approach. An applicable example in the form of an embedded control system illustrates the useful of our synthesis method. These results can be nicely used to investigate dynamic properties of large embedded systems.

Keywords: Petri nets, synthesis, liveness, property preservation, modeling

1. INTRODUCTION

Embedded systems have lots of applications, such as automotive controllers, cellular phones, network switches, consumer electronics, and household appliances, in our everyday life. Petri net based Representation for Embedded Systems (PRES+) is an extension to the classical Petri net model. It captures timing information, allows tokens to carry information explicitly, and supports a precise representation of embedded systems [1, 2]. PRES+ is an important methodology for modeling, verification, analysis and control of embedded systems [1-9].

State space explosion problem of PRES+ is somewhat awful to model and analyze large complex embedded systems. In order to solve state space explosion problem of Petri nets, many researchers have attempted transformation approaches in several ways [21]. It is important that a transformation should preserve some properties under investigation. There are three popular transformations, namely synthesis, refinement and reduction [10].

Petri net based synthesis is an important method to system design which can compose a system from several component modules in such a way that the system can be effectively analyzed for design correctness.

In the following a brief overview of synthesis methods for Petri nets is provided. Synthesis methods can be classified into following groups: bottom-up techniques, top-down techniques, hybrid approach, and knitting technique [11]. To present a modeling framework for semi-automated hyperflexible robotic cells in the aircraft industry, Basile [12] proposed a synthesis approach of a Colored Modified Hybrid Petri Nets (CMHPN) model. Hu [13] focused on the synthesis of distributed liveness-enforcing supervisors of AMSs allowing both flexible process routes and multiple resource acquisition operations. Finkbeiner [14] introduced Petri games, defined as place/transition Petri nets, where the finite set of places is portioned into a set of system places and a set of environment places. The concurrency-preserved parallel composition of two nets is obtained by taking component union [14]. Zhou [15] proposed a hybrid synthesis method for manufacturing systems using sequential and parallel mutual

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exclusions. Xia [16] proposed a shared pb-type subnet synthesis method for place/transition Petri nets that ensure the preservations of liveness and boundedness. Best [17] characterized the reachability graph of a (weakly live, connected) Petri net marked graph and described procedures to synthesize a (minimal) marked graph solving a labe1led transition system enjoying the characterizing properties. For simple sequential processes with resources systems, Liu [18] presented some algorithms to synthesize recovery subnets and monitors. Hu [19] proposed a synthesis method for logic Petri nets. Basile [20] proposed an effective approach for the synthesis of compact and decentralized supervisors for PN systems. For systems specified in P/T nets, Xia [21, 22] proposed a shared pp-type subnet synthesis method and investigated property preservations of this method. Liveness, boundedness, reversibility and some structural properties are preserved under some additional conditions.

In order to solve the state space explosion problem for PRES+, we presented a technique for the stepwise refinement for PRES+ model [7]. This refinement method can be used as a top-down approach for growing a PRES+ model of a system from an abstract level to a desired level of detail. During this refinement process, we prescribed reachability, functionality, timing, liveness and boundedness. Another property preservation refinement approach for PRES+ was proposed in [8]. To improve the verification efficiency of PRES+, we proposed reduction rules for PRES+ [9]. Under some conditions, these reduction rules preserve reachability, functionality, and timing of PRES+. In this paper, we will investigate another transformation—synthesis to solve the state space explosion problem. Firstly, we propose a sharing synthesis operation for PRES+. Then, we study liveness and boundedness preservations of the sharing synthesized PRES+ net system. Lastly, we demonstrate the effectiveness of our synthesis approach in the design process for embedded systems.

The rest of this paper is organized as follows. Some basic definitions of PRES+ are proposed in Section 2. The sharing synthesis operation of PRES+ is presented in Section 3. Preservations of liveness and boundedness are investigated in Section 4. An applicable modeling example of this synthesis method is presented in Section 5. Conclusions are given in Section 6.

2. BASIC DEFINITIONS OF PRES+

In this section we will provide some fundamentals for Petri net based Representation for Embedded Systems (PRES+). A more detailed and rigorous discussion on PRES+ can be found in [1, 2].

Definition 2.1[1] A PRES+ model is a five-tuple $N = (P, T, I, O, M_0)$ where, $P = \{p_1, p_2, ..., p_m\}$ is a finite non-empty set of places; $T = \{t_1, t_2, ..., t_n\}$ is a finite non-empty set of transitions; $I \subseteq P \times T$ is a finite non-empty set of input arcs which define the flow relation between places and transitions; $O \subseteq T \times P$ is a finite non-empty set of output arcs which define the flow relation between transitions and places; M_0 is the initial marking of the net.

