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Bayesian Group Chain Sampling Plan for Poisson Distribution with Gamma Prior

Waqar Hafeez[1], Nazrina Aziz[1,2,*], Zakiyah Zain[1,2] and Nur Azulia Kamarudin[1]

[1]School of Quantitative Sciences, Universiti Utara Malaysia, UUM Sintok, 06010, Kedah, Malaysia
[2]Institute of Strategic Industrial Decision Modelling (ISIDM), Universiti Utara Malaysia, UUM Sintok, 06010, Kedah, Malaysia
[*]Corresponding Author: Nazrina Aziz. Email: nazrina@uum.edu.my
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Abstract: Acceptance sampling is a statistical quality control technique that consists of procedures for sentencing one or more incoming lots of finished products. Acceptance or rejection is based on the inspection of sampled products drawn randomly from the lot. The theory of previous acceptance sampling was built upon the assumption that the process from which the lots are produced is stable and the process fraction nonconforming is a constant. Process variability is inevitable due to random fluctuations, which may inadvertently lead to quality variation. As an alternative to traditional sampling plans, Bayesian approach can be used by considering prior information of the process. Using different combinations of design parameters, this study introduces a Bayesian group chain sampling plan (BGChSP). For the first time in group chain sampling plan, the probability of lot acceptance is derived by using Poisson distribution to estimate an average number of defectives. Gamma distribution is used as a prior distribution with Poisson distribution. Taking into account both consumer's and producer's risks, this research considers two quality regions namely, probabilistic quality region (PQR) and indifference quality region (IQR). By minimizing consumer's and producer's risks, BGChSP can be used to minimize the average number of defective products in industry.

Keywords: Bayesian; acceptance sampling; gamma; produce risk; consumer risk

1 Introduction

There is tough competition in the industry by rapidly increasing the needs of statistical and analytical techniques towards the improvement of product quality [1–3]. This study is related to Bayesian group chain sampling plan (BGChSP) by using a novel approach called quality region. Instead of considering a point estimate of the quality of a process, this plan is based on a range of quality levels. The producer's risk and consumer's risk are associated with every sampling plan [4]. A good plan delivers decision rules of acceptance for both producer and consumer to meet the present quality condition of the product [5]. Improvement in technology

is rapidly increasing with the passage of time and supplier needs high-quality products with low defective fraction [6]. Unfortunately, in some situations, traditional methods cannot detect the defective product with such quality level. Thus, quality regions were introduced to overcome such problems [7,8]. By involving probabilistic quality region (PQR) and indifference quality region (IQR), this study designs the parameters for the plan indexed with quality region.

Bayesian approach is based on the combination of current and prior information for the selection of distribution. In acceptance sampling plan, Bayesian approach is required to specify a distribution of defectives from lot to lot [7]. Prior distribution is the expected distribution of the lot quality, that is under inspection. Prior distribution is formulated before taking the sample and empirical knowledge is based on the sample under study called the distribution of sample or data. The combination of prior and empirical information leads to a decision about the current lot either it will be accepted or rejected.

2 Literature Review

Epstein [9] was the first who introduced the idea of single sampling plan (SSP), to distinguish between good and bad lot for zero and one acceptance number. Dodge [10] introduced the chain sampling plan to overcome the deficiency of SSP, with more than one acceptance number and for the general family of chain sampling inspection plan [11]. Under an assumption that cost is linear in p that is the fraction of defectives. Hald [12] provides a system of attribute single sampling plan obtained by minimizing average cost. Later, single sampling plan was modified by using acceptable quality level (AQL) and limiting quality level (LQL) for Bayesian single sampling plan with weighted Poisson distribution [13] and Poisson distribution [14]. Later, in the construction and performance measure, a Bayesian chain sampling plan was discussed for gamma prior [15]. For Beta prior, a Bayesian double sampling plan was considered for the construction and performance measure through quality region [16]. Mathew et al. [17] also work in two-sided complete Bayesian chain sampling plan with Poisson and gamma as prior distribution.

