

Radiation and Chemical Reaction Effects on Nanofluid Flow Over a Stretching Sheet

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Abstract: The present research aims to examine the steady state of the two-dimensional incompressible magnetohydrodynamics (MHD) flow of a micropolar nanofluid over a stretching sheet in the presence of chemical reactions, radiation and viscous dissipation. The effect of particle rotation is taken into account. A conducting fluid passes over a semi-infinite plate with variable temperature while a magnetic field is directed in the transverse direction. Results for velocity, angular momentum, temperature and concentration profiles are obtained for various values of Eckert number, Schmidt number, Prandtl number, thermophoresis parameter and Brownian motion parameters. A similarity approach is used to transform the original set of two-dimensional partial differential equations into a set of highly nonlinear-coupled differential equations in dimensionless form. A numerical solution is obtained with the help of the COMSOL multiphysics software in the framework of a finite element method. Our findings indicate that on increasing Brownian motion and the chemical reaction rate, the fluid temperature becomes higher. An increase in the values of other physical parameters has the opposite effect. A variation in the boundary layer thickness typically results in changes in the concentration distribution in the flow. The angular velocity is deeply affected by the Eckert number, material parameter and magnetic number.

Keywords: Nanofluid, magnetohydrodynamics, rotation, temperature, concentration.

Nomenclature

b Constant

B_0 Magnetic field (T)

C Concentration of the fluid inside the boundary layer (kg / m^2)

c_p Specific heat at constant pressure ($J / kg K$)

C_w Concentration at the wall surface (kg / m^3)

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C_∞ Concentration of the fluid outside the boundary layer (kg / m^3)

D_B Brownian diffusion coefficient

D_T Thermophoresis diffusion coefficient

Ec Eckert Number

f Reduced stream function

i Reference length

j Microinertia per unit mass m^2

k Thermal conductivity ($W / m k$)

K Material Parameter ($J / kg k$)

M Magnetic parameter

Nu_x Nusselt number

Nb Brownian motion parameter

Nt Thermophoresis parameter

Re_w Local Reynolds number

Ω Angular velocity

Pr Prandtl Number

q_w Heat transfer rate (W)

Sc Schimidt Number

T Boundary layer fluid temperature (k)

T_w Surface temperature (k)

T_∞ Temperature of free stream fluid outside the boundary layer (k)

u Fluid velocity component in x direction (m / s)

u_w Characteristic velocity (m / s)

v Fluid velocity component in y direction (m / s)

x, y Coordinate direction (m)

v_w Rate of stretching (m / s)

$\theta(\xi)$ Dimensionless temperature

- $\varphi(\xi)$ Dimensionless concentration
- ν Kinematic viscosity ($1 / m^2 s$)
- μ Dynamic viscosity ($kg / m s$)
- ρ Fluid density (kg / m^3)
- ρc_p Heat capacity of fluid
- $(\rho c_p)_p$ Heat capacity of the nanoparticle
- Φ_1 Volume fraction
- Ψ Effective magnetic parameter for viscosity
- τ_B Brownian relaxation time
- Ω_p Angular velocity of the particles
- γ_F Spin gradient viscosity
- γ Dimensionless chemical reaction parameter
- Q_0 Heat absorption coefficient
- Q_1 Radiation absorption coefficient
- K_1 Chemical reaction parameter
- Gr Thermal Grashof number
- Gm Solutal Grashof number
- ω Rotation parameter
- χ Dimensionless heat generation/absorption parameter
- Q'_1 Dimensionless radiation absorption coefficient

1 Introduction

Incompressible magnetohydrodynamics (MHD) flow plays an important role in various industrial applications. MHD flow may be applicable in the extrusion of polymer sheet from a die or in drawing of plastic films. The heat and mass transfer for micropolar MHD flow past a stretching sheet has the varieties of applications such as polymer blends, porous rocks, aerogels, alloys and microemulsions. Due to a large number of applications of magnetic fluid, the researchers of many disciplines have been attracted in the field of MHD during the last few decades. In the early stage of MHD research, the problems of MHD flow were solved through analytical method. However, time to time, new methods of solutions came into existence. Now a days, the numerical methods have become the strong tool to solve a set of nonlinear coupled differential equations of MHD

flow. Sometimes the numerical approach of the solution is a big challenge, if the differential equations involved in the problems are highly nonlinear. Therefore, the researchers are finding numerical solution through mathematical modeling without sacrificing the relevant phenomena. In fact, results obtained through mathematical modeling of the system of nonlinear coupled differential equations are very reliable than numerical methods only.

A problem of MHD flow with magnetic and viscous dissipation effects towards the stretching sheet has been studied [Hsiao (2017)]. MHD flow of micropolar nanofluid flow in the presence of some physical parameters have been investigated [Ramzan, Ullah, Chung et al. (2017); Pal and Mandal (2017); Tabassum, Mehmood and Akbar (2017)]. The unsteady electrically conducting magnetic fluid flow past over an oscillating vertical plate is investigated [Sheikholeslami, Kataria and Mittal (2018)]. The boundary layer and stagnation-point flow of nanofluid over a stretching sheet and Soret effects on viscoelastic nanofluid flow over vertical stretching surface have been studied in the presence of viscous dissipation and radiation effects [Imtiaz, Hayat and Alsaedi (2016); Anwar, Shafie, Hayat et al. (2017); Ramzan, Yousaf, Farook et al. (2016)]. Various types of nanofluid with heat and mass transfer have been investigated through some physical properties of nanofluid [Sheikholeslami (2018); Sheikholeslami and Rokni (2017); Sheikholeslami, Haq, Shafee et al. (2019); Sheikholeslami (2019); Lu, Ramzan, Huda et al. (2018)]. Lattice Boltzmann method and Brownian motion were the key parts for the study of nanofluid flow through numerical simulation [Sheikholeslami (2018); Sheikholeslami, Shehzad and Li (2018)]. Two-dimensional MHD stagnation point flow is presented to study the heat transfer of a viscous incompressible non-Newtonian nanofluid [Gupta, Kumar and Singh (2018)].

A three-dimensional nanofluid flow over a nonuniform sheet and an exponentially stretching surface in a porous medium with variable thermal conductivity have been investigated [Subhani and Nadeem (2017); Kumar, Raju, Sekhar et al. (2017); Ramzan, Bilal, Chung et al. (2017)]. The effect of acoustic streaming on nanoparticle motion and morphological evolution inside an acoustically levitated droplet using an analytical approach coupled with experiments is investigated [Saha, Basu and Kumar (2012)]. A mixed stagnation point flow of nanofluids over a stretching surface with thermal radiation and viscous dissipation effects have been analyzed [Pal, Vajravelu and Mandal (2014)]. Heat and mass transfer of nanofluid with motile gyrotactic effects have been investigated [Ramzan, Chung and Ullah (2017)]. Jeffery nanofluid flow over a linearly stretched surface and inclined stretched cylinder with chemical reaction and slip conditions have been studied [Ramzan, Bilal, Chung et al. (2018, 2017)]. Effects of Hall current on MHD unsteady flow through a vertical plate in the presence of radiation and chemical reaction is investigated [Biswas and Ahmed (2018)]. The entropy generation of micropolar fluid flow in a rectangular duct subjected to slip and convective boundary conditions is reported [Srinivasacharya and Himabindu (2017)]. Heat and mass transfer analysis on magnetohydrodynamic flow of viscous fluid by curved surface is presented with joule heating [Hayat Qayyum, Imtiaz et al. (2018)]. Laminar flow of nanofluid has been investigated numerically through Homotopy Analysis Method [Lu, Farooq, Hayat et al. (2018)].

