MHD Boundary Layer Flow of a Power-Law Nanofluid Containing Gyrotactic Microorganisms Over an Exponentially Stretching Surface

Mohamed Abd El-Aziz[1, 2] and A. M. Aly[1, 3, *]

Abstract: This study focusses on the numerical investigations of boundary layer flow for magnetohydrodynamic (MHD) and a power-law nanofluid containing gyrotactic microorganisms on an exponentially stretching surface with zero nanoparticle mass flux and convective heating. The nonlinear system of the governing equations is transformed and solved by Runge-Kutta-Fehlberg method. The impacts of the transverse magnetic field, bioconvection parameters, Lewis number, nanofluid parameters, Prandtl number and power-law index on the velocity, temperature, nanoparticle volume fraction, density of motile microorganism profiles is explored. In addition, the impacts of these parameters on local skin-friction coefficient, local Nusselt, local Sherwood numbers and local density number of the motile microorganisms are discussed. The results reveal that the power law index is considered an important factor in this study. Due to neglecting the buoyancy force term, the bioconvection and nanofluid parameters have slight effects on the velocity profiles. The resultant Lorentz force, from increasing the magnetic field parameter, try to decrease the velocity profiles and increase the rescaled density of motile microorganisms, temperature and nanoparticle volume fraction profiles. Physically, an augmentation of power-law index drops the reduced local skin-friction and reduced Sherwood number, while it increases reduced Nusselt number and reduced local density number of motile microorganisms.

Keywords: Bioconvection, gyrotactic microorganisms, magnetohydrodynamic, nanofluid, boundary layer, power-law.

Nomenclature

B	applied magnetic field	q_w	surface heat flux
B_0	magnitude of magnetic field	q_m	surface mass flux
C	concentration	q_n	surface motile microorganisms flux

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C_{fx}	local skin friction coefficient	Sh_x	local Sherwood number
C_∞	outer concentration	T	temperature
D_B	Brownian diffusion coefficient	T_f	local fluid temperature
D_T	thermophoresis diffusion coefficient	T_∞	ambient temperature
D_n	diffusivity of microorganisms	$u\,,v$	velocity components
f	dimensionless stream function	U_∞	external velocity
h_f	heat transfer coefficient	$x\,,y$	Cartesian coordinates
K	Consistency coefficient	Greek symbols	
k	effective thermal conductivity	α	thermal diffusivity
Le	regular Lewis number	χ	rescaled density of motile microorganisms
Lb	Bioconvection Lewis number	γ	generalized Biot number
n	power-law index	β	bioconvection constant
Nb	Brownian motion parameter	ϕ	rescaled nanoparticle volume fraction
Nt	thermophoresis parameter	θ	dimensionless temperature
Nn_x	local density number of motile microorganisms	τ_w	shear stress
Nu_x	local Nusselt number	ψ	Stream function
Pe	bioconvection Péclet number	σ	Electrical conductivity
Pr	Prandtl number	ρ	density

1 Introduction

After nanofluids in 1995 [Choi and Eastman (1995)], the study of enhancement heat transfer by adding suitable nanoparticles has received numerous attentions due to its wide engineering applications. The review articles for enhancement heat transfer using nanofluids and their applications can be found in Daungthongsuk et al. [Daungthongsuk and Wongwises (2007); Trisaksri and Wongwises (2007); Wang and Mujumdar (2007); Kakaç and Pramuanjaroenkij (2009); Özerinç, Kakaç and Yazıcıoğlu (2010); Kleinstreuer and Feng (2011); Sarkar (2011); Sridhara and Satapathy (2011); Haddad, Oztop and Abu-Nada et al. (2012)].

The study of boundary-layer MHD flow which is resulting from the presence of magnetic fields controls many systems using electrically conducting fluids. Moreover, there are several applications for MHD flow including nuclear reactors, MHD generators and geothermal energy extractions. Sparrow et al. [Sparrow and Cess (1961)] firstly introduced the effects of magnetic field on natural convection flow. Chamkha et al. [Chamkha and Aly (2010)] studied the MHD natural convection flow on boundary layer flow of a nanofluid along a permeable plate. Moreover, Chamkha et al. [Chamkha, Mansour and Aly (2011)] investigated the presence of transverse magnetic field and Hall current with the effects of chemical reaction and heat generation on unsteady free convective along a porous plate.

Uddin et al. [Uddin, Khan and Ismail (2012)] studied numerically MHD laminar boundary layer flows of an electrically conducting Newtonian nanofluid over a solid stationary plate. Mabood et al. [Mabood, Khan and Ismail (2015)] adopted Runge-Kutta Fehlberg fourth-fifth order method to study the MHD laminar boundary layer flow of a nanofluid over a nonlinear stretching sheet.

The macroscopic convection motion of a fluid caused by the density gradient created by collective swimming of motile microorganisms is called bioconvection [Avramenko and Kuznetsov (2004); Hill and Pedley (2005); Kuznetsov (2006, 2011); Nield and Kuznetsov (2006)]. Kuznetsov et al. [Kuznetsov (2010)] first introduced the bioconvection term for nanofluid. Siddiqa et al. [Siddiqa, Gul and Begum et al. (2016)] studied the bioconvection flow of a nanofluid and gyrotactic microorganisms along a wavy cone. Khan [Khan (2018)] introduced the second grade nanofluid thin film flow with bioconvection.

Recently, the collections of the magnetic field, microorganisms and nanoparticles present interesting fluid dynamics problems [Khan and Makinde (2014); Khan, Makinde and Khan (2014); Ahmed and Mahdy (2016); Makinde and Animasaun (2016); Alsaedi, Khan and Farooq et al. (2017); Khan, Waqas, Hayat et al. (2017); Ramzan, Chung and Ullah (2017); Waqas, Hayat and Shehzad et al. (2018)]. Ahmed et al. [Ahmed and Mahdy (2016)] studied the MHD natural convection flows in a nanofluid and gyrotactic microorganisms over a wavy surface.

For non-Newtonian fluids, Raju et al. [Raju and Sandeep (2016)] carried our numerical analysis using Runge-Kutta-Felhberg method for MHD double-diffusive of gyrotactic microorganisms in a Casson fluid along a vertical rotating cone/plate. Khan et al. [Khan and Khan (2016)] studied the MHD boundary layer flow of power-law nanofluid over a non-linear stretching sheet. Salem et al. [Salem and Abd El-Aziz (2013)] studied the impacts of the temperature jump on the hydromagnetic non-Darcian free-convective flow of a non-Newtonian fluid. Abd El-Aziz et al. [Abd El-Aziz and Afify (2016); Afify and Abd El-Aziz (2017)] adopted Lie group analysis for heat transfer of non-Newtonian nanofluid and a power-law fluid over a stretching surface with different boundary conditions. Very recent studies, Salem et al. [Salem and Abd El-Aziz (2019)] studied the convective heat transfer and entropy generation of the radiated non-Newtonian power-law fluid past an exponentially moving surface in the presence of slip effects. Abd El-Aziz et al. [Abd El-Aziz and Saleem (2019)] simulated the impacts of variable heat source and heat flux on entropy generation of a power-Law flow over a permeable exponential stretched surface.

From these investigations, it is noted that there are no studies related to MHD boundary layer flow of a power-law nanofluid containing gyrotatic microorganisms in a stretching surface. Then, the results of the current work are new and this work is an extension for the recent studies of Saleem et al. [Saleem and Abd El-Aziz (2019)]. Runge-Kutta-Fehlberg method will be used to study the MHD boundary layer flow of a power-law nanofluid containing gyrotatic microorganisms over an exponentially stretching surface. It is found that, the power law index is considered an important factor in this study. The rescaled density of motile microorganisms increases as the bioconvection Péclet number, bioconvection constant and magnetic field parameter are increase. The nanoparticle volume fraction increases as thermophoresis parameter, generalized Biot number and magnetic field parameter are increase. The reduced local skin-friction coefficient has the higher values at lower magnetic field parameter and power law index. The reduced Nusselt number is decreasing with an increase on the magnetic field parameter and thermophoresis parameter. The reduced Sherwood number is increasing according to an increase on the magnetic field parameter, Lewis number and Brownian motion parameter. The reduced local density number of the motile microorganisms increases as the Prandtl number, magnetic field parameter and power law index are increase.

