Block Stratification of Sedimenting Granular Matter in a Vessel due to Vertical Vibrations

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Abstract: Sedimentation of granular matter in a vertical channel filled with a viscous liquid and subject to longitudinal translational vibration is studied, starting from a compact suspension. A new vibrational effect is found experimentally and described theoretically; it is the formation of blocks (with a relatively high density) of sedimenting granular matter with stable lower and upper horizontal demarcations and a sharp density discontinuity. Owing to this phenomenon the sedimentation velocity of such granular matter is reduced. A new theoretical model based on viscous vibrational particle interaction in the limit of concentrated suspensions is elaborated, assuming particle-particle attraction in direction parallel to vibration and particle-particle repulsion in the direction perpendicular to vibration. The related theoretical analysis proves the vibrational condensation phenomenon to be due to the mean viscous hydrodynamic interaction of particles (attraction in the direction of vibration). The described effect has an important practical application for the management of liquefied and diluted granular matter (including situations in which microgravity conditions are attained).

keyword: Vibration, Heterogeneous hydrodynamic system, Suspension, Mean dynamics, Stability.

1 Introduction

Study of granular matter behaviour under vibrations is an actual problem from a practical point of view; a large amount of articles have been devoted to the behaviour of dry non-cohesive granular matter under vibrations (Behringer & Jenkins, 1997); few articles have focused on "wet" sand. However, the vibrational dynamics of "wet" sand in a fluid exhibits qualitatively new effects. Among them, the formation of a quasi-stationary spatial relief at the interface of liquefied sand and pure fluid in

a cavity which is subject to intense horizontal vibrations (Kozlov, 1992; Ivanova & Kozlov, 2002; Ivanova, et al., 1996). This relief, which is "static" in mean in the cavity frame, is made of a series of hills and valleys arranged periodically in the direction of the vibration. It exhibits some steep oblique parts delimiting the quasi-steady interface between diluted granular matter and pure liquid; the stability of grains at the interface is not ensured by dry friction in the gravity field, but by mean hydrodynamic mutual interaction of particles. As found in (Kozlov, 1992) the quasi-steady relief can be somehow described by a Kelvin–Helmholtz mechanism averaged upon time, and the mean dynamics of the interface is similar to the one between two immiscible fluids with a negligible surface tension. The important feature of these vibrational phenomena in the case of small particles is the formation of narrow vertical hills of dense suspension of sand, which expands practically over the whole vertical section of the horizontal channel when the vibration speed is large. In this case the distance between the steady hills, corresponding to the typical relief wavelength, can be few times smaller than the "hill" height (which is equal to or is smaller than the channel height).

Another interesting mean effect of such a spontaneous organisation is the formation of horizontal thin clouds during the sedimentation of granular matter (Evesque, 1997) which has been observed in a flat, i.e. nearly 2D, geometry, in a hourglass filled with liquid and subject to intensive vertical vibration. The formation of vibrational patterns strongly modifies the sedimentation process. Experimental study on the sedimentation process under vibration can be found in Kozlov, et al. (2003), with different inclinations of the channel. These experiments demonstrate the existence of some specific mean vibrational hydrodynamic interaction of particles under the effect of vibrations. The present article is devoted to further investigate and understand these interactions; it uses and extends the experimental study Kozlov, et al. (2003) and develops a theoretical analysis focusing

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on the mean vibrational interaction between particles in a concentrated suspension during the sedimentation process in a vertical channel.

2 Experimental technique

Experiments have been carried out on a home-made mechanical vibrator, which provides harmonic translational oscillation of the cavity. The frequency $f = \Omega/(2\pi)$ of vibration can be smoothly adjusted within a relative accuracy $\delta f/f = 0.5$ % in the interval f = 1-50 Hz, and measured within a 0.1% accuracy using either an optoelectronic tachometer or a stroboscope. The amplitude of vibration has been measured within a 5% relative accuracy from direct measurement on a video screen and using a camera; it can be varied in the range b = 0-1.5 cm, that allows to generate accelerations as large as 100g. The sedimentation process has been studied in a relatively long rectangular metal channel (L = 28 cm long, 2 cm wide and 3.0 mm thick) and glass tubes (25 cm long

tively long rectangular metal channel ($L=28 \, \mathrm{cm} \, \mathrm{long}, 2 \, \mathrm{cm}$ wide and 3.0 mm thick) and glass tubes (25 cm long and various diameters $d=3-6 \, \mathrm{mm}$). The plane channel has a plexiglass transparent cover that allows visual observation; the internal part of the back wall has been painted in black to improve the image contrast of particles. The cavities are filled with liquid and sand, with a typical amount of sand of $1/10-1/20 \, \mathrm{part}$ of the total cavity volume about.

