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ARTICLE

# Dombi-Normalized Weighted Bonferroni Mean Operators with Novel Multiple-Valued Complex Neutrosophic Uncertain Linguistic Sets and Their Application in Decision Making

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## ABSTRACT

Although fuzzy set concepts have evolved, neutrosophic sets are attracting more attention due to the greater power of the structure of neutrosophic sets. The ability to account for components that are true, false or neither true nor false is useful in the resolution of real-life problems. However, simultaneous variations render neutrosophic sets unsuitable in specific circumstances. To enable the management of these sorts of issues, we combine the principle of multi-valued neutrosophic uncertain linguistic sets and complex fuzzy sets to develop the principle of multi-valued complex neutrosophic uncertain linguistic sets. Multi-valued complex neutrosophic uncertain linguistic sets can contain grades of truth, abstinence, and falsity, and uncertain linguistic terms, which are expressed as complex numbers whose real and imaginary parts are limited to the unit interval. Some important Dombi laws are elaborated along with Bonferroni mean operators, which offer a flexible general structure with modifiable factors. Bonferroni means aggregation operators perform a significant role in conveying the magnitude level of options and characteristics. To determine relationships among any number of attributes, we develop multi-valued complex neutrosophic uncertain linguistic Dombi-normalized weighted Bonferroni mean operators and discuss their important properties with some special cases. By using these laws, we can deploy the multi-attribute decision-making (MADM) technique using the novel principle of multi-valued complex neutrosophic uncertain linguistic sets. To determine the power and flexibility of the elaborated approach, we resolve some numerical examples based on the proposed operator. Finally, the work is validated with the help of comparative analysis, a discussion of its advantages, and geometric expressions of the elaborated theories.

## KEYWORDS

Multi-valued complex neutrosophic uncertain linguistic sets; Dombi normalized weighted Bonferroni mean operators; multi-attribute decision-making methods

## 1 Introduction

MADM is widely applied to real-world problems, but due to the complicated and inconsistent information acquired, the use of crisp sets in these contexts often has limitations. To resolve the problems which occur in certain issues, Zadeh [1] elaborated the principle of the fuzzy set (FS). FSs are more powerful and effective than crisp sets and can contain the truth grade (TG) belonging to the unit interval. But in certain situations, the FS can fail. FSs do not work effectively when an element can either belong or not belong to the set. To deal with these situations, the intuitionistic FS (IFS) was elaborated by Atanassov [2]. IFS extends the FS to include the falsity grade (FG). The main advantage of the IFS is that the sum of both values belongs to the unit interval. IFS has modified FS to enable the management of inconsistent and awkward information in real-world problems. The powerful structure of IFS has been utilized by various scholars. Liu et al. [3] examined a viable weighted-based hybrid approach using interval-valued IFSs; Garg et al. [4] presented a similarity measure using right-angled triangles based on IFSs; Ejegwa et al. [5] initiated a statistical correlation algorithm using IFSs; Xue et al. [6] elaborated measure-based belief functions using IFSs; Aydin et al. [7] explored interval-valued intuitionistic parameterized interval-valued intuitionistic fuzzy soft sets; Szmidt et al. [8] proposed certain measures based on IFSs, and Ghosh et al. [9] elaborated a fixed charge solid transportation problem based on IFSs.

In several scenarios, the conception of IFS has been neglected if an intellectual faces data in the form of yes, abstinence, or no, then the use of IFSs is not effective. To deal with these situations, Smarandache elaborated the neutrosophic set (NS) [10] by extending the IFS to include the abstinence grade (AG). The main advantage of the NS is that sum of triplet values can belong to the unit interval $[0, 3]$. NS has modified IFS to enable the handling of inconsistent and incongruous information. The NS has been used by various scholars. Zavadskas et al. [11] initiated the MULTIMOORA method by using interval-valued NSs; Tan [12] proposed entropy measures using redefined single-valued NSs; Ye [13] investigated entropy measures by using simplified NSs; Abdullah et al. [14] developed the DEMATEL method using single-valued NSs; Tufail et al. [15] proposed the investigation of brain cancer using NSs on MRI scans; Du et al. [16] explored aggregation operators using neutrosophic Z-numbers; Wang et al. [17] proposed aggregation operators using single-valued NSs; Wei et al. [18] proposed the COPRAS method using single-valued neutrosophic 2-tuple linguistic sets; Jana et al. [19] investigated Dombi power aggregation operators using single-valued NSs, and Zhao et al. [20] elaborated the TODIM method by using 2-tuple linguistic NSs.

FS has typically failed when applied to information in the form of two-dimensions in a single set. To briefly explain two-dimensional information with the help of an example, let us assume an individual who needs to buy a vehicle and the crucial factors are the model and year of manufacture. Since the vehicle model changes with the year of manufacture, the decision-making procedure frequently changes. These issues cannot be demonstrated precisely with conventional speculations. Ramot et al. [21] elaborated the principle of complex FS (CFS) to resolve these types of problems. CFS is powerful and more effective than FS and covers the TG whose real and unreal parts belong to the unit interval. But in certain situations, the CFS can fail. CFS is not effective when a decision-maker faces a choice in the form of yes or no. To deal with these situations, Alkouri et al. [22] elaborated the principle of complex IFS (CIFS) by extending CFS to include the FG. The main advantage of CIFS is that the sum of the real part (also for the unreal part) of both values belongs to the unit interval. CIFS has enabled CFS to handle inconsistent and difficult information. Various scholars have utilized the powerful structure of CIFS.

Rani et al. [23] initiated distance measures using CIFSs; Kumar et al. [24] developed complex intuitionistic soft sets; Garg et al. [25] elaborated information measures based on CIFSs; Ngan et al. [26] proposed quaternion numbers using CIFSs; Garg et al. [27] elaborated a correlation coefficient by using CIFSs; Rani et al. [28] explored power aggregation operators based on CIFSs; Quek et al. [29] initiated the algebraic structure of complex intuitionistic fuzzy soft sets, and Garg et al. [30] initiated Heronian mean operators using complex intuitionistic uncertain linguistic sets.

Yet, in certain situations, the principle of CIFS can fail. CIFS is not effective when responses can take the form of yes, abstinence, and no. To deal with these situations, Ali et al. elaborated the principle of complex NS (CNS) [31] by extending CIFS to include the AG. The main advantage of the CNS is that some of the real part (also unreal part) of triplet values belong to the interval unit $[0, 3]$. CNS enables CIFS to manage inconsistent and difficult information. The CNS has been utilized by various scholars. Broumi et al. [32] proposed bipolar complex NSs; Dat et al. [33] initiated linguistic approaches using interval complex NSs; Singh [34] proposed lattice-based CNSs; Quek et al. [35] explored graph theory using CNSs; Manna et al. [36] developed the VIKOR method based on CNSs; Li et al. [37] explored generalized hybrid weighted averaging operators using interval-valued complex single-valued NSs, and Ali et al. [38] explored complex neutrosophic generalized dice similarity measures and their applications.

Nevertheless, in real-world problems, it is not unusual for decision-makers to express their thoughts as quantifiable interpretations. When a consultant assesses a client's opinion, he may consider it suitable to employ linguistic phrases such as "very good", "good", or "medium", to express an estimation. To manage these problems, Zadeh [39,40] probed the principle of the linguistic variable (LV) to illustrate the inclinations of decision-makers. Furthermore, the principle of 2-tuple LV was created by Herrera et al. [41]. Xu [42] initiated aggregation operators for uncertain linguistic sets (ULSs). Peng et al. [43] initiated power aggregation operators for multi-valued neutrosophic sets (MVNSs); Liu et al. [44] developed the PROMERHEE method using probability MVNSs; Peng et al. [45] explored MVNSs and their applications in decision-making techniques; Liu et al. [46] proposed an extension of the ARAS method using probability MVNSs; Ye et al. [47] initiated a correlation coefficient regarding MVNSs, and Yang et al. [48] developed the Dombi normal weighted Bonferroni mean operator for MVNULSs.

From current accomplishments, we know that operators dependent on Dombi operations have been proposed and applied to combine intuitionistic components, complex intuitionistic components, single-valued neutrosophic components, interval neutrosophic components, and neutrosophic cubic components. They have not been applied to complex MVNSs and MVCNULSs. There has been no exploration of the use of Dombi normalized weighted Bonferroni mean (DNWBM) operators on novel MVCNULNs data. Collection administrators perform numerical tasks like normal, total, count, max, min, and total, on the numeric property of the components in a set. Archimedean Bonferroni mean administrators are the summed-up types of basic collection administrators inferred to adapt to abnormal and convoluted data in genuine issues. Accumulation administrators are numerical capacities that are utilized to consolidate data. That is, they are utilized to consolidate N information (for instance, N mathematical qualities) in a solitary datum. The math mean and the weighted mean are the most notable collection administrators. The middle and the mode can likewise be collection administrators. The primary distinction between the number-crunching means and the weighted mean is that the last option grants us to weight vector various information as per their pertinence. There exist various conglomeration administrators that are applied relying upon the suppositions in the (information types) and the kind of data that we can consolidate in the model. For instance, fluffy integrals license us to allocate pertinence to sets of data sources and not exclusively to individual sources just like the case for the weighted mean. Math and mathematical accumulation administrators are extraordinary sorts of Archimedean Bonferroni mean administrators.

It is important to expand DNWBM dependent on Dombi activities to MVCNULNs. In general, a DNWBM operator has the following qualities. Initially, it has greater flexibility with general boundaries. Then, it can consider both the relationship and the weight of many contentions. Based on the above analysis, the main advantages of the initiated MVCNULSs are discussed below:

(1) If we choose the value of TG, AG, and FG in the form of singleton sets in the initiated MVCNULSs, then the MVCNULSs are changed for complex neutrosophic uncertain linguistic sets.

(2) If we choose the value of TG, AG, and FG in the form of singleton sets and if the value of an uncertain linguistic set is zero in the initiated MVCNULSs, then the MVCNULSs are changed for complex neutrosophic sets.

(3) If we choose the value of the uncertain linguistic set is zero in the initiated MVCNULSs, then the MVCNULSs are changed for multi-valued complex neutrosophic sets.

(4) If we choose the value of AG in the form of zero in the initiated MVCNULSs, then the MVCNULSs are changed for multi-valued complex intuitionistic uncertain linguistic sets.

(5) If the value of AG and FG is zero in the initiated MVCNULSs, then the MVCNULSs are changed for multi-valued complex fuzzy uncertain linguistic sets.

The graphical expressions of the initiated works in this study are presented Fig. 1.

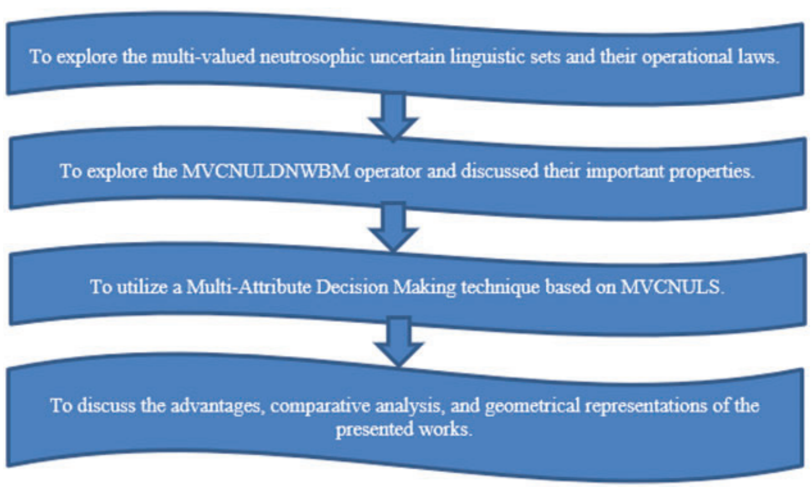


**Figure 1:** Expressions of the presented approaches

MVCNULSs are more suitable for dealing with quantitative or subjective data in addressing MADM and MAGDM issues. Considering these benefits, the objectives of the paper are the following:

(1) To develop the principle of MVCNULS and their important Dombi laws are also elaborated.

(2) To determine the relationship among any number of attributes, we develop the multi-valued complex neutrosophic uncertain linguistic Dombi normalized weighted Bonferroni mean (MVCNULDNWBM) operator and illustrate its important properties with some specific cases.
(3) To utilize a MADM technique using the novel principle of MVCNULS. To determine the power and flexibility of the elaborated approaches, we resolve some numerical examples using the proposed operator.
(4) The priority of the elaborated work is to determine the advantages and geometric expression of the elaborated theories with the help of comparative analysis.

This study proceeds as follows: In Section 2, we briefly recall some prevailing ideas such as MVNSs and their algebraic laws. The principle and laws of ULSs are also revised with Dombi t-norm (DTN) and Dombi t-conorm (DTCN). The notion of the NWBM operator is also discussed. In Section 3, we combine the principle of MVNULSs and CFSs to develop the principle of MVCNULSs. In Section 4, some important Dombi laws are elaborated. We present the multi-valued complex neutrosophic uncertain linguistic Dombi-normalized weighted Bonferroni mean (MVCNULDNWBM) operator and discuss important properties using specific cases. In Section 5, we utilize the MADM technique with the novel principle of MVCNULS. To determine the strength and flexibility of the elaborated approach, we resolve some numerical examples using the proposed operator. Finally, the work is validated with the help of comparative analysis, advantages, and geometric expression of the elaborated theories. The conclusions drawn are presented in Section 6.

## 2 Preliminaries

Certain extensions of the FS have been proposed and utilized in the context of aggregation operators, measures, and methods to easily determine the reliability and consistency of the prevailing ideas. The theory of the MVNS is an important and useful principle for the management of awkward and inconclusive information. We will briefly recall some prevailing ideas such as MVNSs and their algebraic laws. The principle of ULSs and their laws are also revised with Dombi t-norm (DTN) and Dombi t-conorm (DTCN). The notion of the NWBM operator is also discussed at the end of this section. In the overall study, the universal set is specified by $\mathfrak{X}_{uni}$ with TG, AG, and FG specified by $\mathfrak{W}^{i}_{\mathcal{M}_{mn}}$, $\mathfrak{A}^{j}_{\mathcal{M}_{mn}}$, and $\mathfrak{N}^{k}_{\mathcal{M}_{mn}}$.