2 Problem formulation

The two-dimensional steady MHD boundary layer flow of a power-law nanofluid containing gyrotactic microorganisms over an exponentially stretching surface is considered. The flow is originated by virtue of exponentially stretching of the sheet. At a lower surface, the sheet is heated convectively with temperature T_f and a heat transfer coefficient h_f. The ambient temperature and concentration are T_∞ and C_∞. Fig. 1 presents the initial schematic diagram of the current problem.

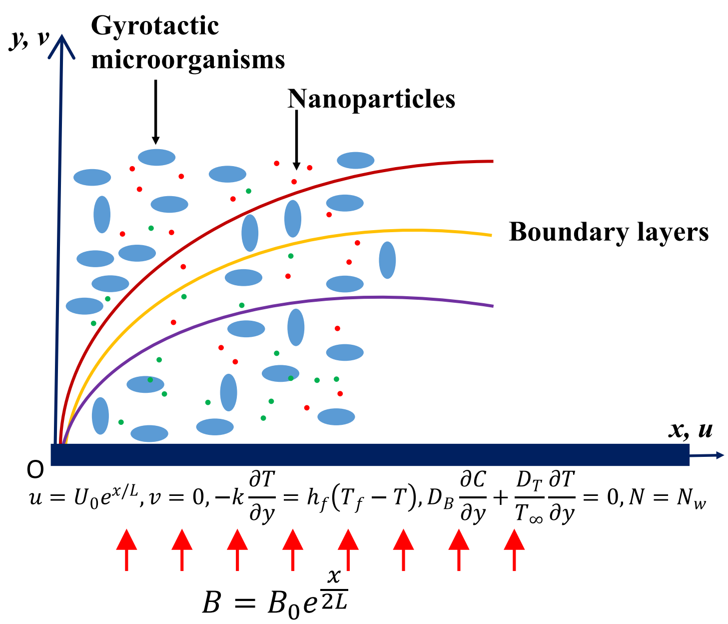


Figure 1: Initial schematic diagram of the problem

Here, the *x*-axis is taken along the exponentially stretching surface and *y*-axis is normal to it. As shown in this figure, the transverse non-uniform magnetic field with strength is taken as parallel to the *y*-axis. The governing equation are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{K}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)-\frac{\sigma B^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}+\frac{b\,W_c}{C_\infty}\left(\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right)\right)=D_n\frac{\partial^2 N}{\partial y^2} \tag{5}$$

The imposed boundary conditions are:

$$\begin{gathered}u=u_w=U_0e^{x/L}, v=0, -k\frac{\partial T}{\partial y}=h_f\left(T_f-T\right)\\ D_B\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\frac{\partial T}{\partial y}=0, N=N_w \quad \text{at } y=0\\ u\to 0, \qquad T\to T_\infty, \qquad C\to C_\infty, \qquad N\to N_\infty \quad \text{as } y\to\infty\end{gathered} \tag{6}$$

Introducing the following similarity transformations [Abd El-Aziz and Afify (2016); Afify and Abd El-Aziz (2017)]:

$$\begin{gathered}\psi=\left(\frac{2KLU_o^{\,2n-1}}{\rho}\right)^{\frac{1}{n+1}}e^{\left(\frac{2n-1}{n+1}\right)\frac{x}{L}}f(\eta),\ \ \eta=y\left(\frac{\rho U_o^{\,2-n}}{2KL}\right)^{\frac{1}{n+1}}e^{\left(\frac{2-n}{n+1}\right)\frac{x}{L}}\\ \theta=\frac{T-T_\infty}{T_f-T_\infty}, \qquad \phi=\frac{C-C_\infty}{C_\infty}, \qquad \chi=\frac{N-N_\infty}{N_w-N_\infty}\end{gathered} \tag{7}$$

Then, the velocity components are:

$$\begin{gathered}u=\frac{\partial\psi}{\partial y}=U_oe^{\frac{x}{L}}f'(\eta),\\ v=-\frac{\partial\psi}{\partial x}=-\left(\frac{2KU_o^{\,2n-1}}{\rho L^n}\right)e^{\left(\frac{2n-1}{n+1}\right)\frac{x}{L}}\left(\left(\frac{2n-1}{n+1}\right)f(\eta)+\left(\frac{2-n}{n+1}\right)\eta f'(\eta)\right)\end{gathered} \tag{8}$$

The dimensionless forms of the governing equations are:

$$n|f''|^{n-1}f'''+2\left(\frac{2n-1}{n+1}\right)ff''-2f'^2-2Mf'=0, \tag{9}$$

$$\theta''+Pr\left(\frac{2n-1}{n+1}\right)f\theta'+Nb\,\theta'\phi'+Nt\,\theta'^2=0, \tag{10}$$

$$\phi''+LePr\left(\frac{2n-1}{n+1}\right)f\phi'+\left(\frac{Nt}{Nb}\right)\theta''=0, \tag{11}$$

$$\chi'' + Lb\,Pr\left(\frac{2n-1}{n+1}\right)f\chi' - Pe(\chi'\phi' + (\chi+\beta)\phi'') = 0, \tag{12}$$

With the boundary conditions:

$$f'(0) = 1, f(0) = 0, \theta'(0) = -\gamma(1-\theta(0)),$$
$$Nb\,\phi'(0) + Nt\theta'(0) = 0, \chi(0) = 1$$
$$f'(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0, \chi(\infty) \to 0 \tag{13}$$

where, $B = B_0 e^{\frac{x}{2L}}$ is applied magnetic field, $M = \frac{\sigma B_0^2 L}{\rho U_o}$ is magnetic field parameter, $Pr = \frac{U_w L}{\alpha}\left(\frac{Re_x}{2}\right)^{\frac{-2}{n+1}}$ is Prandtl number, $Le = \frac{\alpha}{D_B}$ is Lewis number, $Pe = \frac{b\,W_c}{D_n}$ is bioconvection Péclet number, $Nb = \frac{\tau D_B C_\infty}{\alpha}$ is Brownian motion parameter, $Nt = \frac{\tau D_T\,\Delta T}{\alpha T_\infty}$ is thermophoresis parameter, $Lb = \frac{\alpha}{D_n}$ is Bioconvection Lewis number, $\beta = \frac{N_\infty}{\Delta N}$ is the bioconvection constant, $\gamma = \frac{h_f L}{k}\left(\frac{Re_x}{2}\right)^{-\frac{1}{n+1}}$ is the generalized Biot number.

The skin fraction coefficient, local Nusselt number, local Sherwood number and local density number of the motile microorganisms are given by:

$$C_{fx} = \frac{2\tau_w}{\rho_f u_w^2}, \qquad \tau_w = \left(K\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)_{y=0}$$
$$C_{fx}\left(\frac{Re_x}{2}\right)^{\frac{1}{n+1}} = |f''(0)|^{n-1}f''(0) \tag{14}$$

$$Nu_x = \frac{Lq_w}{k\Delta T}, \;\; Nu_x\left(\frac{Re_x}{2}\right)^{\frac{-1}{n+1}} = -\theta'(0) \tag{15}$$

$$Sh_x = \frac{Lq_m}{D_B C_\infty}, \;\; Sh_x\left(\frac{Re_x}{2}\right)^{\frac{-1}{n+1}} = -\phi'(0) \tag{16}$$

$$Nn_x = \frac{Lq_n}{D_n \Delta N}, \;\; Nn_x\left(\frac{Re_x}{2}\right)^{\frac{-1}{n+1}} = -\chi'(0) \tag{17}$$

where, $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$ is surface heat flux, $q_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}$ is the surface mass flux and $q_n = -D_n\left(\frac{\partial N}{\partial y}\right)_{y=0}$ is the surface motile microorganisms flux. τ_w is shear stress.