Experiments have been carried out with water and silicate glass beads of three different sizes ($d=0.36\pm0.03$, 0.09 ± 0.01 and 0.07 ± 0.01 mm) as the granular matter. The dynamics of sand has been observed using stroboscopic enlightenment and has been recorded on video.

In case of relatively large beads (d=0.36 mm) the vibrational experiments has been started from a horizontal position of the vibrator. The cavity with sand (previously packed in one side of it) is fastened on the vibrator in horizontal position; first the vibration of definite frequency and amplitude is imposed, then the vibrator is set in the vertical position. This technique is required owing to the relatively high speed of gravitational sedimentation of grains (it should be noted that the vibrations considerably reduce the sedimentation speed, so that it is possible to set the vibrator with the cavity in the vertical position before the occurrence of massive sedimentation).

A different technique has been used in experiments with small particles (d=0.09 and 0.07 mm). In this case, the granular matter is preliminary well packed under vi-

brations in one side of the channel. Then the cavity is turned over. The process of particles sedimentation from the lower boundary of packed grains is relatively slow or even absent (in case of d=0.07 mm), thus the experimenter has enough time to fasten the cavity on the vertical vibrator and impose the vibration after it.

3 Experimental results

So, experiments start with the sand compacted in the top part of the cavity. We begin with the description of the experiments with the beads of diameter d=0.09 mm in water, performed in the rectangular channel. In the absence of vibration, the process of sand spreading (which depends on the grains diameter and on the liquid viscosity) runs relatively slowly at a speed much slower than the normal speed of sedimentation of a single particle.

Nevertheless, the spreading speed of the sand increases considerably with vibration because of the fluidization of the sand at the lower boundary of the packed sand layer, which allows also the forcing of parametrical oscillations and/or the local organization of flows, with local inhomogeneities. When the vibration is increased further, locally dense zones are stretched horizontally, that reduces the mean vertical speed of sedimentation of free particles. However such flat formations are not steady and collapse rapidly. At some threshold intensity (frequency) of vibrations the flat cloud of sand becomes able to block the whole cross-section of the cavity; a layer is formed in this case with a stable horizontal lower boundary between the liquefied sand and the liquid below it. This layer slowly moves down. The spreading of sand from the lower boundary of this layer is practically absent when vibration is strong enough.

At the top of the cavity one observes that the sand tends to remain packed for a quite long time. One observes also jumps of concentration, when the complete blocking of the channel occurs. The formation of several steady layers at the same time has been observed during experiments; in this case one observes the layers slowly moving downwards at a constant and equal speed, with a constant vibration rate.

The spreading of separate particles from the layers increases, when the frequency of vibration is decreased; some layers can become suddenly unstable below some threshold frequency of vibration and fall off rapidly. The threshold frequency at which a layer becomes unstable

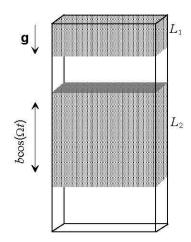


Figure 1: Sketch of the plane cavity.

is notably lower than the frequency needed for the creation of a stable layer. The destruction of a peculiar layer has no considerable effect upon the stability of the other ones, when they are few, irrespectively of their position. In some cases the layer located just below a collapsing layer remains steady after the upper one has collapsed and has fallen down, or vice versa, i.e. the disturbances caused by the collapse of a layer do not influence the stability of the upper ones.

With a significant decrease of vibration intensity the destruction of all steady sedimenting layers occurs (one observes the formation of a rather homogeneous suspension). The speed of sedimentation in this case is much higher in comparison with the case of vibrational condensation of sand into blocks. It is worth noting that the increase of vibration rate does not allow the formation of a new bedded structure from a homogeneous suspension.