***Definition 1: [43]*** A MVNS $\mathcal{M}_{mn}$ is specified by:

$$\mathcal{M}_{mn} = \left\{\left(\left(\mathfrak{W}^{i}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}), \mathfrak{A}^{j}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}), \mathfrak{N}^{k}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el})\right)\right), i, j, k = 1, 2, \ldots, \mathfrak{r}, \mathfrak{s}, \mathfrak{t} \colon \mathfrak{X}_{el} \in \mathfrak{X}_{uni}\right\} \tag{1}$$

where $\mathfrak{W}^{i}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}) = \left\{\mathfrak{W}^{1}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}), \mathfrak{W}^{2}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}), \ldots, \mathfrak{W}^{\mathfrak{r}}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el})\right\}$, $\mathfrak{A}^{j}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}) = \left\{\mathfrak{A}^{1}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}), \mathfrak{A}^{2}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}), \ldots, \mathfrak{A}^{\mathfrak{s}}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el})\right\}$ and $\mathfrak{N}^{k}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}) = \left\{\mathfrak{N}^{1}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}), \mathfrak{N}^{2}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}), \ldots, \mathfrak{N}^{\mathfrak{t}}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el})\right\}$ with a restriction such that $0 \leq \sup\left(\mathfrak{W}^{i}_{\mathcal{M}_{mn}}\right) + \sup\left(\mathfrak{A}^{j}_{\mathcal{M}_{mn}}\right) + \sup\left(\mathfrak{N}^{k}_{\mathcal{M}_{mn}}\right) \leq 3$, where each $\mathfrak{W}^{i}_{\mathcal{M}_{mn}}, \mathfrak{A}^{j}_{\mathcal{M}_{mn}}, \mathfrak{N}^{k}_{\mathcal{M}_{mn}} \in [0, 1]$. Additionally, multi-valued neutrosophic numbers (MVNNs) are shown by $\boldsymbol{\mathcal{M}}_{mn-\jmath}=\left(\mathfrak{W}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\mathfrak{A}^{j}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\mathfrak{N}^{k}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}}\right),\jmath=1,2,\ldots,z$. For any two MVNNs $\boldsymbol{\mathcal{M}}_{mn-\jmath}=\left(\mathfrak{W}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\mathfrak{A}^{j}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\mathfrak{N}^{k}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}}\right),\jmath=1,2$, we specify some algebraic laws, such that

$$\boldsymbol{\mathcal{M}}_{mn-1}\oplus\boldsymbol{\mathcal{M}}_{mn-2}=\cup_{\begin{pmatrix}\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}},\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}}\in\mathfrak{W}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\\ \mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}},\mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}}\in\mathfrak{A}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\\ \mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}},\mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}}\in\mathfrak{N}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}}\end{pmatrix}}\begin{pmatrix}\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}+\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}}-\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}},\\ \mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}},\mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}}\end{pmatrix} \tag{2}$$

$$\boldsymbol{\mathcal{M}}_{mn-1}\otimes\boldsymbol{\mathcal{M}}_{mn-2}=\cup_{\begin{pmatrix}\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}},\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}}\in\mathfrak{W}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\\ \mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}},\mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}}\in\mathfrak{A}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\\ \mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}},\mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}}\in\mathfrak{N}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}}\end{pmatrix}}\begin{pmatrix}\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}},\\ \mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}+\mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}}-\mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}},\\ \mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}+\mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}}-\mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-2}}\end{pmatrix} \tag{3}$$

$$\Xi_{SC}\boldsymbol{\mathcal{M}}_{mn-1}=\cup_{\begin{pmatrix}\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\in\mathfrak{W}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\\ \mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\in\mathfrak{A}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\\ \mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\in\mathfrak{N}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}}\end{pmatrix}}\left(\left(1-\left(1-\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\right)^{\Xi_{SC}}\right),\mathfrak{A}^{1\,\Xi_{SC}}_{\boldsymbol{\mathcal{M}}_{mn-1}},\mathfrak{N}^{1\,\Xi_{SC}}_{\boldsymbol{\mathcal{M}}_{mn-1}}\right) \tag{4}$$

$$\boldsymbol{\mathcal{M}}^{\Xi_{SC}}_{mn-1}=\cup_{\begin{pmatrix}\mathfrak{W}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\in\mathfrak{W}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\\ \mathfrak{A}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\in\mathfrak{A}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\\ \mathfrak{N}^{1}_{\boldsymbol{\mathcal{M}}_{mn-1}}\in\mathfrak{N}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}}\end{pmatrix}}\left(,\mathfrak{W}^{1\,\Xi_{SC}}_{\boldsymbol{\mathcal{M}}_{mn-1}},\left(1-\left(1-\mathfrak{A}_{\boldsymbol{\mathcal{M}}_{mn-1}}\right)^{1\,\Xi_{SC}}\right),\left(1-\left(1-\mathfrak{N}_{\boldsymbol{\mathcal{M}}_{mn-1}}\right)^{1\,\Xi_{SC}}\right)\right) \tag{5}$$

Moreover, by using any MVNN $\boldsymbol{\mathcal{M}}_{mn-\jmath}=\left(\mathfrak{W}^{i}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\mathfrak{A}^{j}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}},\mathfrak{N}^{k}_{\boldsymbol{\mathcal{M}}_{mn-\jmath}}\right),\jmath=1$, we specify the principle of score and accuracy function, such that

$$\boldsymbol{\mathcal{S}}^{sv}\left(\boldsymbol{\mathcal{M}}_{mn-1}\right)=\frac{1}{3}\left(\sum_{i=1}^{\mathfrak{r}}\mathfrak{W}^{i}_{\boldsymbol{\mathcal{M}}_{mn-1}}-\sum_{j=1}^{\mathfrak{s}}\mathfrak{A}^{j}_{\boldsymbol{\mathcal{M}}_{mn-1}}-\sum_{k=1}^{\mathfrak{t}}\mathfrak{N}^{k}_{\boldsymbol{\mathcal{M}}_{mn-1}}\right) \tag{6}$$

$$\boldsymbol{\mathcal{H}}^{av}\left(\boldsymbol{\mathcal{M}}_{mn-1}\right)=\frac{1}{3}\left(\sum_{i=1}^{\mathfrak{r}}\mathfrak{W}^{i}_{\boldsymbol{\mathcal{M}}_{mn-1}}+\sum_{j=1}^{\mathfrak{s}}\mathfrak{A}^{j}_{\boldsymbol{\mathcal{M}}_{mn-1}}+\sum_{k=1}^{\mathfrak{t}}\mathfrak{N}^{k}_{\boldsymbol{\mathcal{M}}_{mn-1}}\right) \tag{7}$$

To determine relationships among any number of attributes, we define the ordered relations which are stated below:

1. When $\boldsymbol{\mathcal{S}}^{sv}\left(\boldsymbol{\mathcal{M}}_{mn-1}\right)>\boldsymbol{\mathcal{S}}^{sv}\left(\boldsymbol{\mathcal{M}}_{mn-2}\right)$, then $\boldsymbol{\mathcal{M}}_{mn-1}>\boldsymbol{\mathcal{M}}_{mn-2}$;
2. When $\boldsymbol{\mathcal{S}}^{sv}\left(\boldsymbol{\mathcal{M}}_{mn-1}\right)<\boldsymbol{\mathcal{S}}^{sv}\left(\boldsymbol{\mathcal{M}}_{mn-2}\right)$, then $\boldsymbol{\mathcal{M}}_{mn-1}<\boldsymbol{\mathcal{M}}_{mn-2}$;
3. When $\boldsymbol{\mathcal{S}}^{sv}\left(\boldsymbol{\mathcal{M}}_{mn-1}\right)=\boldsymbol{\mathcal{S}}^{sv}\left(\boldsymbol{\mathcal{M}}_{mn-2}\right)$, then

1) When $\mathcal{H}^{av}(\mathcal{M}_{mn-1}) > \mathcal{H}^{av}(\mathcal{M}_{mn-2})$, then $\mathcal{M}_{mn-1} > \mathcal{M}_{mn-2}$;
2) When $\mathcal{H}^{av}(\mathcal{M}_{mn-1}) < \mathcal{H}^{av}(\mathcal{M}_{mn-2})$, then $\mathcal{M}_{mn-1} < \mathcal{M}_{mn-2}$;
3) When $\mathcal{H}^{av}(\mathcal{M}_{mn-1}) = \mathcal{H}^{av}(\mathcal{M}_{mn-2})$, then $\mathcal{M}_{mn-1} = \mathcal{M}_{mn-2}$.

***Definition 2:* [42]** For a ULS $\mathfrak{L}_{ul} = \left[\mathfrak{L}_{\alpha(\mathfrak{x}_{el})}, \mathfrak{L}_{\beta(\mathfrak{x}_{el})}\right]$, where $\mathfrak{L}_{\alpha(\mathfrak{x}_{el})}, \mathfrak{L}_{\beta(\mathfrak{x}_{el})} \in \ddot{\mathfrak{L}}_{ul} = \{\mathfrak{L}_{\alpha} : \alpha \in R\}$, the upper and lower boundary is called LTS. For any two ULSs $\mathfrak{L}_{ul-1} = \left[\mathfrak{L}_{\alpha_1}, \mathfrak{L}_{\beta_1}\right]$ and $\mathfrak{L}_{ul-2} = \left[\mathfrak{L}_{\alpha_2}, \mathfrak{L}_{\beta_2}\right]$, then

$$\mathfrak{L}_{ul-1} \oplus \mathfrak{L}_{ul-2} = \left[\mathfrak{L}_{\alpha_1}, \mathfrak{L}_{\beta_1}\right] \oplus \left[\mathfrak{L}_{\alpha_2}, \mathfrak{L}_{\beta_2}\right] = \left[\mathfrak{L}_{\alpha_1+\alpha_2}, \mathfrak{L}_{\beta_1+\beta_2}\right] \tag{8}$$

$$\mathfrak{L}_{ul-1} \otimes \mathfrak{L}_{ul-2} = \left[\mathfrak{L}_{\alpha_1}, \mathfrak{L}_{\beta_1}\right] \otimes \left[\mathfrak{L}_{\alpha_2}, \mathfrak{L}_{\beta_2}\right] = \left[\mathfrak{L}_{\alpha_1*\alpha_2}, \mathfrak{L}_{\beta_1*\beta_2}\right] \tag{9}$$

$$\Xi_{SC}\mathfrak{L}_{ul-1} = \Xi_{SC}\left[\mathfrak{L}_{\alpha_1}, \mathfrak{L}_{\beta_1}\right] = \left[\mathfrak{L}_{\Xi_{SC}*\alpha_1}, \mathfrak{L}_{\Xi_{SC}*\beta_1}\right] \tag{10}$$

$$\mathfrak{L}_{ul-1}^{\Xi_{SC}} = \left[\mathfrak{L}_{\alpha_1}, \mathfrak{L}_{\beta_1}\right]^{\Xi_{SC}} = \left[\mathfrak{L}_{\alpha_1^{\Xi_{SC}}}, \mathfrak{L}_{\beta_1^{\Xi_{SC}}}\right] \tag{11}$$

Numerous scholars have utilized different sorts of t-norm and t-conorm, but the DTN and DTVN offer a flexible arrangement with modifiable factors. On the other hand, DTN and DTVN also perform a significant role in conveying the magnitude level of options and characteristics. Further, the DTN and DTVN are illustrated in the shape of (12) and (13), such that

***Definition 3:* [48]** For any two-real numbers $g$ and $h$ with $\varrho \geq 0$, the DTN and DTVN are specified by:

$$D(g,h) = \frac{1}{1 + \left(\left(\frac{1-g}{g}\right)^{\varrho} + \left(\frac{1-h}{h}\right)^{\varrho}\right)^{\frac{1}{\varrho}}} \tag{12}$$

$$D'(g,h) = 1 - \frac{1}{1 + \left(\left(\frac{g}{1-g}\right)^{\varrho} + \left(\frac{h}{1-h}\right)^{\varrho}\right)^{\frac{1}{\varrho}}} \tag{13}$$

***Definition 4:* [48]** Let $\mathcal{M}_{mn-j}, j = 1, 2, \ldots, z$ be a set of positive numbers. Then, the normalized weighted Bonferroni mean (NWBM) operator is specified by:

$$NWBM(\mathcal{M}_{mn-1}, \mathcal{M}_{mn-2}, \ldots, \mathcal{M}_{mn-z}) = \left(\sum_{\substack{j,k=1 \\ j,\neq k}}^{z} \frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}} \left(\mathcal{M}_{mn-j}\mathcal{M}_{mn-k}\right)\right)^{\frac{1}{\mathbb{p}+\mathbb{q}}} \tag{14}$$

where $\Omega^{w_j}$ expresses the weight vector with a restriction such that $\sum_{j=1}^{z}\Omega^{w_j} = 1, \Omega^{w_j} \in [0,1]$ with $\mathbb{p}, \mathbb{q} \geq 0$. Furthermore, we present the novel idea of MVCNULSs and their algebraic and Dombi laws.

## 3 Multi-Valued Complex Neutrosophic Uncertain Linguistic Sets

The principle of FSs has been modified, but the principle of NSs has received more attention due to its powerful structure that includes the TG, AG, and FG which fulfill a need in real-world problems but fail in the face of variations at specific times. Consequently, to manage these issues, we combine the principle of MVNULSs and CFSs to develop the MVCNULS. The MVCNULS contains the TG, AG, FG, and uncertain linguistic terms in the form of complex numbers whose real and unreal parts are limited to the unit interval.

***Definition 5:*** A MVCNULS $\mathcal{M}_{mn}$ is specified by:

$$\mathcal{M}_{mn}=\left\{\left(\left[\mathfrak{L}_{\alpha(\mathfrak{X}_{el})},\mathfrak{L}_{\beta(\mathfrak{X}_{el})}\right],\left(\mathfrak{W}^{i}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}),\mathfrak{A}^{j}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}),\mathfrak{N}^{k}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el})\right)\right),i,j,k=1,2,\ldots,\mathfrak{r},\mathfrak{s},\mathfrak{t}\colon \mathfrak{X}_{el}\in\mathfrak{X}_{uni}\right\} \tag{15}$$

where $\mathfrak{W}^{i}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}) = \mathfrak{W}^{i}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{W}^{i}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)} = \left\{\mathfrak{W}^{1}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{W}^{1}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)},\mathfrak{W}^{2}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{W}^{2}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)},\ldots,\mathfrak{W}^{\mathfrak{r}}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{W}^{\mathfrak{r}}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)}\right\}$, $\mathfrak{A}^{j}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}) = \mathfrak{A}^{j}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{A}^{j}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)} = \left\{\mathfrak{A}^{1}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{A}^{1}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)},\mathfrak{A}^{2}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{A}^{2}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)},\ldots,\mathfrak{A}^{\mathfrak{s}}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{A}^{\mathfrak{s}}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)}\right\}$ and $\mathfrak{N}^{k}_{\mathcal{M}_{mn}}(\mathfrak{X}_{el}) = \mathfrak{N}^{k}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{N}^{k}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)} = \left\{\mathfrak{N}^{1}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{N}^{1}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)},\mathfrak{N}^{2}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{N}^{2}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)},\ldots,\mathfrak{N}^{\mathfrak{t}}_{\mathcal{M}_{RP}}(\mathfrak{X}_{el})\,e^{i2\pi\left(\mathfrak{N}^{\mathfrak{t}}_{\mathcal{M}_{IP}}(\mathfrak{X}_{el})\right)}\right\}$ with a restriction such that $0 \le \sup\left(\mathfrak{W}^{i}_{\mathcal{M}_{mn}}\right) + \sup\left(\mathfrak{A}^{j}_{\mathcal{M}_{mn}}\right) + \sup\left(\mathfrak{N}^{k}_{\mathcal{M}_{mn}}\right) \le 3$, where each $\mathfrak{W}^{i}_{\mathcal{M}_{mn}},\mathfrak{A}^{j}_{\mathcal{M}_{mn}},\mathfrak{N}^{k}_{\mathcal{M}_{mn}} \in [0,1]$ and $\left[\mathfrak{L}_{\alpha(\mathfrak{X}_{el})},\mathfrak{L}_{\beta(\mathfrak{X}_{el})}\right]$, where $\mathfrak{L}_{\alpha(\mathfrak{X}_{el})},\mathfrak{L}_{\beta(\mathfrak{X}_{el})} \in \ddot{\mathfrak{L}}_{ul} = \{\mathfrak{L}_{\alpha} \colon \alpha \in R\}$ expresses the ULS. Additionally, the MVCNULNs are shown by $\mathcal{M}_{mn-\jmath} = \left(\left[\mathfrak{L}_{\alpha_{\jmath}},\mathfrak{L}_{\beta_{\jmath}}\right]\left(\mathfrak{W}^{i}_{\mathcal{M}_{RP-\jmath}}e^{i2\pi\left(\mathfrak{W}^{i}_{\mathcal{M}_{IP-\jmath}}\right)},\mathfrak{A}^{j}_{\mathcal{M}_{RP-\jmath}}e^{i2\pi\left(\mathfrak{A}^{j}_{\mathcal{M}_{IP-\jmath}}\right)},\mathfrak{N}^{k}_{\mathcal{M}_{RP-\jmath}}e^{i2\pi\left(\mathfrak{N}^{k}_{\mathcal{M}_{IP-\jmath}}\right)}\right)\right), \jmath = 1,2,\ldots,z.$