3 Numerical method

In this section, the numerical procedure for solving the similar nonlinear Eqs. (9)-(12) with boundary conditions (13) are introduced. The Runge-Kutta-Fehlberg method with shooting technique, which introduced by Abd El-Aziz et al. [Abd El-Aziz and Saleem (2019); Saleem and Abd El-Aziz (2019)] is applied to solve the nonlinear equations as followings:

The nonlinear Eqs. (9)-(12) are transformed into set of first-order ordinary differential equations as:

$$\begin{aligned} &y_1 = f, \quad y_2 = f', \quad y_3 = f'' \\ &y_4 = \theta, \quad y_5 = \theta' \\ &y_6 = \phi, \quad y_7 = \phi' \\ &y_8 = \chi, \quad y_9 = \chi' \end{aligned} \tag{18}$$

Using Eq. (18) into the system (9)-(12), hence the nonlinear equations are converted to the first order differential equations as:

$$y_3' = \frac{2}{n|\,y_3|^{n-1}}\left(\left(\frac{1-2n}{n+1}\right)y_1\,y_3 + {y_2}^2 + My_2\right) \tag{19}$$

$$y_5' = Pr\left(\frac{1-2n}{n+1}\right)y_1\,y_5 - Nb\;\,y_5\,y_7 - Nt\;\,{y_5}^2 \tag{20}$$

$$y_7' = LePr\left(\frac{1-2n}{n+1}\right)y_1\,y_7 - \left(\frac{Nt}{Nb}\right)\,y_5' \tag{21}$$

$$y_9' = Lb\;Pr\left(\frac{1-2n}{n+1}\right)y_1\,y_9 + Pe(\,y_7\,y_9 + (\,y_8 + \beta)y_7') \tag{22}$$

with the boundary conditions:

$$\begin{aligned} &y_1(0) = 0,\; y_2(0) = 1, y_3(0) = s_1, \\ &y_5(0) = -\gamma\big(1 - \;y_4(0)\big), y_4(0) = s_2, \\ &Nb\,y_7(0) + Nt\,y_5(0) = 0,\; y_6(0) = s_3, \\ &y_8(0) = 1,\; y_9(0) = s_4. \end{aligned} \tag{23}$$

Finally, the shooting technique is used to estimate disappeared initial conditions s_1, s_2, s_3 and s_4 by a stepwise process.

The step size in Runge-Kutta-Fehlberg method for solving initial value problem (Eqs. (19)-(22)) is $\Delta\eta = 0.001$.

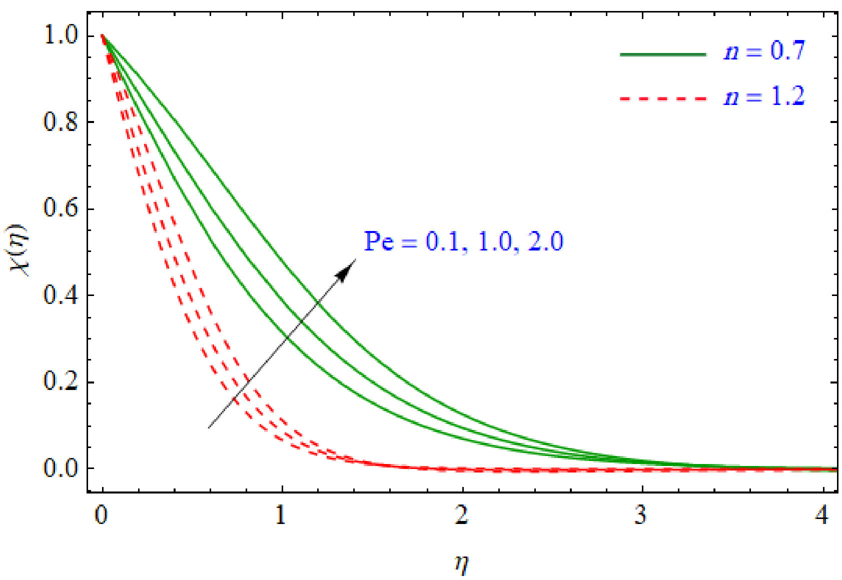
The computed values at η_∞ with boundary conditions at ∞, are fixed by Newton-Raphson method to give a superior estimation for the required solution.

The iterative process is repeated until getting the results with correction up to 10^{-6}.

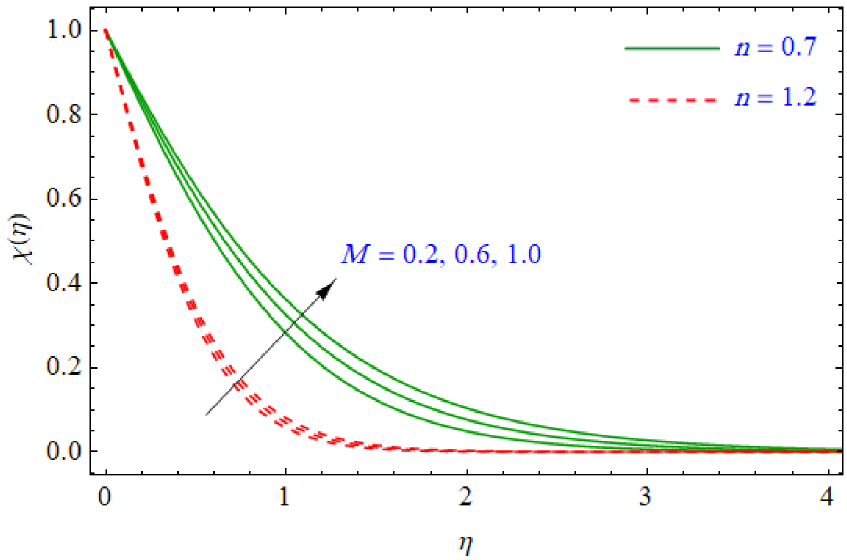
4 Results and discussion

For getting clear insight of the current physical problem, the graphical illustrations were displayed for the numerical results. The computations of the physical parameters were carried out including magnetic field parameter $(0.2 \le M \le 1.0)$, bioconvection Péclet number $(0.1 \le Pe \le 2.0)$, bioconvection constant $(0.1 \le \beta \le 0.9)$, Brownian motion parameter$(0.1 \le Nb \le 0.6)$, thermophoresis parameter $(0.1 \le Nt \le 0.9)$, bioconvection Lewis number $(0.5 \le Lb \le 2.0)$, generalized Biot number $(0.1 \le \gamma \le 3.0)$, Prandtl number $(1.0 \le Pr \le 3.0)$, Lewis number $(1.0 \le Le \le 5.0)$, and power-law index $(0.2 \le n \le 1.2)$.

The profiles of the rescaled density of motile microorganisms under the effects of bioconvection Péclet number, and magnetic field parameter M at two values of a power law index n=0.7 and n=1.2 have been shown in Figs. 2 (a) and 2(b). It is observed that, the rescaled density of motile microorganisms within the boundary layer increases as both of the bioconvection Péclet number and magnetic field parameter are increase. The rescaled density of motile microorganisms is lower at higher value of power law index n=1.2. Moreover, the boundary layer thickness is shrinking as power index n is increasing from 0.7 to 1.2. Fig. 3 presents the rescaled density of motile microorganisms under the impacts of bioconvection Lewis number Lp, and bioconvection constant β at two values of power law index n=0.5 and n=1.2. The rescaled density of motile microorganisms decreases as Lp increases from 0.5 to 2. In addition, the rescaled density of motile microorganisms increases as bioconvection constant β increases. Moreover, the boundary layer thickness is shrinking as power law index n is increasing from 0.5 to 1.2.

