Intelligent Automation & Soft Computing
DOI:10.32604/iasc.2021.016491
Article

Control Charts for the Shape Parameter of Skewed Distribution

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Received: 03 January 2021; Accepted: 14 March 2021

Abstract: The weighted distributions are useful when the sampling is done using an unequal probability of the sampling units. The Weighted Power function distribution (WPFD) has applications in the fields of reliability engineering, management sciences and survival analysis. WPFD is more beneficial in Statistical process control (SPC). SPC is defined as the use of statistical techniques to control a process or production method. SPC tools and procedures can help to monitor process behaviour, discover problems in internal systems, and find solutions for production issues. To identify and remove the variation in different reliability processes and also to monitor the reliability of machines where the number of errors follows WPFD, we develop control charts to keep the process in control. A memory-based control chart like an exponentially weighted moving average (EWMA) control chart and an extended exponentially weighted moving average (EEWMA) control chart are discussed and compared each other. The proposal of these control charts is based on the modified maximum likelihood estimator (MMLE) under the shape parameter of WPFD. We have presented Monte Carlo simulation technique and a real-life application to compare the proposed control charts. This study shows that an EEWMA control chart based on MMLE performs better than EWMA control chart, when the underlying distribution of the errors in process monitoring follows WPFD. These findings can be useful for researchers and practitioners in dealing with production errors and optimizing the output.

Keywords: Weighted Power function distribution; EWMA control chart; EEWMA control chart; modified maximum likelihood estimator; management sciences; quality charts

1 Introduction

The different production processes in industries face commonly two types of variations: common cause variation and special cause variation. Common cause variation always exists even if the process is designed very well and maintained very carefully. This variation should be relatively small in magnitude and is uncontrollable and due to many small unavoidable causes. A process is said to be in statistical control if only common cause variation is present. The variations outside this common cause pattern are called special cause variations. These variations are subject to some problem in the system, like the poor tuning of equipment, controller fell asleep or got absent, the computer stopped working, a poor lot of raw material, machine break down. A process working under both types of variation is said to be out of control.

These variations may affect the production process and ultimately result in a defective output. The consequences of it are pretty severe. Companies have to face high cost as well as their reputation will be at stake. Therefore, in order to avoid variations in the process, there is a need to use control charts to keep the process under control. Zaka et al. [1] introduced a new class of distribution called WPFD in order to have more application in applied sciences such as biosciences engineering, management sciences. The weighted distributions are helpful when the sampling is done using an unequal probability of the sampling units. WPFD is more useful in Statistical process control. Statistical process control (SPC) is defined as the use of statistical techniques to control a process or production method. SPC tools and procedures can help to monitor process behavior, discover issues in internal systems, and find solutions for production problems. Two types of control charts are commonly studied, memory less control chart i.e., Shewhart control chart and memory based control charts such as EWMA and HEWMA etc. Many works have been done in this regard, such as the EWMA control charts firstly by Roberts [2], and recently by Li et al. [3] and Nguyen et al. [4]. The cumulative-sum (CUSUM) control charts firstly by Page [5], Sanusi et al. [6], Haq et al. [7] and Hossain et al. [8]. The mixed EWMA-CUSUM control charts by Abbas et al. [9], Ajadi and Riaz [10]. The hybrid exponential weighted moving average (HEWMA) control charts due to Shamma et al. [11] and Haq [12].

In real life scenario, this is not always possible to fulfil the normality assumption for the distribution of error during the process. A very few works in literature is about this situation of not normal process distribution including Noorossana et al. [13], Lin et al. [14], Erto et al. [15], Liang et al. [16] and Ahmed et al. [17]. The EWMA statistic is widely used for shift detection in the ongoing process either to monitor qualitative or quantitative physical phenomena. The generalized form of the existing EWMA statistic was introduced by Naveed et al. [18] and named it as EEWMA.

2 The Conventional EWMA Control Chart

Let the distribution of the underlying process having the sequence $\{X_t\}$ is normal. Also, let the $0 \leq \lambda \leq 1$, is a known constant. Now EWMA statistics is given by

$$W_t = \lambda X_t + (1-\lambda)W_{t-1}.$$

The smoothing constant λ plays a very important here. As it approaches to zero, it becomes sensitive for a small and moderate shift in mean and as it becomes close to one. It approaches to Shewart control chart. The control limits for EWMA are given below

$$UCL_{W_t} = \mu + L * \sqrt{V(W_t)}$$

$$LCL_{W_t} = \mu - L * \sqrt{V(W_t)}$$

$$E(W_t) = \mu$$

$$V(W_t) = \sigma^2\left(1-(1-\lambda)^{2t}\right)\left(\frac{\lambda}{2-\lambda}\right)$$

3 The Traditional Extended Exponentially Weighted Moving Averages Control chart

When the distribution of the process is normal, the EEWMA control chart was introduced by Naveed et al. [18]. The EEWMA control chart by Naveed et al. [18] is given as

$$Z_t = \lambda_1 T_t - \lambda_2 T_{t-1} + (1 - \lambda_1 + \lambda_2) Z_{t-1}$$

where $0 \leq \lambda_1 \leq 1$ and $0 \leq \lambda_2 \leq \lambda_1$. T_{t-1} is represents the previous value of the variable and Z_{t-1} denotes the previous value of a statistic.