For any two MVCNULNs $\mathcal{M}_{mn-\jmath} = \left(\left[\mathfrak{L}_{\alpha_{\jmath}},\mathfrak{L}_{\beta_{\jmath}}\right]\left(\mathfrak{W}^{i}_{\mathcal{M}_{RP-\jmath}}e^{i2\pi\left(\mathfrak{W}^{i}_{\mathcal{M}_{IP-\jmath}}\right)},\mathfrak{A}^{j}_{\mathcal{M}_{RP-\jmath}}e^{i2\pi\left(\mathfrak{A}^{j}_{\mathcal{M}_{IP-\jmath}}\right)},\mathfrak{N}^{k}_{\mathcal{M}_{RP-\jmath}}e^{i2\pi\left(\mathfrak{N}^{k}_{\mathcal{M}_{IP-\jmath}}\right)}\right)\right), \jmath = 1,2,$ we can specify some algebraic laws, such that

$$\mathcal{M}_{mn-1} \oplus \mathcal{M}_{mn-2} = \left( \bigcup_{\substack{\mathfrak{W}^1_{\mathcal{M}_{RP-1}}, \mathfrak{W}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{RP-1}}, \mathfrak{A}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{RP-1}}, \mathfrak{N}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{W}^1_{\mathcal{M}_{IP-1}}, \mathfrak{W}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{IP-1}}, \mathfrak{A}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{IP-1}}, \mathfrak{N}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{IP-j}}}} \begin{matrix} \left[\mathfrak{L}_{\alpha_1+\alpha_2}, \mathfrak{L}_{\beta_1+\beta_2}\right], \\ \begin{pmatrix} \left(\mathfrak{W}^1_{\mathcal{M}_{RP-1}} + \mathfrak{W}^1_{\mathcal{M}_{RP-2}} - \mathfrak{W}^1_{\mathcal{M}_{RP-1}} \mathfrak{W}^1_{\mathcal{M}_{RP-2}}\right) \\ e^{i2\pi\left(\mathfrak{W}^1_{\mathcal{M}_{IP-1}} + \mathfrak{W}^1_{\mathcal{M}_{IP-2}} - \mathfrak{W}^1_{\mathcal{M}_{IP-1}} \mathfrak{W}^1_{\mathcal{M}_{IP-2}}\right)}, \\ \mathfrak{A}^1_{\mathcal{M}_{RP-1}} \mathfrak{A}^1_{\mathcal{M}_{RP-2}} e^{i2\pi\left(\mathfrak{A}^1_{\mathcal{M}_{IP-1}} \mathfrak{A}^1_{\mathcal{M}_{IP-2}}\right)}, \\ \mathfrak{N}^1_{\mathcal{M}_{RP-1}} \mathfrak{N}^1_{\mathcal{M}_{RP-2}} e^{i2\pi\left(\mathfrak{N}^1_{\mathcal{M}_{IP-1}} \mathfrak{N}^1_{\mathcal{M}_{IP-2}}\right)} \end{pmatrix} \end{matrix} \right) \quad (16)$$

$$\mathcal{M}_{mn-1} \otimes \mathcal{M}_{mn-2} = \left( \bigcup_{\substack{\mathfrak{W}^1_{\mathcal{M}_{RP-1}}, \mathfrak{W}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{RP-1}}, \mathfrak{A}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{RP-1}}, \mathfrak{N}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{W}^1_{\mathcal{M}_{IP-1}}, \mathfrak{W}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{IP-1}}, \mathfrak{A}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{IP-1}}, \mathfrak{N}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{IP-j}}}} \begin{matrix} \left[\mathfrak{L}_{\alpha_1\alpha_2}, \mathfrak{L}_{\beta_1\beta_2}\right], \\ \begin{pmatrix} \mathfrak{W}^1_{\mathcal{M}_{RP-1}} \mathfrak{W}^1_{\mathcal{M}_{RP-2}} e^{i2\pi\left(\mathfrak{W}^1_{\mathcal{M}_{IP-1}} \mathfrak{W}^1_{\mathcal{M}_{IP-2}}\right)}, \\ \left(\mathfrak{A}^1_{\mathcal{M}_{RP-1}} + \mathfrak{A}^1_{\mathcal{M}_{RP-2}} - \mathfrak{A}^1_{\mathcal{M}_{RP-1}} \mathfrak{A}^1_{\mathcal{M}_{RP-2}}\right) \\ e^{i2\pi\left(\mathfrak{A}^1_{\mathcal{M}_{IP-1}} + \mathfrak{A}^1_{\mathcal{M}_{IP-2}} - \mathfrak{A}^1_{\mathcal{M}_{IP-1}} \mathfrak{A}^1_{\mathcal{M}_{IP-2}}\right)}, \\ \left(\mathfrak{N}^1_{\mathcal{M}_{RP-1}} + \mathfrak{N}^1_{\mathcal{M}_{RP-2}} - \mathfrak{N}^1_{\mathcal{M}_{RP-1}} \mathfrak{N}^1_{\mathcal{M}_{RP-2}}\right) \\ e^{i2\pi\left(\mathfrak{N}^1_{\mathcal{M}_{IP-1}} + \mathfrak{N}^1_{\mathcal{M}_{IP-2}} - \mathfrak{N}^1_{\mathcal{M}_{IP-1}} \mathfrak{N}^1_{\mathcal{M}_{IP-2}}\right)} \end{pmatrix} \end{matrix} \right) \quad (17)$$

$$\Xi_{SC}\mathcal{M}_{mn-1} = \left( \bigcup_{\substack{\mathfrak{W}^1_{\mathcal{M}_{RP-1}} \in \mathfrak{W}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{RP-1}} \in \mathfrak{A}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{RP-1}} \in \mathfrak{N}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{W}^1_{\mathcal{M}_{IP-1}} \in \mathfrak{W}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{IP-1}} \in \mathfrak{A}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{IP-1}} \in \mathfrak{N}^i_{\mathcal{M}_{IP-j}}}} \begin{matrix} \left[\mathfrak{L}_{\Xi_{SC}*\alpha_1}, \mathfrak{L}_{\Xi_{SC}*\beta_1}\right], \\ \begin{pmatrix} \left(1 - \left(1 - \mathfrak{W}^1_{\mathcal{M}_{RP-1}}\right)^{\Xi_{SC}}\right) e^{i2\pi\left(\left(1-\left(1-\mathfrak{W}^1_{\mathcal{M}_{IP-1}}\right)^{\Xi_{SC}}\right)\right)}, \\ \mathfrak{A}^{1\,\Xi_{SC}}_{\mathcal{M}_{RP-1}} e^{i2\pi\left(\mathfrak{A}^{1\,\Xi_{SC}}_{\mathcal{M}_{IP-1}}\right)}, \\ \mathfrak{N}^{1\,\Xi_{SC}}_{\mathcal{M}_{RP-1}} e^{i2\pi\left(\mathfrak{N}^{1\,\Xi_{SC}}_{\mathcal{M}_{IP-1}}\right)} \end{pmatrix} \end{matrix} \right) \quad (18)$$

$$\mathcal{M}_{mn-1}^{\Xi_{SC}} = \left( \cup_{\begin{pmatrix} \mathfrak{W}^1_{\mathcal{M}_{RP-1}} \in \mathfrak{W}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{RP-1}} \in \mathfrak{A}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{RP-1}} \in \mathfrak{N}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{W}^1_{\mathcal{M}_{IP-1}} \in \mathfrak{W}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{IP-1}} \in \mathfrak{A}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{IP-1}} \in \mathfrak{N}^i_{\mathcal{M}_{IP-j}} \end{pmatrix}} \begin{matrix} \left[ \mathfrak{L}_{\alpha_1^{\Xi_{SC}}}, \mathfrak{L}_{\beta_1^{\Xi_{SC}}} \right], \\ \begin{pmatrix} \mathfrak{W}^{1\,\Xi_{SC}}_{\mathcal{M}_{RP-1}} e^{i2\pi\left(\mathfrak{W}^{1\,\Xi_{SC}}_{\mathcal{M}_{IP-1}}\right)}, \\ \left(1-\left(1-\mathfrak{A}^1_{\mathcal{M}_{RP-1}}\right)^{\Xi_{SC}}\right) e^{i2\pi\left(\left(1-\left(1-\mathfrak{A}^1_{\mathcal{M}_{IP-1}}\right)^{\Xi_{SC}}\right)\right)}, \\ \left(1-\left(1-\mathfrak{N}^1_{\mathcal{M}_{RP-1}}\right)^{\Xi_{SC}}\right) e^{i2\pi\left(\left(1-\left(1-\mathfrak{N}^1_{\mathcal{M}_{IP-1}}\right)^{\Xi_{SC}}\right)\right)} \end{pmatrix} \end{matrix} \right) \tag{19}$$

Moreover, by using any MVCNULN $\mathcal{M}_{mn-j} = \left(\left[\mathfrak{L}_{\alpha_j}, \mathfrak{L}_{\beta_j}\right] \left(\mathfrak{W}^i_{\mathcal{M}_{RP-j}} e^{i2\pi\left(\mathfrak{W}^i_{\mathcal{M}_{IP-j}}\right)}, \mathfrak{A}^j_{\mathcal{M}_{RP-j}} e^{i2\pi\left(\mathfrak{A}^j_{\mathcal{M}_{IP-j}}\right)}, \mathfrak{N}^k_{\mathcal{M}_{RP-j}} e^{i2\pi\left(\mathfrak{N}^k_{\mathcal{M}_{IP-j}}\right)}\right)\right), j = 1$, we can specify score and accuracy function, such that

$$\mathcal{S}^{sv}\left(\mathcal{M}_{mn-1}\right) = \frac{\left(\alpha_j + \beta_j\right)}{O\left(\ddot{\mathfrak{L}}_{ul}\right)} * \frac{1}{6} \begin{pmatrix} \sum_{i=1}^{\mathfrak{r}} \mathfrak{W}^i_{\mathcal{M}_{RP-1}} + \sum_{i=1}^{\mathfrak{r}} \mathfrak{W}^i_{\mathcal{M}_{IP-1}} - \sum_{j=1}^{\mathfrak{s}} \mathfrak{A}^j_{\mathcal{M}_{RP-1}} \\ - \sum_{j=1}^{\mathfrak{s}} \mathfrak{A}^j_{\mathcal{M}_{IP-1}} - \sum_{k=1}^{\mathfrak{t}} \mathfrak{N}^k_{\mathcal{M}_{RP-1}} - \sum_{k=1}^{\mathfrak{t}} \mathfrak{N}^k_{\mathcal{M}_{IP-1}} \end{pmatrix} \tag{20}$$

$$\mathcal{H}^{av}\left(\mathcal{M}_{mn-1}\right) = \frac{\left(\alpha_j + \beta_j\right)}{O\left(\ddot{\mathfrak{L}}_{ul}\right)} * \frac{1}{6} \begin{pmatrix} \sum_{i=1}^{\mathfrak{r}} \mathfrak{W}^i_{\mathcal{M}_{RP-1}} + \sum_{i=1}^{\mathfrak{r}} \mathfrak{W}^i_{\mathcal{M}_{IP-1}} + \sum_{j=1}^{\mathfrak{s}} \mathfrak{A}^j_{\mathcal{M}_{RP-1}} \\ + \sum_{j=1}^{\mathfrak{s}} \mathfrak{A}^j_{\mathcal{M}_{IP-1}} + \sum_{k=1}^{\mathfrak{t}} \mathfrak{N}^k_{\mathcal{M}_{RP-1}} + \sum_{k=1}^{\mathfrak{t}} \mathfrak{N}^k_{\mathcal{M}_{IP-1}} \end{pmatrix} \tag{21}$$

where $O\left(\ddot{\mathfrak{L}}_{ul}\right)$ expresses the total number of linguistic sets in the sets concerned. To determine the relationship among any number of attributes, we define the ordered relations stated below:

1. When $\mathcal{S}^{sv}\left(\mathcal{M}_{mn-1}\right) > \mathcal{S}^{sv}\left(\mathcal{M}_{mn-2}\right)$, then $\mathcal{M}_{mn-1} > \mathcal{M}_{mn-2}$;
2. When $\mathcal{S}^{sv}\left(\mathcal{M}_{mn-1}\right) < \mathcal{S}^{sv}\left(\mathcal{M}_{mn-2}\right)$, then $\mathcal{M}_{mn-1} < \mathcal{M}_{mn-2}$;
3. When $\mathcal{S}^{sv}\left(\mathcal{M}_{mn-1}\right) = \mathcal{S}^{sv}\left(\mathcal{M}_{mn-2}\right)$, then

1) When $\mathcal{H}^{av}\left(\mathcal{M}_{mn-1}\right) > \mathcal{H}^{av}\left(\mathcal{M}_{mn-2}\right)$, then $\mathcal{M}_{mn-1} > \mathcal{M}_{mn-2}$;
2) When $\mathcal{H}^{av}\left(\mathcal{M}_{mn-1}\right) < \mathcal{H}^{av}\left(\mathcal{M}_{mn-2}\right)$, then $\mathcal{M}_{mn-1} < \mathcal{M}_{mn-2}$;
3) When $\mathcal{H}^{av}\left(\mathcal{M}_{mn-1}\right) = \mathcal{H}^{av}\left(\mathcal{M}_{mn-2}\right)$, then $\mathcal{M}_{mn-1} = \mathcal{M}_{mn-2}$.

## 4 Dombi Normalized Weighted Bonferroni Mean Operators Based on MVCNULSs

When aggregating any two MVCNULNs, the BM operators are more flexible than other operators. Different operators have been utilized in the environment of fuzzy sets and their generalizations. The goal of this study is to elaborate some important Dombi laws. Additionally, BM operators offer a flexible arrangement with modifiable factors because of Bonferroni's general structure. On the other hand, BM aggregation operators perform a significant role in conveying the magnitude level of options and characteristics. To determine the relationship among any number of attributes, we present the MVCNULDNWBM operator and discuss its important properties in some special cases.