Aslam et al. [18] developed a group sampling plan (GSP), by considering multiple testers at the same time. In GSP the sample size is divided into g number of groups based on the available number of testers $n = r * g$. Aslam et al. [19] designed group acceptance sampling plan (GASP) for truncated life tests, with the product lifetime following Pareto distribution of the second kind. Later, for a family of Pareto distribution, an efficient GASP was introduced by Mughal et al. [20]; they assumed the total number of defectives as groups. The number of defectives in the proposed plan was recorded based on all groups instead of an individual group. Moreover, for a family of Pareto distribution, an economic reliability acceptance sampling plan was proposed for an efficient group sampling technique [21]. By satisfying design parameters for a given group and acceptance number, a minimum termination time was required for the proposed plan. Using GASP for Pareto distribution of 2nd kind, an economic reliability group acceptance sampling plan was developed by Mughal et al. [22]. For the biased data theory to find the required design parameters, they used Poisson and weighted Poisson distributions. It was proven that their proposed plans required minimum testing time.

Mughal et al. [23] developed a GChSP plan for the lifetime of a product following Pareto distribution of 2nd kind. Satisfying pre-assumed design parameters at several quality levels, probability of lot acceptance was obtained. Based on their sampling plan, Mughal [24] extended and proposed a generalized GChSP. By considering several values of the proportion of defectives, the minimum sample size and probability of lot acceptance were found to satisfy pre-specified

consumer's risk. Recently, Hafeez et al. [25] also worked on this plan and estimated AQL and LQL for the average proportion of defective products in their study.

Based on Mughal et al. [23] plan, this study concerns with the development of a Bayesian group chain sampling plan (BGChSP). In group chain sampling plan, the probability of lot acceptance is derived by using Poisson distribution and gamma is used as a prior distribution. For PQR and IQR, this study designs the plan indexed parameters which are: AQL, LQL, producer's risk α, consumer's risk β, for the specified values of the number of testers r and shape parameter s. Also, the numerical illustrations are provided for prior distribution parameters.

3 Methodology

The operating procedure for the proposed plan is based on the following steps:

(1) Select an ideal number of g groups for each lot and assign r items to each group with the sample size required $n = g * r$.
(2) Count the number of defectives d.
(3) If $d = 0$, accept the lot.
(4) If $d > 1$, reject the lot.
(5) If $d = 1$, and no defectives was found in the immediately preceding i samples, then accept the lot.

All the above steps can be summarized in a flow chart presented in Fig. 1.

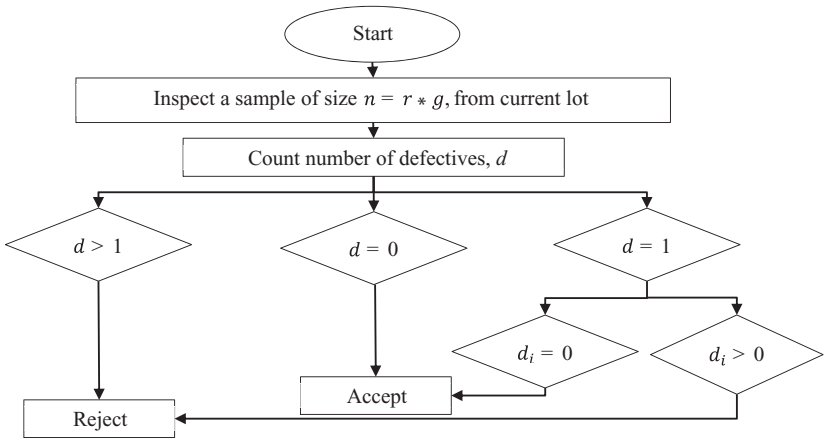

Figure 1: Operating procedure of the proposed sampling plan

In attribute acceptance sampling, lot inspection is based on the number of defectives (discrete variable). The procedure of GChSP for $i = 2$, is illustrated through a tree diagram, represented in Fig. 2, where defective and non-defective products are denoted by D and $\overline{D}$, respectively. From Fig. 2, for $i = 2$ it can observed that GChSP has five acceptance criteria (AC) $\{DD\overline{D}, D\overline{D}D, \overline{D}DD, \overline{D}\overline{D}D, \overline{D}D\overline{D}\}$. Here $\{DD\overline{D}, D\overline{D}D, \overline{D}DD, \overline{D}D\overline{D}\}$ concern with the current lot, hence the probability of lot acceptance is P_0 and $\{\overline{D}\overline{D}D\}$ has a probability of lot acceptance $P_0P_0P_1$. Finally in the group chain, the probability of lot acceptance can be written as:

$$L(p)_{GChSP} = P_0 + P_0P_0P_1 \tag{1}$$

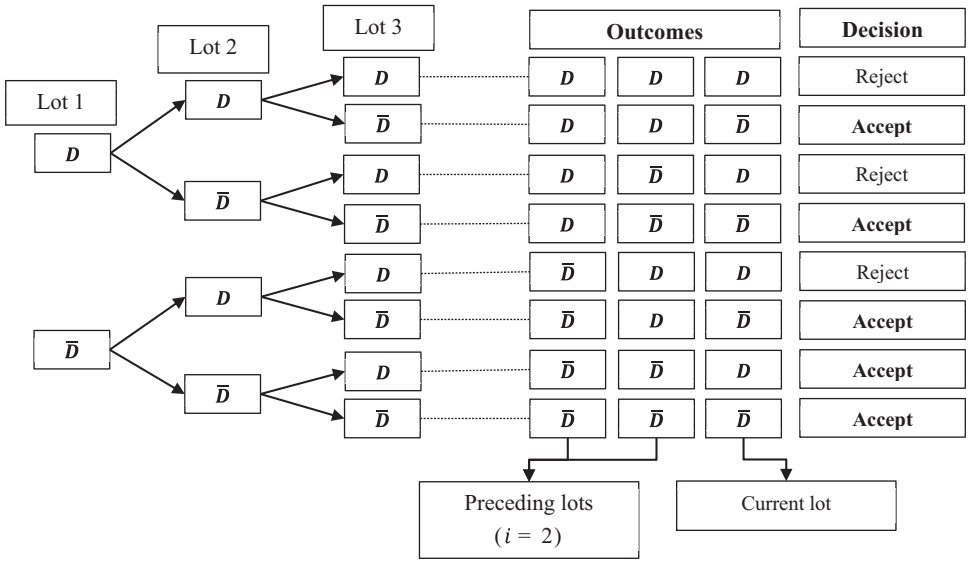

Figure 2: Tree diagram for the proposed sampling plan

$$L(p)_{GChSP} = P_0 + P_1(P_0)^2 \tag{2}$$

The general expression for $i = 2$ is like the operating characteristics (OC) function of ChSP-1 given by Dodge [10].

$$L(p)_{GChSP} = P_0 + P_1(P_0)^i \tag{3}$$

For the average number of defectives, consider Poisson distribution with mean $\mu = np$ and $n = r * g$, to achieve the probability of lot acceptance for zero and one defective product which are respectively:

$$P_0 = e^{-(r*g)p}, \tag{4}$$

$$P_1 = (r*g)\,pe^{-(r*g)p}. \tag{5}$$

After replacing Eqs. (4) and (5) in Eq. (3), we get:

$$L(p)_{GChSP} = e^{-(r*g)p}\left\{1 + (r*g)\,pe^{-(r*g)pi}\right\}. \tag{6}$$

Let us consider gamma distribution as a suitable prior for the Poisson distribution [26,27], with pdf:

$$f(p) = \frac{t^s}{\Gamma(s)} p^{s-1} e^{-tp}, \tag{7}$$

where shape parameter, $s > 0$ and rate parameter, $t > 0$ with mean $\mu = \frac{s}{t}$ under the proposed sampling plan. For the average probability of lot acceptance, the general expression used in Bayesian is as follows:

$$\overline{P} = \int_0^\infty L(p) f(p)\, dp. \tag{8}$$

After replacing Eqs. (6) and (7) in Eq. (8) and then from simplification we get:

$$\overline{P} = \frac{t^s}{\Gamma(s)}\left[\frac{\Gamma(s)}{(r*g+t)^s} + (r*g)\frac{\Gamma(s+1)}{(r*g*(1+i)+t)^{s+1}}\right], \tag{9}$$

$$\overline{P} = \left(\frac{t}{r*g+t}\right)^s + (r*g)\frac{st^s}{(r*g(1+i)+t)^{s+1}}. \tag{10}$$