As explained before, for large enough vibrations the spreading of sand from the lower boundary of the liquefied suspension (cloud of suspended granular matter) is absent and the speed of sedimentation is quite slow. In this case a single layer of diluted sand is ever formed during the continuous liquefaction of sand compacted in the top of the cavity. The sand concentration in the block (thickness L_2 in Fig. 1) is much lower than in initial compacted state (L_1).

Figure 2 reports the dynamics of such a layer of sand under vertical vibrations for combination of water – glass beads d=0.09 mm. The shaded area between the two curves corresponds to sand suspension, falling downwards as a whole.

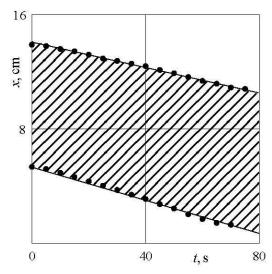


Figure 2: Longitudinal distribution of granular matter in vertical channel under vibration versus the time (d = 0.09mm, b = 1.1cm, $f \equiv \Omega/(2\pi) = 30.4$ Hz).

The speed of downward motion of blocks is rather small and does not change with time, but it depends on the intensity of vibration, the stronger the vibrations – the lower the sedimentation velocity. As the block speed is determined by the mean velocity of filtration through the porous granular matter, this slowing down effect indicates an increase of the particle concentration in the falling block at large vibration intensities. Keeping viscosity and particles size constant, one finds that the dimensionless vibrational acceleration $\Gamma \equiv b\Omega^2/g$ determines the velocity of the block sedimentation, indicating conversely a constant and stable suspension concentration in the block for each set of parameters. The sedimentation velocity (as a function of the dimensionless vibrational acceleration) exhibits a gradual decrease when the vibration intensity $\Gamma \equiv b\Omega^2/g$ is increased (Fig. 3, marks 1, the typical discrepancy of velocity is shown by error

The mentioned effect of vibrational "condensation" of sedimenting granular medium into blocks with steady lower boundaries was not found in experiments with large beads (d = 0.36 mm) and has some specific features in case of smaller beads (d = 0.07 mm). The experiments with the last ones (which were performed in cylindrical glass tubes filled with water) have shown the formation of a system of suspension layers (columns) (from 3 to 5 homogeneous layers) with different sand concentration.

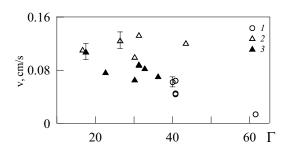


Figure 3: Block sedimentation velocity (glass beads in water) versus the dimensionless acceleration $\Gamma = b\Omega^2/g$. Mark 1 corresponds $d=0.09\mathrm{mm}$ (single layer), 2 and $3-d=0.07\mathrm{mm}$ (velocity of lower and upper layers in multilayer system).

The height of the layers could be about ten tube diameters. The effect, however, does not depend on the tube diameter. The layers have sharp and steady horizontal interfaces with a jump of concentration. The lower layer (which has the stable boundary with pure water from below) has the lowest concentration of sand and its boundary moves quicker than all the others, so the vertical size of this block grows with time. One can see that the velocity of its lower boundary practically does not depend on the vibration intensity (Fig. 3, marks 2). The upper layer (which has a steady size and is neighbouring pure water on its top) is characterised by the highest concentration of granular matter and the lowest speed of sedimentation.

4 Theory of vibrational interaction between particles

Consider a concentrated suspension as a homogeneous porous medium with permeability K, obeying some Carman – Kozeny relationship:

$$K = \frac{D^2 \varphi^3}{180(1 - \varphi)^2} \tag{1}$$

Here φ is the porosity, D — the hydraulic diameter (which is equal to the diameter d of the spherical beads when particle diameters fall within a narrow range). Darcy law determines the relative motion of suspension phase and liquid in the case of a slow viscous flow:

$$\nabla P = -\frac{\eta}{K} \upsilon \tag{2}$$

Here υ is the filtration velocity (the liquid volume flowing per unit cross-section per second), ∇P — the pres-

sure gradient and η — the dynamic viscosity of the fluid. The intrinsic average velocity \mathbf{V}_{in} (average velocity in porous) is related to υ by the Dupuit - Forchheimer relationship

$$\upsilon = \phi V_{\it in}$$

Equation (2) describes the viscous motion of the liquid. So, if the cavity frame undergoes a vertical harmonic oscillation with amplitude b and cyclic frequency $\Omega = 2\pi f$, the system feels a modulated gravitational force field in the cavity frame, which reads $\mathbf{g} + b\Omega^2 \cos \Omega t \mathbf{n}$. According to the made assumptions the equation of liquid and suspension relative motion projected along the vertical axis reads:

$$\upsilon = \frac{(\rho - 1)D^2\varphi^3}{180\nu(1 - \varphi)}(g + b\Omega^2\cos\Omega t)$$
 (3)