Definition 2.2[1] A token is a park $k =< v, r >$ where v is the token value. The type of this value is referred to as token type; r is the token time, a non-negative real number representing the time stamp of the token.

Definition 2.3[1] The pre-set ${}^\bullet t = \{p \in P|(p, t) \in I\}$ of a transition $t \in T$ is the set of input places of t. Similarly, the post-set $t^\bullet = \{p \in P|(t, p) \in O\}$ of a transition $t \in T$ is the set of output places of t. The pre-set ${}^\bullet p$ and the post-set $p^\bullet$ of a place $p \in P$ are given by ${}^\bullet p = \{t \in T|(t, p) \in O\}$ and $p^\bullet = \{t \in T|(p, t) \in I\}$ respectively.

Definition 2.4[1] For every transition $t \in T$ there exists a transition function f associated to t, that is, for all $t \in T$ there exists $f : \tau(p_1) \times \tau(p_2) \times ... \times \tau(p_a) \to \tau(q)$, where ${}^\bullet t = \{p_1, p_2, ..., p_a\}, q \in t^\bullet$.

Definition 2.5[1] For every transition $t \in T$ there exists a minimum transition delay d^- and a maximum transition delay d^+, which are non-negative real numbers such that $d^- \leqslant d^+$ and represent, respectively, the lower and upper limits for the execution time of the function associated to the transition.

Definition 2.6[1] The firing of an enabled transition $t \in T$, for a binding $b = (k_1, k_2, ..., k_a)$ changes a marking M into a new marking M'. As a result of firing the transition t, the following occurs:

(i) Tokens from its pre-set ${}^\bullet t$ are removed, that is, $M'(p_i) = M(p_i) - \{k_i\}$ for all $p_i \in^\bullet t$;

(ii) One new token $k =< v, r >$ is added to each place of its post-set $t^\bullet$, that is, $M'(p) = M(p)+\{k\}$ for all $p \in t^\bullet$. The token value of k is calculated by evaluating the transition function f with token values of tokens in the binding $b$as arguments, that is$v = f(v_1, v_2, ..., v_a)$. The token time of k is the instant at which the transition t fires, that is, $r = tt^*$ where $tt^* \in [tt^-, tt^+]$.

(iii) The marking of places different from input and output places of t remain unchanged, that is, $M'(p) = M(p)$ for all $p \in P \backslash {}^\bullet t \backslash t^\bullet$.

3. SHARING SYNTHESIS OPERATION FOR PRES+

In this section, we will propose the sharing synthesis operation for PRES+. In the following, suppose $N_1 = (P_1, T_1, I_1, O_1, M_{10})$ and $N_2 = (P_2, T_2, I_2, O_2, M_{20})$ are two PRES+ models.

Definition 3.1 For transition $t_{1i} \in^\bullet p_{1i}$ of N_1 (where $p_{1i} \in P_1$, $1 \leq i \leq k$) there exists transition function $f_{1i} : \tau(p_{1i1}) \times \tau(p_{1i2}) \times \cdots \times \tau(p_{1is}) \to \tau(p_{1i})$ (where ${}^\bullet({}^\bullet p_{1i}) = \{p_{1i1}, p_{1i2}, \ldots, p_{1is}\}$). For transition $t_{2i} \in^\bullet p_{2i}$ of N_2 (where, $p_{2i} \in P_2$, $1 \leq i \leq k$) there exists transition function $f_{2i} : \tau(p_{2i1}) \times \tau(p_{2i2}) \times \cdots \times \tau(p_{2is}) \to \tau(p_{2i})$ (where ${}^\bullet({}^\bullet p_{2i}) = \{p_{2i1}, p_{2i2}, \ldots, p_{2it}\}$). $f_{1i} = f_{2i} (1 \leq i \leq k)$.

Definition 3.2 For transition $t_{1i} \in^\bullet p_{1i}$ of N_1 (where $p_{1i} \in P_1$, $1 \leq i \leq k$) there exists transition delay $[d_{2i}^-, d_{2i}^+]$. For transition $t_{2i} \in^\bullet p_{2i}$ of N_2 (where $p_{2i} \in P_2$, $1 \leq i \leq k$) there exists transition delay $[d_{2i}^-, d_{2i}^+]$. Where $d_{1i}^-, d_{1i}^+, d_{2i}^-, d_{2i}^+ \in R^+$ such that $d_{1i}^- \leq d_{1i}^+$ and $d_{2i}^- \leq d_{2i}^+$ (where R^+ is the set of non-negative real numbers). $[d_{1i}^-, d_{1i}^+] = [d_{2i}^-, d_{2i}^+]$ $(1 \leq i \leq k)$.