The mean and variance are given as

$$E(Z_t) = \mu$$

And

$$var(Z_t) = \sigma^2 \left[\left({\lambda_1}^2 + {\lambda_2}^2\right) \left\{ \frac{1-(1-\lambda_1+\lambda_2)^{2t}}{2(\lambda_1-\lambda_2)-(\lambda_1-\lambda_2)^2} \right\} - 2a\lambda_1\lambda_2 \left\{ \frac{1-(1-\lambda_1+\lambda_2)^{2t-2}}{2(\lambda_1-\lambda_2)-(\lambda_1-\lambda_2)^2} \right\} \right]$$

4 Proposed Extended Exponentially Weighted Moving Averages Control Chart Under Non-Normality

Following Zaka et al. [1], it is assumed that the process random variable $x_1, x_2, x_3, \ldots, x_t$ are independently and identically distributed following WPFD with probability density function (pdf) and cumulative density function (cdf) for the WPFD are given respectively by

$$f(x) = \frac{2\gamma x^{2\gamma-1}}{\beta^{2\gamma}}; 0 < x < \beta$$

and

$F(x) = \left(\frac{x}{\beta}\right)^{2\gamma}$, where "$\beta$" and "$\gamma$" are the scale and shape parameters.

Modified Maximum Likelihood Estimator (MMLE) is used to construct memory less and memory-based control charts to monitor the shape parameter of a process that follows a WPFD. The estimator for the shape parameter of WPFD defined by Zaka et al. [1] is given as,

$$\widehat{\gamma}_{\mathrm{MMLE}} = \frac{-1 + \sqrt{1 + \frac{\bar{x}^2}{S^2}}}{2}. \tag{1}$$

The variance of the $\widehat{\gamma}_{\mathrm{MMLE}}$ is given by

$$\mathrm{Var}(\widehat{\gamma}_{\mathrm{MMLE}}) = \mathrm{E}(\widehat{\gamma}_{\mathrm{MMLE}} - \gamma)^2. \tag{2}$$

Now using Zaka et al. [1] and Naveed et al. [18], the EEWMA statistic is given as

$$EEW_t = \lambda_1 \widehat{\gamma}_{\mathrm{MMLE(t)}} - \lambda_2 \widehat{\gamma}_{\mathrm{MMLE(t-1)}} + (1 - \lambda_1 + \lambda_2) EEW_{t-1}$$

For t = 1

$$EEW_1 = \lambda_1 \widehat{\gamma}_{\mathrm{MMLE(1)}} - \lambda_2 \widehat{\gamma}_{\mathrm{MMLE(0)}} (1 - \lambda_1 + \lambda_2) EEW_0$$

For t = 2

$$EEW_2 = \lambda_1 \widehat{\gamma}_{\mathrm{MMLE(2)}} - \lambda_2 \widehat{\gamma}_{\mathrm{MMLE(1)}} (1 - \lambda_1 + \lambda_2) EEW_1$$

$$EEW_2 = \lambda_1 \widehat{\gamma}_{\mathrm{MMLE(2)}} - \lambda_2 \widehat{\gamma}_{\mathrm{MMLE(1)}} + (1 - \lambda_1 + \lambda_2) \left(\lambda_1 \widehat{\gamma}_{\mathrm{MMLE(1)}} - \lambda_2 \widehat{\gamma}_{\mathrm{MMLE(0)}} + (1 - \lambda_1 + \lambda_2) EEW_0 \right)$$

Let $a = (1 - \lambda_1 + \lambda_2)$

$$EEW_2 = \lambda_1 \widehat{\gamma}_{\mathrm{MMLE(2)}} - \lambda_2 \widehat{\gamma}_{\mathrm{MMLE(1)}} + a\lambda_1 \widehat{\gamma}_{\mathrm{MMLE(1)}} - a\lambda_2 \widehat{\gamma}_{\mathrm{MMLE(0)}} + a^2 EEW_0$$

$$EEW_2 = \lambda_1\widehat{\gamma}_{\text{MMLE(2)}} + (a\lambda_1 - \lambda_2)\widehat{\gamma}_{\text{MMLE(1)}} - a\lambda_2\widehat{\gamma}_{\text{MMLE(0)}} + a^2 EEW_0$$

Let $b = (a\lambda_1 - \lambda_2)$

$$EEW_2 = \lambda_1\widehat{\gamma}_{\text{MMLE(2)}} + b\widehat{\gamma}_{\text{MMLE(1)}} - a\lambda_2\widehat{\gamma}_{\text{MMLE(0)}} + a^2 EEW_0$$

$$EEW_3 = \lambda_1\widehat{\gamma}_{\text{MMLE(3)}} + b\widehat{\gamma}_{\text{MMLE(2)}} + ab\widehat{\gamma}_{\text{MMLE(1)}} - a^2\lambda_2\widehat{\gamma}_{\text{MMLE(0)}} + a^3 EEW_0$$

$$EEW_4 = \lambda_1\widehat{\gamma}_{\text{MMLE(4)}} + b\widehat{\gamma}_{\text{MMLE(3)}} + ab\widehat{\gamma}_{\text{MMLE(2)}} - a^2 b\lambda_2\widehat{\gamma}_{\text{MMLE(1)}} + a^3\lambda_2\widehat{\gamma}_{\text{MMLE(0)}} + a^4 EEW_0$$