Effects of viscous dissipation and Joule heating in MHD flow by rotating disk of variable thickness are investigated [Hayat, Qayyum, Khan et al. (2018)]. MHD mixed convection flow and heat transfer in a porous enclosure filled with a Cu-water nanofluid in the presence of partial slip effect are investigated in the presence of heat sink and source parameters [Chamkha, Rashad, Mansour et al. (2017)]. MHD viscous two-phase dusty fluid flow and heat transfer over permeable stretching sheet is studied taking linear deformation in the wall boundary [Turkyilmazoglu (2017)]. The heat and mass transfer of a MHD nanofluid on a stretching sheet and stretching cylinder have been analyzed with Brownian motion and thermophoresis effects [Atif, Hussain and Sagheer (2018); Hayat, Nasseem, Khan et al. (2018)]. A mathematical model has been developed to examine the MHD micropolar nanofluid flow with buoyancy effects in the presence of nonlinear thermal radiation and dual stratification [Ramzan, Ullah, Chung et al. (2017)]. The problems on Casson nanofluid over a stretching surface with various physical parameters have been investigated by the researchers [Rana, Mehmood, Narayana et al. (2016); Rana, Mehmood and Akbar (2016); Iqbal, Mehmood, Azhar et al. (2017); Mehmood and Iqbal (2017)]. Kellor box algorithm has been developed to study the behavior of nanofluid in the presence of various physical parameters [Mehmood, Rana and Maraj (2018)]. The velocity and temperature distribution are studied on two-dimensional oblique Oldroyd-B flow on a stretching heated sheet [Mehmood and Rana (2018)]. The stagnation point flow using Jeffery nanofluid is studied in the presence of various physical parameters [Mehmood, Nadeem, Saleem et al. (2017)]. Homotopy perturbation transform method is also useful to solve Navier-Stokes equation for MHD flow [Kumar, Singh and Kumar (2015)]. Homotopy perturbation Sumudu mehod has been used to study two-dimensional viscous flow over a shrinking sheet [Rathore, Shisodia and Singh (2013)].

In the present problem, two dimesional MHD flow of micropolar nanofluid over a semi-infinite stretching plate is considered subjected to a transverse magnetic filed with variable temperature. The fluid is electrically conducting and flow is an incompressible. Thermal and concentration buyoyancy effects and heat bsorption and radiation effects are also presnt in the nanofluid. The chemical concentration effect is also considered in the fluid. The nonlinear coupled differential equations involed in the problem are solved numerically using finite element method through mathematical modeling in COMSOL.

The micropolar nanofluids are special kind of fluid which exhibits the microscopic and nano effects due to micromotion and Brownian motion of the fluid particles. The model for micropolar nanofluid are useful to study the behavior of collodial suspensions, liquid crystels, polymeric fluid, liquid crystals and animal bloods.

2 Mathematical formulation

Two dimensional incompressible flow of micropolar nanofluid due to moving surface is considered here. The magnetic filed is directed perpendicular to the streching sheet and the x axis and y axis are along the surfacce and perpendicular to the surface, respectively [Hiao (2017) and Singh and Kumar (2016)]. The governing equations are as follows:

The equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

The equation of motion

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\eta + \frac{k}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho} \frac{\partial \Omega}{\partial y} - \frac{\sigma B_0^2}{\rho} u + g\beta(T - T_\infty) + g\beta'(C - C_\infty) + \frac{I}{2\tau_B} \left(\frac{\partial \Omega_1}{\partial y} - \frac{\partial \Omega}{\partial y} \right) \quad (2)$$

The equation of angular momentum

$$u \frac{\partial \Omega}{\partial x} + v \frac{\partial \Omega}{\partial y} = \frac{\gamma_F}{j\rho} \frac{\partial^2 \Omega}{\partial y^2} + \frac{k}{j\rho} \left(2\Omega + \frac{\partial u}{\partial y} \right) - \frac{1}{\tau_B} (\Omega_p - \Omega) \quad (3)$$

The energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{(\mu + k)}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{\rho c_p} u^2 + \frac{(\rho c_p)_p}{\rho c_p} \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] - \frac{Q_0}{\rho c_p} (T - T_\infty) + Q_1 (C - C_\infty) \quad (4)$$

The equation of concentration

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{D_\infty} \frac{\partial^2 T}{\partial y^2} - K_1 (C - C_\infty) \quad (5)$$

In Eq. (2) (Momentum equation), the expressions $g\beta(T - T_\infty)$ and $g\beta'(C - C_\infty)$ indicate the thermal and concentration buoyancy effects, respectively. In Eq. (4) (Temperature equation), the expressions $\frac{Q_0}{\rho c_p}(T - T_\infty)$ and $Q_1(C - C_\infty)$ denote the heat

absorption and radiation effects, respectively. In Eq. (5) (Concentration equation), the expression $K_1(C - C_\infty)$ represents the chemical concentration buoyancy effects. The

expression $\frac{(\mu + k)}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2$ represent viscous dissipation, $\frac{\sigma B_0^2}{\rho c_p} u^2$ denote Ohmic heating

effect, $\frac{(\rho c_p)_p}{\rho c_p} D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y}$ represents the Brownian motion effect,

$\frac{(\rho c_p)_p}{\rho c_p} \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2$ indicates the thermophoresis effects. In Eq. (3), $\gamma_F = \left(\mu + \frac{k}{2} \right) i$ and

$i = \frac{m}{b}$ is taken as reference length. The expression $\frac{I}{2\tau_B} \left(\frac{\partial \Omega_1}{\partial y} - \frac{\partial \Omega}{\partial y} \right) = \frac{3}{2} \eta \Phi_1 \Psi \frac{\partial^2 u}{\partial y^2}$

represent the force due rotational motion.

The boundary conditions are as follows:

$$\begin{aligned} y=0: \quad u=v_w=bx, \quad v=0, \quad \Omega=0, \quad T=T_w, \quad C=C_w \\ y \rightarrow \infty: \quad u=0, \quad \Omega=0, \quad T=T_\infty, \quad C=C_\infty \end{aligned} \quad (6)$$

Eqs. (1)-(6) can be written in the dimensionsless form as:

$$\left(1 + \frac{3}{2}\Phi_1\Psi + K\right) \frac{d^4 f}{d\xi^4} + f \frac{d^2 f}{d\xi^2} - \left(\frac{df}{d\xi}\right)^2 + K \frac{dg}{d\xi} - M \frac{df}{d\xi} - Gr\theta - Gm\varphi = 0 \quad (7)$$

$$\left(1 + \frac{K}{2}\right) \frac{d^2 g}{d\xi^2} + f \frac{dg}{d\xi} - \frac{df}{d\xi} g - K \left(2g + \frac{d^2 f}{d\xi^2}\right) - \omega g = 0 \quad (8)$$

$$\begin{aligned} \frac{1}{Pr} \frac{d^2 \theta}{d\xi^2} + f \frac{d\theta}{d\xi} + (1+K)Ec \frac{d^2 f}{d\xi^2} + Ec M \left(\frac{df}{d\xi}\right)^2 + Nb \frac{d\theta}{d\xi} \frac{d\varphi}{d\xi} + Nt \left(\frac{d\theta}{d\xi}\right)^2 \\ - \chi\theta + Q_1' \varphi = 0 \end{aligned} \quad (9)$$

$$\frac{d^2 \varphi}{d\xi^2} + Sc f \frac{d\varphi}{d\xi} + \frac{Nt}{Nb} \frac{d^2 \theta}{d\xi^2} - Sc \gamma \varphi = 0 \quad (10)$$

The dimensionless quantities are presented as follwos:

$$\begin{aligned} \xi = \sqrt{\left(\frac{b}{\eta}\right)} y, \quad u = bx \frac{df}{d\xi}, \quad v = -\sqrt{(b\eta)} f(\xi), \quad \Omega = \sqrt{\left(\frac{b^3}{\eta}\right)} x g(\xi), \\ \theta(\xi) = \frac{T - T_\infty}{T_w - T_\infty}, \quad T = T_\infty + Ax\theta(\xi), \quad \varphi(\xi) = \frac{C - C_\infty}{C_w - C_\infty}, \quad C = C_\infty + bx\varphi(\xi), \\ Gr = \frac{\mu g \beta (T_w - T_\infty)}{\rho \nu^3}, \quad Gm = \frac{\mu g \beta' (C_w - C_\infty)}{\rho \nu^3}, \quad Q_1' = \frac{\mu Q_1 (C_w - C_\infty)}{\rho \nu^2 (T_w - T_\infty)}, \\ Pr = \frac{\mu c_p}{k}, \quad K = \frac{k}{\mu}, \quad \chi = \frac{Q_0 \mu}{\rho^2 c_p \nu^2}, \quad \gamma = \frac{K_1 \mu}{\rho \nu^2}, \quad Sc = \frac{\mu}{\rho D}, \quad M = \frac{\sigma B_0^2}{\rho b}, \\ Ec = \frac{u_w^2}{c_p (T_w - T_\infty)}, \quad Nb = \frac{(\rho c_p)_p D_B (C_w - C_\infty)}{\rho c_p \nu}, \quad Nt = \frac{(\rho c_p)_p D_T (T_w - T_\infty)}{\rho c_p \nu T_\infty} \end{aligned} \quad (11)$$