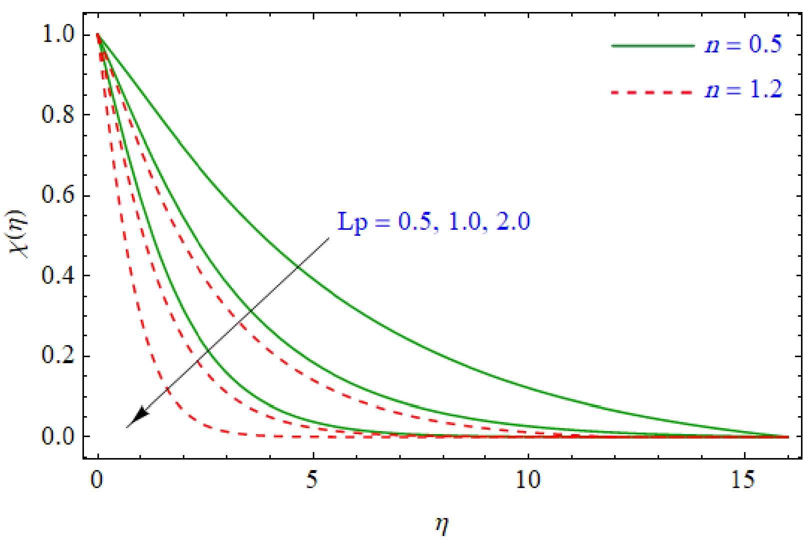


(a) Effects of bioconvection Péclet number

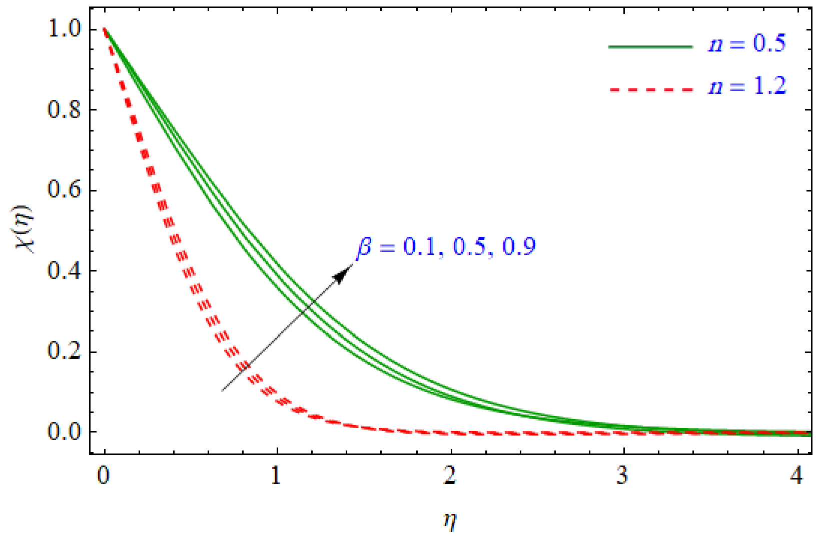


(b) Effects of magnetic field parameter ***M***

Figure 2: Profiles of rescaled density of motile microorganisms under the effects of (a) bioconvection Péclet number, and (b) magnetic field parameter ***M*** at two values of a power law index *n*=0.7 and *n*=1.2

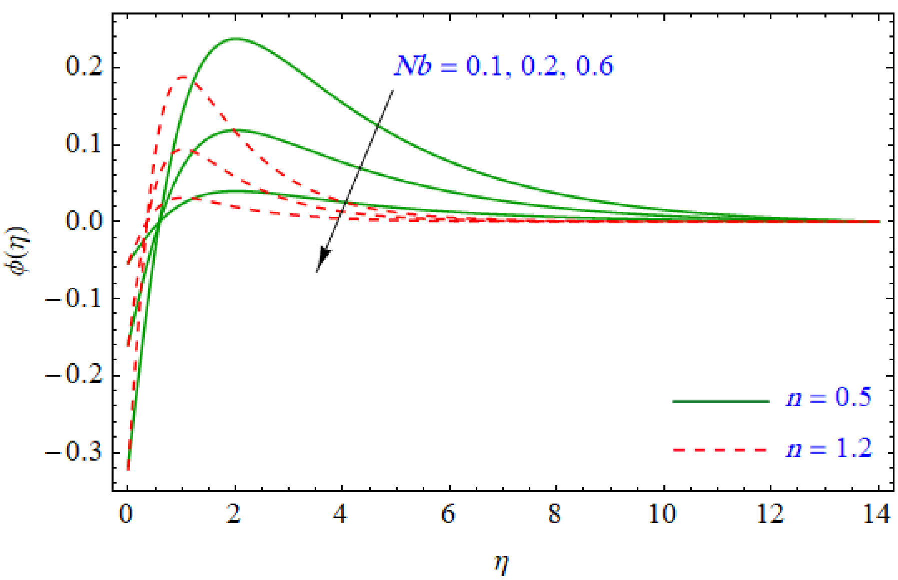


(a) Effects of bioconvection Lewis number

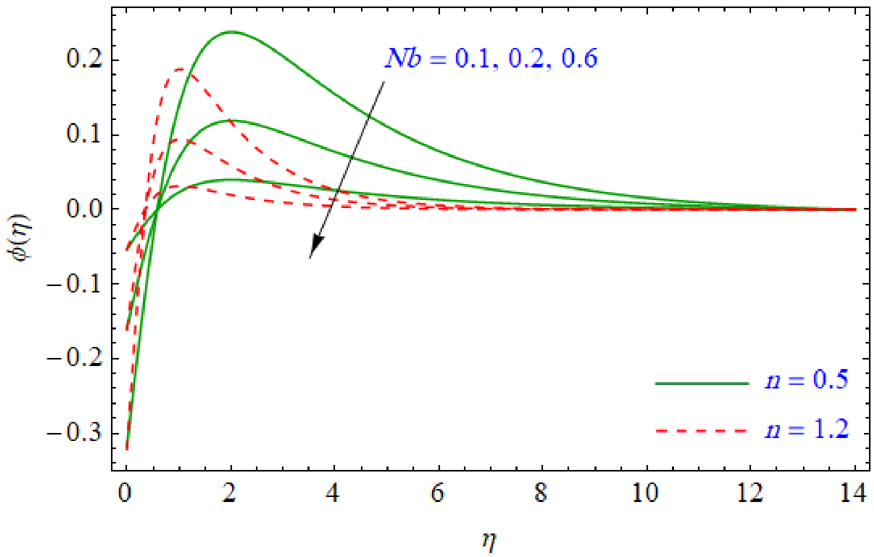


(b) Effects of bioconvection constant β

Figure 3: Profiles of rescaled density of motile microorganisms under the effects of (a) bioconvection Lewis number Lp, and (b) bioconvection constant β at two values of power law index n=0.5 and n=1.2