$$\begin{aligned} EEW_t = {} & \lambda_1\widehat{\gamma}_{\text{MMLE(t)}} + b\widehat{\gamma}_{\text{MMLE(t-1)}} + ab\widehat{\gamma}_{\text{MMLE(t-2)}} + a^2 b\lambda_2\widehat{\gamma}_{\text{MMLE(t-3)}} + a^3\lambda_2\widehat{\gamma}_{\text{MMLE(0)}} + \ldots \\ & + a^{t-2} b\widehat{\gamma}_{\text{MMLE(1)}} - a^{t-1}\lambda_2\widehat{\gamma}_{\text{MMLE(0)}} + a^t EEW_0 \end{aligned}$$

By taking expectation, we get

$$E(EEW_t) = \lambda_1\gamma + b\gamma + ab\gamma + a^2 b\lambda_2\gamma + a^3\lambda_2\gamma + \ldots + a^{t-2}b\gamma - a^{t-1}\lambda_2\gamma + a^t\gamma$$

Replacing $b = (a\lambda_1 - \lambda_2)$

$$E(EEW_t) = \gamma\big(\lambda_1 + (a\lambda_1 - \lambda_2) + a(a\lambda_1 - \lambda_2) + a^2(a\lambda_1 - \lambda_2) + \ldots + a^{i-2}a^2(a\lambda_1 - \lambda_2) - a^{t-1}\lambda_2 + a^t\big)$$

$$E(EEW_t) = \gamma\big\{\lambda_1\big(1 + a + a^2 + a^3 + \ldots + a^{t-1}\big) - \lambda_2\big(1 + a + a^2 + a^3 + \ldots + a^{t-1}\big) + a^t\big\}$$

$$E(EEW_t) = \gamma\big\{\lambda_1\big(1 + a + a^2 + a^3 + \ldots + a^{t-1}\big) - \lambda_2\big(1 + a + a^2 + a^3 + \ldots + a^{t-1}\big) + a^t\big\}$$

$$E(EEW_t) = \gamma\big\{(\lambda_1 - \lambda_2)\big(1 + a + a^2 + a^3 + \ldots + a^{t-1}\big) + a^t\big\}$$

By using geometric series, we get

$$E(EEW_t) = \gamma\left\{(\lambda_1 - \lambda_2)\left(\frac{1 - a^t}{1 - a}\right) + a^t\right\}$$

So,

$$E(EEW_t) = \gamma\{1 - a^t + a^t\}$$

$$E(EEW_t) = \gamma$$

Let $Var\left(\widehat{\gamma}_{\text{MMLE(t)}}\right) = \upsilon = E(\widehat{\gamma}_{\text{MMLE}} - \gamma)^2$, we get

$$Var(EEW_t) = {\lambda_1}^2 v + b^2 v + a^2 b^2 v + a^4 b^2 \lambda_2^2 v + \ldots + a^{2(t-2)} b^2 v - a^{2(t-1)} {\lambda_2}^2 v$$

Let $b = (a\lambda_1 - \lambda_2)$

$$Var(EEW_t) = v\left\{{\lambda_1}^2 + (a\lambda_1 - \lambda_2)^2 + a^2(a\lambda_1 - \lambda_2)^2 + a^4(a\lambda_1 - \lambda_2)^2 + \ldots + a^{2(t-2)}(a\lambda_1 - \lambda_2)^2 - a^{2(t-1)}{\lambda_2}^2\right\}$$

$$\begin{aligned} Var(EEW_t) = v\Big\{ & {\lambda_1}^2 + a^2{\lambda_1}^2 - 2a\lambda_1 + {\lambda_2}^2 + a^4{\lambda_1}^2 - 2a^3\lambda_1 + a^2{\lambda_2}^2 + \big(a^6{\lambda_1}^2 - 2a^5{\lambda_1}^2 + a^4{\lambda_2}^2\big) + \ldots \\ & + a^{2(t-2)}{\lambda_1}^2 - 2a^{2t-3}\lambda_1 + a^{2(t-2)}{\lambda_2}^2 + a^{2(t-1)}{\lambda_2}^2\Big\} \end{aligned}$$

$$\begin{aligned} Var(EEW_t) = v\Big\{ & {\lambda_1}^2 + a^2{\lambda_1}^2 - 2a\lambda_1 + {\lambda_2}^2 + a^4{\lambda_1}^2 - 2a^3\lambda_1 + a^2{\lambda_2}^2 + \big(a^6{\lambda_1}^4 - 2a^5{\lambda_1}^3 + a^4{\lambda_2}^4\big) + \ldots \\ & + a^{2(t-2)}{\lambda_1}^2 - 2a^{2t-3}\lambda_1 + a^{2(t-2)}{\lambda_2}^2 + a^{2(t-1)}{\lambda_2}^2\Big\} \end{aligned}$$

$$Var(EEW_t) = v\Big\{\lambda_1{}^2+a^2\lambda_1{}^2-2a\lambda_1+\lambda_2{}^2+a^4\lambda_1{}^2-2a^3\lambda_1+a^2\lambda_2{}^2+\left(a^6\lambda_1{}^4-2a^5\lambda_1{}^3+a^4\lambda_2{}^4\right)+\ldots$$
$$+a^{2(t-2)}\lambda_1{}^2-2a^{2t-3}\lambda_1+a^{2(t-2)}\lambda_2{}^2+a^{2(t-1)}\lambda_2{}^2\Big\}$$