The boundary conditions are as follows:

$$\begin{aligned} f(0)=0, \quad f'(0)=0, \quad g(0)=1, \quad \varphi(0)=1, \\ f'(\infty)=0, \quad g(\infty)=0, \quad \theta(\infty)=0, \quad \varphi(\infty)=0 \end{aligned} \quad (12)$$

3 Numerical solution

Eqs. (7)-(10) are solved numerically through mathematical modeling in COMSOL. Under PDE interface, COMSOL solves only second order partial differntial equations. Therefore, the following transformation has been used to reduce the above system of nonlinear equations into second order differential equations:

$$f = p, \quad \frac{df}{d\xi} = q, \quad \frac{d^2 f}{d\xi^2} = r \quad (13)$$

Hence, Eqs. (8)-(12) can be written as

$$\frac{dp}{d\xi} = q \quad (14)$$

$$\frac{dq}{d\xi} = r \quad (15)$$

$$\left(1 + \frac{3}{2}\Phi_1\Psi + K\right)\frac{d^2 r}{d\xi^2} + pr - q^2 + K\frac{dg}{d\xi} - Mq - Gr\theta - Gm\varphi = 0 \quad (16)$$

$$\left(1 + \frac{K}{2}\right)\frac{d^2 g}{d\xi^2} + p\frac{dg}{d\xi} - qg - K(2g + r) - \omega g = 0 \quad (17)$$

$$\frac{1}{Pr}\frac{d^2 \theta}{d\xi^2} + p\frac{d\theta}{d\xi} + (1 + K)Ecr + EcMq^2 + Nb\frac{d\theta}{d\xi}\frac{d\varphi}{d\xi} + Nt\left(\frac{d\theta}{d\xi}\right)^2 - \chi\theta + Q'_1\varphi = 0 \quad (18)$$

$$\frac{d^2 \varphi}{d\xi^2} + Scp\frac{d\varphi}{d\xi} + \frac{Nt}{Nb}\frac{d^2 \theta}{d\xi^2} - Sc\gamma\varphi = 0 \quad (19)$$

$$\begin{aligned} p(0) &= 0, \quad q(0) = 0, \quad g(0) = 0, \quad \theta(0) = 1, \quad \varphi(0) = 1, \\ q(\infty) &= 0, \quad g(\infty) = 0, \quad \theta(\infty) = 0, \quad \varphi(\infty) = 0 \end{aligned} \quad (20)$$

Eqs. (14)-(19) with the help of Eq. (20) has been modled in COMSOL and this solution is based on finite element method. These equations have been solved in COMSOL under Dirichlet boundary conditions. Extremely fine mesh has been used during the solution. Maximum size of elemet is 0.045 and the minimum element size is 0.00009. The shear stress at the surface can be calculated as:

$$q_w(x) = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k(T_w - T_\infty)\sqrt{\left(\frac{b\rho}{\mu}\right)}\theta'(0) \quad (21)$$

Using Eq. (11),

$$\tau_w = (\mu + k)bx\sqrt{\left(\frac{b\rho}{\mu}\right)}|f''(0)| \quad (22)$$

The relation $u_w = bx$ represents the characteristic velocity, the skin friction velocity C_f can be expressed as:

$$C_f = \frac{\tau_w}{\rho u_w^2} \quad (23)$$

Eq. (23) can be written with the help of Eqs. (11), (21) and (22) as:

$$C_f = \sqrt{\text{Re}_w} (1 + K) |f''(0)| \quad (24)$$

In Eq. (24), Re_w is the local Reynolds number. Sherwood number can be calculated as:

$$\frac{Sh_x}{\sqrt{\text{Re}_w}} = -\theta'(0) \quad (25)$$

The local Nusselt number can be calculated as:

$$Nu_x = -\sqrt{\text{Re}_w} \theta'(0) \quad (26)$$

4 Results and discussion

The main aim of this work is to study the heat and mass transfer on MHD flow of a micropolar nanfluid over a stretching sheet with various physical parameters. In this study, the effects of material parameter (K), Chemical reaction parameter (γ), Prandtl number (Pr), Eckert number (Ec), Brwonian motion parameter (Nb), thermophoresis parameter (Nt), magnetic parameter (M), thermal Grashof number (Gr), Solutal Grashof number (Gm), Schmidt number (Sc), heat generation/ absorption parameter (χ), radiation absorption parameter (Q'_1) on the temperature, concentration and velocity distributions are investigated. Rotation of the particle has also been taken into consideration. Due to the difference between the rotation of the particle and the rotation of the fluid, a viscous torque is generated which is being equilibrated by magnetic torque. The rotation of the particles in the presence of magnetic field generate additional resistance in the flow. Due to which, additional viscosity known as rotational viscosity is generated in the flow.

Figs. 1-7 represent the temperature distribution with the variation of different parameters. Fig. 1 is obtained for different values of radiation absorption coefficient. Increasing the value of this parameter, the heat will be absorbed. Therefore, it decreases the temperature; however, other parameters are taken same. Fig. 2 is obtained for various values of Thermal Grashof number (Gr). In the result, increasing in the values of thermal Grashof number increases the temperature distribution in the flow. It is also clear from Fig. 3, the temperature increases also for increasing the values of chemical reaction parameter (γ). Increasing the values of Thermal Grashof number makes the bond between the fluid to become weaker. This parameter decreases the internal friction resulting the gravity becomes stronger enough.

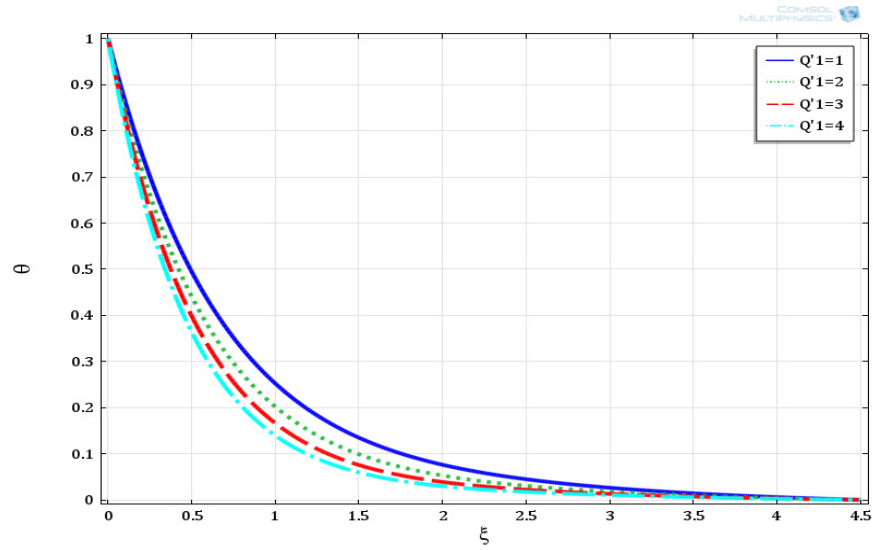


Figure 1: Temperature profile for $K = \gamma = Sc = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nt = Nb = 0.1$, $M = Gr = Gm = \chi = 2$

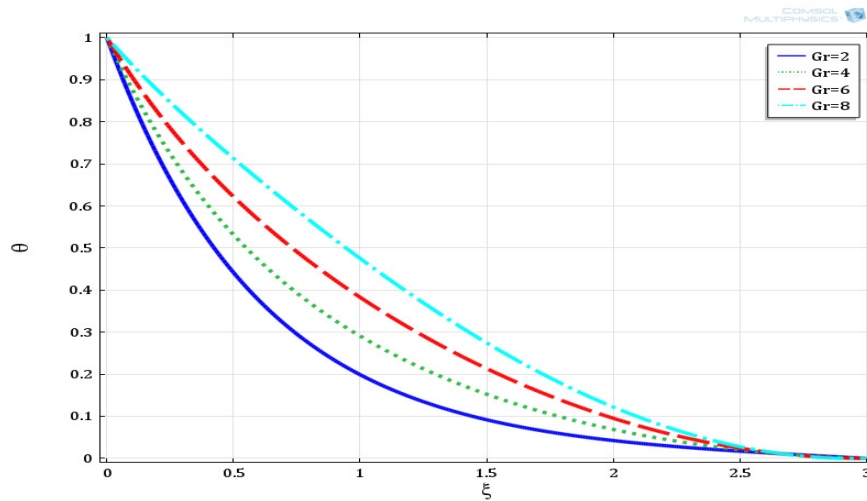


Figure 2: Temperature profile for $K = \gamma = Sc = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nt = Nb = 0.1$, $M = Gm = \chi = Q'1 = 2$