***Definition 6:*** For any two MVCNULNs $\boldsymbol{\mathcal{M}}_{mn-\jmath} = \left(\left[\mathfrak{L}_{\alpha_\jmath}, \mathfrak{L}_{\beta_\jmath}\right]\left(\mathfrak{W}^{i}_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{W}^{i}_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}, \mathfrak{A}^{j}_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{A}^{j}_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}, \mathfrak{N}^{k}_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{N}^{k}_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}\right)\right)$, $\jmath = 1,2$, with $\rho_{SC} \geq 0$ we can specify some Dombi laws, such that

$$\boldsymbol{\mathcal{M}}_{mn-1} \oplus \boldsymbol{\mathcal{M}}_{mn-2}$$

$$= \left( \begin{array}{c} \left[\mathfrak{L}_{\alpha_1+\alpha_2}, \mathfrak{L}_{\beta_1+\beta_2}\right], \\ \bigcup_{\begin{pmatrix} \mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}, \mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{RP-2}} \in \mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{RP-\jmath}}, \\ \mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}, \mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{RP-2}} \in \mathfrak{A}^i_{\boldsymbol{\mathcal{M}}_{RP-\jmath}}, \\ \mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}, \mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{RP-2}} \in \mathfrak{N}^i_{\boldsymbol{\mathcal{M}}_{RP-\jmath}}, \\ \mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}, \mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{IP-2}} \in \mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}, \\ \mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}, \mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{IP-2}} \in \mathfrak{A}^i_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}, \\ \mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}, \mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{IP-2}} \in \mathfrak{N}^i_{\boldsymbol{\mathcal{M}}_{IP-\jmath}} \end{pmatrix}} \left( \begin{array}{c} \left(1 - \frac{1}{1+\left(\left(\frac{\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}{1-\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}\right)^{\rho_{SC}} + \left(\frac{\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{RP-2}}}{1-\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{RP-2}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right) e^{i2\pi\left(1-\frac{1}{1+\left(\left(\frac{\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}{1-\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}\right)^{\rho_{SC}} + \left(\frac{\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{IP-2}}}{1-\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{IP-2}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right)}, \\ \left(\frac{1}{1+\left(\left(\frac{1-\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}{\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}\right)^{\rho_{SC}} + \left(\frac{1-\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{RP-2}}}{\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{RP-2}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right) e^{i2\pi\left(\frac{1}{1+\left(\left(\frac{1-\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}{\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}\right)^{\rho_{SC}} + \left(\frac{1-\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{IP-2}}}{\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{IP-2}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right)}, \\ \left(\frac{1}{1+\left(\left(\frac{1-\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}{\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}\right)^{\rho_{SC}} + \left(\frac{1-\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{RP-2}}}{\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{RP-2}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right) e^{i2\pi\left(\frac{1}{1+\left(\left(\frac{1-\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}{\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}\right)^{\rho_{SC}} + \left(\frac{1-\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{IP-2}}}{\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{IP-2}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right)} \end{array} \right) \end{array} \right) \tag{22}$$

$$
\begin{aligned}
&\mathcal{M}_{mn-1} \otimes \mathcal{M}_{mn-2} \\
&= \left( \bigcup_{\begin{pmatrix} \mathfrak{W}^1_{\mathcal{M}_{RP-1}}, \mathfrak{W}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{RP-1}}, \mathfrak{A}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{RP-1}}, \mathfrak{N}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{W}^1_{\mathcal{M}_{IP-1}}, \mathfrak{W}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{IP-1}}, \mathfrak{A}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{IP-1}}, \mathfrak{N}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{IP-j}} \end{pmatrix}}
\begin{pmatrix} \left[\mathfrak{L}_{\alpha_1\alpha_2}, \mathfrak{L}_{\beta_1\beta_2}\right], \\
\left( \begin{array}{c}
\left( \dfrac{1}{1+\left( \left( \dfrac{1-\mathfrak{W}^1_{\mathcal{M}_{RP-1}}}{\mathfrak{W}^1_{\mathcal{M}_{RP-1}}} \right)^{\rho SC} + \left( \dfrac{1-\mathfrak{W}^1_{\mathcal{M}_{RP-2}}}{\mathfrak{W}^1_{\mathcal{M}_{RP-2}}} \right)^{\rho SC} \right)^{\frac{1}{\rho SC}}} \right) \\
e^{i2\pi \left( \dfrac{1}{1+\left( \left( \dfrac{1-\mathfrak{W}^1_{\mathcal{M}_{IP-1}}}{\mathfrak{W}^1_{\mathcal{M}_{IP-1}}} \right)^{\rho SC} + \left( \dfrac{1-\mathfrak{W}^1_{\mathcal{M}_{IP-2}}}{\mathfrak{W}^1_{\mathcal{M}_{IP-2}}} \right)^{\rho SC} \right)^{\frac{1}{\rho SC}}} \right)}, \\
\left( 1 - \dfrac{1}{1+\left( \left( \dfrac{\mathfrak{A}^1_{\mathcal{M}_{RP-1}}}{1-\mathfrak{A}^1_{\mathcal{M}_{RP-1}}} \right)^{\rho SC} + \left( \dfrac{\mathfrak{A}^1_{\mathcal{M}_{RP-2}}}{1-\mathfrak{A}^1_{\mathcal{M}_{RP-2}}} \right)^{\rho SC} \right)^{\frac{1}{\rho SC}}} \right) \\
e^{i2\pi \left( 1 - \dfrac{1}{1+\left( \left( \dfrac{\mathfrak{A}^1_{\mathcal{M}_{IP-1}}}{1-\mathfrak{A}^1_{\mathcal{M}_{IP-1}}} \right)^{\rho SC} + \left( \dfrac{\mathfrak{A}^1_{\mathcal{M}_{IP-2}}}{1-\mathfrak{A}^1_{\mathcal{M}_{IP-2}}} \right)^{\rho SC} \right)^{\frac{1}{\rho SC}}} \right)}, \\
\left( 1 - \dfrac{1}{1+\left( \left( \dfrac{\mathfrak{N}^1_{\mathcal{M}_{RP-1}}}{1-\mathfrak{N}^1_{\mathcal{M}_{RP-1}}} \right)^{\rho SC} + \left( \dfrac{\mathfrak{N}^1_{\mathcal{M}_{RP-2}}}{1-\mathfrak{N}^1_{\mathcal{M}_{RP-2}}} \right)^{\rho SC} \right)^{\frac{1}{\rho SC}}} \right) \\
e^{i2\pi \left( 1 - \dfrac{1}{1+\left( \left( \dfrac{\mathfrak{N}^1_{\mathcal{M}_{IP-1}}}{1-\mathfrak{N}^1_{\mathcal{M}_{IP-1}}} \right)^{\rho SC} + \left( \dfrac{\mathfrak{N}^1_{\mathcal{M}_{IP-2}}}{1-\mathfrak{N}^1_{\mathcal{M}_{IP-2}}} \right)^{\rho SC} \right)^{\frac{1}{\rho SC}}} \right)}
\end{array} \right)
\end{pmatrix} \right)
\end{aligned} \tag{23}
$$

$$\Xi_{SC}\boldsymbol{\mathcal{M}}_{mn-1} = \left( \cup_{\begin{pmatrix} \mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{RP-1}} \in \mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{RP-j}}, \\ \mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{RP-1}} \in \mathfrak{A}^i_{\boldsymbol{\mathcal{M}}_{RP-j}}, \\ \mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{RP-1}} \in \mathfrak{N}^i_{\boldsymbol{\mathcal{M}}_{RP-j}}, \\ \mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{IP-1}} \in \mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{IP-j}}, \\ \mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{IP-1}} \in \mathfrak{A}^i_{\boldsymbol{\mathcal{M}}_{IP-j}}, \\ \mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{IP-1}} \in \mathfrak{N}^i_{\boldsymbol{\mathcal{M}}_{IP-j}} \end{pmatrix}} \begin{pmatrix} \left[\mathfrak{L}_{\Xi_{SC}*\alpha_1}, \mathfrak{L}_{\Xi_{SC}*\beta_1}\right], \\ \left( \left(1 - \frac{1}{1+\left(\Xi_{SC}\left(\frac{\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}{1-\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right) e^{i2\pi\left(1-\frac{1}{1+\left(\Xi_{SC}\left(\frac{\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}{1-\mathfrak{W}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right)}, \\ \left(\frac{1}{1+\left(\Xi_{SC}\left(\frac{1-\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}{\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right) e^{i2\pi\left(\frac{1}{1+\left(\Xi_{SC}\left(\frac{1-\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}{\mathfrak{A}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right)}, \\ \left(\frac{1}{1+\left(\Xi_{SC}\left(\frac{1-\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}{\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{RP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right) e^{i2\pi\left(\frac{1}{1+\left(\Xi_{SC}\left(\frac{1-\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}{\mathfrak{N}^1_{\boldsymbol{\mathcal{M}}_{IP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}\right)} \right) \end{pmatrix} \right) \tag{24}$$

$$\mathcal{M}_{mn-1}^{\Xi_{SC}} = \left( \begin{array}{c} \left[\mathfrak{L}_{\alpha_1^{\Xi_{SC}}}, \mathfrak{L}_{\beta_1^{\Xi_{SC}}}\right], \\ \bigcup_{\left(\begin{array}{c} \mathfrak{W}^1_{\mathcal{M}_{RP-1}} \in \mathfrak{W}^i_{\mathcal{M}_{RP-\jmath}}, \\ \mathfrak{A}^1_{\mathcal{M}_{RP-1}} \in \mathfrak{A}^i_{\mathcal{M}_{RP-\jmath}}, \\ \mathfrak{N}^1_{\mathcal{M}_{RP-1}} \in \mathfrak{N}^i_{\mathcal{M}_{RP-\jmath}}, \\ \mathfrak{W}^1_{\mathcal{M}_{IP-1}} \in \mathfrak{W}^i_{\mathcal{M}_{IP-\jmath}}, \\ \mathfrak{A}^1_{\mathcal{M}_{IP-1}} \in \mathfrak{A}^i_{\mathcal{M}_{IP-\jmath}}, \\ \mathfrak{N}^1_{\mathcal{M}_{IP-1}} \in \mathfrak{N}^i_{\mathcal{M}_{IP-\jmath}} \end{array}\right)} \left( \begin{array}{c} \left( \frac{1}{1+\left(\Xi_{SC}\left(\frac{1-\mathfrak{W}^1_{\mathcal{M}_{RP-1}}}{\mathfrak{W}^1_{\mathcal{M}_{RP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}} \right) e^{i2\pi\left( \frac{1}{1+\left(\Xi_{SC}\left(\frac{1-\mathfrak{W}^1_{\mathcal{M}_{IP-1}}}{\mathfrak{W}^1_{\mathcal{M}_{IP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}} \right)}, \\ \left( 1-\frac{1}{1+\left(\Xi_{SC}\left(\frac{\mathfrak{A}^1_{\mathcal{M}_{RP-1}}}{1-\mathfrak{A}^1_{\mathcal{M}_{RP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}} \right) e^{i2\pi\left( 1-\frac{1}{1+\left(\Xi_{SC}\left(\frac{\mathfrak{A}^1_{\mathcal{M}_{IP-1}}}{1-\mathfrak{A}^1_{\mathcal{M}_{IP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}} \right)}, \\ \left( 1-\frac{1}{1+\left(\Xi_{SC}\left(\frac{\mathfrak{N}^1_{\mathcal{M}_{RP-1}}}{1-\mathfrak{N}^1_{\mathcal{M}_{RP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}} \right) e^{i2\pi\left( 1-\frac{1}{1+\left(\Xi_{SC}\left(\frac{\mathfrak{N}^1_{\mathcal{M}_{IP-1}}}{1-\mathfrak{N}^1_{\mathcal{M}_{IP-1}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}} \right)} \end{array} \right) \end{array} \right) \tag{25}$$

***Theorem 1:*** For any two MVCNULNs $\mathcal{M}_{mn-\jmath} = \left(\left[\mathfrak{L}_{\alpha_\jmath}, \mathfrak{L}_{\beta_\jmath}\right]\left(\mathfrak{W}^i_{\mathcal{M}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{W}^i_{\mathcal{M}_{IP-\jmath}}\right)}, \mathfrak{A}^j_{\mathcal{M}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{A}^j_{\mathcal{M}_{IP-\jmath}}\right)}, \mathfrak{N}^k_{\mathcal{M}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{N}^k_{\mathcal{M}_{IP-\jmath}}\right)}\right)\right), \jmath = 1, 2,$ with $\rho_{SC} \geq 0,$ then

(1) $\mathcal{M}_{mn-1} \oplus \mathcal{M}_{mn-2} = \mathcal{M}_{mn-2} \oplus \mathcal{M}_{mn-1}$;
(2) $\mathcal{M}_{mn-1} \otimes \mathcal{M}_{mn-2} = \mathcal{M}_{mn-2} \otimes \mathcal{M}_{mn-1}$;
(3) $\Xi_{SC}\left(\mathcal{M}_{mn-1} \oplus \mathcal{M}_{mn-2}\right) = \Xi_{SC}\mathcal{M}_{mn-1} \oplus \Xi_{SC}\mathcal{M}_{mn-2}$;
(4) $\Xi_{SC-1}\mathcal{M}_{mn-1} \oplus \Xi_{SC-2}\mathcal{M}_{mn-1} = \left(\Xi_{SC-1} + \Xi_{SC-2}\right)\mathcal{M}_{mn-1}$;
(5) $\mathcal{M}_{mn-1}^{\Xi_{SC-1}} \otimes \mathcal{M}_{mn-1}^{\Xi_{SC-2}} = \mathcal{M}_{mn-1}^{\left(\Xi_{SC-1}+\Xi_{SC-2}\right)}$;
(6) $\mathcal{M}_{mn-1}^{\Xi_{SC}} \otimes \mathcal{M}_{mn-2}^{\Xi_{SC}} = \left(\mathcal{M}_{mn-1} \otimes \mathcal{M}_{mn-2}\right)^{\Xi_{SC}}$

***Proof:*** Straightforward.

***Definition 7:*** For any family of MVCNULNs $\mathcal{M}_{mn-\jmath} = \left(\left[\mathfrak{L}_{\alpha_\jmath}, \mathfrak{L}_{\beta_\jmath}\right]\left(\mathfrak{W}^{i}_{\mathcal{M}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{W}^{i}_{\mathcal{M}_{IP-\jmath}}\right)}, \mathfrak{A}^{j}_{\mathcal{M}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{A}^{j}_{\mathcal{M}_{IP-\jmath}}\right)}, \mathfrak{N}^{k}_{\mathcal{M}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{N}^{k}_{\mathcal{M}_{IP-\jmath}}\right)}\right)\right), \jmath = 1, 2, \ldots, z$, with $\rho_{SC} \geq 0$, then the MVNULD-NWBM operator is specified by:

$$MVCNULDNWBM\left(\mathcal{M}_{mn-1}, \mathcal{M}_{mn-2}, \ldots, \mathcal{M}_{mn-z}\right)$$

$$= \left(\oplus^{z}_{\substack{\jmath, \Bbbk = 1 \\ \jmath, \neq \Bbbk}} \frac{\Omega^{w_\jmath}\Omega^{w_\Bbbk}}{1 - \Omega^{w_\jmath}}\left(\mathcal{M}_{mn-\jmath} \otimes \mathcal{M}_{mn-\Bbbk}\right)\right)^{\frac{1}{\mathbb{p}+\mathbb{q}}} \tag{26}$$

where $\Omega^{w_\jmath}$ expresses the weight vector with a restriction such that $\sum_{\jmath=1}^{z}\Omega^{w_\jmath} = 1, \Omega^{w_\jmath} \in [0, 1]$ with $\mathbb{p}, \mathbb{q} \geq 0$.