Definition 3.3 For transition $t_{1i} \in^{\bullet} p_{1i}$ of N_1 (where $p_{1i} \in P_1$, $1 \le i \le k$) there exists transition enable time et_{1i}. et_{1i} is obtained by the maximum token time of the tokens in ${}^{\bullet}t_{1i}$, i.e. $et_{1i} = \max(\tau(p_{1i1}), \tau(p_{1i2}), \ldots, \tau(p_{1ir}))$ (where ${}^{\bullet}({}^{\bullet}p_{1i}) = \{p_{1i1}, p_{1i2}, \ldots, p_{1ir}\}$, $1 \le i \le k$). For transition $t_{2i} \in^{\bullet} p_{2i}$ of N_2 (where $p_{2i} \in P_2$, $1 \le i \le k$) there exists transition enable time et_{2i}. $et_{2i} = \max(\tau(p_{2i1}), \tau(p_{2i2}), \ldots, \tau(p_{2is}))$ (where ${}^{\bullet}({}^{\bullet}p_{2i}) = \{p_{2i1}, p_{2i2}, \ldots, p_{2is}\}$, $1 \le i \le k$). $et_{1i} = et_{2i}$ $(1 \le i \le k)$.

Definition 3.4 For transition $t_{1i} \in^{\bullet} p_{1i}$ (where $p_{1i} \in P_1$, $1 \le i \le k$) there exist earliest trigger time $d_{1ie} = et_{1i} + d^{-}_{1i}$ and latest trigger time $d_{1i1} = et_{1i} + d^{+}_{1i}$. d_{1ie} and d_{1i1} are the lower and upper time limits for the firing of t_{1i}. There exist earliest trigger time $d_{2ie} = et_{2i} + d^{-}_{2i}$ and latest trigger time $d_{2il} = et_{2i} + d^{+}_{2i}$ for $t_{2i} \in^{\bullet} p_{2i}$ (where $p_{2i} \in P_2$, $1 \le i \le k$). d_{2ie} and d_{2il} are the lower and upper time limits for the firing of t_{2i}. $d_{1ie} = d_{2ie}$ and $d_{1il} = d_{2il}$ $(1 \le i \le k)$.

Definition 3.5 Suppose N_1 and N_2 are two PRES+ models, where $P_1 \cap P_2 = P_m \ne \Phi$, and $T_1 \cap T_2 = \Phi$. $N = (P, T, I, O, M_0)$ is said to be a sharing synthesis PRES+ model of N_1 and N_2 if the following conditions are satisfied,

(i) $P = P_1 \cup P_2$; (ii) $T = T_1 \cup T_2$; (iii) $I = I_1 \cup I_2$; (iv) $O = O_1 \cup O_2$;

(v) $\forall p \in P_m$, $M_{10}(p) = M_{20}(p)$, $M_0 = \begin{cases} M_{10}(p), & p \in P_1 \\ M_{20}(p), & p \in P_2 \end{cases}$;

(vi) Place $p_{1i} \in P_1$ of N_1 and $p_{2i} \in P_2$ of N_2 (where $1 \le i \le k$) have the same type (i.e. the same number of tokens, token time and token type).

(vii) Place $p_{1i} \in P_1$ of N_1 and $p_{2i} \in P_2$ of N_2 (where $1 \le i \le k$) are merged as place $p_{mi} \in P_m$ $(1 \le i \le k)$ of N, i.e. place set $\{p_{11}, p_{12}, \ldots, p_{1k}\}$ of N_1 and $\{p_{21}, p_{22}, \ldots, p_{2k}\}$ are merged as one place set $\{p_{m1}, p_{m2}, \ldots, p_{mk}\}$.

4. LIVENESS AND BOUNDEDNESS PRESERVATIONS OF SHARING SYNTHESIS NET SYSTEM OF PRES+

In this section we will investigate liveness and boundedness preservations of this sharing synthesis approach for PRES+.

We recall concepts from PRES+ theory [1, 7]. Suppose $N = (P, T, I, O, M_0)$ is a PRES+ model. For each transition $t \in T$, there exists $d : T \to R^{+} \times (R^{+} \cup \{\infty\})$, where $d(t) = (d^{-}(t), d^{+}(t))$ and $d^{-}(t) \le d^{+}(t)$. d is the interval function of N. $d^{-}(t)$ and $d^{+}(t)$ are the earliest firing time of t and the latest firing time of t, respectively.

A state of PRES+ may be represented as $S = (M, J)$, where M is a reachable marking of $R(M_0)$, and $J : T \to R^{+} \cup \{\#\}$ (where # is a symbol which describes unusable status). $S_0 = (M_0, J_0)$ is the initial state of the PRES+ N, where

$$J_0(t) = \begin{cases} 0 & M_0(p) \ge W(p, t) \quad \forall p \in^{\bullet} t \\ \# & \text{otherwise} \end{cases}$$

$$\sum = (Z, S_0) \text{ is the PRES+net system of } N$$

where $Z = (P, T, I, O)$ is the skeleton of N.