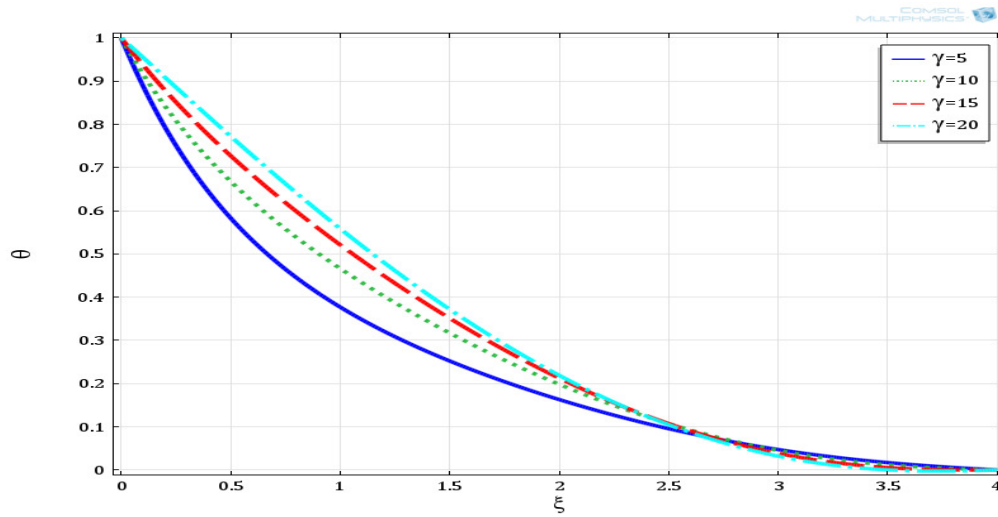


Figure 3: Temperature profile for $K = Sc = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nt = Nb = 0.1$,
 $M = Gm = \chi = Q'_1 = Gr = 2$

Fig. 4 is obtained for different values of Prandtl number (Pr). For increasing the values of Pr , a significant change in the temperature profile can be observed. Large values of Prandtl number indicate that the viscous force is dominant over thermal force result in temperature distribution decreases. Small values of Prandtl number the thermal diffusivity dominates whereas with the large values of Prandtl number the momentum diffusivity dominates in the flow. Fig. 5 indicates the temperature profile for different values of Solutal Grashof number (Gm). This parameter is generated due to concentration effects. For increasing values of Gm decreases the temperature in the flow. Fig. 6 depicts the temperature profile for different values of Brownian motion parameter (Nb). Its impact in the temperature depends on the motion of the particles and concentration in the flow. Temperature distribution for different values thermophoresis parameter (Nt) is given in Fig. 7. For increasing values of Nt decreases the temperature in the nanofluid flow for this problem.

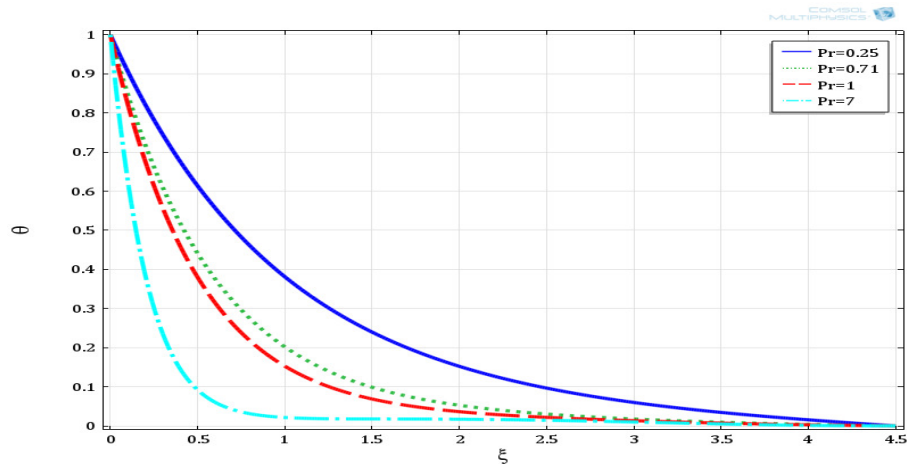


Figure 4: Temperature profile for $K = Sc = \gamma = 0.2$, $Ec = 0.02$, $Nt = Nb = 0.1$, $M = Gm = \chi = Q'_1 = Gr = 2$

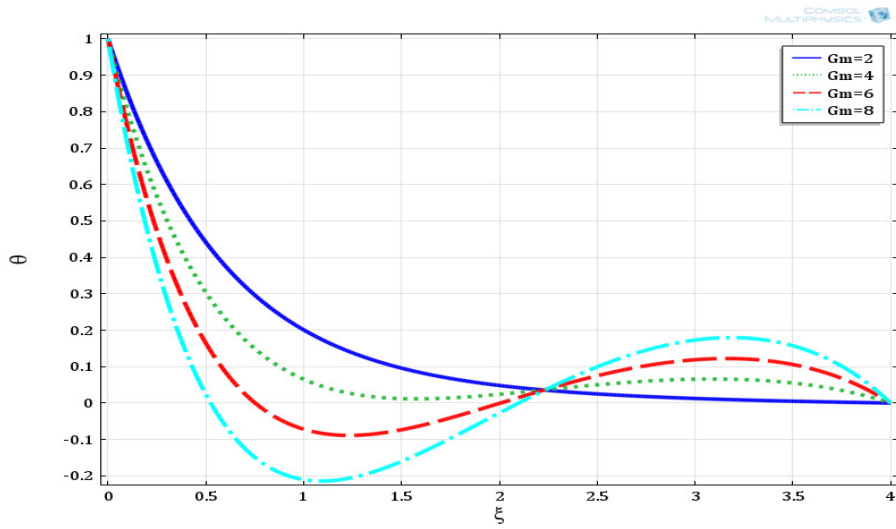


Figure 5: Temperature profile for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nt = Nb = 0.1$, $M = \chi = Q'_1 = Gr = 2$

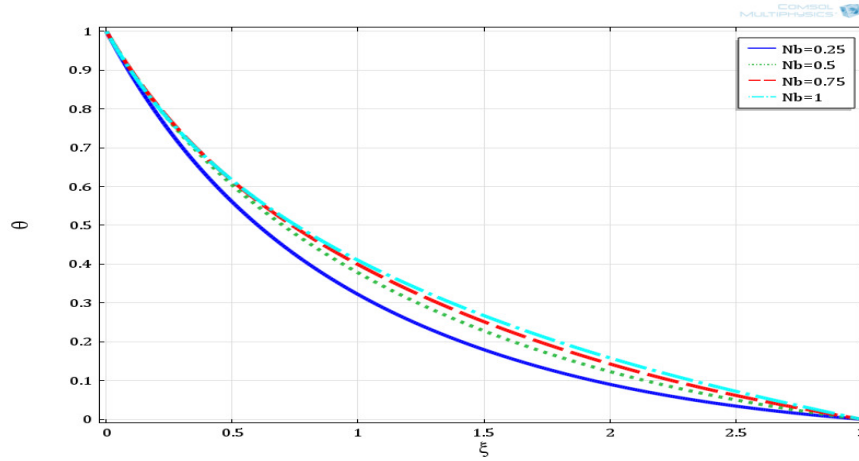


Figure 6: Temperature profile for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nt = 0.1$,
 $M = \chi = Q'_1 = Gr = Gm = 2$

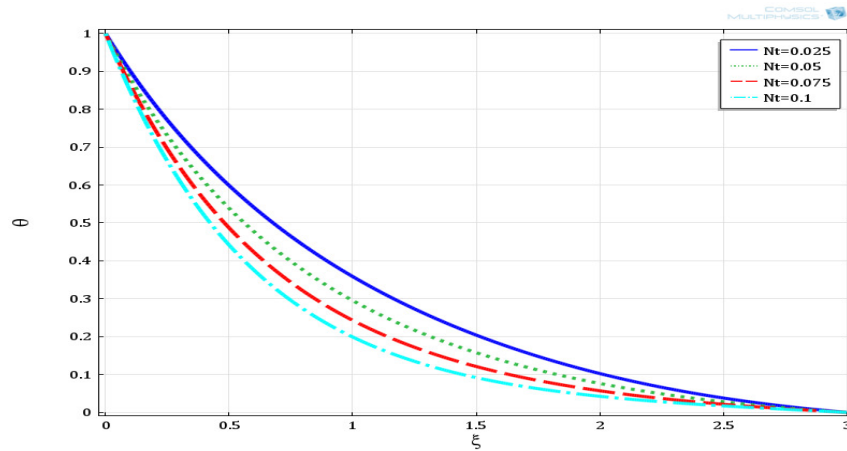


Figure 7: Temperature profile for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = 0.1$,
 $M = \chi = Q'_1 = Gr = Gm = 2$