***Theorem 2:*** For any family of MVCNULNs $\mathcal{M}_{mn-\jmath} = \left(\left[\mathfrak{L}_{\alpha_\jmath}, \mathfrak{L}_{\beta_\jmath}\right]\left(\mathfrak{W}^{i}_{\mathcal{M}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{W}^{i}_{\mathcal{M}_{IP-\jmath}}\right)}, \mathfrak{A}^{j}_{\mathcal{M}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{A}^{j}_{\mathcal{M}_{IP-\jmath}}\right)}, \mathfrak{N}^{k}_{\mathcal{M}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{N}^{k}_{\mathcal{M}_{IP-\jmath}}\right)}\right)\right), \jmath = 1, 2, \ldots, z$, with $\rho_{SC} \geq 0$, then by using Eq. (26), we obtain the following result, such that

$$MVCNULDNWBM\left(\mathcal{M}_{mn-1}, \mathcal{M}_{mn-2}, \ldots, \mathcal{M}_{mn-z}\right)$$

$$= \left(\oplus^{z}_{\substack{\jmath, \Bbbk = 1 \\ \jmath, \neq \Bbbk}} \frac{\Omega^{w_\jmath}\Omega^{w_\Bbbk}}{1 - \Omega^{w_\jmath}}\left(\mathcal{M}_{mn-\jmath} \otimes \mathcal{M}_{mn-\Bbbk}\right)\right)^{\frac{1}{\mathbb{p}+\mathbb{q}}}$$

$$= \left(\textit{liguistic terms}, \ \textit{truthgrade}, \ \textit{abstinence grade}, \ \textit{falsity grade}\right) \tag{27}$$

where *linguistic terms* $= \left[\mathfrak{L}_{\left(\oplus^{z}_{\substack{j,k=1\\ j,\neq k}}\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\left(\alpha_j^{\mathbb{p}}{}''*\alpha_k^{\mathbb{q}}\right)\right)^{\frac{1}{\mathbb{p}+\mathbb{q}}}}, \mathfrak{L}_{\left(\oplus^{z}_{\substack{j,k=1\\ j,\neq k}}\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\left(\beta_j^{\mathbb{p}}{}''*\beta_k^{\mathbb{q}}\right)\right)^{\frac{1}{\mathbb{p}+\mathbb{q}}}}\right]$, where

$\begin{pmatrix} \mathfrak{W}^1_{\mathcal{M}_{RP-1}}, \mathfrak{W}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{RP-1}}, \mathfrak{A}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{RP-1}}, \mathfrak{N}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{W}^1_{\mathcal{M}_{IP-1}}, \mathfrak{W}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{IP-1}}, \mathfrak{A}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{IP-1}}, \mathfrak{N}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{IP-j}} \end{pmatrix}$, then

*real part of truth grade*

$$= \left(1 \left| 1 + \left(\frac{1}{+}\left(\frac{1-\left(1-\frac{1}{1+\left(\sum^{z}_{\substack{j,k=1\\ j,\neq k}}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\left(\frac{1-\mathfrak{W}^1_{\mathcal{M}_{RP-j}}}{\mathfrak{W}^1_{\mathcal{M}_{RP-j}}}\right)^{\rho_{SC}}+\left(\frac{1-\mathfrak{W}^1_{\mathcal{M}_{RP-k}}}{\mathfrak{W}^1_{\mathcal{M}_{RP-k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}\right)}{1-\frac{1}{1+\left(\sum^{z}_{\substack{j,k=1\\ j,\neq k}}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\left(\frac{1-\mathfrak{W}^1_{\mathcal{M}_{RP-j}}}{\mathfrak{W}^1_{\mathcal{M}_{RP-j}}}\right)^{\rho_{SC}}+\left(\frac{1-\mathfrak{W}^1_{\mathcal{M}_{RP-k}}}{\mathfrak{W}^1_{\mathcal{M}_{RP-k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right),$$

*imaginary part of truth grade*

$$= e^{i2\pi\left(1\Bigg|1+\left(\frac{1}{\mathbb{p}+\mathbb{q}}\left(\frac{1-\left(1-\frac{1}{\left(1+\left(\sum_{\jmath,\,\kappa=1,\ \jmath\neq\kappa}^{z}\left(\frac{\Omega^{w_\jmath}\Omega^{w_\kappa}}{1-\Omega^{w_\jmath}}\cdot\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{IP-\jmath}}{\mathfrak{W}^{1}\mathcal{M}_{IP-\jmath}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{IP-\kappa}}{\mathfrak{W}^{1}\mathcal{M}_{IP-\kappa}}\right)^{\rho_{SC}}}\right)\right)\right)^{\rho_{SC}}}\right)}{1-\frac{1}{\left(1+\left(\sum_{\jmath,\,\kappa=1,\ \jmath\neq\kappa}^{z}\left(\frac{\Omega^{w_\jmath}\Omega^{w_\kappa}}{1-\Omega^{w_\jmath}}\cdot\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{IP-\jmath}}{\mathfrak{W}^{1}\mathcal{M}_{IP-\jmath}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{IP-\kappa}}{\mathfrak{W}^{1}\mathcal{M}_{IP-\kappa}}\right)^{\rho_{SC}}}\right)\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right)},$$

*real part of abstinence grade*

$$=\left(1-1\Bigg|1+\left(\frac{1}{\mathbb{p}+\mathbb{q}}\left(\frac{\left(\frac{1}{\left(1+\left(\sum_{\jmath,\,\kappa=1,\ \jmath\neq\kappa}^{z}\left(\frac{\Omega^{w_\jmath}\Omega^{w_\kappa}}{1-\Omega^{w_\jmath}}\cdot\frac{1}{\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{RP-\jmath}}{\mathfrak{A}^{1}\mathcal{M}_{RP-\jmath}}\right)^{\rho_{SC}}+\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{RP-\kappa}}{\mathfrak{A}^{1}\mathcal{M}_{RP-\kappa}}\right)^{\rho_{SC}}}\right)\right)\right)^{\rho_{SC}}}\right)}{1-\frac{1}{\left(1+\left(\sum_{\jmath,\,\kappa=1,\ \jmath\neq\kappa}^{z}\left(\frac{\Omega^{w_\jmath}\Omega^{w_\kappa}}{1-\Omega^{w_\jmath}}\cdot\frac{1}{\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{RP-\jmath}}{\mathfrak{A}^{1}\mathcal{M}_{RP-\jmath}}\right)^{\rho_{SC}}+\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{RP-\kappa}}{\mathfrak{A}^{1}\mathcal{M}_{RP-\kappa}}\right)^{\rho_{SC}}}\right)\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right)$$

*imaginary part of abstinence*

$$= e^{i2\pi\left(1-1\left|1+\left(\frac{1}{+}\left(\left(\frac{\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\left(\frac{1-\mathfrak{A}^1\mathcal{M}_{IP-j}}{\mathfrak{A}^1\mathcal{M}_{IP-j}}\right)^{\rho_{SC}}+\left(\frac{1-\mathfrak{A}^1\mathcal{M}_{IP-k}}{\mathfrak{A}^1\mathcal{M}_{IP-k}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}{1-\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\left(\frac{1-\mathfrak{A}^1\mathcal{M}_{IP-j}}{\mathfrak{A}^1\mathcal{M}_{IP-j}}\right)^{\rho_{SC}}+\left(\frac{1-\mathfrak{A}^1\mathcal{M}_{IP-k}}{\mathfrak{A}^1\mathcal{M}_{IP-k}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right.\right)}$$

*real part of falsity grade*

$$= \left(1-1\left|1+\left(\frac{1}{\mathbb{p}+\mathbb{q}}\left(\left(\frac{\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\left(\frac{1-\mathfrak{N}^1\mathcal{M}_{RP-j}}{\mathfrak{N}^1\mathcal{M}_{RP-j}}\right)^{\rho_{SC}}+\left(\frac{1-\mathfrak{N}^1\mathcal{M}_{RP-k}}{\mathfrak{N}^1\mathcal{M}_{RP-k}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}{1-\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\left(\frac{1-\mathfrak{N}^1\mathcal{M}_{RP-j}}{\mathfrak{N}^1\mathcal{M}_{RP-j}}\right)^{\rho_{SC}}+\left(\frac{1-\mathfrak{N}^1\mathcal{M}_{RP-k}}{\mathfrak{N}^1\mathcal{M}_{RP-k}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right.\right)$$

*imaginary part of falsity grade*

$$
= e^{i2\pi\left(1-1\left|1+\left(\frac{1}{+}\left(\left(\frac{\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}\right)^{\rho_{SC}}+\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}{1-\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}\right)^{\rho_{SC}}+\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right)}
$$

***Proof:*** See the Appendix section.

By choosing the value of weight vectors $\Omega^{w} = \left(\frac{1}{z}, \frac{1}{z}, \ldots, \frac{1}{z}\right)$, then the principle of the MVCNULDNWBM operator is converted for the multi-valued complex neutrosophic uncertain linguistic Dombi BM operator, which is discussed below:

$$
MVCNULDNWBM\left(\mathcal{M}_{mn-1}, \mathcal{M}_{mn-2}, \ldots, \mathcal{M}_{mn-z}\right)
$$

$$
= \left(\oplus_{\substack{j,k=1\\ j,\neq k}}^{z} \frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\left(\mathcal{M}_{mn-j}\otimes\mathcal{M}_{mn-k}\right)\right)^{\frac{1}{\mathrm{p}+\mathrm{q}}} = \left(\oplus_{\substack{j,k=1\\ j,\neq k}}^{z} \frac{\frac{1}{z}\frac{1}{z}}{1-\frac{1}{z}}\left(\mathcal{M}_{mn-j}\otimes\mathcal{M}_{mn-k}\right)\right)^{\frac{1}{\mathrm{p}+\mathrm{q}}}
$$

$$
= \left(\oplus_{\substack{j,k=1\\ j,\neq k}}^{z} \frac{\frac{1}{z^2}}{1-\frac{1}{z}}\left(\mathcal{M}_{mn-j}\otimes\mathcal{M}_{mn-k}\right)\right)^{\frac{1}{\mathrm{p}+\mathrm{q}}} = \left(\frac{1}{z\left(z-1\right)}\oplus_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\mathcal{M}_{mn-j}\otimes\mathcal{M}_{mn-k}\right)\right)^{\frac{1}{\mathrm{p}+\mathrm{q}}}
$$

$$
= MVCNULDBM\left(\mathcal{M}_{mn-1}, \mathcal{M}_{mn-2}, \ldots, \mathcal{M}_{mn-z}\right)
$$

***Property 1:*** For any family of MVCNULNs $\boldsymbol{\mathcal{M}}_{mn-\jmath} = \left(\left[\mathfrak{L}_{\alpha_\jmath}, \mathfrak{L}_{\beta_\jmath}\right]\left(\mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}, \mathfrak{A}^j_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{A}^j_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}, \mathfrak{N}^k_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{N}^k_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}\right)\right), \jmath = 1, 2, \ldots, z$, with $\rho_{SC} \geq 0$, if $\boldsymbol{\mathcal{M}}_{mn-\jmath} = \boldsymbol{\mathcal{M}}_{mn}$ then

$$MVCNULDNWBM\left(\boldsymbol{\mathcal{M}}_{mn-1}, \boldsymbol{\mathcal{M}}_{mn-2}, \ldots, \boldsymbol{\mathcal{M}}_{mn-z}\right) = \boldsymbol{\mathcal{M}}_{mn} \tag{28}$$

***Proof:*** See the Appendix section.

***Property 2:*** For any family of MVCNULNs $\boldsymbol{\mathcal{M}}_{mn-\jmath} = \left(\left[\mathfrak{L}_{\alpha_\jmath}, \mathfrak{L}_{\beta_\jmath}\right]\left(\mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}, \mathfrak{A}^j_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{A}^j_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}, \mathfrak{N}^k_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{N}^k_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}\right)\right), \jmath = 1, 2, \ldots, z$, with $\rho_{SC} \geq 0$, then

$$\min_{\jmath}\left\{\boldsymbol{\mathcal{M}}_{mn-\jmath}\right\} \leq MVCNULDNWBM\left(\boldsymbol{\mathcal{M}}_{mn-1}, \boldsymbol{\mathcal{M}}_{mn-2}, \ldots, \boldsymbol{\mathcal{M}}_{mn-z}\right) \leq \max_{\jmath}\left\{\boldsymbol{\mathcal{M}}_{mn-\jmath}\right\} \tag{29}$$

***Proof:*** See the Appendix section.

Based on the ideas proposed above, we will develop the multi-attribute decision-making technique to determine the reliability and consistency of the elaborated operators.

## 5 MADM Method Based on Proposed MVCNULSs

Aggregation operators, measures, and methods have been used in the environments of IFSs, PFSs, IVIFSs, IVPFSs, CIFSs, CIVIFSs, NSs, CNSs, and applied in the MADM technique to determine the consistency and strength of current works. But to date, no one has proposed the application of any kind of operator to MVCNULSs in a MADM-based model. The goal of this manuscript is to utilize DNWBM operators based on MVCNULSs to determine the strength of the elaborated work. For this, we chose a group of alternatives $\boldsymbol{\mathcal{F}}_{Al-1}, \boldsymbol{\mathcal{F}}_{Al-2}, \ldots, \boldsymbol{\mathcal{F}}_{Al-n}$ and their attributes $\check{\boldsymbol{\mathcal{F}}}_{At-1}, \check{\boldsymbol{\mathcal{F}}}_{At-2}, \ldots, \check{\boldsymbol{\mathcal{F}}}_{At-m}$ concerning weight vector $\Omega = \left\{\Omega_1, \Omega_2, \ldots, \Omega_{le}\right\}$ with a rule that is $\sum_{j=1}^{le} \Omega_j = 1$. To evaluate the above issues, we constructed a decision matrix whose items are in the form of complex interval-valued Pythagorean fuzzy numbers such that $\boldsymbol{\mathcal{F}}_{cp-\jmath} = \left(\left[\mathfrak{L}_{\alpha_\jmath}, \mathfrak{L}_{\beta_\jmath}\right], \left(\mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}, \mathfrak{A}^j_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{A}^j_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}, \mathfrak{N}^k_{\boldsymbol{\mathcal{M}}_{RP-\jmath}} e^{i2\pi\left(\mathfrak{N}^k_{\boldsymbol{\mathcal{M}}_{IP-\jmath}}\right)}\right)\right)$, $\jmath = 1, 2, \ldots, z$ with certain rules that are $0 \leq \sup\left(\mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{mn}}\right) + \sup\left(\mathfrak{A}^j_{\boldsymbol{\mathcal{M}}_{mn}}\right) + \sup\left(\mathfrak{N}^k_{\boldsymbol{\mathcal{M}}_{mn}}\right) \leq 3$, where each $\mathfrak{W}^i_{\boldsymbol{\mathcal{M}}_{mn}}, \mathfrak{A}^j_{\boldsymbol{\mathcal{M}}_{mn}}, \mathfrak{N}^k_{\boldsymbol{\mathcal{M}}_{mn}} \in [0,1]$ and $\left[\mathfrak{L}_{\alpha(\mathfrak{x}_{el})}, \mathfrak{L}_{\beta(\mathfrak{x}_{el})}\right]$, and where $\mathfrak{L}_{\alpha(\mathfrak{x}_{el})}, \mathfrak{L}_{\beta(\mathfrak{x}_{el})} \in \ddot{\mathfrak{L}}_{ul} = \{\mathfrak{L}_\alpha : \alpha \in R\}$ expresses the ULS. Then by using the above family of $n$ alternatives and $m$ attributes, we developed an algorithm whose steps are as follows:

***Step 1:*** Develop the decision matrix, whose every item is in the form of MVCNULNs.

***Step 2:*** Use the MVCNULDNWBM operator to aggregate the entries of the decision matrix.

***Step 3:*** By using the score function, we find the Score values of the aggregated values of ***Step 2***.

***Step 4:*** Rank all alternatives and examine the best one.

As shown above, we illustrate certain numerical examples to determine the consistency and validity of the elaborated operators.

***Example 1:*** With the rapid advance of financial globalization, and the developing climate of competition, the rivalry between ventures has become a contest between supply chains. The variety of items entering the market is expanding, and the life cycles of new items are becoming shorter. The instability of the market and other elements drives the search for viable inventory networks, and partnerships with different ventures are essential to improve focus and resist external risks. The critical measure to accomplish this objective is provider choice. Hence, the provider choice issue has acquired a great deal of importance, whether in respect of inventory network or the executive's decision. To delineate our proposed technique in this article, we use MVCNULNs to give a mathematical guide to choosing green providers in a green inventory network. For this, we choose a family of five possible green suppliers in green supplier chain management $\mathcal{F}_{Al-1}, \mathcal{F}_{Al-2}, \mathcal{F}_{Al-3}, \mathcal{F}_{Al-4}, \mathcal{F}_{Al-5}$ and their attributes are in the form of selection factors whose expressions are as follows:

$\mathcal{F}_{At-1}$: Expresses the product quality factor.

$\mathcal{F}_{At-2}$: Expresses the environmental factor.

$\mathcal{F}_{At-3}$: Expresses the delivery factor.

$\mathcal{F}_{At-4}$: Expresses the price factor.