Transition $\hat{t}$ is ready to fire at state S, denoted by $S \xrightarrow{\hat{t}}$, iff $W(p, \hat{t}) \le M$ and $d(\hat{t}) \ge d^{-}(\hat{t})$. State $S' = (M', J')$ is got from $S = (M, J)$ by firing $\hat{t}$, denoted by $S \xrightarrow{\hat{t}} S'$, iff $\hat{t}$ is ready to fire at S, $M' = M - W(p_i, \hat{t}) + W(\hat{t}, p_j)$ (where $p_i \in^{\bullet} \hat{t}$ and $p_j \in \hat{t}^{\bullet}$). $S' = (M', J')$ is got from $S = (M, J)$ by the time elapsing $\tau \in R^{+}$, denoted by $S \xrightarrow{\tau} S'$, iff $M' = M$,

$$\forall t (t \in T \wedge d(t) \ne \# \to d(t) + \tau \le d^{+}(t)),$$

and

$$\forall t \left(t \in T \to d'(t) = \begin{cases} d(t) + \tau & W(p, t) \le M' \\ \# & W(p, t) > M' \end{cases} \right).$$

$S = (M, J)$, is reachable in $\sum$ iff there exist $S_1, S'_1, S_2, S'_2, \ldots, S_n, S'_n$, and times $\tau_i \in R^{+}_{0}$, $i = 1, 2, \ldots, n+1$ and it holds $S_0 \xrightarrow[\tau_1]{} S_1 \xrightarrow{t_1} S'_1 \xrightarrow{\tau_2} S_2 \xrightarrow{t_2} \ldots S'_2 \ldots \xrightarrow{\tau_n} S_n \xrightarrow{t_n} S'_n \xrightarrow{\tau_{n+1}} S$, denoted by $S_0 \xrightarrow{\sigma, \tau} S$, where $\sigma = t_1 t_2 \ldots t_n$, $\tau = \tau_1 + \tau_2 + \cdots + \tau_{n+1}$. Set R_N of all reachable states in $\sum$ is the state space of $\sum$.

Definition 4.1[7] Let $\sum = (Z, S_0)$ be a PRES+ net system and S a reachable state.

(i) A transition t is live at state S iff $\forall S'(S' \in R_N(S) \to \exists S''(S'' \in R_N(S') \wedge S'' \xrightarrow{t}))$.

(ii) A PRES+ net system $\sum$ is live iff all transitions are live in S.

Definition 4.2[7] Let $\sum = (Z, S_0)$ be a PRES+ net system and S_0 an initial state.

(i) A place $p \in P$ is called bounded iff there exists a natural number $K > 0$ with $M(p) \le K$ for each $S \in R_N(S_0)$.

(ii) PRES+ net system $\sum$ is bounded iff all places are bounded.

Definition 4.3 Suppose $\sum = (Z, S_0)$ is a PRES+ net system and $P = P_1 \cup P_2$, $P_1 \cap P_2 = \Phi$. (p_r, p_s) is said to be a place ordered pair of $\sum$ on P_1 if the following conditions are satisfied,

(i) $p_r, p_s \in P_1$ (where $r \ne s$),

(ii) if $\exists M_1 \in R(M_0)$ such that $M_1(p_r) = 0$ and $M_1(p_s) > 0$, then for all $\sigma \in (T - p_r^{\bullet})^{*}$, $M_1[\sigma > M_2]$ and $M(p_s) = 0$.

Definition 4.4 Suppose (p_r, p_s) is a place ordered pair of $\sum_1$ on P_m and (p_s, p_r) is a place ordered pair of $\sum_2$ on P_m. Then (p_r, p_s) is said to be an inter-reciprocal place ordered pair of $\sum_1$ and $\sum_2$ on P_m.

Definition 4.5 Suppose $\sum$ is the sharing synthesis PRES+ net system of $\sum_1$ and $\sum_2$. Then (p_r, p_s) (where $p_r, p_s \in P_m$) is said to be a sharing place pair iff

(i) $\forall S \in R_N(S_0)$ (where $S = (M, J)$), $M(p_r) > 0$ and $M(p_s) > 0$.

(ii) If $S \xrightarrow{t'} S_1$ (where $t' \in T_1 \cap P_r^{\bullet}$), then $\forall \sigma \in (T - T_1 \cap p_s^{\bullet})^{*}$, $S_1 \xrightarrow{\sigma} S_2$, and $\forall t \in T \cap p_s^{\bullet}$, $\neg S_2 \xrightarrow{t}$.