The Equation (3) is valid in the limiting case of low dimensionless frequencies

$$\frac{\Omega l^2}{\rm v} << 1 \tag{4}$$

when the distance l between the particles is much shorter than the radius of viscous interaction $\delta \equiv \sqrt{2\nu/\Omega}$. In this case the quasi-steady approach is valid and the oscillatory liquid flow is completely determined by viscosity and Darcy law (here also ν is the kinematic viscosity of the liquid, $\rho \equiv \rho_S/\rho_L$ — the relative density of inclusions).

In the case of large vibrational acceleration Γ which corresponds to the conditions of the present experiment

$$\Gamma \equiv \frac{b\Omega^2}{g} >> 1$$

the relative velocity of granular matter oscillations in liquid considerably surpasses the speed related to the slow sedimentation of the particles block in the gravity field.

Consider now a block of granular matter sedimenting through the liquid; it has a lower border, the stability of which is now considered. The condition of stabilization of this bottom border is obtained when writing that the motion of a typical particle pertaining to the front and falling with the speed of the block is equal to the speed of a particle falling freely but which is also subject to an attraction force from the block. As a matter of fact, in the

case of normal sedimentation the speed of sedimentation of a single particle is always faster than a dense packing. So, the force of attraction (elevating force), directed vertically upwards which maintains the considered particle in contact with the border of the block moving with speed v_{sed} , should be:

$$\Delta F = (\rho_S - \rho_L) \frac{\pi d^3}{6} g - 3\pi \eta dv_{sed}$$
 (5)

Such a force is of a vibrational nature. In fact the calculation shall consider the case of the vibrational hydrodynamic interaction of fine particles in suspension and in a vibrational force field in the limiting case of low dimensionless frequencies (4) when it is possible to apply a quasi steady approach and the solutions found in a stationary case.

Let us now consider the suspension block as a homogeneous system of spherical bodies of a relatively small size (d << l), arranged as a spatial regular lattice with spatial period l. Suppose that hydrodynamic interaction occurs only between neighboring particles. In Vasseur & Cox (1977) the case of the interaction between two spherical bodies in a stationary stream has been examined and it has been found that the flow generates a repulsion force between the two particles when the pair forms a vector perpendicular to the liquid flow, and an attraction force when the pair vector is parallel to the velocity vector. The force of attraction is found to vary as in the limit $lV_{in}/v << 1$:

$$F_{attr} = \frac{9}{8} \pi \rho_L d^2 V_{in}^2 \tag{6}$$

The nature of this interaction is related to relatively weak inertial effects. Starting from Eq. (6) and considering the case of an oscillating flow, one gets a mean contribution after averaging Eq. (6) over time: in other words, a mean force of attraction appears between a pair of neighboring particles oriented in the direction of oscillation; and one gets in the case of large vibrational acceleration $\Gamma >> 1$:

$$\overline{F_{attr}} = \frac{9}{16} \pi \rho_L d^2 V_0^2 \tag{7}$$

Here V_0 is the amplitude of the oscillating velocity components, which is given by Eq. (3) in the case of the present experiment:

$$V_0 = \frac{(\rho - 1)d^2\phi^2}{180\nu(1 - \phi)}b\Omega^2$$
 (8)

At the same time the repulsion mean force acts on pairs of particles oriented perpendicularly to the oscillating liquid flow

$$\overline{F_{rep}} = \frac{9\sqrt{2}}{16} \pi \frac{\eta d^2 V_0}{l}$$

Considering now a whole block, one can conclude owing to symmetry consideration that the resulting force acting on a single particle located inside the homogeneous block is equal to zero. On the contrary, when considering a particle on the frontier, it interacts hydrodynamically with four lateral particles and with one particle above; as a consequence the particle at the frontier is attracted towards the internal layer, with a mean force which could be estimated as a half (due to break of symmetry) of the one determined by Eq. (7). This force ensures the cohesion of the block if it is large enough, i.e. if it is larger than ΔF given by Eq. (5). The limit of stability is obtained by writing equality instead of inequalities, so that one finds the limit of stability of the border as given by:

$$\frac{v_{sed}^2}{gd} \frac{\Gamma^2}{\varphi^2} = \frac{16}{27} (\rho - 1) - \frac{32}{3} \frac{v v_{sed}}{d^2 g}$$
 (9)