Fig. 8 shows the concentration distribution for different values of Schmidt number (Sc). Schmidt number relates the relative thickness of the hydrodynamic layer and mass-transfer boundary layer. For increasing values of Sc , the concentration motion decreases in the nanofluid. Fig. 9 indicates concentration distribution for different values of chemical reaction parameter (γ). It is noticed that for increasing the values of the chemical reaction parameter decreases the concentration of species in the boundary layer. The effect of radiation absorption parameter is shown in Fig. 10. For increasing the heat absorption parameter (Q'_1) the concentration increases. The concentration distribution with the variation of material parameter is shown in Fig. 11. There is no significant

impact of this parameter in concentration motion. However, at large values of ξ , the material parameter increases the concentration effects. Fig. 12 depicts the effect of Brownian motion parameter on the concentration profile. It is clearly observable from the results that increasing the Brownian motion parameter increases the concentration and temperature effects in the flow.

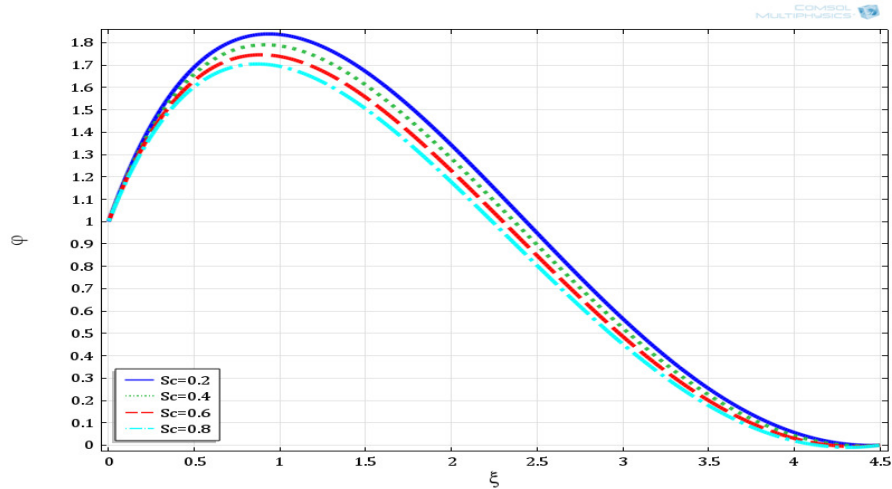


Figure 8: Concentration distribution for $K = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = Nt = 0.1$, $M = \chi = Q'_1 = Gr = Gm = 2$

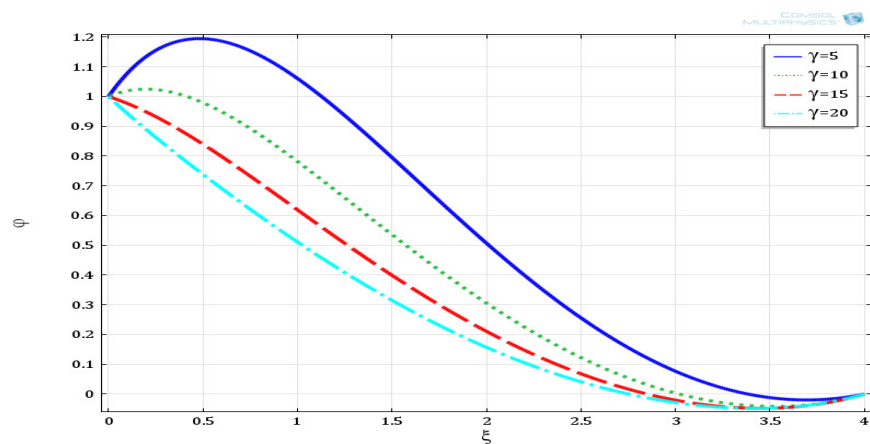


Figure 9: Concentration distribution for $K = Sc = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = Nt = 0.1$, $M = \chi = Q'_1 = Gr = Gm = 2$

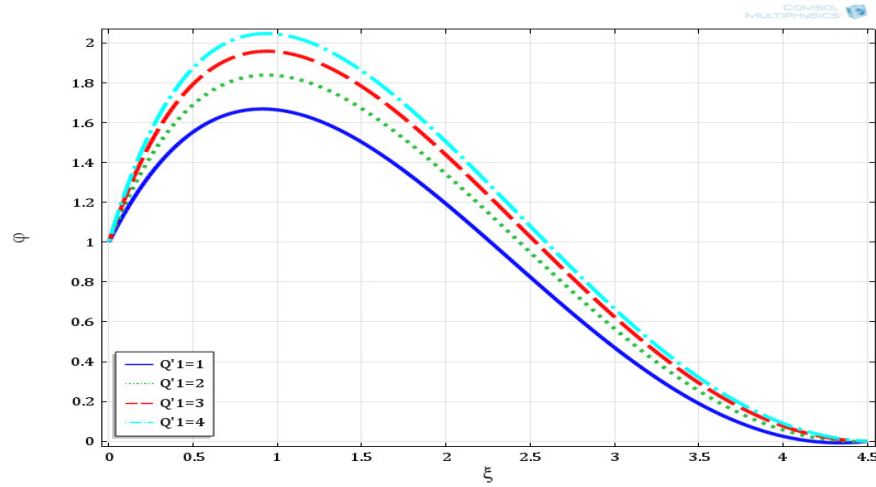


Figure 10: Concentration distribution for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = Nt = 0.1$, $M = \chi = Gr = Gm = 2$

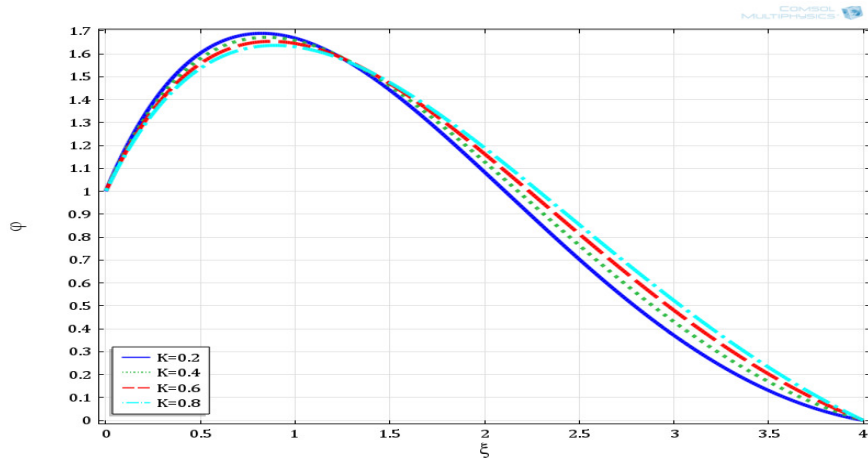


Figure 11: Concentration distribution for $Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = Nt = 0.1$, $M = \chi = Q'_1 = Gr = Gm = 2$

Fig. 13 depicts the concentration profile for various values of Prandtl number (Pr). The concentration effects are larger when the thermal diffusivity dominates, however it decreases when momentum diffusivity dominates. Fig. 14 shows the relation between magnetic number (M) and concentration profile. Increasing magnetic number plays an important role in decreasing the concentration motion in a micropolar nanofluid. Magnetic number represents the magnetic field intensity effects in concentration motion. Effects of thermophoresis parameter on concentration is shown in Fig. 15. This parameter also includes the effect of motion of the fluid particles in the concentration. It is seen that for increasing values of Nt , the concentration effects decrease in the flow. Fig. 16 is

given to see the concentration effects in the flow due to Eckert number (Ec). Initially, the Eckert number decreases the concentration effect, however for larger boundary layer thickness, the concentration effects increases for increasing values of Ec . The concentration distribution is also shown in Figs. 17-18 for different values of thermal Grashof number (Gr) and solutal Grashof number (Gm). Both the parameters increase the concentration effects in the flow. It is noticeable that Gm has larger effects in concentration distribution than Gm .