Upon substituting mean $\mu = \frac{s}{t}$ in Eq. (10) and simplifying:

$$\overline{P} = \left(\frac{s}{rg\mu+s}\right)^s + \frac{rg\mu s^{s+1}}{(rg\mu(1+i)+s)^{s+1}}. \tag{11}$$

Further simplification of Eq. (11), for $s = 1, 2, 3$ gives:

$$\overline{P} = \frac{1}{(rg\mu+1)} + \frac{rg\mu}{(rg\mu(1+i)+1)^2}, \tag{12}$$

$$\overline{P} = \frac{4}{(rg\mu+2)^2} + \frac{8rg\mu}{(rg\mu(1+i)+2)^3}, \tag{13}$$

$$\overline{P} = \frac{27}{(rg\mu+3)^3} + \frac{81rg\mu}{(rg\mu(1+i)+3)^4}. \tag{14}$$

It is to be noted that Newton's approximation is employed in Eqs. (12)–(14) to find the quality regions of BGChSP, where μ is used as a point of control by reducing $\overline{P}$. Tab. 1 presents the average number of defectives for the specified values of shape parameter $s = 1, 2, 3$; the number of testers $r = 2, 3, 4$ and number of preceding lots $i = 1, 2, 3, 4$. Operating ratios for different values of consumer's risk and producer's risk are presented in Tab. 2.

Example 1: In Tab. 1, for $s = 1, r = 2$, $i = 3$ and $\overline{P} = 0.50$ the corresponding value of the average number of defectives is 0.5788 and for $s = 2, r = 3$ and $i = 4$, the corresponding value is 0.3017. From Tab. 1, it can be concluded that as the values of s, r and i are increased the average number of defective products decreases.

3.1 Designing Sampling Plans for Given AQL and LQL

For the selection of BGChSP, Tabs. 1–2 are used for specified AQL, LQL, α and β by using the following three steps: (i) construct a plan for the given points (AQL, $1-\alpha$) and (LQL, β), then compute the desired operating ratio; (ii) from Tab. 2, find the value of the operating ratio that is equal to or just less than the desired operating ratio in the desired columns for α, β; (iii) obtain the corresponding values of s, r, g and i, from Tab. 1 for the corresponding located value of the operating ratio given in Tab. 2.

3.2 Construction of Quality Regions

3.2.1 Probabilistic Quality Region (PQR)

In this quality region, the product is accepted with the maximum probability of 0.95 and minimum probability of 0.10, where 0.95 corresponds to (AQL, $1-\alpha$) and 0.10 corresponds to (LQL, β). In other words, PQR (R_1) is exactly the conventional setting of AQL $= \mu_1$ and LQL $= \mu_2$. In Fig. 3, we can see the PQR lies between $\mu_1 < R_1 < \mu_2$.

Table 1: For specified values of s, r, i and $\overline{P}$, the average number of defectives for BGChSP