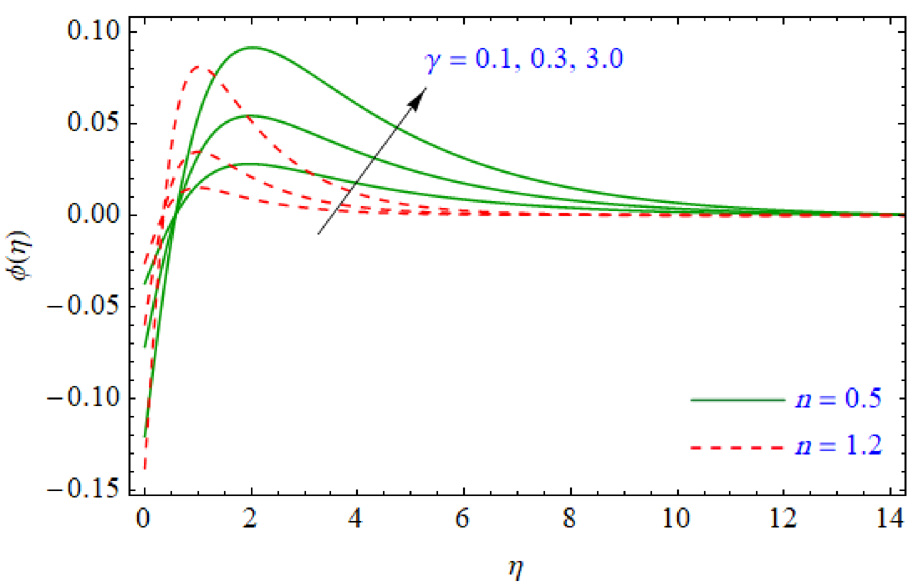


(a) Effects of Brownian motion parameter

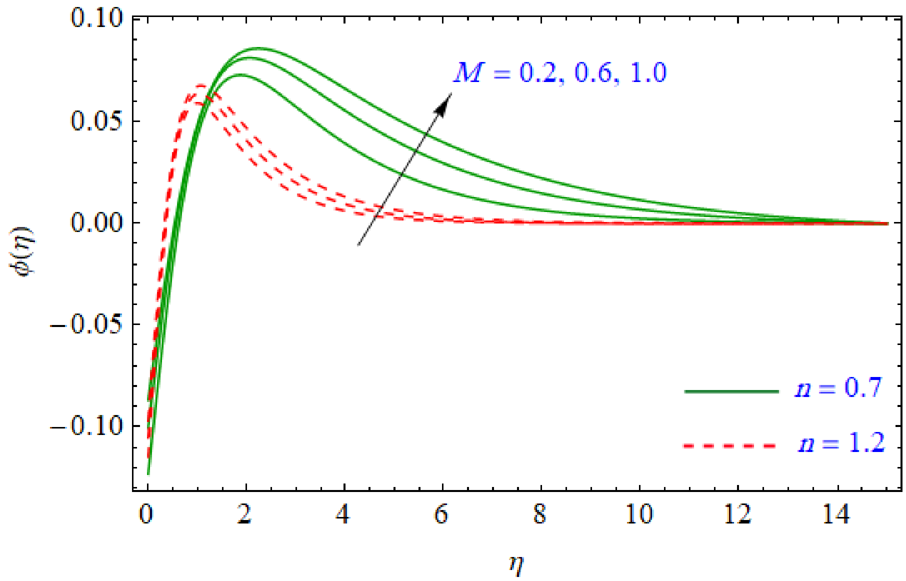


(b) Effects of thermophoresis parameter

Figure 4: Profiles of nanoparticle volume fraction under the effects of (a) Brownian motion parameter, and (b) thermophoresis parameter at two values of a power law index n=0.5 and n=1.2



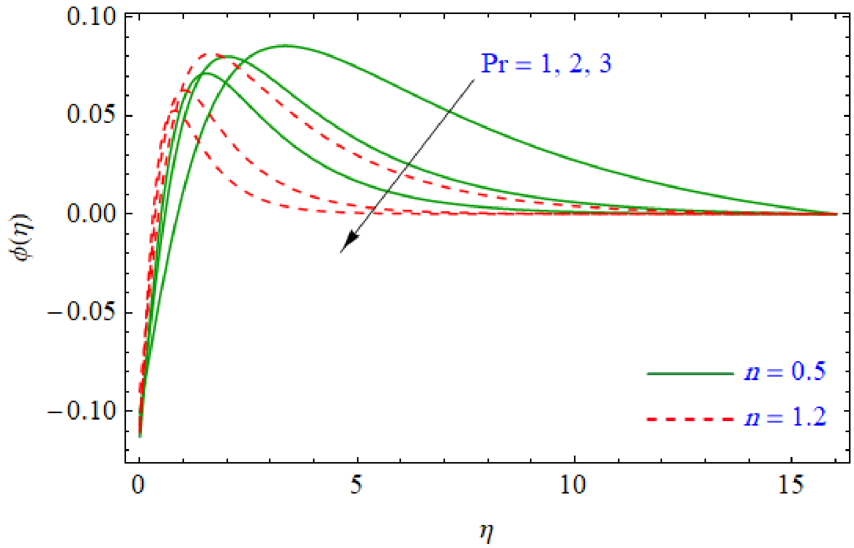
(a) Effects of generalized Biot number γ



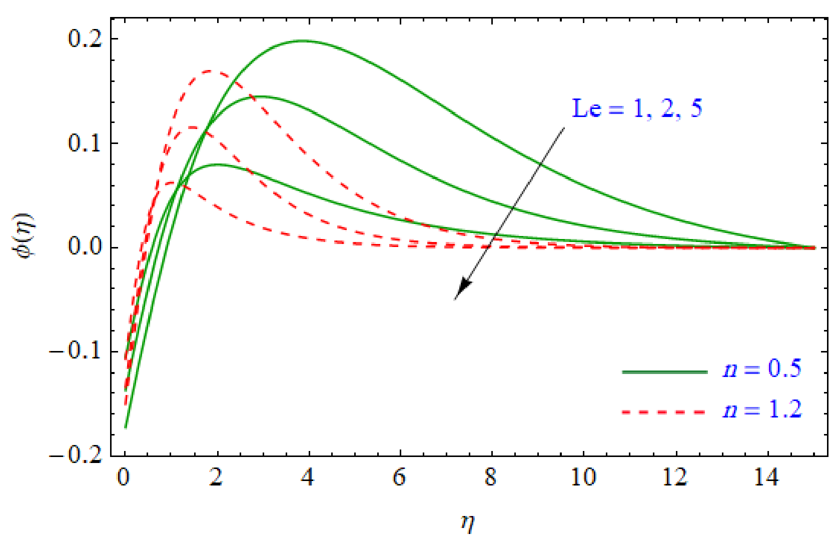
(b) Effects of magnetic field parameter M

Figure 5: Profiles of nanoparticle volume fraction under the effects of (a) generalized Biot number γ at two values of power law index n=0.5 and n=1.2, and (b) magnetic field parameter M at two values of power law index n=0.7 and n=1.2

The profiles of nanoparticle volume fraction under the effects of Brownian motion parameter Nb, thermophoresis parameter Nt, generalized Biot number γ, and magnetic field parameter M at two values of power law index have been shown in Figs. 4 and 5. The nanoparticle volume fraction increases as thermophoresis parameter Nt, generalized Biot number γ, and magnetic field parameter M are increase and it decreases as Brownian motion parameter Nb increases. In addition, the nanoparticle volume fraction has the higher values at lowest values of a power law index n for all of the cases. In addition, Fig. 6 shows the profiles of the nanoparticle volume fraction under the effects of Prandtl number and Lewis number at two values of power law index n=0.5 and n=1.2. The profiles of the nanoparticle volume fraction are decrease as the Prandtl number and Lewis number are increase.



(a) Effects of Prandtl number

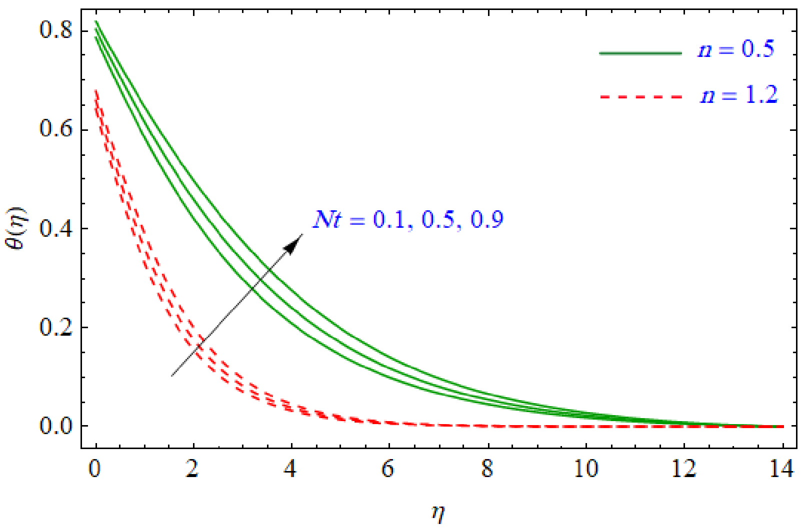


(b) Effects of Lewis number

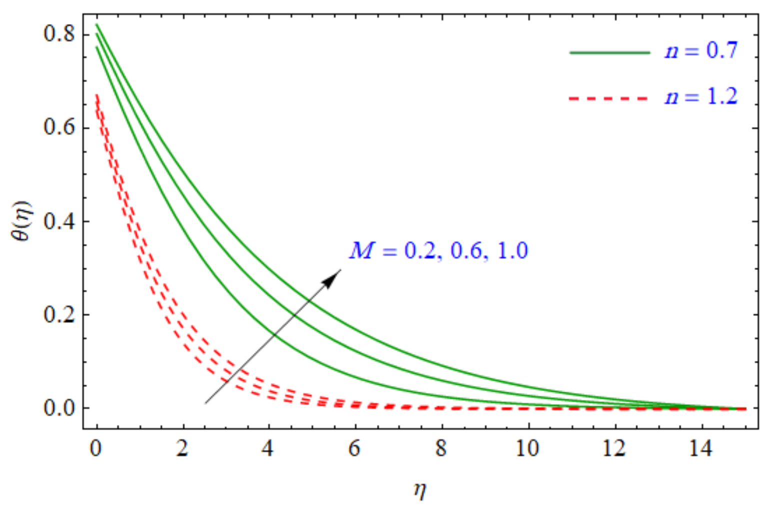
Figure 6: Profiles of nanoparticle volume fraction under the effects of (a) Prandtl number, and (b) Lewis number at two values of power law index n=0.5 and n=1.2

Figs. 7 and 8 present the temperature profiles under the effects of thermophoresis parameter, magnetic field parameter M, generalized Biot number γ and Prandtl number at two values of power law index. Here, the temperature profiles as well as isothermal boundary layer thickness are increase according to an increase on thermophoresis parameter, magnetic field parameter and generalized Biot number. While, the temperature profiles are decrease as the

Prandtl number increases. The reason return to less thermal conductivity at higher Prandtl number. In addition, the power law n index plays an important role on controlling the thickness of the thermal boundary layer and their profiles. The thermal boundary layer thickness decreases as the power law index increases to n=1.2. The temperature profiles have the higher values at lower power law index n at all cases.