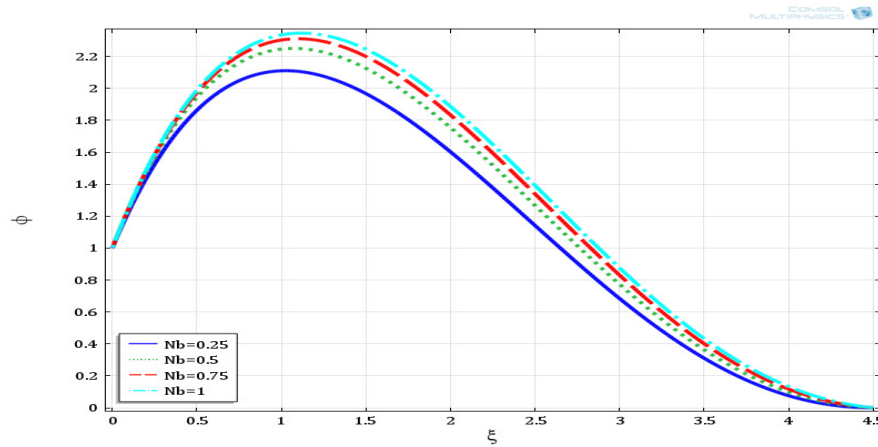


Figure 12: Concentration distribution for $Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = Nt = 0.1$,

$$M = Q'_1 = \chi = Gr = Gm = 2$$

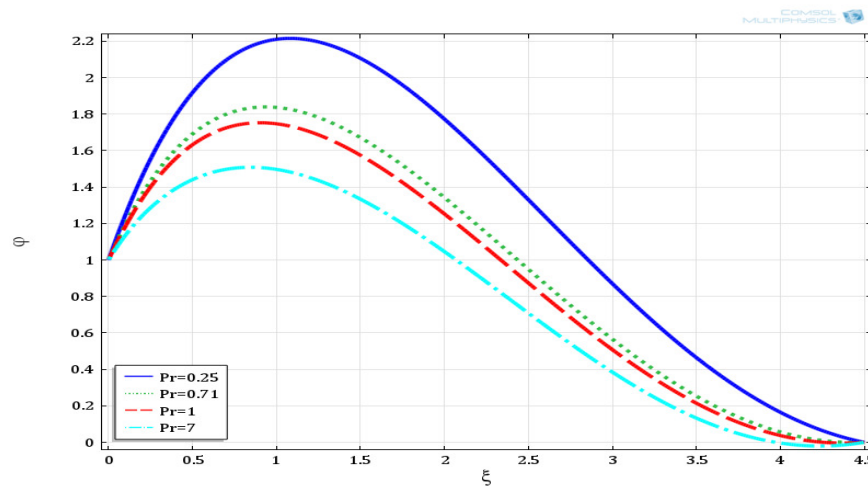


Figure 13: Concentration distribution for $K = Sc = \gamma = 0.2$, $Ec = 0.02$, $Nb = Nt = 0.1$,

$$M = Q'_1 = \chi = Gr = Gm = 2$$

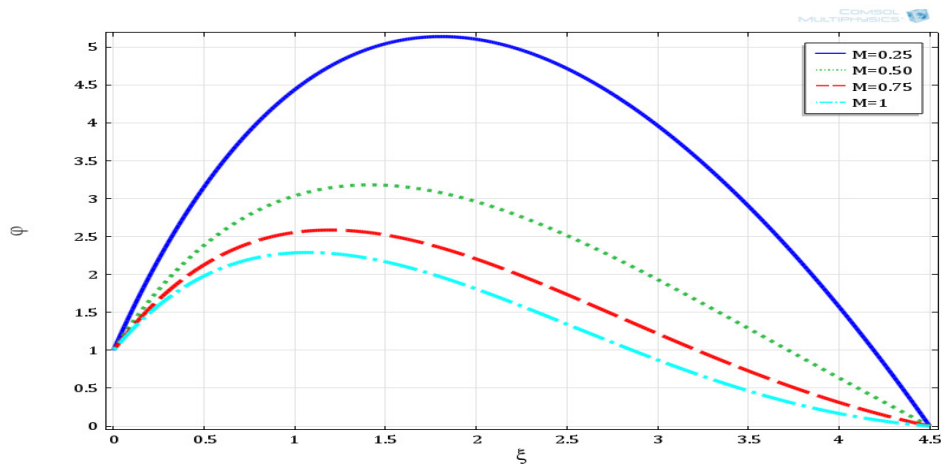


Figure 14: Concentration distribution for $K = Sc = \gamma = 0.2$, $Ec = 0.02$, $Nb = Nt = 0.1$, $Q'_1 = \chi = Gr = Gm = 2$

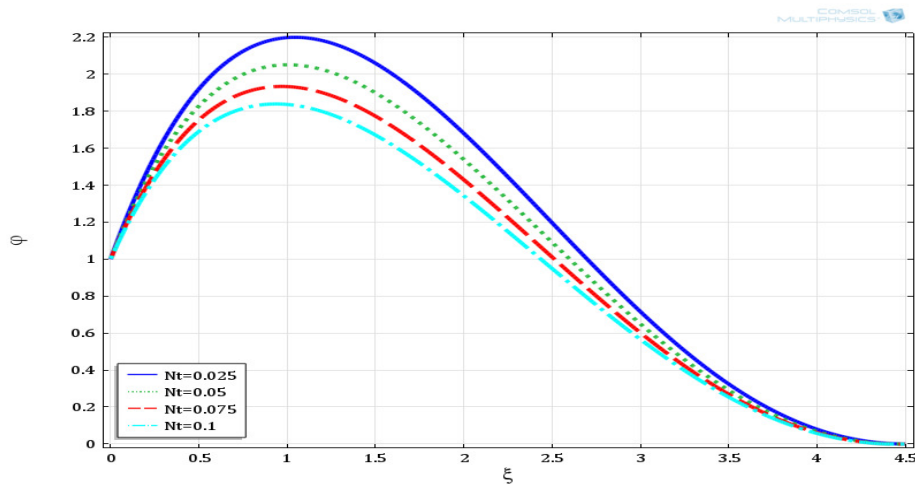


Figure 15: Concentration distribution for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = 0.1$, $M = Q'_1 = \chi = Gr = Gm = 2$

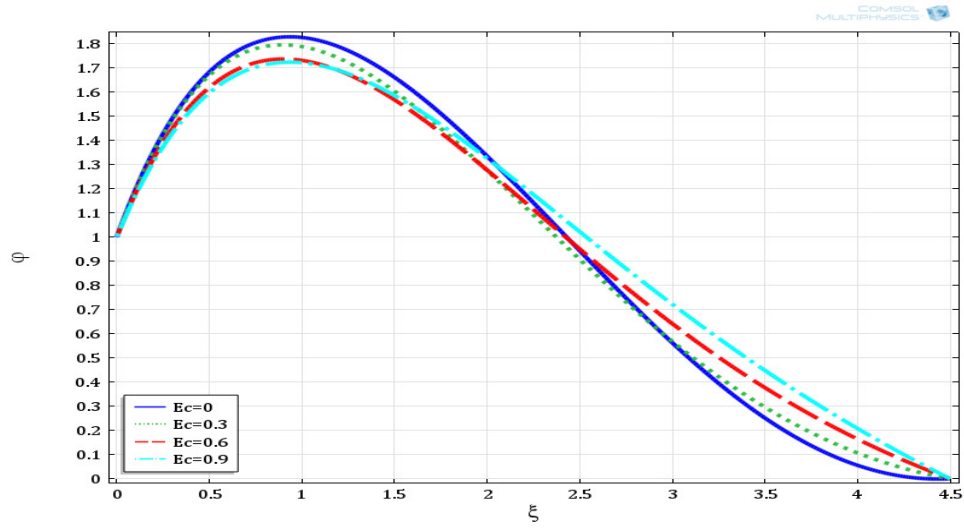


Figure 16: Concentration distribution for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Nb = Nt = 0.1$, $M = Q'_1 = \chi = Gr = Gm = 2$