To resolve the above problem, we choose a family of four weight vectors, $0.3, 0.2, 0.3, 0.2$. Then, by using the above family of $n$ alternatives and $m$ attributes, we develop an algorithm whose steps are as follows:

***Step 1:*** We develop the decision matrix, whose every item is expressed in the form of MVCNULNs in Table 1.

***Step 2:*** The MVCNULDNWBM operators used to aggregate the entries of the decision matrix are discussed below:

$$\mathcal{F}_{Al-1} = MVCNULDNWBM\left(\mathcal{F}_{At-1}, \mathcal{F}_{At-2}, \mathcal{F}_{At-3}, \mathcal{F}_{At-4}\right)$$

$$= \left(\left(\begin{array}{c} [\mathfrak{L}_{0.8}, \mathfrak{L}_{1.6}], \\ \left\{0.572e^{i2\pi(0.332)}, 0.652e^{i2\pi(0.412)}\right\}, \\ \left\{0.412e^{i2\pi(0.332)}, 0.252e^{i2\pi(0.732)}\right\}, \\ \left\{0.412e^{i2\pi(0.252)}, 0.492e^{i2\pi(0.332)}, 0.572e^{i2\pi(0.412)}\right\} \end{array}\right)\right)$$

$$\mathcal{F}_{Al-2} = MVCNULDNWBM\left(\mathcal{F}_{At-1}, \mathcal{F}_{At-2}, \mathcal{F}_{At-3}, \mathcal{F}_{At-4}\right)$$

$$= \left(\left(\begin{array}{c} [\mathfrak{L}_{1.6}, \mathfrak{L}_{2.4}], \\ \left\{0.492e^{i2\pi(0.252)}, 0.572e^{i2\pi(0.332)}\right\}, \\ \left\{0.332e^{i2\pi(0.252)}, 0.172e^{i2\pi(0.652)}\right\}, \\ \left\{0.332e^{i2\pi(0.172)}, 0.412e^{i2\pi(0.252)}, 0.492e^{i2\pi(0.332)}\right\} \end{array}\right)\right)$$

**Table 1:** The original decision matrix that covers the complex interval-valued Pythagorean fuzzy numbers

| | $\mathcal{F}_{At-1}$ | $\mathcal{F}_{At-2}$ |
|---|---|---|
| $\mathcal{F}_{Al-1}$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_2],\left\{0.7e^{i2\pi(0.4)},0.8e^{i2\pi(0.5)}\right\},\left\{0.5e^{i2\pi(0.4)},0.3e^{i2\pi(0.9)}\right\},\left\{0.5e^{i2\pi(0.3)},0.6e^{i2\pi(0.4)},0.7e^{i2\pi(0.5)}\right\}\right)\right)$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_2],\left\{0.71e^{i2\pi(0.41)},0.81e^{i2\pi(0.51)}\right\},\left\{0.51e^{i2\pi(0.41)},0.31e^{i2\pi(0.91)}\right\},\left\{0.51e^{i2\pi(0.31)},0.61e^{i2\pi(0.41)},0.71e^{i2\pi(0.51)}\right\}\right)\right)$ |
| $\mathcal{F}_{Al-2}$ | $\left(\left([\mathfrak{L}_2,\mathfrak{L}_3],\left\{0.6e^{i2\pi(0.3)},0.7e^{i2\pi(0.4)}\right\},\left\{0.4e^{i2\pi(0.3)},0.2e^{i2\pi(0.8)}\right\},\left\{0.4e^{i2\pi(0.2)},0.5e^{i2\pi(0.3)},0.6e^{i2\pi(0.4)}\right\}\right)\right)$ | $\left(\left([\mathfrak{L}_2,\mathfrak{L}_3],\left\{0.61e^{i2\pi(0.31)},0.71e^{i2\pi(0.41)}\right\},\left\{0.41e^{i2\pi(0.31)},0.21e^{i2\pi(0.81)}\right\},\left\{0.41e^{i2\pi(0.21)},0.51e^{i2\pi(0.31)},0.61e^{i2\pi(0.41)}\right\}\right)\right)$ |
| $\mathcal{F}_{Al-3}$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_3],\left\{0.8e^{i2\pi(0.5)},0.9e^{i2\pi(0.6)}\right\},\left\{0.6e^{i2\pi(0.5)},0.4e^{i2\pi(0.8)}\right\},\left\{0.6e^{i2\pi(0.4)},0.7e^{i2\pi(0.5)},0.8e^{i2\pi(0.6)}\right\}\right)\right)$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_3],\left\{0.81e^{i2\pi(0.51)},0.91e^{i2\pi(0.61)}\right\},\left\{0.61e^{i2\pi(0.51)},0.41e^{i2\pi(0.81)}\right\},\left\{0.61e^{i2\pi(0.41)},0.71e^{i2\pi(0.51)},0.81e^{i2\pi(0.61)}\right\}\right)\right)$ |
| $\mathcal{F}_{Al-4}$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_2],\left\{0.6e^{i2\pi(0.3)},0.7e^{i2\pi(0.4)}\right\},\left\{0.5e^{i2\pi(0.4)},0.3e^{i2\pi(0.9)}\right\},\left\{0.6e^{i2\pi(0.4)},0.7e^{i2\pi(0.5)},0.8e^{i2\pi(0.6)}\right\}\right)\right)$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_2],\left\{0.61e^{i2\pi(0.31)},0.71e^{i2\pi(0.41)}\right\},\left\{0.51e^{i2\pi(0.41)},0.31e^{i2\pi(0.91)}\right\},\left\{0.61e^{i2\pi(0.41)},0.71e^{i2\pi(0.51)},0.81e^{i2\pi(0.61)}\right\}\right)\right)$ |
| $\mathcal{F}_{Al-5}$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_4],\left\{0.5e^{i2\pi(0.4)},0.7e^{i2\pi(0.5)}\right\},\left\{0.6e^{i2\pi(0.7)},0.3e^{i2\pi(0.9)}\right\},\left\{0.7e^{i2\pi(0.3)},0.8e^{i2\pi(0.4)},0.7e^{i2\pi(0.5)}\right\}\right)\right)$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_4],\left\{0.51e^{i2\pi(0.41)},0.71e^{i2\pi(0.51)}\right\},\left\{0.61e^{i2\pi(0.71)},0.31e^{i2\pi(0.91)}\right\},\left\{0.71e^{i2\pi(0.31)},0.81e^{i2\pi(0.41)},0.71e^{i2\pi(0.51)}\right\}\right)\right)$ |
| | $\mathcal{F}_{At-3}$ | $\mathcal{F}_{At-4}$ |
| $\mathcal{F}_{Al-1}$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_2],\left\{0.72e^{i2\pi(0.42)},0.82e^{i2\pi(0.52)}\right\},\left\{0.52e^{i2\pi(0.42)},0.32e^{i2\pi(0.92)}\right\},\left\{0.52e^{i2\pi(0.32)},0.62e^{i2\pi(0.42)},0.72e^{i2\pi(0.52)}\right\}\right)\right)$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_2],\left\{0.73e^{i2\pi(0.43)},0.83e^{i2\pi(0.53)}\right\},\left\{0.53e^{i2\pi(0.43)},0.33e^{i2\pi(0.93)}\right\},\left\{0.53e^{i2\pi(0.33)},0.63e^{i2\pi(0.43)},0.73e^{i2\pi(0.53)}\right\}\right)\right)$ |
| $\mathcal{F}_{Al-2}$ | $\left(\left([\mathfrak{L}_2,\mathfrak{L}_3],\left\{0.62e^{i2\pi(0.32)},0.72e^{i2\pi(0.42)}\right\},\left\{0.42e^{i2\pi(0.32)},0.22e^{i2\pi(0.82)}\right\},\left\{0.42e^{i2\pi(0.22)},0.52e^{i2\pi(0.32)},0.62e^{i2\pi(0.42)}\right\}\right)\right)$ | $\left(\left([\mathfrak{L}_2,\mathfrak{L}_3],\left\{0.63e^{i2\pi(0.33)},0.73e^{i2\pi(0.43)}\right\},\left\{0.43e^{i2\pi(0.33)},0.23e^{i2\pi(0.83)}\right\},\left\{0.43e^{i2\pi(0.23)},0.53e^{i2\pi(0.33)},0.63e^{i2\pi(0.43)}\right\}\right)\right)$ |
| $\mathcal{F}_{Al-3}$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_3],\left\{0.82e^{i2\pi(0.52)},0.92e^{i2\pi(0.62)}\right\},\left\{0.62e^{i2\pi(0.52)},0.42e^{i2\pi(0.82)}\right\},\left\{0.62e^{i2\pi(0.42)},0.72e^{i2\pi(0.52)},0.82e^{i2\pi(0.62)}\right\}\right)\right)$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_3],\left\{0.83e^{i2\pi(0.53)},0.93e^{i2\pi(0.63)}\right\},\left\{0.63e^{i2\pi(0.53)},0.43e^{i2\pi(0.83)}\right\},\left\{0.63e^{i2\pi(0.43)},0.73e^{i2\pi(0.53)},0.83e^{i2\pi(0.63)}\right\}\right)\right)$ |
| $\mathcal{F}_{Al-4}$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_2],\left\{0.62e^{i2\pi(0.32)},0.72e^{i2\pi(0.42)}\right\},\left\{0.52e^{i2\pi(0.42)},0.32e^{i2\pi(0.92)}\right\},\left\{0.62e^{i2\pi(0.42)},0.72e^{i2\pi(0.52)},0.82e^{i2\pi(0.62)}\right\}\right)\right)$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_2],\left\{0.63e^{i2\pi(0.33)},0.73e^{i2\pi(0.43)}\right\},\left\{0.53e^{i2\pi(0.43)},0.33e^{i2\pi(0.93)}\right\},\left\{0.63e^{i2\pi(0.43)},0.73e^{i2\pi(0.53)},0.83e^{i2\pi(0.63)}\right\}\right)\right)$ |
| $\mathcal{F}_{Al-5}$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_4],\left\{0.52e^{i2\pi(0.42)},0.72e^{i2\pi(0.52)}\right\},\left\{0.62e^{i2\pi(0.72)},0.32e^{i2\pi(0.92)}\right\},\left\{0.72e^{i2\pi(0.32)},0.82e^{i2\pi(0.42)},0.72e^{i2\pi(0.52)}\right\}\right)\right)$ | $\left(\left([\mathfrak{L}_1,\mathfrak{L}_4],\left\{0.53e^{i2\pi(0.43)},0.73e^{i2\pi(0.53)}\right\},\left\{0.63e^{i2\pi(0.73)},0.33e^{i2\pi(0.93)}\right\},\left\{0.73e^{i2\pi(0.33)},0.83e^{i2\pi(0.43)},0.73e^{i2\pi(0.53)}\right\}\right)\right)$ |

$$\mathcal{F}_{Al-3} = MVCNULDNWBM\left(\mathcal{F}_{At-1}, \mathcal{F}_{At-2}, \mathcal{F}_{At-3}, \mathcal{F}_{At-4}\right)$$

$$= \left(\left(\begin{array}{c} [\mathfrak{L}_{0.8}, \mathfrak{L}_{2.4}], \\ \left\{0.652e^{i2\pi(0.412)}, 0.732e^{i2\pi(0.492)}\right\}, \\ \left\{0.492e^{i2\pi(0.412)}, 0.332e^{i2\pi(0.652)}\right\}, \\ \left\{0.492e^{i2\pi(0.332)}, 0.572e^{i2\pi(0.412)}, 0.652e^{i2\pi(0.412)}\right\} \end{array}\right)\right)$$

$$\mathcal{F}_{Al-4} = MVCNULDNWBM\left(\mathcal{F}_{At-1}, \mathcal{F}_{At-2}, \mathcal{F}_{At-3}, \mathcal{F}_{At-4}\right)$$

$$= \left(\left(\begin{array}{c} [\mathfrak{L}_{0.8}, \mathfrak{L}_{1.6}], \\ \left\{0.492e^{i2\pi(0.252)}, 0.572e^{i2\pi(0.332)}\right\}, \\ \left\{0.412e^{i2\pi(0.332)}, 0.252e^{i2\pi(0.732)}\right\}, \\ \left\{0.492e^{i2\pi(0.332)}, 0.572e^{i2\pi(0.412)}, 0.652e^{i2\pi(0.492)}\right\} \end{array}\right)\right)$$

$$\mathcal{F}_{Al-5} = MVCNULDNWBM\left(\mathcal{F}_{At-1}, \mathcal{F}_{At-2}, \mathcal{F}_{At-3}, \mathcal{F}_{At-4}\right)$$

$$= \left(\left(\begin{array}{c} [\mathfrak{L}_{0.8}, \mathfrak{L}_{3.2}], \\ \left\{0.412e^{i2\pi(0.332)}, 0.572e^{i2\pi(0.412)}\right\}, \\ \left\{0.492e^{i2\pi(0.572)}, 0.252e^{i2\pi(0.732)}\right\}, \\ \left\{0.572e^{i2\pi(0.252)}, 0.652e^{i2\pi(0.332)}, 0.572e^{i2\pi(0.412)}\right\} \end{array}\right)\right)$$

***Step 3:*** By using the score function, we find the Score values of the aggregated values of ***Step 2***, which are shown below:

$$\mathcal{S}^{sv}\left(\mathcal{F}_{Al-1}\right) = -0.0563, \quad \mathcal{S}^{sv}\left(\mathcal{F}_{Al-2}\right) = -0.0725, \quad \mathcal{S}^{sv}\left(\mathcal{F}_{Al-3}\right) = -0.0808,$$

$$\mathcal{S}^{sv}\left(\mathcal{F}_{Al-4}\right) = -0.0819, \quad \mathcal{S}^{sv}\left(\mathcal{F}_{Al-5}\right) = -0.1454$$

***Step 4:*** By using the above Score values, we rank all alternatives and examine the best one such that

$$\mathcal{S}^{sv}\left(\mathcal{F}_{Al-1}\right) \geq \mathcal{S}^{sv}\left(\mathcal{F}_{Al-2}\right) \geq \mathcal{S}^{sv}\left(\mathcal{F}_{Al-3}\right) \geq \mathcal{S}^{sv}\left(\mathcal{F}_{Al-4}\right) \geq \mathcal{S}^{sv}\left(\mathcal{F}_{Al-5}\right)$$

or,

$$\mathcal{F}_{Al-1} \geq \mathcal{F}_{Al-2} \geq \mathcal{F}_{Al-3} \geq \mathcal{F}_{Al-4} \geq \mathcal{F}_{Al-5}$$

Therefore, from the above analysis, $\mathcal{F}_{Al-1}$ is the best option. Moreover, we show the consistency of the parameters $\mathbb{p}$ and $\mathbb{q}$ by using different values. By using the information in Table 1, the influences of the parameters $\mathbb{p}$ and $\mathbb{q}$ are shown in Table 2, for $\rho_{SC} = 2$.

As shown above, by using different values of the parameters $\mathbb{p}$ and $\mathbb{q}$, the ranking value is still the same, and the best option is still $\mathcal{F}_{Al-1}$. Furthermore, we will determine the consistency of the different values of the parameter $\rho_{SC}$. The influence of $\rho_{SC}$ is shown in Table 3, using the information from Table 1.

The graphical expression of the information in Table 2 is presented in Fig. 2.