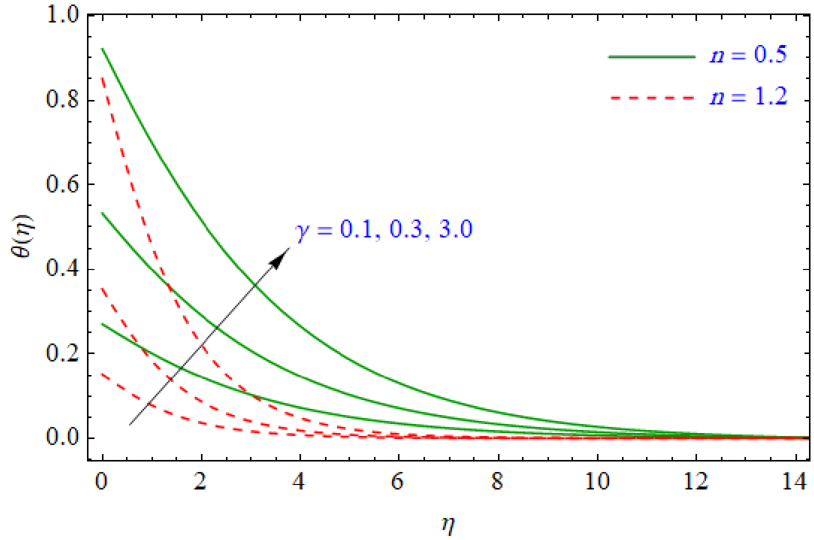


(a) Effects of thermophoresis parameter

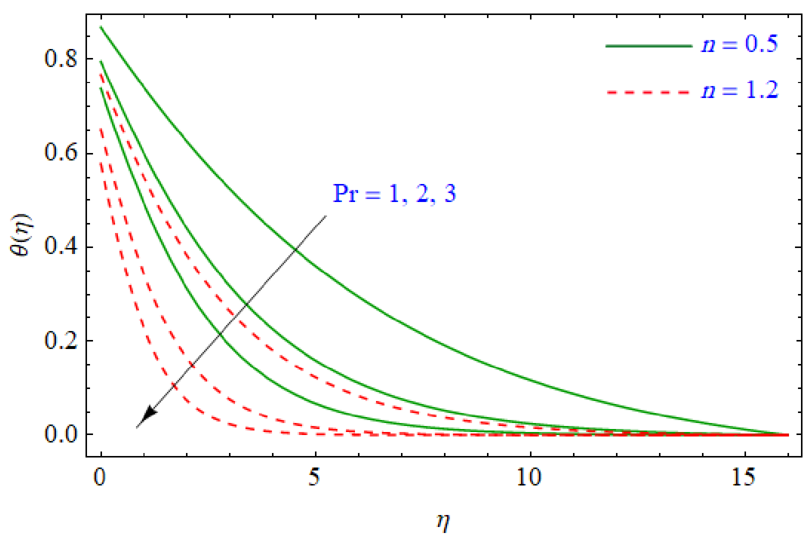


(b) Effects of magnetic field parameter M

Figure 7: Profiles of temperature under the effects of (a) thermophoresis parameter at two values of power law index n=0.5 and n=1.2, and (b) magnetic field parameter M at two values of power law index n=0.7 and n=1.2



(a) Effects of generalized Biot number γ



(b) Effects of Prandtl number

Figure 8: Profiles of temperature under the effects of (a) generalized Biot number γ, and (b) Prandtl number at two values of power law index n=0.5 and n=1.2

Fig. 9 shows the velocity profiles under the effects of magnetic field parameter M at two values of power law index n=0.7 and n=1.2. Due to Lorentz force, that suppresses the velocity, then velocity profiles are decrease with an increase on the magnetic field parameter. Moreover, the velocity profiles decrease slightly as power law index increases. In addition, due to neglecting the buoyancy force terms in the momentum equation, the bioconvection parameters and nanofluid parameters have slight effects on the velocity profiles. Fig. 10 depicts the profiles of the rescaled density of motile microorganisms under the effects of Prandtl number at two values of power law index n=0.5 and n=1.2. The rescaled density of motile microorganisms reduces as Prandtl number increases and it has the lowest values at higher power law index n=1.2.

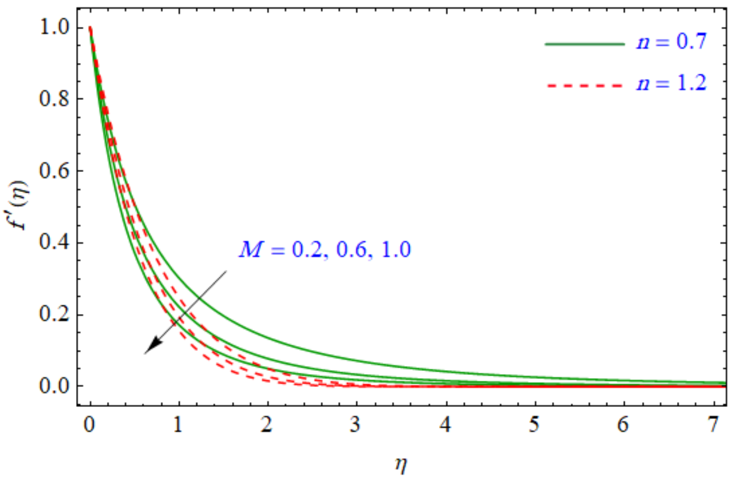


Figure 9: Profiles of velocity under the effects of magnetic field parameter M at two values of power law index n=0.7 and n=1.2

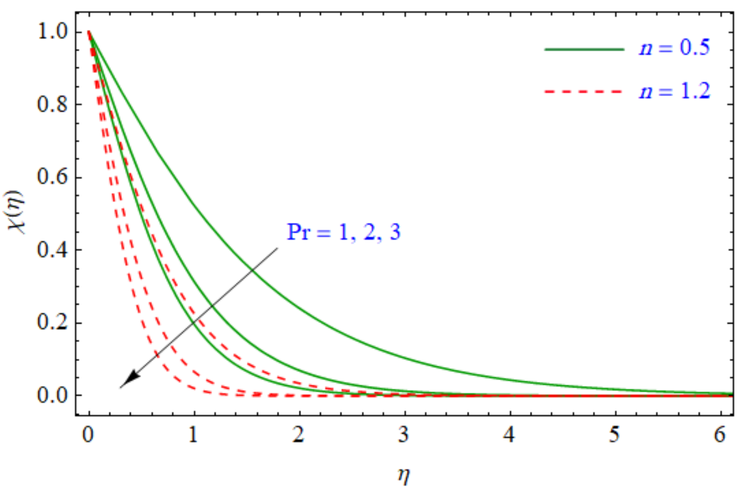


Figure 10: Profiles of rescaled density of motile microorganisms under the effects of Prandtl number at two values of power law index n=0.5 and n =1.2

Fig. 11 shows the variations of the reduced local skin-friction coefficient with magnetic field, power law index along thermophoresis parameter. It is observed that, the reduced local skin-friction coefficient has the higher values at lower magnetic field parameter $M = 0.2$ and lower power law index n=0.7. The reduced local skin-friction coefficient is slightly change according to an increase on thermophoresis parameter.