In the following, we propose some conditions for the liveness preservation and boundedness preservation of PRES+ synthesis net system.

Theorem 4.1 Suppose $\sum_1$ and $\sum_2$ are two live and bounded PRES+ net systems, $\sum$ is the sharing synthesis PRES+ net system of $\sum_1$ and $\sum_2$. Then $\sum$ is bounded.

Proof. Since $\sum_1$ and $\sum_2$ are live and bounded, then in $\sum_1$, there exists an integer $k_1 > 0$, $\forall p_1 \in P_1$, $\forall S_{11} \in R_N(S_{10})$ (where $S_{11} = (M_{11}, J_{11})$), such that $M_{11}(p_1) \leq k_1$; in $\sum_2$, there exists an integer $k_2 > 0$, $\forall p_2 \in P_2$, $\forall S_{21} \in R_N(S_{20})$, such that $M_{21}(p_2) \leq k_2$.

Since $\sum$ is the sharing synthesis PRES+ net system of $\sum_1$ and $\sum_2$, by Definition 3.5, in $\sum$, $\forall p \in P$, then $p \in P_m$ or $p \in P_1 - P_m$ or $p \in P_2 - P_m$. If $p \in P_m$, $\forall S \in R_N(S_0)$ (where $S = (M, J)$, $S_0 = (M_0, J_0)$), $M(p) \leq \max_{M_{11} \in R(M_{10})} M_{11}(p) + \max_{M_{21} \in R(M_{20})} M_{21}(p) \leq k_1 + k_2$.

If $p \in P_1 - P_m$, then $\forall S \in R_N(S_0)$, $M(p) \leq \max_{M_{11} \in R(M_{10})} + \sum_{p' \in P_m} \max_{M_{21} \in R(M_{20})}(M_{21}(p')) \leq k_1 + lk_2$ (where l is the number of places of P_m). If $p \in P_2 - P_m$, $\forall S \in R_N(S_0)$, $M(p) \leq k_2 + lk_1$. Let $k = (1 + l)(k_1 + k_2)$, then $\forall p \in P$, $\forall S \in R_N(S_0)$, $M(p) \leq k$, by Definition 4.2, $\sum$ is bounded.

Theorem 4.2 Suppose $\sum_1$ and $\sum_2$ are two live and bounded PRES+ net systems, $\sum$ is the sharing synthesis PRES+ net system of $\sum_1$ and $\sum_2$. If for all $p_r, p_s \in P_m$, (p_r, p_s) is not an inter-reciprocal place pair of $\sum_1$ and $\sum_2$ on P_m, then $\sum$ is live.

Proof. Suppose $\sum = (Z, S_0)$ is not live, by Definition 3.1–3.5 and Definition 4.1, then $\exists S_1 \in R_N(S_0)$ and $t' \in T$, for all $S \in R_N(S_1)$, $\neg S \xrightarrow{t'}$. It is easy to see that $t' \in \{t | t \in p^\bullet, \forall p \in P_m\}$. Without loss of generality, suppose $t' \in T_1$. Since $\sum_1$ is bounded, $\exists S_2 \in R_N(S_1)$, $p_r \in P_m$, $t' \in p_s^\bullet$ such that $M_2(p_r) = 0$ (where $S_2 = (M_2, J_2)$) and $\forall p \in^\bullet t' \cap P_1$, $M_2(p) \geq 1$. Since $\sum_2$ is live and bounded, and $\sum_2$ has obtained the resources of p_r, if $\forall t \in T_2 \cap \{t | t \in p^\bullet, \forall p \in P_m, p \neq p_r\}$, $\exists \sigma_1 \in (T_2)^*$ such that $S_2 \xrightarrow{\sigma_1} S' \xrightarrow{t}$, then $\exists t_r' \in^\bullet p_r \cap T_2$, and $\exists \sigma_2 \in (T_2)^*$ such that $S' \xrightarrow{\sigma_2} S'' \xrightarrow{t_r'}$. This contradicts with the fact that $\neg S \xrightarrow{t'}$ for all $S \in R_N(S_1)$. Then $\exists p_r \in p_m$ (where $p_r \neq p_s$), for all $\sigma \in (T_2)^*$, $t'' \in T_2 \cap p_r^\bullet$ such that $S_2 \xrightarrow{\sigma} S_3 \xrightarrow{t''}$. Since $\forall p \in^\bullet t'' \cap P_2$, $M_3(p) \geq 1$ and $M_3(p_s) = 0$. This means that the resources of p_r are used by $\sum_1$ and not given back to p_s, and the resources of p_s are used by $\sum_2$ and not given back to p_r. By Definition 4.4, (p_r, p_s) is an inter-reciprocal place ordered pair of $\sum_1$ and $\sum_2$ on P_m. This contradicts with the fact that for all $p_r, p_s \in P_m$, (p_r, p_s) is not an inter-reciprocal place pair of $\sum_1$ and $\sum_2$ on P_m. So $\sum$ is live.