In Eq. (9), the variables v_{sed} and φ are not independent since:

$$v_{sed} = \frac{(\rho - 1)d^2 \varphi^3 g}{180 v (1 - \varphi)} \tag{10}$$

Let us also introduce the dimensionless speed of subsidence

$$V = \frac{\mathbf{v}}{d^2 g} \mathbf{v}_{sed} \tag{11}$$

in which the normalized velocity d^2g/v characterizes the speed of sedimentation of a single isolated spherical particle due to gravity g.

And the dimensionless equation of granular matter block motion writes:

$$V^{2}W_{p} + \frac{32}{3}V - \frac{16}{27}(\rho - 1) = 0$$
 (12)

Where the dimensionless vibrational parameter W_p has been introduced in (12), and reads:

(8)
$$W_p \equiv \frac{v_{osc}^2}{gd} \equiv \frac{1}{gd} \left(\frac{d^2 b \Omega^2}{v \varphi}\right)^2$$
 (13)

This parameter is similar to the parameter

$$W_d = \frac{b^2 \Omega^2}{gd}$$

which has been introduced in earlier works considering problems of high-frequency (nonviscous limit) vibrational mechanics of heterogeneous systems (Kozlov, 1992; Kozlov, 1996) and having the meaning of a vibrational Froude number.

One can see also from Eq. (13) that v_{osc} characterizes the amplitude of the relative velocity between the viscous liquid and the particles in an oscillating field. Equation (12) determines the steady state of granular matter falling as a block with a stable lower boundary in the cavity subject to intensive vertical vibrations.

5 Discussion

The dependence of $V(W_n)$, Eq. (12), for a relative density $\rho = 2.6$, that corresponds to the conditions of experiments is drown in Fig. 4, with the marks being the experimental data points and the curve the theoretical prediction. So, the larger the vibrational parameter $W_p = d^3b^2\Omega^4/(gv^2\varphi^2)$, the denser the suspension and the lower the speed of sedimentation. The results of the theoretical analysis are in agreement with the experimental ones (that corroborate the theoretical description). It is worth recalling that the assumption of a regular arrangement of particles in the block is not necessary: for instance, if one considers the case of the attraction force, that occurs between two grains in the direction of vibration, the case of a random but homogeneous distribution of particles leads only to a minor alteration of the prefactor in Eq. (7), since the attraction vibrational force (7) does not depend on the distance between the particles (within the limit of distance of viscous interaction). However, with a change of the distance between the particles the permeability of granular matter varies, that results in a change of the oscillating velocity component V_0 (at definite Γ), and thus it changes the mean interaction forces.

The repulsion force (which is generated in the direction perpendicular to the liquid oscillations) behaves in a different way: this force decreases with an increase of the distance between the particles. This fact explains why the homogeneous distribution of granular matter in all the cross-section of the vertical channel is forced when imposing longitudinal vibrations.

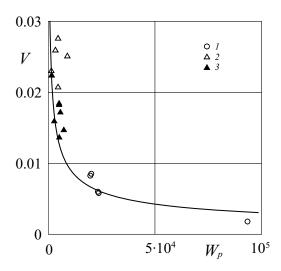


Figure 4: Dependence of the dimensionless speed V of sedimentation of a block of particles as a function of the vibrational parameter $W_p = d^3b^2\Omega^4/(gv^2\varphi^2)$. Solid line - theory, Eq. (12); marks - experiment (correspond to Fig. 3).

The above analysis demonstrates that the mean vibrational interaction between particles in a concentrated suspension results in a compression of granular matter in the direction of vibrations and in its extension in the direction perpendicular to the vibration axes.

It is worth noting that in case of nonviscous liquid the mean hydrodynamic interaction of particles in the oscillating flow has the opposite sign (Lamb, 1932). The change of interaction and attraction of particles in the vibration direction takes place only in viscous liquid at short distances (Tabakova & Zapruanov, 1982a, b; similar effects, with a sign-change of the interaction according to the particle distance, have been also found by Thomson et al., 1997).