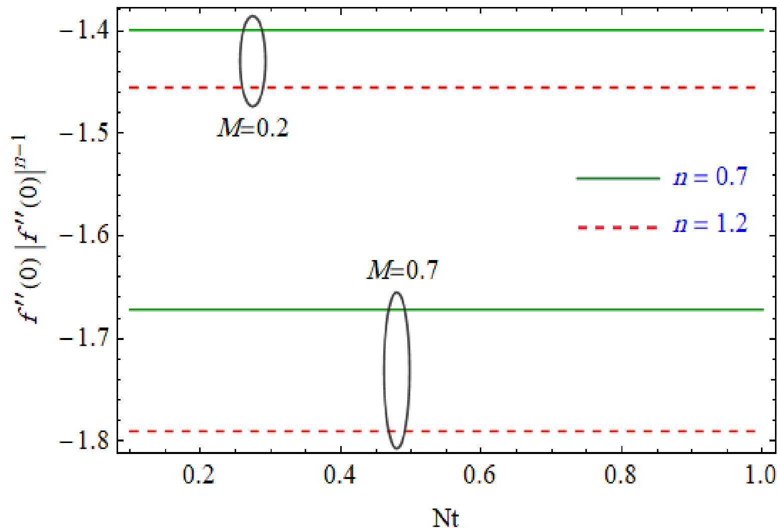
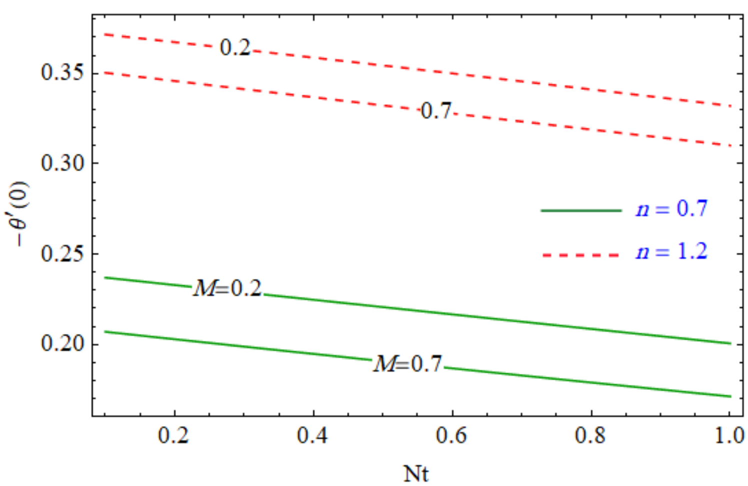
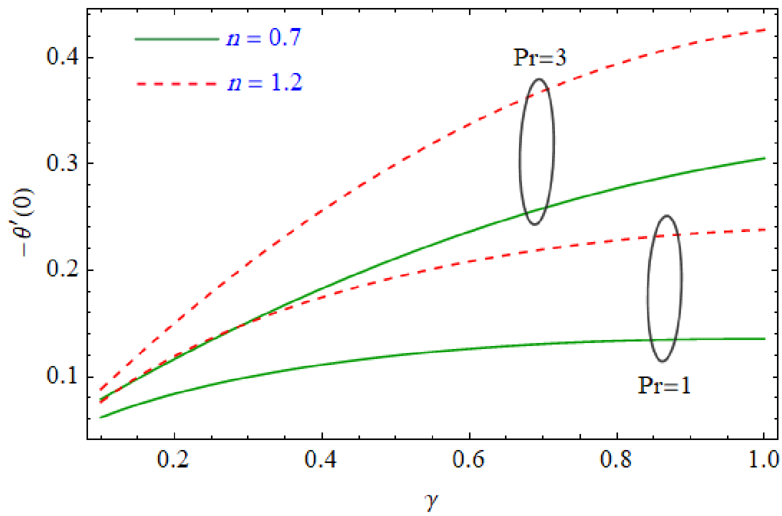


Figure 11: Variation of reduced local skin-friction coefficient with magnetic field, power law index along thermophoresis parameter

The variations on the reduced Nusselt numbers under the effects of several parameters are shown in Fig. 12. In Fig. 12(a), the reduced Nusselt number is decreasing with an increase on the magnetic field parameter and thermophoresis parameter. This is relevant to the fact that magnetic field parameter reduce the velocity and increase the temperature within the boundary layer and then the reduced Nusselt number decreases as magnetic field parameter increases. In addition, the reduced Nusselt number has the higher values at higher power law index *n*=1.2. In Fig. 2(b), the reduced Nusselt number is increasing as the Prandtl number, generalized Biot number γ and power law index are increase.



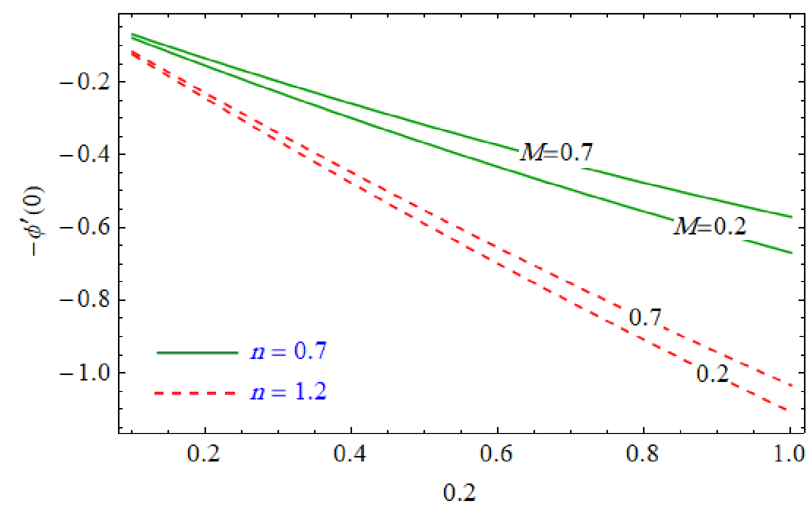
(a)



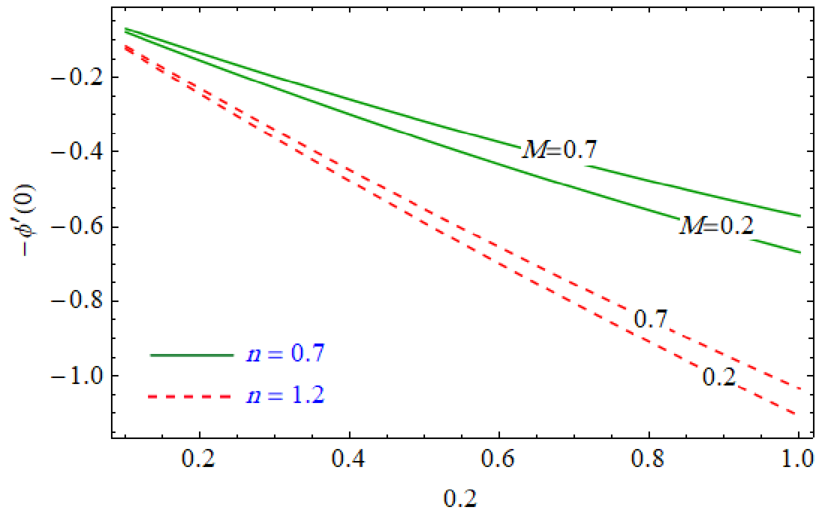
(b)