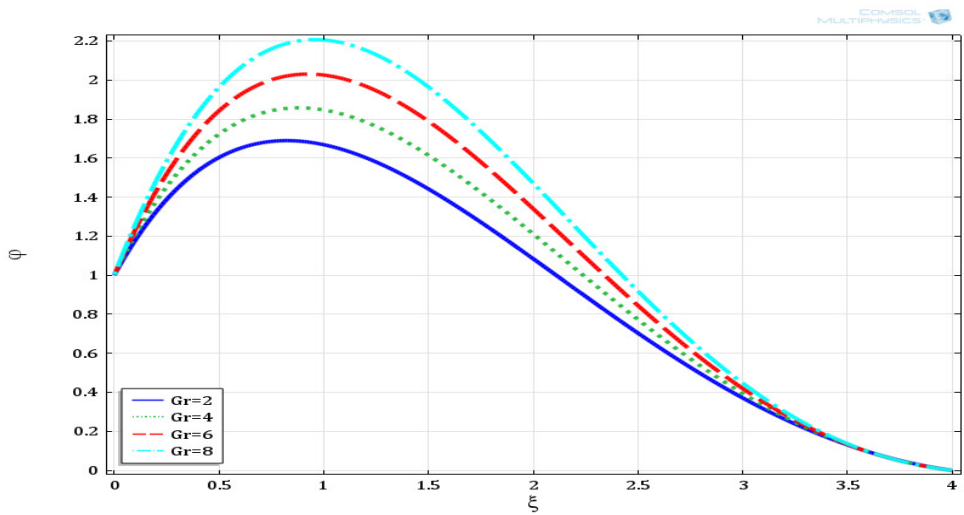


Figure 17: Concentration distribution for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = Nt = 0.1$, $M = Q'_1 = \chi = Gm = 2$

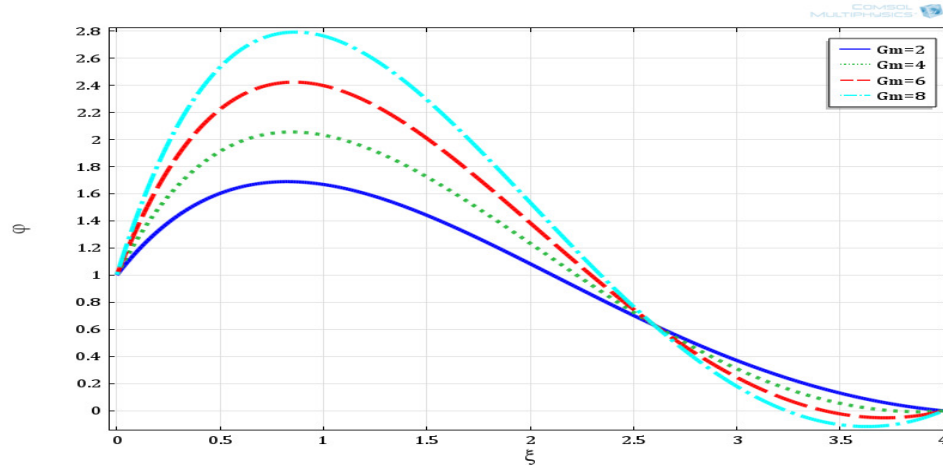


Figure 18: Concentration distribution for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = Nt = 0.1$,

$$M = Q'_1 = \chi = Gr = 2$$

Fig. 19 depicts the angular velocity distribution for increasing values of Eckert number (Ec). From the curve, the angular velocity is lower for higher values of Ec . The effect of material parameter (K) in the angular velocity is given in Fig. 20. The angular velocity decreases for increasing values of the material parameter. However, for increasing values of the magnetic parameter, the angular velocity increases as shown in Fig. 21. Magnetic field tries to rotate the particle in the fluid resulting an additional resistance is created in the flow, but it increases angular velocity.

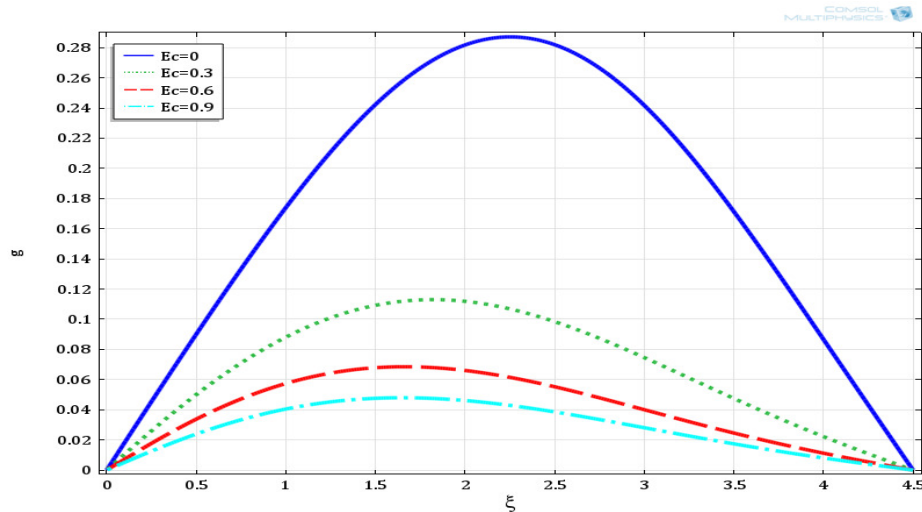


Figure 19: Angular velocity distribution for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Nb = Nt = 0.1$,

$$M = Q'_1 = \chi = Gr = Gm = 2$$

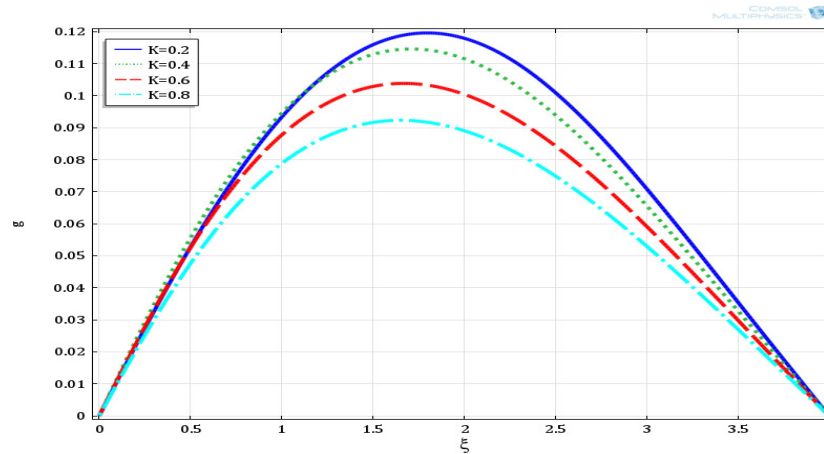


Figure 20: Angular velocity distribution for $Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = Nt = 0.1$, $M = Q' = \chi = Gr = Gm = 2$

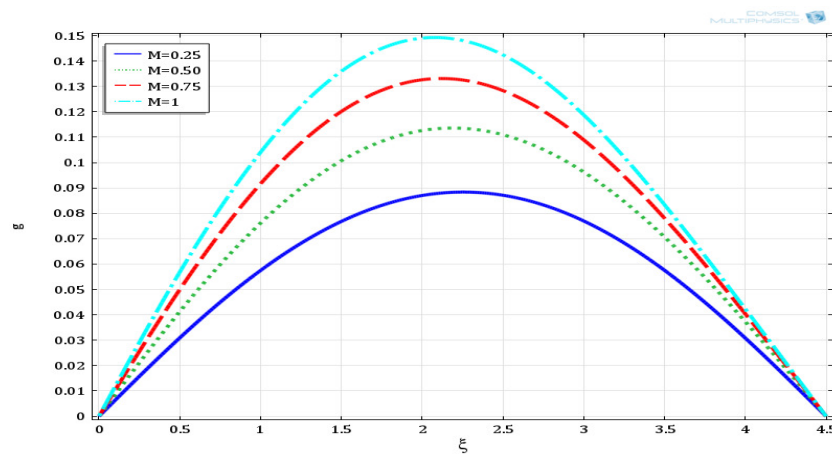


Figure 21: Angular velocity distribution for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Nb = Nt = 0.1$, $Ec = 0.02$, $Q' = \chi = Gr = Gm = 2$

Figs. 22-25 show the velocity profile for different values of physical parameters. Fig. 22 is the velocity profile for different values magnetic number (M). For small values of M , the particle moves randomly in the fluid. However, for increasing the magnetic parameter tries to stable the pattern of velocity profile. Fig. 23 shows the velocity profile for different values of Ec . Increasing values of Ec decreases the velocity. Fig. 24 indicates the velocity distribution for different values of material parameter. For increasing values of the parameter K , the fluctuations in the fluid velocity can be observed clearly. Fig. 25 indicate the velocity distribution for different values of Schmidt number(Sc). Negative values of velocity indicate that the flow is directed towards the stretching sheet. It is also

clear from the result that there is no effect of Sc in the velocity distribution. The velocity distribution remains same for the other physical parameter presented in the model. Therefore, these results are not presented in the graphical form. If the effects of radiation and chemical reaction parameters is excluded, the results reduce to the previous published model [Hsaio (2017)].