The proposed present model of vibrational viscous interaction between particles differs essentially from the known models of mean behavior of granular inclusions in vibrational fields (Lobov, et al., 1999), which consider the suspensions at low concentration and exclude the grain-grain force of interaction from consideration. In practice, the above mechanism works only if the condition of viscous hydrodynamic interaction between particles is ensured during oscillations, which reads $\Omega l^2/\nu < 1$. It means that the distance l between the particles has to be shorter than the thickness of the Stokes layer

 $\delta \equiv \sqrt{2\nu/\Omega}$; otherwise (at larger l) the hydrodynamic interaction is negligible or, it can be replaced by the opposite effect as shown in Ivanova et al. (2005). Probably, that is why the effect of "particles condensation" under vibrations was not found in experiments with grains of larger sizes.

The importance of the mean viscous vibration-induced hydrodynamic interaction between close particles was also pointed in earlier investigations dealing with problems quite different with respect to the one treated here. For instance, Langbein (1991) showed the onset of possible "edge" effects due to such an interaction. It was demonstrated that the application of forced periodic vibrations to an ensemble of uniformly distributed and close particles in viscous liquid tends to perturb the distribution regularity close to the solid boundaries.

Along these lines, it is worth noting that the present theoretical description, based on a viscous vibrational hydrodynamic interaction between particles in an oscillating force field (with particles having a density different from surrounding liquid) can be used to explain a number of experimental effects, which have been found out earlier.

For instance in the case of the study of vibrational liquefaction of granular matter in a container filled with a viscous liquid and subject to vertical vibrations (Kozlov, et al., 1998; Ivanova, et al., 2000), one has observed just before the liquefaction the excitation of intense parametrical oscillations located at the top sand layer; these parametrical oscillations lead to the flattening of the sand surface destroying the cone distribution which is obtained just below this threshold. It means from one side that the upper sand layer makes vertical oscillations of appreciable amplitude (breathes) in the cavity frame, but from the other side, it demonstrates also that the sand upper layer has the property of an elastic membrane which tends to keep it flat. This effect can be likely understood within the present theory, since the viscous interaction generates the equivalent of a surface tension force that tends to flatten the surface.

Another probable example where the mechanism of hydrodynamic viscous mean interaction between particles manifests itself is the pattern formed by particles near the surface in the viscous liquid (air or water) of spherical particles in the hills extended across the direction of their oscillations (Reis & Mullin, 2002; Wunenburger, et al., 2002).

The phenomena of mean viscous vibrational interaction of particles could be also responsible for acoustic condensation of dust in air (Krasil'nikov & Krilov, 1984).

The vibrational viscous force which results in an attraction between the particles in the direction of vibrations and a repulsion between the particles in the direction perpendicular to the vibrations direction sheds a new light on the alignment of particles into strata during vibrating a hourglass filled with liquid (Evesque, 1997).

6 Conclusion

A new mean vibrational effect has been found in the experimental study concerning a suspension sedimenting in a vertical channel subject to intensive longitudinal oscillations: it is the stabilisation of a sharp potentially unstable horizontal demarcation between a zone of dense mixture (made of heavy granular matter and liquid intermixed) and a zone of pure liquid. The situation with the dense suspension above pure liquid has been found to be stable (so that formation of blocks of concentrated suspension is achieved with sharp and stable boundaries perpendicular to the vibration axis). The suspension clouds completely expand over the whole cross section of the channel and fall slowly down in gravity field. The phenomenon puts in evidence the existence of a mean short range effect due to a vibrational hydrodynamic interaction (attraction) between particles in an oscillating force field. Another effect consists of the strong reduction of the sedimentation speed. The speed of sedimentation of diluted granular matter is few times lower than without vibrations and decreases with the increase of the dimensionless acceleration.

A theoretical model has been elaborated that takes into account the mean interactions of solid fine inclusions suspended in a viscous liquid in an oscillating force fields with the assumption of a relatively high suspension concentration (i.e. with the particles at the distance of viscous interaction from each other). This vibrational mechanism between particles in hydrodynamic interaction is valid in concentrated suspensions; it is new and qualitatively differs from already known ones elaborated for suspensions with a low concentration.

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