**Table 2:** The influences of the parameters $\mathbb{p}, \mathbb{q}$ for $\boldsymbol{\rho_{SC}=2}$

| Parameter | Score value | Ranking value |
|---|---|---|
| $\mathbb{p}=\mathbf{1}, \mathbb{q}=\mathbf{1}$ | $\mathcal{S}^{sv}(\mathcal{F}_{Al-1})=-0.0563, \mathcal{S}^{sv}(\mathcal{F}_{Al-2})=-0.0725,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-3})=-0.0808, \mathcal{S}^{sv}(\mathcal{F}_{Al-4})=-0.0819,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-5})=-0.1454$ | $\mathcal{F}_{Al-1}\geq\mathcal{F}_{Al-2}\geq\mathcal{F}_{Al-3}$ $\geq\mathcal{F}_{Al-4}\geq\mathcal{F}_{Al-5}$ |
| $\mathbb{p}=\mathbf{1}, \mathbb{q}=\mathbf{2}$ | $\mathcal{S}^{sv}(\mathcal{F}_{Al-1})=-0.0452, \mathcal{S}^{sv}(\mathcal{F}_{Al-2})=-0.0614,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-3})=-0.0707, \mathcal{S}^{sv}(\mathcal{F}_{Al-4})=-0.0708,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-5})=-0.1343$ | $\mathcal{F}_{Al-1}\geq\mathcal{F}_{Al-2}\geq\mathcal{F}_{Al-3}$ $\geq\mathcal{F}_{Al-4}\geq\mathcal{F}_{Al-5}$ |
| $\mathbb{p}=\mathbf{2}, \mathbb{q}=\mathbf{3}$ | $\mathcal{S}^{sv}(\mathcal{F}_{Al-1})=-0.0674, \mathcal{S}^{sv}(\mathcal{F}_{Al-2})=-0.0836,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-3})=-0.0919, \mathcal{S}^{sv}(\mathcal{F}_{Al-4})=-0.0921,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-5})=-0.1565$ | $\mathcal{F}_{Al-1}\geq\mathcal{F}_{Al-2}\geq\mathcal{F}_{Al-3}$ $\geq\mathcal{F}_{Al-4}\geq\mathcal{F}_{Al-5}$ |
| $\mathbb{p}=\mathbf{4}, \mathbb{q}=\mathbf{5}$ | $\mathcal{S}^{sv}(\mathcal{F}_{Al-1})=-0.0796, \mathcal{S}^{sv}(\mathcal{F}_{Al-2})=-0.1058,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-3})=-0.1141, \mathcal{S}^{sv}(\mathcal{F}_{Al-4})=-0.1152,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-5})=-0.1787$ | $\mathcal{F}_{Al-1}\geq\mathcal{F}_{Al-2}\geq\mathcal{F}_{Al-3}$ $\geq\mathcal{F}_{Al-4}\geq\mathcal{F}_{Al-5}$ |
| $\mathbb{p}=\mathbf{7}, \mathbb{q}=\mathbf{8}$ | $\mathcal{S}^{sv}(\mathcal{F}_{Al-1})=-0.0897, \mathcal{S}^{sv}(\mathcal{F}_{Al-2})=-0.1169,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-3})=-0.1252, \mathcal{S}^{sv}(\mathcal{F}_{Al-4})=-0.1263,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-5})=-0.1898$ | $\mathcal{F}_{Al-1}\geq\mathcal{F}_{Al-2}\geq\mathcal{F}_{Al-3}$ $\geq\mathcal{F}_{Al-4}\geq\mathcal{F}_{Al-5}$ |
| $\mathbb{p}=\mathbf{9}, \mathbb{q}=\mathbf{10}$ | $\mathcal{S}^{sv}(\mathcal{F}_{Al-1})=-0.1008, \mathcal{S}^{sv}(\mathcal{F}_{Al-2})=-0.1280,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-3})=-0.1363, \mathcal{S}^{sv}(\mathcal{F}_{Al-4})=-0.1374,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-5})=-0.1909$ | $\mathcal{F}_{Al-1}\geq\mathcal{F}_{Al-2}\geq\mathcal{F}_{Al-3}$ $\geq\mathcal{F}_{Al-4}\geq\mathcal{F}_{Al-5}$ |

**Table 3:** The influence of the parameter $\boldsymbol{\rho_{SC}}$ for $\mathbb{p}=\mathbb{q}=\mathbf{2}$

| Parameter | Score value | Ranking value |
|---|---|---|
| $\boldsymbol{\rho_{SC}=2}$ | $\mathcal{S}^{sv}(\mathcal{F}_{Al-1})=-0.0563, \mathcal{S}^{sv}(\mathcal{F}_{Al-2})=-0.0725,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-3})=-0.0808, \mathcal{S}^{sv}(\mathcal{F}_{Al-4})=-0.0819,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-5})=-0.1454$ | $\mathcal{F}_{Al-1}\geq\mathcal{F}_{Al-2}\geq\mathcal{F}_{Al-3}$ $\geq\mathcal{F}_{Al-4}\geq\mathcal{F}_{Al-5}$ |
| $\boldsymbol{\rho_{SC}=3}$ | $\mathcal{S}^{sv}(\mathcal{F}_{Al-1})=-0.0342, \mathcal{S}^{sv}(\mathcal{F}_{Al-2})=-0.0504,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-3})=-0.0607, \mathcal{S}^{sv}(\mathcal{F}_{Al-4})=-0.0608,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-5})=-0.1233$ | $\mathcal{F}_{Al-1}\geq\mathcal{F}_{Al-2}\geq\mathcal{F}_{Al-3}$ $\geq\mathcal{F}_{Al-4}\geq\mathcal{F}_{Al-5}$ |
| $\boldsymbol{\rho_{SC}=4}$ | $\mathcal{S}^{sv}(\mathcal{F}_{Al-1})=-0.0564, \mathcal{S}^{sv}(\mathcal{F}_{Al-2})=-0.0726,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-3})=-0.0809, \mathcal{S}^{sv}(\mathcal{F}_{Al-4})=-0.0811,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-5})=-0.1455$ | $\mathcal{F}_{Al-1}\geq\mathcal{F}_{Al-2}\geq\mathcal{F}_{Al-3}$ $\geq\mathcal{F}_{Al-4}\geq\mathcal{F}_{Al-5}$ |
| $\boldsymbol{\rho_{SC}=5}$ | $\mathcal{S}^{sv}(\mathcal{F}_{Al-1})=-0.0686, \mathcal{S}^{sv}(\mathcal{F}_{Al-2})=-0.1033,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-3})=-0.1031, \mathcal{S}^{sv}(\mathcal{F}_{Al-4})=-0.1032,$ $\mathcal{S}^{sv}(\mathcal{F}_{Al-5})=-0.1677$ | $\mathcal{F}_{Al-1}\geq\mathcal{F}_{Al-2}\geq\mathcal{F}_{Al-3}$ $\geq\mathcal{F}_{Al-4}\geq\mathcal{F}_{Al-5}$ |

(Continued)

**Table 3:** (Continued)

| Parameter | Score value | Ranking value |
|---|---|---|
| $\rho_{SC} = \mathbf{7}$ | $\mathcal{S}^{sv}\left(\mathcal{F}_{Al-1}\right) = -0.0785, \mathcal{S}^{sv}\left(\mathcal{F}_{Al-2}\right) = -0.1158,$ $\mathcal{S}^{sv}\left(\mathcal{F}_{Al-3}\right) = -0.1159, \mathcal{S}^{sv}\left(\mathcal{F}_{Al-4}\right) = -0.1274,$ $\mathcal{S}^{sv}\left(\mathcal{F}_{Al-5}\right) = -0.1687$ | $\mathcal{F}_{Al-1} \geq \mathcal{F}_{Al-2} \geq \mathcal{F}_{Al-3}$ $\geq \mathcal{F}_{Al-4} \geq \mathcal{F}_{Al-5}$ |
| $\rho_{SC} = \mathbf{10}$ | $\mathcal{S}^{sv}\left(\mathcal{F}_{Al-1}\right) = -0.1118, \mathcal{S}^{sv}\left(\mathcal{F}_{Al-2}\right) = -0.1390,$ $\mathcal{S}^{sv}\left(\mathcal{F}_{Al-3}\right) = -0.1473, \mathcal{S}^{sv}\left(\mathcal{F}_{Al-4}\right) = -0.1484,$ $\mathcal{S}^{sv}\left(\mathcal{F}_{Al-5}\right) = -0.1979$ | $\mathcal{F}_{Al-1} \geq \mathcal{F}_{Al-2} \geq \mathcal{F}_{Al-3}$ $\geq \mathcal{F}_{Al-4} \geq \mathcal{F}_{Al-5}$ |

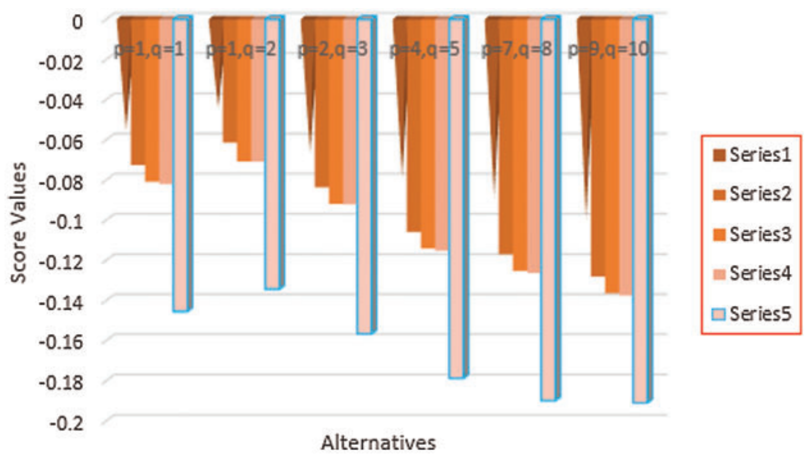


**Figure 2:** Geometrical expressions of the information in Table 2

Similarly, as shown above, by using the same value of parameters $\mathbb{p} = 2$ and $\mathbb{q} = 2$ and changing the value of the parameter $\rho_{SC}$, the ranking value is still the same, and the best option remains $\mathcal{F}_{Al-1}$. The strength and consistency of the new MVCNULDNWBM operators are discussed in the next study with the help of comparative analysis.

### *5.1 Comparative Analysis*

By using the prevailing works in [43,48,49], the sensitive works are diagnosed below:

(1) Power AOs for MVNS was invented by Peng et al. [43], which includes the mixture of power AOs with MVNSs. But a lot of deficiencies exists in [43] under MVNS, because the work in [43] is the particular part of invented works under MVCNULSs. For proposed work is not difficult to handle the data in Peng et al. [43], but the converse is very problematic.

(2) DNWBM for MVNULS was invented by Yang et al. [48], which includes the mixture of BM operators with MVNULS s. But a lot of deficiencies exists in [48] under MVNULS, because the work in [48] is the particular part of invented works under MVCNULSs. For proposed work is not difficult to handle the data in Yang et al. [48], but the converse is very problematic.

(3) Liu et al. [49] stated the BM operators for MVNSs, which discuss the MVNSs and BM works. But a lot of deficiencies exists in [49] under MVNS, because the work in [49] is the particular part of invented works under MVNLSs. For proposed work is not difficult to handle the data in Liu et al. [49], but the converse is very problematic.

A graphical expression of the information in Table 3 is presented in Fig. 3.

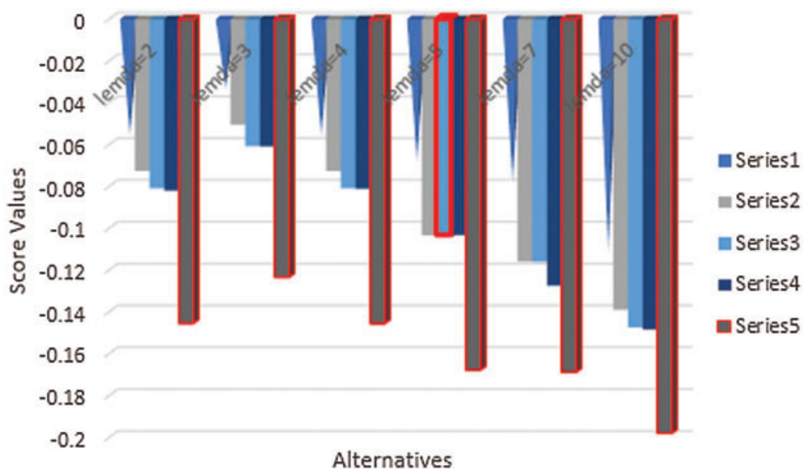


**Figure 3:** Graphical expressions of the information in Table 3

Hence, our considerations works are beneficial for utilizing in the region of medical areas and decision analysis.

## 6 Conclusion

In this study, we developed the principle of multi-valued neutrosophic uncertain linguistic sets and their important Dombi laws were also elaborated by using the investigated multi-valued neutrosophic uncertain linguistic sets. Further, we developed a multi-valued complex neutrosophic uncertain linguistic Dombi-normalized weighted Bonferroni mean operator and discussed important properties of the operator with some specific cases. By using these laws, we deployed the multiple attribute decision making technique under the novel principle of multi-valued neutrosophic uncertain linguistic sets. To determine the strength and flexibility of the elaborated approaches, we resolved some numerical examples based on the proposed operator. Finally, the elaborated work was validated with the help of comparative analysis, a demonstration of its advantages, and geometric expressions.

In the future, we will try to modify the prevailing principle of complex q-rung orthopair fuzzy sets [50], Complex spherical fuzzy sets [51], T-spherical fuzzy sets [52], bipolar soft sets [53], and others [54–57], to generalize and expand the value of this work.

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## Appendix

***Proof of Theorem 2:*** We proved that the Eq. (27), such that if, $\mathcal{M}_{mn-j} \otimes \mathcal{M}_{mn-k}$, then

$$linguistic\ terms = \left[\mathfrak{L}_{(\alpha_j^{\mathbb{P}} " * \alpha_k^{\mathbb{q}})}, \mathfrak{L}_{(\beta_j^{\mathbb{P}} " * \beta_k^{\mathbb{q}})}\right], \text{ where } \begin{pmatrix} \mathfrak{W}^1_{\mathcal{M}_{RP-1}}, \mathfrak{W}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{RP-1}}, \mathfrak{A}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{RP-1}}, \mathfrak{N}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{W}^1_{\mathcal{M}_{IP-1}}, \mathfrak{W}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{IP-1}}, \mathfrak{A}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{IP-1}}, \mathfrak{N}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{IP-j}} \end{pmatrix}, \text{ then}$$

*real part of truth grade*

$$= \left( 1 \left| 1 + \left( \frac{1 - \left( 1 - \frac{1}{1 + \left( \sum_{\substack{j,k=1 \\ j,\neq k}}^{z} \left( \cdot \frac{1}{\mathbb{P}\left(\frac{1-\mathfrak{W}^1_{\mathcal{M}_{RP-j}}}{\mathfrak{W}^1_{\mathcal{M}_{RP-j}}}\right)^{\rho_{SC}} + \mathbb{q}\left(\frac{1-\mathfrak{W}^1_{\mathcal{M}_{RP-k}}}{\mathfrak{W}^1_{\mathcal{M}_{RP-k}}}\right)^{\rho_{SC}}} \right) \right)^{\rho_{SC}}} \right)}{1 - \frac{1}{1 + \left( \sum_{\substack{j,k=1 \\ j,\neq k}}^{z} \left( \cdot \frac{1}{\mathbb{P}\left(\frac{1-\mathfrak{W}^1_{\mathcal{M}_{RP-j}}}{\mathfrak{W}^1_{\mathcal{M}_{RP-j}}}\right)^{\rho_{SC}} + \mathbb{q}\left(\frac{1-\mathfrak{W}^1_{\mathcal{M}_{RP-k}}}{\mathfrak{W}^1_{\mathcal{M}_{RP-k}}}\right)^{\rho_{SC}}} \right) \right)^{\rho_{SC}}}} \right)^{\rho_{SC}\frac{1}{\rho_{SC}}} \right),$$

*imaginary part of truth grade*

$$= e^{i2\pi\left(1\,\Bigg|\,1+\left(\frac{1-\left(1-\frac{1}{\left(1+\left(\sum_{j,k=1,\ j\neq k}^{z}\left(\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{W}^{1}_{\mathcal{M}_{IP-j}}}{\mathfrak{W}^{1}_{\mathcal{M}_{IP-j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{W}^{1}_{\mathcal{M}_{IP-k}}}{\mathfrak{W}^{1}_{\mathcal{M}_{IP-k}}}\right)^{\rho_{SC}}}\right)\right)\right)^{\rho_{SC}}}\right)}{1-\frac{1}{\left(1+\left(\sum_{j,k=1,\ j\neq k}^{z}\left(\cdot\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{W}^{1}_{\mathcal{M}_{IP-j}}}{\mathfrak{W}^{1}_{\mathcal{M}_{IP-j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{W}^{1}_{\mathcal{M}_{IP-k}}}{\mathfrak{W}^{1}_{\mathcal{M}_{IP-k}}}\right)^{\rho_{SC}}}\right)\right)\right)^{\rho_{SC}}}}\right)^{\rho_{SC}\,\frac{1}{\rho_{SC}}}\right)},$$