Corollary 4.1 Suppose $\sum_1$ and $\sum_2$ are two live and bounded PRES+ net systems, $\sum$ is the sharing synthesis PRES+ net system of $\sum_1$ and $\sum_2$. If $\sum_1$ and $\sum_2$ have the same place ordered pair on P_m, or $\sum_1$ and $\sum_2$ don't have a place ordered pair on P_m, then $\sum$ is live and bounded.

Theorem 4.3 Suppose $\sum_1$ and $\sum_2$ are two live and bounded PRES+ net systems, $\sum$ is the sharing synthesis PRES+ net system of $\sum_1$ and $\sum_2$. Then $\sum$ is live iff $\forall t \in \{t \in p^\bullet, \forall p \in P_m\}$, t is live.

Proof. ($\Rightarrow$) Since $\forall t \in \{t | t \in p^\bullet, \forall p \in P_m\}$, t is live, by Definition 3.1–3.5 and Definition 4.1, $\forall S \in R_N(S_0)$, $\exists S_1 \in R_N(S)$ such that $S_1 \xrightarrow{t}$. Without loss of generality, suppose $\forall t \in T_1 - \{t | t \in p^\bullet, \forall p \in P_m\} \cap T_1$. Since $\sum_1$ and $\sum_2$ are two live and bounded, and in $\sum$, $\forall t \in \{t | t \in p^\bullet, \forall p \in P_m\}$, t is live, by Definition 3.5, $\forall S_1 \in R_N(S_0)$, $\exists \sigma_1 \in T^*$, $S_1 \xrightarrow{\sigma_1} S_2$. Suppose S_{12} is the projection of S_2 on $\sum_1$, then $\exists \sigma_2 \in T_1^*$ such that $S_{12} \xrightarrow{\sigma_2} S_3 \xrightarrow{t}$. So $\sum$ is live.

($\Leftarrow$) Since $\sum$ is live, it is obvious that $\forall t \in \{t | t \in p^\bullet, \forall p \in P_m\}$, t is live.

Theorem 4.4 Suppose $\sum_1$ and $\sum_2$ are two live and bounded PRES+ net systems, $\sum$ is the sharing synthesis PRES+ net system of $\sum_1$ and $\sum_2$. Let (p_r, p_s) be an inter-reciprocal place pair of $\sum_1$ and $\sum_2$ on P_m. If (p_r, p_s) and (p_s, p_r) are sharing place pairs of $\sum_1$ and $\sum_2$, then $\sum$ is live and bounded.

Proof. For every inter-reciprocal PRES+ place pair (p_r, p_s) of $\sum_1$ and $\sum_2$ on P_m, if $t' \in p_r^\bullet \cap T_1$, $t'' \in p_s^* \cap T_2$, then t' and t'' don't fire for all $S \in R_N(S_0)$. Since $\sum$ is the sharing synthesis net system of $\sum_1$ and $\sum_2$, and $\sum_1$ and $\sum_2$ are live, by Definition 4.1, if $t' \in p_r^\bullet \cap T_1$, $\forall S_1 \in R_N(S_0)$, $S_1 \xrightarrow{t'} S_2$, then $\forall t \in p_s^\bullet \cap T_1$, $\exists S_3 \in R_N(S_0)$ such that $S_3 \xrightarrow{t}$, that means t' is live. If $t'' \in p_s^\bullet \cap T_2$, $S_1 \xrightarrow{t''} S_2$ for all $S_1 \in R_N(S_0)$, then $\forall t \in p_r^\bullet \cap T$, $\exists S_3 \in R_N(S_0)$ such that $S_3 \xrightarrow{t}$, that means t' is live. Since $p_r, p_s \in P_m$, (p_r, p_s) is not an inter-reciprocal place pair of $\sum_1$ and $\sum_2$ on P_m, by Corollary 4.1, t is live for $\forall t \in \{t | t \in p^\bullet, \forall p \in P_m\}$. By the proofs of Theorem 4.1–4.3, $\sum$ is live and bounded.

5. APPLICATIONS

The design process for embedded systems starts with a system specification. This specification gives the functionality without implementation details. The designer needs to come up with functional aspects of the specification. Sound low level models of computation are very important in the design process. The designer must synthesize these models. This stage is known as synthesis operation. The system total model is obtained after synthesis operations.

The designer should construct the prototype of the embedded system. When the prototype has been produced, it needs to be thoroughly checked to find out whether it functions correctly.

The synthesis operation of our paper is an important operation to be used in the design process.