$$Var(EEW_t) = v\Big\{\lambda_1{}^2+a^2\lambda_1{}^2-2a\lambda_1+\lambda_2{}^2+a^4\lambda_1{}^2-2a^3\lambda_1+a^2\lambda_2{}^2+\left(a^6\lambda_1{}^4-2a^5\lambda_1{}^3+a^4\lambda_2{}^4\right)+\ldots$$
$$+a^{2(t-2)}\lambda_1{}^2-2a^{2t-3}\lambda_1+a^{2(t-2)}\lambda_2{}^2+a^{2(t-1)}\lambda_2{}^2\Big\}$$

$$Var(EEW_t) = v\{\lambda_1{}^2\left(1+a^2+a^4+\ldots.+a^{2(t-1)}\right)+\lambda_2{}^2\left(1+a^2+a^4+\ldots.+a^{2(t-1)}\right)$$
$$-2a\lambda_1\lambda_2\left(1+a^2+a^4+\ldots.+a^{2(t-2)}\right)\Big\}$$

$$Var(EEW_t) = v\Big\{\left(\lambda_1{}^2+\lambda_2{}^2\right)\left(1+a^2+a^4+\ldots.+a^{2(t-1)}\right)-2a\lambda_1\lambda_2\left(1+a^2+a^4+\ldots.+a^{2(t-2)}\right)\Big\}$$

$$Var(EEW_t) = v\left\{\left(\lambda_1{}^2+\lambda_2{}^2\right)\left(\frac{1-a^{2t}}{1-a^2}\right)-2a\lambda_1\lambda_2\left(\frac{1-a^{2t-2}}{1-a^2}\right)\right\}$$

$$Var(EEW_t) = v\left\{\left(\lambda_1{}^2+\lambda_2{}^2\right)\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t}}{1-(1-\lambda_1+\lambda_2)^2}\right)-2a\lambda_1\lambda_2\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t-2}}{1-(1-\lambda_1+\lambda_2)^2}\right)\right\}$$

The control limits are given as

$$UCL = \gamma_0 + Lv\sqrt{\left(\lambda_1{}^2+\lambda_2{}^2\right)\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t}}{1-(1-\lambda_1+\lambda_2)^2}\right)-2a\lambda_1\lambda_2\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t-2}}{1-(1-\lambda_1+\lambda_2)^2}\right)}$$

$$CL = \gamma_0$$

$$LCL = \gamma_0 - Lv\sqrt{\left(\lambda_1{}^2+\lambda_2{}^2\right)\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t}}{1-(1-\lambda_1+\lambda_2)^2}\right)-2a\lambda_1\lambda_2\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t-2}}{1-(1-\lambda_1+\lambda_2)^2}\right)}$$

4.1 Algorithmic Steps

a) Generate a random sample of size n = 150 on "X_t" from the WPFD are $x = \beta R^{\frac{1}{2\gamma}}$ with parameters (β,γ) = (1, 2). Where R: random numbers.
b) Compute $\hat{\gamma}_{\mathrm{MMLE}(t)}$.
c) Repeat steps (a) and (b), for 5000 times and compute $\hat{\gamma}_{\mathrm{MMLE}(t)}$, $E\left(\hat{\gamma}_{\mathrm{MMLE}(t)}\right)$ and $V\left(\hat{\gamma}_{\mathrm{MMLE}(t)}\right)$.
d) Repeat step (c) for 5000 times and compute $\hat{\gamma}_{\mathrm{MMLE}(t)}$.
e) Compute control limits to construct EEWMA control charts based on $\hat{\gamma}_{MMLE(t)}$ (Computed in step (d)).
f) Compute ARL value for each EEWMA control chart that based on $\hat{\gamma}_{MMLE(t)}$ given that process is in-control state.
g) Now fix ARL_0 = 500 for in-control state of the process and search the suitable value of L, so that ARL_0 for in-control state of process is achieved.

h) Now assume if the process parameter "γ" is shifted by from its true value and compute ARL_1. This step is repeated for different shift values 0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.13 and 0.88. Also compute ARL_1 in each case of shift values.
i) Plot ARL_s values against the values of shift that used in step (g) and (h).
j) It is to note that the procedure of EWMA control chart based on $\hat{\gamma}_{MMLE(t)}$ observe whether the process following the WPFD is in-control or out of control. If the process is in-control, go to Step (a). Otherwise, record the Run Length, i.e. the process remained in control before it is declared to be out-of-control.
k) Repeat this process 5000 times to obtain the ARLs, SDRLs and percentiles.

4.2 Results and Discussions

We consider the average run length to compare the performance of the proposed estimators. It is taken as the average number of samples that are used before as the process is out of control. ARL_0 indicates the in control ARL, while the ARL_1 describes the out of control ARL. A chart having a larger ARL_0 and smaller ARL_1 is considered better to be used for the process monitoring.