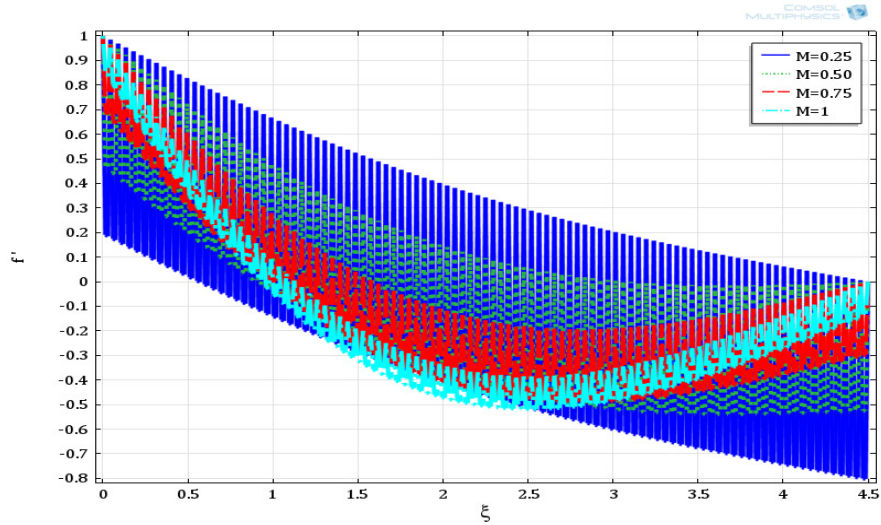


Figure 22: Velocity distribution for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = Nt = 0.1$, $Q'_1 = \chi = Gr = Gm = 2$

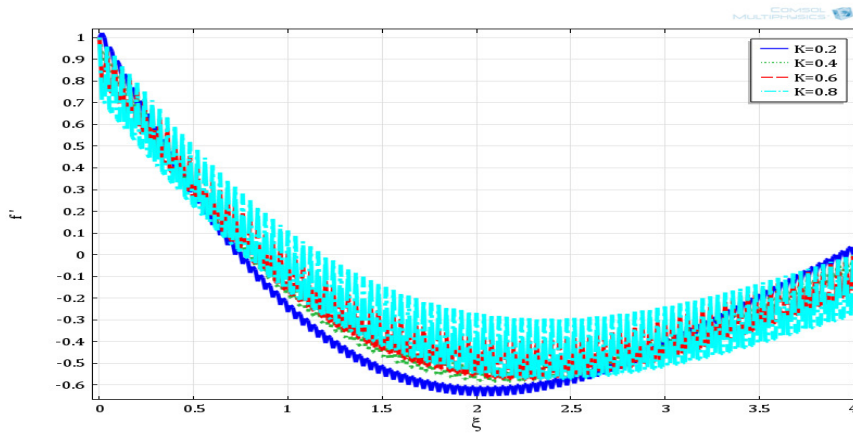


Figure 23: Velocity distribution for $Sc = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = Nt = 0.1$, $Q'_1 = \chi = Gr = Gm = 2$

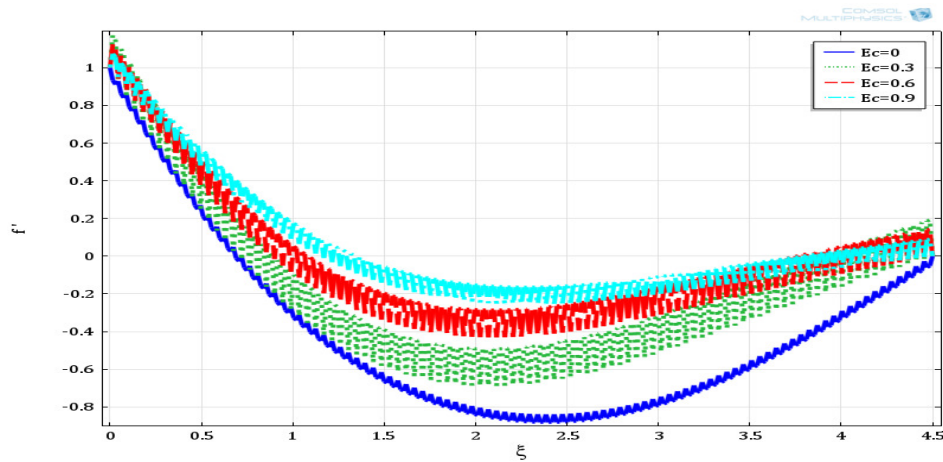


Figure 24: Velocity distribution for $K = Sc = \gamma = 0.2$, $Pr = 0.71$, $Nb = Nt = 0.1$,
 $M = Q'_1 = \chi = Gr = Gm = 2$

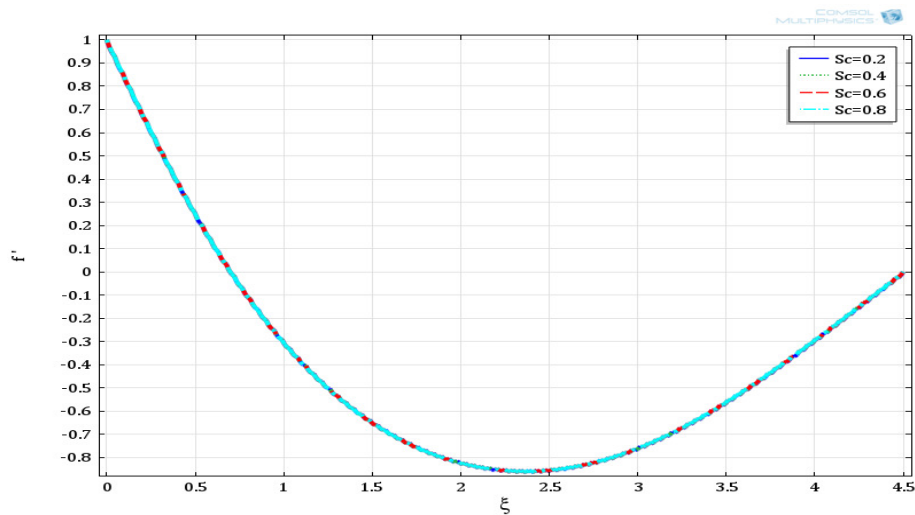


Figure 25: Velocity distribution for $K = \gamma = 0.2$, $Pr = 0.71$, $Ec = 0.02$, $Nb = Nt = 0.1$,
 $M = Q'_1 = \chi = Gr = Gm = 2$

5 Conclusions

In the present problem, a novel study has been carried out for an incompressible flow of a micropolar nanofluid past a stretching surface with the chemical reaction and radiation considering the effect of the particles motion. The following points can be concluded from the results:

1. For increasing the values of the radiation absorption parameter, Prandtl number, Solutal Grashof number, thermophoresis parameter, the temperature in the flow

- decreases. However, temperature increases only for increasing the values of the Brwonian motion parameter and chemical reaction parameter.
2. It is obserable that for incresing the values of Schmidt number, Chemical reaction parameter, Prandtl number, magnetic parameter and thermophoresis parameter decreases the concentration effects in the flow. However, variation in material parameter and Eckert number have mixed effects in the concentration distribution. For small boundary layer thickness, the concentration effects decrease but for large boundary layer, concentration effects increase. It is also noticeable that for increasing the values radiation absorption parameter, Brwonian motion parameter, thermal Grashof number, Solutal Grashof number, the concentration motion also increases.
 3. Angular velocity changes significantly for different values of the Eckert number, material parameter and magnetic number, however other parameters do not have much effect on the angular velocity. For increasing values of the Eckert number and material parameter decrease the angular velocity distribution and the magnetic number increases in the angular velocity.
 4. It is also interesting to notice that only the Eckert number, material number and magnetic number disturbs the velocity distribution. It remains approximately same for other parameters.

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