Verification is also important in the design of synthesized model. We propose a sharing synthesis approach for PRES+, under some constraints, several important properties, such as liveness and boundedness, will be preserved.

In order to demonstrate the effectiveness of our synthesis approach for PRES+, we will use this approach to model and analyze an embedded system based on PRES+.

In the following, we will give models of control systems by using PRES+ sharing synthesis operation, and discuss liveness and boundedness preservations of this synthesis PRES+ net systems.

Firstly, the PRES+ models of two control subsystems are proposed. Then the synthesis PRES+ model is obtained.

Two control subsystems are illustrated in Fig. 1 and Fig. 2 by using PRES+ methods.

$\sum_1$ and $\sum_2$ are two control subsystems. They share 3 kinds of resources ($p_{11} \leftrightarrow p_{21}$, $p_{12} \leftrightarrow p_{22}$, $p_{13} \leftrightarrow p_{23}$). Suppose

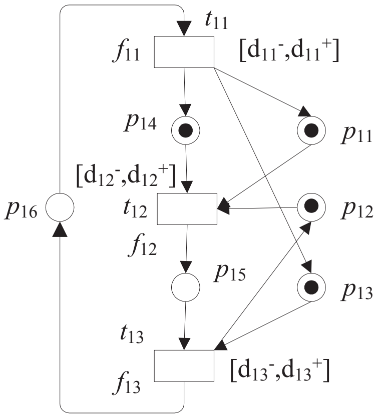

Figure 1 The PRES+ model of control subsystem $\sum_1$.

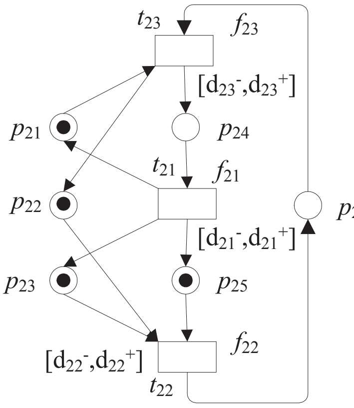

Figure 2 The PRES+ model of control subsystem $\sum_2$.

$[d_{12}^-, d_{12}^+] = [d_{22}^-, d_{22}^+]$, $[d_{11}^-, d_{21}^+] = [d_{21}^-, d_{21}^+]$, $[d_{13}^-, d_{13}^+] = [d_{23}^-, d_{23}^+]$, and $f_{11} = f_{21}$, $f_{12} = f_{22}$, $f_{13} = f_{23}$. The synthesized PRES+ net system $\sum$ (Fig. 3) is obtained by PRES+ sharing synthesis operation, i.e. p_{11}, p_{12} and p_{13} of $\sum_1$ and p_{21}, p_{22} and p_{23} of $\sum_2$ are merged into p_{m1}, p_{m2} and p_{m3} of $\sum$, respectively.

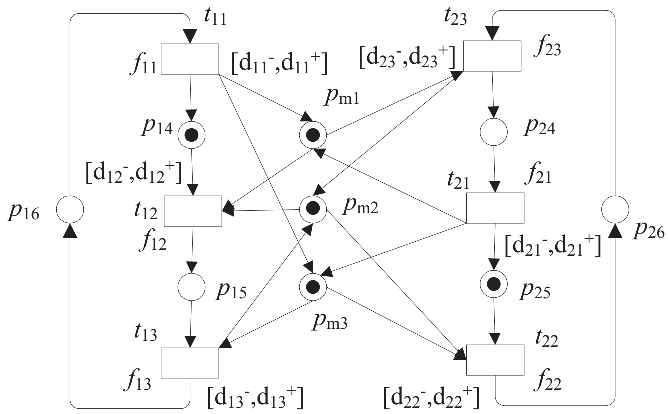

Figure 3 The PRES+ model of synthesis control system $\sum$.

It is easy to see that PRES+ net system $\sum_1$ and $\sum_2$ are live and bounded. Since for all p_r, $p_s \in P_m$ (where $P_m = \{p_{m1}, p_{m2}, p_{m3}\}$), (p_r, p_s) is not an inter-reciprocal place pair of $\sum_1$ and $\sum_2$ on P_m, by Theorem 4.1 and Theorem 4.2, $\sum$ is live and bounded.

6. CONCLUSIONS

Embedded systems have lots of applications in our everyday life. These systems as part of larger systems include both software and hardware elements and interact with their environment. In order to facilitate the design and verification of embedded systems, PRES+ is proposed. PRES+ has an enormous expressive power and lots of applications in embedded system design. But State space explosion problem for PRES+ is somewhat awful to model and analyze large complex embedded systems.