The Tabs. 1–3 is constructed for ARL values when the parameters of underlying process following WPFD using EWMA and EEWMA. Figs. 1–3 presents the ARL values for EWMA and EEWMA. We observed that EEWMA control chart detects earlier on small shifts as compare to EWMA control chart for different choices of λ_1 and λ_2. The same behaviour is observed from Figs. 1–3.

Table 1: Average run length (ARL), Standard deviation Run length and Percentiles for WPFD under the proposed EEWMA ($\lambda_1 = 0.10$, $\lambda_2 = 0.03$) and EWMA ($\lambda = 0.10$)

Shifts	Proposed EEWMA $\lambda_1 = 0.10$, $\lambda_2 = 0.03$ L = 16.65				EWMA $\lambda = 0.10$ L = 16.3			
	ARL	SDRL	P25	P75	ARL	SDRL	P25	P75
0	500.509	497.0645	163.75	647.25	500.415	512.487	145.75	658.25
0.01	297.865	285.0804	91.75	413.25	314.384	313.7296	92.00	438.25
0.02	189.79	179.0556	65.00	250.25	200.772	191.563	70.00	263.25
0.03	129.029	114.8576	46.75	174.00	136.626	129.6943	47.0	183.0
0.04	88.135	73.72819	36.0	117.0	93.874	81.6371	37.00	126.25
0.05	66.053	49.85326	30.0	89.0	69.781	59.8337	28.0	91.0
0.06	52.268	38.99031	24	70	54.712	44.54965	23.0	73.0
0.07	42.435	29.88861	20	58	43.483	33.66992	19.00	59.25
0.08	34.402	24.03956	18	47	35.905	26.73666	18.00	46.25
0.13	15.62	10.37117	10	23	16.98	11.03464	9.0	22.0
0.88	1	0.5663495	1	2	1.35	0.5193937	1	2

Table 2: ARL, Standard deviation Run length and Percentiles for WPFD under the proposed EEWMA ($\lambda_1 = 0.20$, $\lambda_2 = 0.05$) and EWMA ($\lambda = 0.20$)

Shifts	Proposed EEWMA $\lambda_1 = 0.20$, $\lambda_2 = 0.05$ L = 17.245				EWMA $\lambda = 0.20$ L = 16.975			
	ARL	SDRL	P25	P75	ARL	SDRL	P25	P75
0	500.136	493.1804	152.0	667.5	500.606	497.0729	145.75	685.00
0.01	331.041	339.1675	95.75	456.00	351.294	359.168	100.0	480.0
0.02	225.598	222.0587	74.00	296.25	235.775	238.3603	71.75	318.00
0.03	158.066	147.8465	51.0	217.0	170.249	169.09	51.75	228.00
0.04	118.562	109.8235	38.00	163.25	127.716	118.176	40.75	178.50
0.05	86.46	78.38407	30.0	119.0	92.09	84.99995	32	123
0.06	67.112	57.45761	25	90	71.569	64.56178	25	95
0.07	54.088	46.67229	20	74	58.698	53.25019	19.75	79.00
0.08	42.084	35.33439	17	57	46.133	39.63216	18	63
0.13	17.98	14.13157	9	25	19.802	16.26079	9	26
0.88	1	0.5033929	1	2	1.237	0.4461278	1	1

Table 3: ARL, Standard deviation Run length and Percentiles for WPFD under the proposed EEWMA ($\lambda_1 = 0.30$, $\lambda_2 = 0.15$) and EWMA ($\lambda = 0.30$)

Shifts	Proposed EEWMA $\lambda_1 = 0.30$, $\lambda_2 = 0.15$ L = 18.108				EWMA $\lambda = 0.30$ L = 17.39			
	ARL	SDRL	P25	P75	ARL	SDRL	P25	P75
0	500.966	490.7444	143.0	685.0	500.966	497.4742	137.00	684.25
0.01	355.016	360.5669	104.0	477.5	369.121	368.5989	101.00	502.25
0.02	260.056	251.002	83.75	355.00	270.565	270.2252	85.75	371.25
0.03	192.215	190.9364	59.00	258.25	203.551	206.272	66.75	276.50
0.04	144.88	135.8518	47.00	199.25	155.95	152.1694	47.00	213.25
0.05	110.483	103.1633	37.00	152.25	121.669	114.7251	38.0	166.0
0.06	84.995	79.4971	29.00	116.25	90.49	85.77742	30.00	122.25
0.07	67.581	59.47043	25.00	89.25	73.268	68.67861	23.00	102.25
0.08	54.099	48.83753	21.00	72.25	59.065	54.23303	20.0	81.0
0.13	22.414	18.08457	11	31	24.415	21.55605	9	34
0.88	1	0.583489	1	2	1.222	0.430001	1	1