Figure 12: Variations of reduced Nusselt number with different parameters

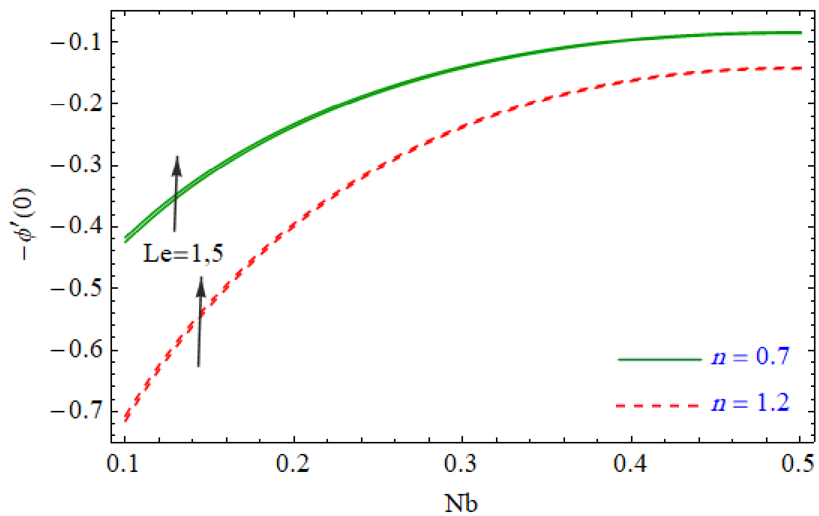
The variations on the reduced Sherwood numbers under the effects of several parameters are shown in Fig. 13. In this figure, the reduced Sherwood numbers are increasing according to an increase on the magnetic field parameter, Lewis number and Brownian motion parameter. Moreover, the reduced Sherwood numbers are decreasing, as the Parndtl number, power law index and generalized Biot number are increase. It is important to observe that, an increment rate on the reduced Sherwood number is higher at Brownian motion parameter (0.1-0.5) compare to Lewis number (1-5).



(a)



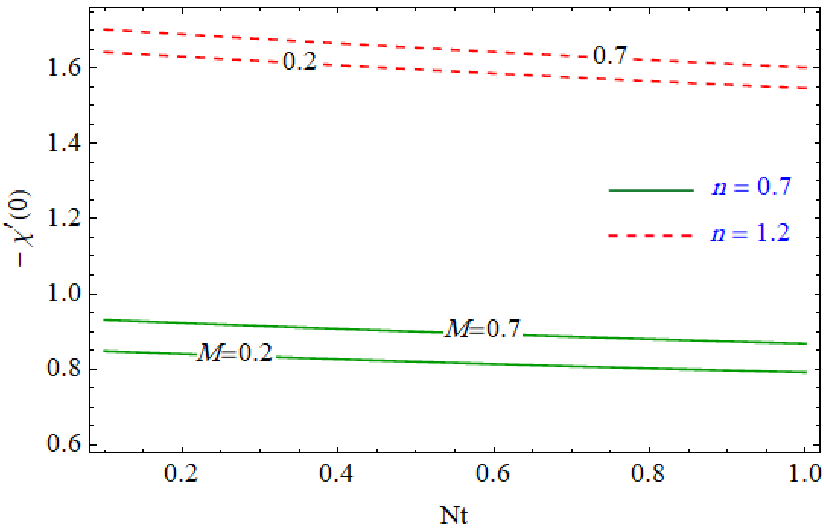
(b)



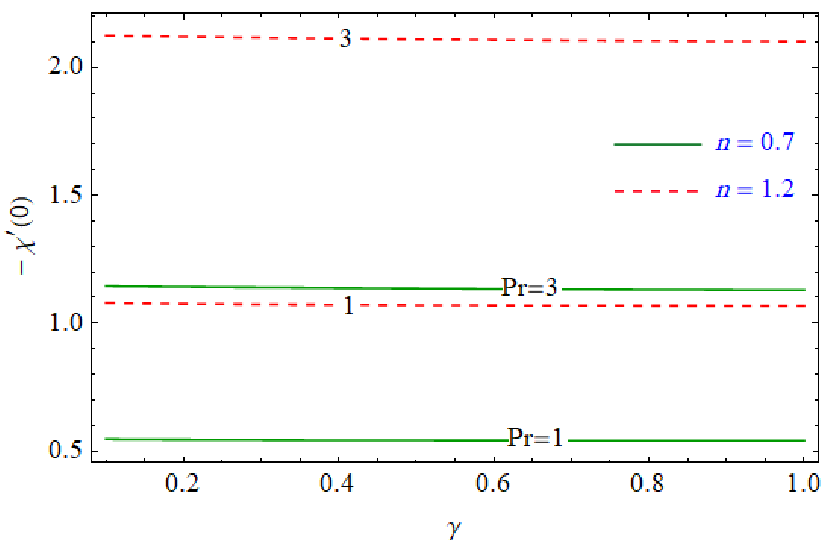
(c)

Figure 13: Variations of reduced Sherwood number with different parameters

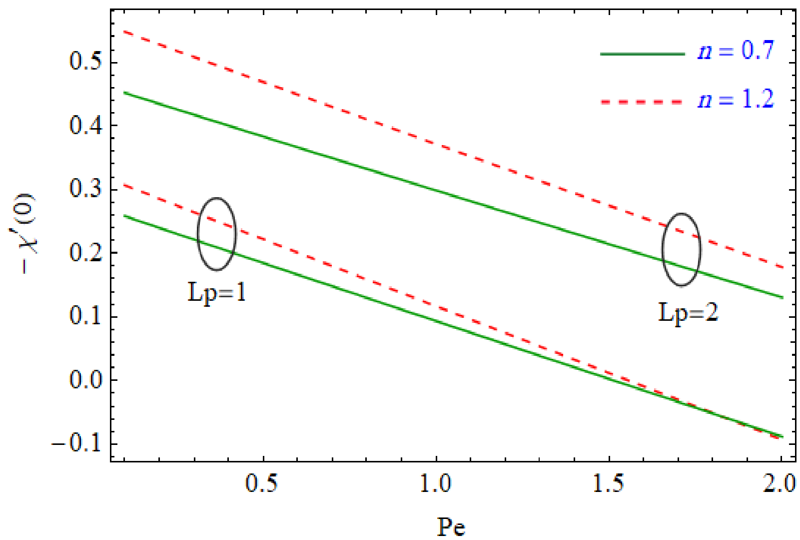
The variations on the reduced local density number of the motile microorganisms under the effects of several parameters are shown in Fig. 14. It is observed that, there are almost no changes on the reduced local density number of the motile microorganisms under the variations of the thermophoresis parameter (0.1-1.0) and generalized Biot number (0.1-1.0). In addition, the reduced local density number of the motile microorganisms increases as the Prandtl number, magnetic field parameter and power law index are increase. The bioconvection parameters impacts on the reduced local density number of the motile microorganisms are shown in Fig. 14(c). It is known that, the bioconvection numbers are related to the diffusivity of microorganisms. In this figure, the reduced local density number of the motile microorganisms decreases as the bioconvection Péclet number increases, while it increases as Bioconvection Lewis number increases.



(a)



(b)



(c)

Figure 14: Variations of reduced local density number of the motile microorganisms with different parameters

5 Conclusion

This study investigated the steady natural convection MHD boundary layer flow of a power-law nanofluid containing gyrotactic microorganisms over an exponentially stretching surface. The nonlinear system of the governing equations is transformed into dimensionless similar equations, which are solved numerically using Runge-Kutta-Fehlberg method. The effects of the governing parameters such as transverse magnetic field, bioconvection parameters, nanofluid parameters, Prandtl number, Lewis number and power-law index on the velocity, temperature, nanoparticle volume fraction, density of motile microorganism's profiles as well as the local skin-friction coefficient, local Nusselt, local Sherwood numbers and local density number of the motile microorganisms are explored. The main findings of this work are reported:

- The rescaled density of motile microorganisms increases as the bioconvection Péclet number, bioconvection constant and magnetic field parameter are increase.
- The nanoparticle volume fraction increases as thermophoresis parameter, generalized Biot number and magnetic field parameter are increase.
- Due to Lorentz force, that suppresses the velocity, velocity profiles are decrease with an increase on the magnetic field parameter.
- The reduced local skin-friction coefficient has higher values at a lower magnetic field parameter and a lower power law index.
- The reduced Nusselt number is decreasing with an increase on the magnetic field parameter and thermophoresis parameter.

- The reduced Sherwood number is increasing according to an increase on the magnetic field parameter, Lewis number and Brownian motion parameter.
- The reduced local density number of the motile microorganisms increases as the Prandtl number, magnetic field parameter and power law index are increase.

Acknowledgment: The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University for funding this work through Big Group Research Project under grant number (R.G.P2/16/40).

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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