*real part of abstinence grade*

$$= \left(1-1\,\Bigg|\,1+\left(\left(\frac{\left(\frac{1}{\left(1+\left(\sum_{j,k=1,\ j\neq k}^{z}\left(\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{A}^{1}_{\mathcal{M}_{RP-j}}}{\mathfrak{A}^{1}_{\mathcal{M}_{RP-j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{A}^{1}_{\mathcal{M}_{RP-k}}}{\mathfrak{A}^{1}_{\mathcal{M}_{RP-k}}}\right)^{\rho_{SC}}}\right)\right)\right)^{\rho_{SC}}}\right)}{1-\frac{1}{\left(1+\left(\sum_{j,k=1,\ j\neq k}^{z}\left(\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{A}^{1}_{\mathcal{M}_{RP-j}}}{\mathfrak{A}^{1}_{\mathcal{M}_{RP-j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{A}^{1}_{\mathcal{M}_{RP-k}}}{\mathfrak{A}^{1}_{\mathcal{M}_{RP-k}}}\right)^{\rho_{SC}}}\right)\right)\right)^{\rho_{SC}}}}\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right)$$

*imaginary part of abstinence*

$$= e^{i2\pi\left(1-1\left|1+\left(\left(\left(\frac{\left(\frac{1}{1+\left(\sum_{\mathscr{j},\mathscr{k}=1,\ \mathscr{j}\neq\mathscr{k}}^{z}\left(\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{A}^1\mathcal{M}_{IP-\mathscr{j}}}{\mathfrak{A}^1\mathcal{M}_{IP-\mathscr{j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{A}^1\mathcal{M}_{IP-\mathscr{k}}}{\mathfrak{A}^1\mathcal{M}_{IP-\mathscr{k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}\right)}{1-\frac{1}{1+\left(\sum_{\mathscr{j},\mathscr{k}=1,\ \mathscr{j}\neq\mathscr{k}}^{z}\left(\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{A}^1\mathcal{M}_{IP-\mathscr{j}}}{\mathfrak{A}^1\mathcal{M}_{IP-\mathscr{j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{A}^1\mathcal{M}_{IP-\mathscr{k}}}{\mathfrak{A}^1\mathcal{M}_{IP-\mathscr{k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right)}$$

*real part of falsity grade*

$$= \left(1-1\left|1+\left(\left(\left(\frac{\left(\frac{1}{1+\left(\sum_{\mathscr{j},\mathscr{k}=1,\ \mathscr{j}\neq\mathscr{k}}^{z}\left(\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{N}^1\mathcal{M}_{RP-\mathscr{j}}}{\mathfrak{N}^1\mathcal{M}_{RP-\mathscr{j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{N}^1\mathcal{M}_{RP-\mathscr{k}}}{\mathfrak{N}^1\mathcal{M}_{RP-\mathscr{k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}\right)}{1-\frac{1}{1+\left(\sum_{\mathscr{j},\mathscr{k}=1,\ \mathscr{j}\neq\mathscr{k}}^{z}\left(\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{N}^1\mathcal{M}_{RP-\mathscr{j}}}{\mathfrak{N}^1\mathcal{M}_{RP-\mathscr{j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{N}^1\mathcal{M}_{RP-\mathscr{k}}}{\mathfrak{N}^1\mathcal{M}_{RP-\mathscr{k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right)$$

*imaginary part of falsity grade*

$$= e^{i2\pi\left(1-1\left|1+\left(\left(\frac{\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}{1-\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right)}$$

If, $\left(\oplus_{\substack{j,k=1\\ j,\neq k}}^{z} \frac{\Omega^{w_j}\,\Omega^{w_k}}{1-\Omega^{w_j}}\left(\mathcal{M}_{mn-j}\otimes\mathcal{M}_{mn-k}\right)\right)^{\frac{1}{\mathbb{p}+\mathbb{q}}}$, then

$$linguistic\ \ terms = \left[\mathfrak{L}_{\left(\oplus_{\substack{j,k=1\\ j,\neq k}}^{z}\frac{\Omega^{w_j}\,\Omega^{w_k}}{1-\Omega^{w_j}}\left(\alpha_j^{\mathbb{p}}{}''*\alpha_k^{\mathbb{q}}\right)\right)^{\frac{1}{\mathbb{p}+\mathbb{q}}}}, \mathfrak{L}_{\left(\oplus_{\substack{j,k=1\\ j,\neq k}}^{z}\frac{\Omega^{w_j}\,\Omega^{w_k}}{1-\Omega^{w_j}}\left(\beta_j^{\mathbb{p}}{}''*\beta_k^{\mathbb{q}}\right)\right)^{\frac{1}{\mathbb{p}+\mathbb{q}}}}\right],$$

where

$$\begin{pmatrix} \mathfrak{W}^1_{\mathcal{M}_{RP-1}}, \mathfrak{W}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{RP-1}}, \mathfrak{A}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{RP-1}}, \mathfrak{N}^1_{\mathcal{M}_{RP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{RP-j}}, \\ \mathfrak{W}^1_{\mathcal{M}_{IP-1}}, \mathfrak{W}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{W}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{A}^1_{\mathcal{M}_{IP-1}}, \mathfrak{A}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{A}^i_{\mathcal{M}_{IP-j}}, \\ \mathfrak{N}^1_{\mathcal{M}_{IP-1}}, \mathfrak{N}^1_{\mathcal{M}_{IP-2}} \in \mathfrak{N}^i_{\mathcal{M}_{IP-j}} \end{pmatrix},$$

then

*real part of truth grade*

$$= \left( 1 \middle| 1 + \left( \frac{1}{\mathbb{p}+\mathbb{q}} \left( \frac{1 - \left( 1 - \frac{1}{1+\left( \sum_{\substack{\jmath, \Bbbk=1 \\ \jmath, \neq \Bbbk}}^{z} \left( \frac{\Omega^{w\jmath}\,\Omega^{w\Bbbk}}{1-\Omega^{w\jmath}} \cdot \frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{RP-\jmath}}{\mathfrak{W}^{1}\mathcal{M}_{RP-\jmath}}\right)^{\rho_{SC}} + \mathbb{q}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{RP-\Bbbk}}{\mathfrak{W}^{1}\mathcal{M}_{RP-\Bbbk}}\right)^{\rho_{SC}}} \right) \right)^{\rho_{SC}}} \right)}{1 - \frac{1}{1+\left( \sum_{\substack{\jmath, \Bbbk=1 \\ \jmath, \neq \Bbbk}}^{z} \left( \frac{\Omega^{w\jmath}\,\Omega^{w\Bbbk}}{1-\Omega^{w\jmath}} \cdot \frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{RP-\jmath}}{\mathfrak{W}^{1}\mathcal{M}_{RP-\jmath}}\right)^{\rho_{SC}} + \mathbb{q}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{RP-\Bbbk}}{\mathfrak{W}^{1}\mathcal{M}_{RP-\Bbbk}}\right)^{\rho_{SC}}} \right) \right)^{\rho_{SC}}}} \right) \right)^{\rho_{SC}} \right)^{\frac{1}{\rho_{SC}}},$$

*imaginary part of truth grade*

$$= e^{i2\pi \left( 1 \middle| 1 + \left( \frac{1}{\mathbb{p}+\mathbb{q}} \left( \frac{1 - \left( 1 - \frac{1}{1+\left( \sum_{\substack{\jmath, \Bbbk=1 \\ \jmath, \neq \Bbbk}}^{z} \left( \frac{\Omega^{w\jmath}\,\Omega^{w\Bbbk}}{1-\Omega^{w\jmath}} \cdot \frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{IP-\jmath}}{\mathfrak{W}^{1}\mathcal{M}_{IP-\jmath}}\right)^{\rho_{SC}} + \mathbb{q}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{IP-\Bbbk}}{\mathfrak{W}^{1}\mathcal{M}_{IP-\Bbbk}}\right)^{\rho_{SC}}} \right) \right)^{\rho_{SC}}} \right)}{1 - \frac{1}{1+\left( \sum_{\substack{\jmath, \Bbbk=1 \\ \jmath, \neq \Bbbk}}^{z} \left( \frac{\Omega^{w\jmath}\,\Omega^{w\Bbbk}}{1-\Omega^{w\jmath}} \cdot \frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{IP-\jmath}}{\mathfrak{W}^{1}\mathcal{M}_{IP-\jmath}}\right)^{\rho_{SC}} + \mathbb{q}\left(\frac{1-\mathfrak{W}^{1}\mathcal{M}_{IP-\Bbbk}}{\mathfrak{W}^{1}\mathcal{M}_{IP-\Bbbk}}\right)^{\rho_{SC}}} \right) \right)^{\rho_{SC}}}} \right) \right)^{\rho_{SC}} \right)^{\frac{1}{\rho_{SC}}}},$$

*real part of abstinence grade*

$$= \left(1-1\left|1+\left(\frac{1}{\mathbb{p}+\mathbb{q}}\left(\frac{\left(\frac{1}{1+\left(\sum_{j,k=1,\ j\neq k}^{z}\left(\frac{\Omega^{wj}\Omega^{wk}}{1-\Omega^{wj}}\cdot\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{RP-j}}{\mathfrak{A}^{1}\mathcal{M}_{RP-j}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{RP-k}}{\mathfrak{A}^{1}\mathcal{M}_{RP-k}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}\right)}{1-\frac{1}{1+\left(\sum_{j,k=1,\ j\neq k}^{z}\left(\frac{\Omega^{wj}\Omega^{wk}}{1-\Omega^{wj}}\cdot\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{RP-j}}{\mathfrak{A}^{1}\mathcal{M}_{RP-j}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{RP-k}}{\mathfrak{A}^{1}\mathcal{M}_{RP-k}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right)$$

*imaginary part of abstinence*

$$= e^{i2\pi\left(1-1\left|1+\left(\frac{1}{\mathbb{p}+\mathbb{q}}\left(\frac{\left(\frac{1}{1+\left(\sum_{j,k=1,\ j\neq k}^{z}\left(\frac{\Omega^{wj}\Omega^{wk}}{1-\Omega^{wj}}\cdot\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{IP-j}}{\mathfrak{A}^{1}\mathcal{M}_{IP-j}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{IP-k}}{\mathfrak{A}^{1}\mathcal{M}_{IP-k}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}\right)}{1-\frac{1}{1+\left(\sum_{j,k=1,\ j\neq k}^{z}\left(\frac{\Omega^{wj}\Omega^{wk}}{1-\Omega^{wj}}\cdot\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{IP-j}}{\mathfrak{A}^{1}\mathcal{M}_{IP-j}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{A}^{1}\mathcal{M}_{IP-k}}{\mathfrak{A}^{1}\mathcal{M}_{IP-k}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}\right)}$$

*real part of falsity grade*

$$= \left(1 - 1 \middle| 1 + \left(\frac{1}{\mathbb{p}+\mathbb{q}}\left(\frac{\left(\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{RP-j}}}{\mathfrak{N}^1_{\mathcal{M}_{RP-j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{RP-k}}}{\mathfrak{N}^1_{\mathcal{M}_{RP-k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}\right)}{1-\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{RP-j}}}{\mathfrak{N}^1_{\mathcal{M}_{RP-j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{RP-k}}}{\mathfrak{N}^1_{\mathcal{M}_{RP-k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}$$

*imaginary part of falsity grade*

$$= e^{i2\pi\left(1 - 1 \middle| 1 + \left(\frac{1}{\mathbb{p}+\mathbb{q}}\left(\frac{\left(\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}\right)}{1-\frac{1}{1+\left(\sum_{\substack{j,k=1\\ j,\neq k}}^{z}\left(\frac{\Omega^{w_j}\Omega^{w_k}}{1-\Omega^{w_j}}\cdot\frac{1}{\mathbb{p}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-j}}}\right)^{\rho_{SC}}+\mathbb{q}\left(\frac{1-\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}{\mathfrak{N}^1_{\mathcal{M}_{IP-k}}}\right)^{\rho_{SC}}}\right)\right)^{\rho_{SC}}}}\right)\right)^{\rho_{SC}}\right)^{\frac{1}{\rho_{SC}}}}$$

$$= \left( \oplus_{\substack{j, k = 1 \\ j, \neq k}}^{z} \frac{\Omega^{w_j} \Omega^{w_k}}{1 - \Omega^{w_j}} \left( \mathcal{M}_{mn-j}^{\mathbb{p}} \otimes \mathcal{M}_{mn-k}^{\mathbb{q}} \right) \right)^{\frac{1}{\mathbb{p}+\mathbb{q}}}$$

$$= MVCNULDNWBM \left( \mathcal{M}_{mn-1}, \mathcal{M}_{mn-2}, \ldots, \mathcal{M}_{mn-z} \right)$$

The result is proved.

***Proof of Property 1:*** By using the idea of the MVCNULDNWBM operator, we have $MVCNULDNWBM \left( \mathcal{M}_{mn-1}, \mathcal{M}_{mn-2}, \ldots, \mathcal{M}_{mn-z} \right)$

$$= \left( \oplus_{\substack{j, k = 1 \\ j, \neq k}}^{z} \frac{\Omega^{w_j} \Omega^{w_k}}{1 - \Omega^{w_j}} \left( \mathcal{M}_{mn-j}^{\mathbb{p}} \otimes \mathcal{M}_{mn-k}^{\mathbb{q}} \right) \right)^{\frac{1}{\mathbb{p}+\mathbb{q}}} = \left( \oplus_{\substack{j, k = 1 \\ j, \neq k}}^{z} \frac{\Omega^{w_j} \Omega^{w_k}}{1 - \Omega^{w_j}} \left( \mathcal{M}_{mn}^{\mathbb{p}} \otimes \mathcal{M}_{mn}^{\mathbb{q}} \right) \right)^{\frac{1}{\mathbb{p}+\mathbb{q}}}$$

$$= \left( \oplus_{\substack{j, k = 1 \\ j, \neq k}}^{z} \frac{\Omega^{w_j} \Omega^{w_k}}{1 - \Omega^{w_j}} \left( \mathcal{M}_{mn}^{\mathbb{p}+\mathbb{q}} \right) \right)^{\frac{1}{\mathbb{p}+\mathbb{q}}} = \left( \left( \mathcal{M}_{mn}^{\mathbb{p}+\mathbb{q}} \right) \right)^{\frac{1}{\mathbb{p}+\mathbb{q}}} = \mathcal{M}_{mn}.$$

***Proof of Property 2:*** Based on Eq. (28), we have

$$MVCNULDNWBM \left( \min_{j} \left\{ \mathcal{M}_{mn-j} \right\}, \min_{j} \left\{ \mathcal{M}_{mn-j} \right\}, \ldots, \min_{j} \left\{ \mathcal{M}_{mn-j} \right\} \right) = \min_{j} \left\{ \mathcal{M}_{mn-j} \right\}$$

and

$$MVCNULDNWBM \left( \max_{j} \left\{ \mathcal{M}_{mn-j} \right\}, \max_{j} \left\{ \mathcal{M}_{mn-j} \right\}, \ldots, \max_{j} \left\{ \mathcal{M}_{mn-j} \right\} \right) = \max_{j} \left\{ \mathcal{M}_{mn-j} \right\}$$

and it is clear that

$$\min_{j} \left\{ \mathcal{M}_{mn-j} \right\} \leq \mathcal{M}_{mn-j} \leq \max_{j} \left\{ \mathcal{M}_{mn-j} \right\}$$

Then, we obtained the result, such that

$$\min_{j} \left\{ \mathcal{M}_{mn-j} \right\} \leq MVCNULDNWBM \left( \mathcal{M}_{mn-1}, \mathcal{M}_{mn-2}, \ldots, \mathcal{M}_{mn-z} \right) \leq \max_{j} \left\{ \mathcal{M}_{mn-j} \right\}.$$