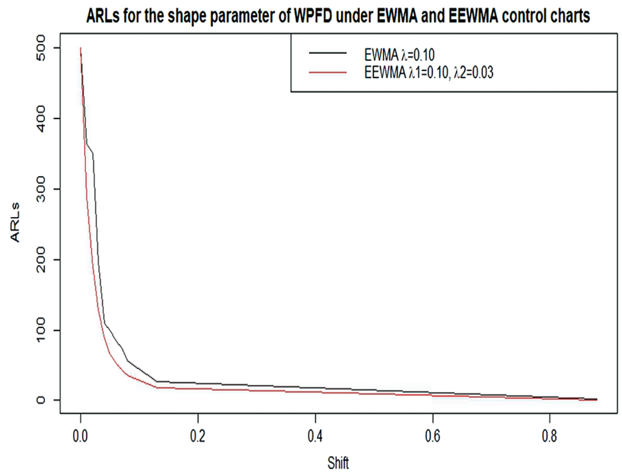

Figure 1: ARL for the shape parameter of WPFD using EWMA and EEWMA

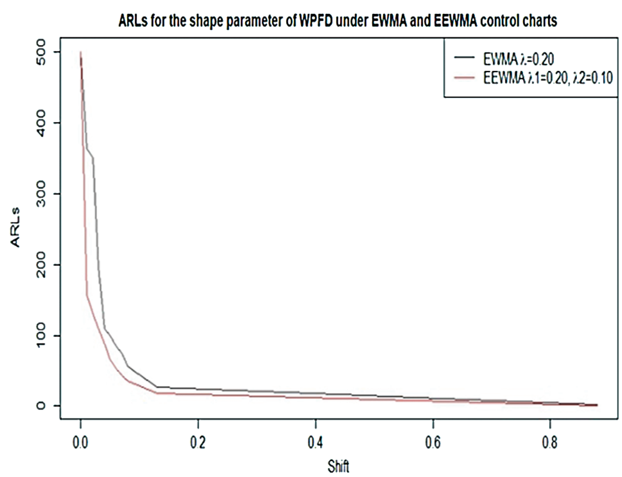

Figure 2: ARL for the shape parameter of WPFD using EWMA and EEWMA

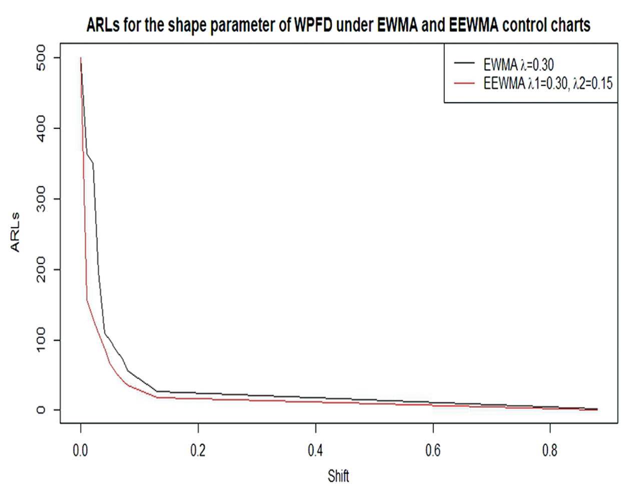

Figure 3: ARL for the shape parameter of WPFD using EWMA and EEWMA

5 Simulation Study

In order to see the working procedure of the proposed control charts, a simulation study was carried out. For this purpose, we generated 25 observations from a WPFD for in-control process, and the next 25 observations were generated from the shifted process with 0.01. The estimated values of the proposed EWMA statistic under MMLE were computed for the selected levels of the proposed control charts parameters with $\lambda = 0.20$ and L = 16.975. The data and values of the proposed and existing statistic are listed in Tab. 4, and plotted values of these statistics are shown in Fig. 4. Further, the estimated values of the proposed EEWMA statistic under MMLE were computed for the selected levels of the proposed control charts parameters with $\lambda_1 = 0.20, \lambda_2 = 0.05\, and\, L = 17.245$. The data and values of the proposed and existing statistic are listed in Tab. 4, and plotted values of these statistics are shown in Fig. 5.

Table 4: Simulated data

EWMA			EEWMA		
$\lambda = 0.20$, L = 16.975			$\lambda_1 = 0.20$, $\lambda_2 = 0.05$, L = 17.245		
EW_t	LCL	UCL	EEW_t	LCL	UCL
1.999742	1.958025	2.081975	1.999922	2.001297	2.038704
2.032648	1.936621	2.103379	2.009740	1.984710	2.055290
2.022034	1.922676	2.117324	2.013428	1.969518	2.070482
2.001997	1.912700	2.127300	2.009999	1.956267	2.083733
2.000829	1.905254	2.134746	2.007248	1.945017	2.094983
2.026739	1.899560	2.140440	2.013095	1.935621	2.104379
2.038168	1.895137	2.144863	2.020617	1.927854	2.112146
2.027747	1.891667	2.148333	2.022756	1.921479	2.118521
2.034679	1.888923	2.151077	2.026333	1.916270	2.123730
2.021543	1.886742	2.153258	2.024896	1.912028	2.127972
2.007367	1.885002	2.154998	2.019637	1.908582	2.131418
1.991564	1.883608	2.156392	2.011215	1.905786	2.134214
1.958908	1.882489	2.157511	1.995523	1.903520	2.136480
1.987081	1.881590	2.158410	1.992991	1.901684	2.138316
1.974873	1.880866	2.159134	1.987555	1.900198	2.139802
1.990290	1.880282	2.159718	1.988376	1.898995	2.141005
1.965196	1.879811	2.160189	1.981422	1.898022	2.141978
1.987566	1.879430	2.160570	1.983265	1.897234	2.142766
1.984756	1.879123	2.160877	1.983712	1.896597	2.143403
2.001607	1.878874	2.161126	1.989081	1.896081	2.143919
1.993854	1.878673	2.161327	1.990513	1.895664	2.144336
2.050537	1.878511	2.161489	2.008520	1.895326	2.144674
2.002203	1.878379	2.161621	2.006625	1.895052	2.144948

(Continued)

Table 4 (continued).