To solve this problem, in this paper we have investigated property preservations of synthesis PRES+ net system. Sharing synthesis operation for PRES+ is proposed. Under some conditions, liveness and boundedness are preserved after merging some places of PRES+ net systems. As a consequence, these results can be applied to solve design problems in control system, manufacturing engineering and intelligent buildings. Further research is needed to investigate subnet-sharing synthesis operations of PRES+ and their applications.

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REFERENCES

1. L. A. Cortés, P. Eles, Z. Peng. Modelling and formal verification of embedded systems based on a Petri net based representation. Journal of Systems Architecture, 2003, Vol. 49, pp. 571-598.
2. L. A. Cortés, P. Eles, Z. Peng. Definitions of equivalence for transformation synthesis of embedded systems. the Sixth IEEE International Conference on Engineering of Complex Computer Systems, 2000, pp.134-142.
3. D. Karlsson, P. Eles, Z. Peng. Formal verification of component-based designs. Journal of Design Automation for Embedded Systems , 2007, Vol.11, pp. 49–90.
4. D. Karlsson, P. Eles, Z. Peng. Model validation for embedded systems using formal method-aided simulation. LET Computer & Digital Techniques, 2008, Vol. 2, No.6, pp. 413-433.
5. S. Bandyopadhyay, K. Banerjee, D. Sarkar, C. R. Mandal. Translation validation for PRES+ models of parallel behaviours via an FSMD equivalence checker. H. Rahaman et al. (Eds.): VDAT 2012, LNCS 7373, pp. 69–78, 2012.
6. L. A. Cortés, P. Eles, Z. Peng. Hierarchical modeling and verification of embedded systems. Proc. Euromicro Symposium on Digital Systems Design, 2001, pp. 63-70.
7. **C. Xia**. Property preservation of refinement for Petri net based representation for embedded systems. Cluster Computing (2016) 19: 1373-1384.

8. **C. Xia**, Z. Zhang, Z. Wang. Property analysis of refinement of Petri net based representation for embedded systems. Open Automation and Control Systems Journal, 2013, Vol. 5, pp. 214-218.
9. **C. Xia**. Reduction rules for Petri net based representation for embedded systems. Journal of Frontiers of Computer Science and Technology, 2008, Vol. 2, No. 6, pp.614-626.
10. L. Jiao, T. Y. Cheung, W. M. Lu. On liveness and boundedness of asymmetric choice nets. Theoretical Computer Science 311 (2004) 165-197.
11. A. A. Pouyan, H. T. Shandiz, S. Arastehfar. Synthesis a Petri net based control model for a FMS cell . Computers in Industry, 2011, Vol. 62, pp. 501–508.
12. F. Basile, F. Caccavale, P. Chiacchio, J. Coppola, A. Marino, D. Gerbasio. Automated synthesis of hybrid Petri net models for robotic cells in the aircraft industry. Control Engineering Practice 31(2014) 35-49.
13. H. Hu, R. Su, M. C. Zhou, Y. Liu. Polynomially complex synthesis of distributed supervisors for large-scale AMSs using Petri nets. IEEE Transactions on Control Systems Technology, 2016, Vol. 24, No. 5, pp. 1610-1622.
14. B. Finkbeiner, E. –R. Olderog. Petri games: synthesis of distributed systems with causal memory. Information and Computation 253 (2017) 181-203.
15. M. C. Zhou, F. DiCesare. A hybrid methodology for synthesis of Petri net models for manufacturing systems. IEEE Transactions on Robotics and Automation 8 (1992) 350-361.
16. **C. Xia**. Liveness and boundedness analysis of Petri net synthesis. Methematical Structures in Computer Sciences, 2014, Vol.24, iss.5, pp. 1-19. Cambridge University Press 2014.
17. E. Best, R. Devillers. Characterisation of the state spaces of marked graph Petri nets. Information and Computation, 253 (2017) 399-410.
18. G. Y. Liu, Z. W. Li, K. Barkaoui, A. M. Al-Ahmari. Roubustness of deadlock control for a class of Petri nets with unreliable resources. Information Sciences, 2013, Vol. 235, pp. 259-279.
19. Q. Hu, Y. Y. Du, S. Yu. Service net algebra based on logic Petri nets. Information Sciences, 2014, Vol.268, pp. 271-289.
20. F. Basile, R. Cordone, L. Piroddi. A branch and bound approach for the design of decentralized supervisors in Petri net models. Automata, 2015, Vol.52, pp. 322-333.
21. **C. Xia.** Analysis of properties of Petri synthesis net. J. –Y. Cai, S. B. Cooper, and A. Li (Eds.): TAMC 2006, LNCS 3959, pp. 576-587.
22. **C. Xia**. Structural property analysis of Petri net synthesis shared pp subnet. Journal of Computers, 2012, Vol. 7, No. 1, pp. 292-300.