EWMA			EEWMA		
$\lambda = 0.20$, L = 16.975			$\lambda_1 = 0.20$, $\lambda_2 = 0.05$, L = 17.245		
EW_t	LCL	UCL	EEW_t	LCL	UCL
2.018215	1.878273	2.161727	2.010102	1.894831	2.145169
2.035307	1.878186	2.161814	2.017664	1.894652	2.145348
2.041714	1.878117	2.161883	2.027329	1.894507	2.145493
2.081554	1.878060	2.161940	2.048474	1.894389	2.145611
2.075656	1.878014	2.161986	2.063326	1.894294	2.145706
2.059292	1.877977	2.162023	2.070312	1.894217	2.145783
2.062785	1.877947	2.162053	2.077926	1.894155	2.145845
2.094064	1.877923	2.162077	2.094428	1.894104	2.145896
2.109171	1.877903	2.162097	2.111844	1.894063	2.145937
2.100904	1.877888	2.162112	2.122475	1.894030	2.145970
2.111034	1.877875	2.162125	2.134089	1.894003	2.145997
2.100044	1.877864	2.162136	2.139766	1.893981	2.146019
2.087353	1.877856	2.162144	2.140586	1.893964	2.146036
2.073221	1.877849	2.162151	2.137590	1.893949	2.146051
2.040269	1.877843	2.162157	2.125775	1.893938	2.146062
2.072167	1.877839	2.162161	2.128186	1.893929	2.146071
2.060706	1.877835	2.162165	2.126809	1.893921	2.146079
2.077973	1.877832	2.162168	2.131641	1.893915	2.146085
2.052867	1.877830	2.162170	2.127647	1.893910	2.146090
2.077280	1.877828	2.162172	2.132809	1.893906	2.146094
2.075126	1.877826	2.162174	2.136058	1.893903	2.146097
2.093397	1.877825	2.162175	2.144269	1.893900	2.146100
2.085676	1.877824	2.162176	2.147808	1.893898	2.146102
2.146482	1.877823	2.162177	2.169608	1.893896	2.146104
2.096218	1.877822	2.162178	2.169412	1.893895	2.146105
2.113459	1.877822	2.162178	2.174811	1.893894	2.146106
2.131736	1.877821	2.162179	2.184422	1.893893	2.146107

In Fig. 5, we noted that the proposed EEWMA control chart under MMLE detected a shift at the 32th sample, while in Fig. 4; the EWMA control chart under MMLE could not detect the shift. Hence, this shows that the proposed EEWMA control chart under MMLE has a greater ability to detect smaller shifts earlier, as compared to the EWMA control chart.

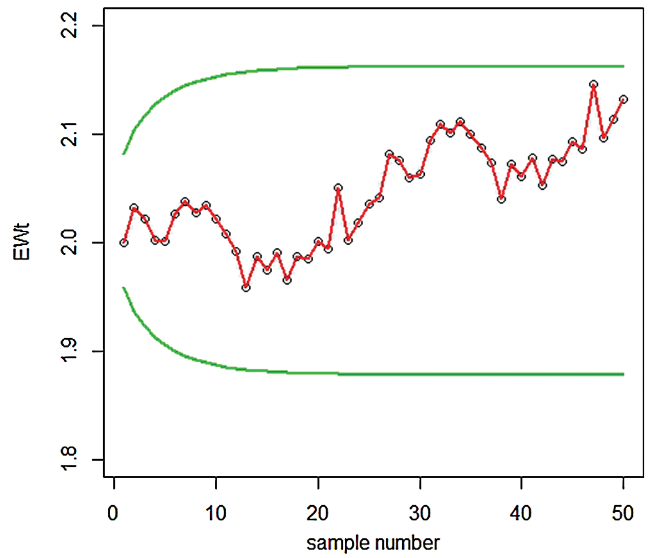

Figure 4: Graph of simulated data of the proposed EWMA control chart under MMLE

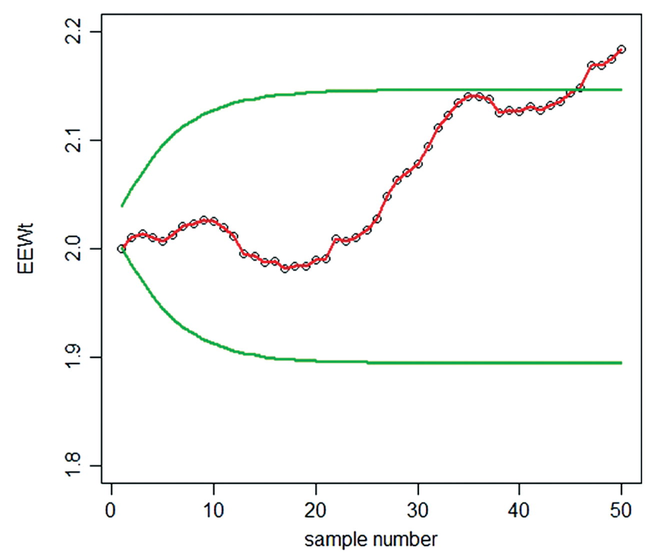

Figure 5: Graph of simulated data of the proposed EEWMA control chart under MMLE

6 Conclusions

We have discussed the process monitoring for WPFD. In real life, we may face the situation that any specific process does not follow the normal distribution. But the distribution of the errors in the process becomes WPFD. In the current work, we first estimate the parameters of the distribution of the errors during any process by using MMLE, which was claimed better to estimate the parameters of WPFD. By using the MMLE, we then modified the quality control charts used in literature, such as EWMA and EEWMA control charts. We see that EEWMA control chart under MMLE can be used to monitor the process when the underlying distribution of the errors in process monitoring follows WPFD. It is therefore hoped that the findings of this study will be useful for researchers in different fields of applied sciences. It will be helpful to identify the error in time and making strategies to deal with it. Companies can control their cost and improve product quality by applying the proposed quality chart processes.

Funding Statement: The authors received no specific funding for this study.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References

[1] A. Zaka, A. S. Akhter and R. Jabeen, "Asymmetry approach to study for chemotherapy treatment and devices failure time's data using modified Power function distribution with some modified estimators," *International Journal of Computing Science and Mathematics,* (Accepted Paper), 2020. https://www.inderscience.com/info/ingeneral/forthcoming.php?jcode=ijcsm.

[2] S. W. Roberts, "Control chart tests based on geometric moving averages," *Technometrics*, vol. 1, pp. 239–250, 1959.

[3] Z. Li, M. Xie and M. Zhou, "Rank-based EWMA procedure for sequentially detecting changes of process location and variability," *Quality Technology and Quantitative Management*, vol. 15, no. 3, pp. 354–373, 2018.

[4] H. D. Nguyen, K. P. Tran and C. Heuchenne, "Monitoring the ratio of two normal variables using variable sampling interval exponentially weighted moving average control charts," *Quality and Reliability Engineering*, vol. 35, no. 1, pp. 439–460, 2019.

[5] E. S. Page, "Continuous inspection scheme," *Biometrika*, vol. 41, no. 1, pp. 100–115, 1954.

[6] R. A. Sanusi, M. Riaz and N. Abbas, "Combined Shewhart CUSUM charts using auxiliary variable," *Computers and Industrial Engineering*, vol. 105, pp. 329–337, 2017.

[7] A. Haq and W. Munir, "Improved CUSUM charts for monitoring process mean," *Journal of Statistical Computation and Simulation*, vol. 88, no. 9, pp. 1684–1701, 2018.

[8] M. P. Hossain, R. A. Sanusi, M. H. Omar and M. Riaz, "On designing Maxwell CUSUM control chart: An efficient way to monitor failure rates in boring processes," *International Journal of Advanced Manufacturing Technology*, vol. 100, no. 5–8, pp. 1923–1930, 2019.

[9] N. Abbas, M. Riaz and R. J. M. M. Does, "Mixed exponentially weighted moving average-cumulative sum charts for process monitoring," *Quality and Reliability Engineering International*, vol. 29, no. 3, pp. 345–356, 2013.

[10] J. O. Ajadi and M. Riaz, "Mixed multivariate EWMA-CUSUM control charts for an improved process monitoring," *Communications in Statistics Theory and Methods*, vol. 46, no. 14, pp. 6980–6993, 2017.

[11] S. E. Shamma and A. K. Shamma, "Development and evaluation of control charts using double exponentially weighted moving averages," *International Journal of Quality and Reliability Management*, vol. 9, no. 6, pp. 18–25, 1992.

[12] A. Haq, "A new hybrid exponentially weighted moving average control chart for monitoring process mean," *Quality and Reliability Engineering International*, vol. 29, pp. 1015–1025, 2013.

[13] R. Noorossana, S. Fathizadan and M. R. Nayebpour, "EWMA Control Chart performance with estimated parameters under non-normality," *Quality Technology and Quantitative Management*, vol. 32, no. 5, pp. 1638–1654, 2016.

[14] Y. C. Lin, C. Y. Chou and C. H. Chen, "Robustness of the EWMA median control chart to non- normality," *International Journal of Industrial and Systems Engineering*, vol. 25, no. 1, pp. 35–58, 2017.

[15] P. Erto, G. Pallotta, B. Palumbo and C. M. Mastrangelo, "The performance of semi empirical bayesian control charts for monitoring Weibull data," *Quality Technology and Quantitative Management*, vol. 15, no. 1, pp. 69–86, 2018.

[16] W. Liang, D. Xiang, X. Pu, Y. Li and L. Jin, "A robust multivariate sign control chart for detecting shifts in covariance matrix under the elliptical directions distributions," *Quality Technology and Quantitative Management*, vol. 16, no. 1, pp. 113–127, 2019.

[17] A. Ahmed, A. Sanaullah and M. Hanif, "A robust alternate to the HEWMA control chart under non-normality," *Quality Technology and Quantitative Management*, vol. 17, no. 4, pp. 423–447, 2019.

[18] M. Naveed, M. Azam, N. Khan and M. Aslam, "Design of a control chart using extended EWMA statistic," *Technologies*, vol. 6, no. 108, pp. 2–15, 2018.