s	*r*	*i*	0.99	0.95	0.9	0.5	0.25	0.1	0.05	0.01
1	2	1	0.0323	0.0843	0.1363	0.7339	1.9938	5.7479	11.9990	62.0005
	2	2	0.0253	0.067	0.1096	0.6247	1.7383	5.0722	10.6279	55.0740
	2	3	0.0216	0.0581	0.0963	0.5788	1.6408	4.8277	10.1401	52.6412
	2	4	0.0193	0.0526	0.0881	0.5546	1.5931	4.7121	9.9119	51.5148
	3	1	0.0215	0.0562	0.0908	0.4892	1.3292	3.8319	7.9994	41.3347
	3	2	0.0168	0.0447	0.0731	0.4164	1.1588	3.3814	7.0852	36.7168
	3	3	0.0144	0.0388	0.0642	0.3859	1.0939	3.2185	6.7600	35.0959
	3	4	0.0128	0.0351	0.0587	0.3697	1.0620	3.1414	6.6079	34.3417
	4	1	0.0161	0.0422	0.0681	0.3669	0.9969	2.8739	5.9995	31.0002
	4	2	0.0126	0.0335	0.0548	0.3123	0.8691	2.5361	5.3140	27.5374
	4	3	0.0108	0.0291	0.0481	0.2894	0.8204	2.4139	5.0700	26.3206
	4	4	0.0096	0.0263	0.0440	0.2773	0.7965	2.3561	4.9560	25.7573
2	2	1	0.0363	0.0914	0.1429	0.6055	1.2760	2.5818	4.0480	10.2300
	2	2	0.0284	0.0722	0.1141	0.5123	1.1180	2.3188	3.6742	9.4007
	2	3	0.0242	0.0624	0.0995	0.4730	1.0615	2.2367	3.5647	9.1760
	2	4	0.0215	0.0561	0.0905	0.4526	1.0361	2.2032	3.5219	9.0921
	3	1	0.0242	0.0609	0.0953	0.4037	0.8506	1.7212	2.6987	6.8200
	3	2	0.0189	0.0482	0.0761	0.3415	0.7454	1.5458	2.4494	6.2671
	3	3	0.0161	0.0416	0.0663	0.3153	0.7077	1.4911	2.3765	6.1173
	3	4	0.0143	0.0374	0.0603	0.3017	0.6907	1.4688	2.3479	6.0614
	4	1	0.0182	0.0457	0.0715	0.3027	0.6380	1.2909	2.0240	5.1149
	4	2	0.0142	0.0361	0.0571	0.2561	0.5590	1.1594	1.8371	4.7002
	4	3	0.0121	0.0312	0.0498	0.2365	0.5308	1.1184	1.7823	4.5880
	4	4	0.0108	0.0281	0.0452	0.2263	0.5180	1.1016	1.7609	4.5461
3	2	1	0.0381	0.0946	0.1461	0.5690	1.1049	2.0096	2.8989	5.9575
	2	2	0.0298	0.0746	0.1164	0.4798	0.9698	1.8209	2.6659	5.5848
	2	3	0.0253	0.0643	0.1012	0.4421	0.9234	1.7682	2.6076	5.5052
	2	4	0.0225	0.0577	0.0917	0.4226	0.9037	1.7492	2.5881	5.4810
	3	1	0.0254	0.0631	0.0974	0.3794	0.7366	1.3397	1.9326	3.9717
	3	2	0.0198	0.0498	0.0776	0.3199	0.6465	1.2139	1.7773	3.7232
	3	3	0.0169	0.0429	0.0674	0.2947	0.6156	1.1788	1.7384	3.6702
	3	4	0.0150	0.0385	0.0611	0.2817	0.6024	1.1661	1.7254	3.6540
	4	1	0.0191	0.0473	0.0731	0.2845	0.5525	1.0048	1.4494	2.9788
	4	2	0.0149	0.0373	0.0582	0.2399	0.4849	0.9104	1.3329	2.7924
	4	3	0.0127	0.0321	0.0506	0.2210	0.4617	0.8841	1.3038	2.7526
	4	4	0.0113	0.0289	0.0459	0.2113	0.4518	0.8746	1.2941	2.7405

The range of probability is $R_1 = \mu_2 - \mu_1$ which is derived from the equation of the average probability of acceptance Eq. (11). This Eq. (11) is used in the same way to find AQL, LQL and operating ratios represented in Tab. 3.

Table 2: For specified α and β in BGChSP, operating ratio μ_2/μ_1 values are tabulated against s, r and i

s	r	i	$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
			$\beta = 0.01$	$\beta = 0.05$	$\beta = 0.10$	$\beta = 0.01$	$\beta = 0.05$	$\beta = 0.10$	$\beta = 0.01$	$\beta = 0.05$	$\beta = 0.10$
1	2	1	1919.5201	371.4861	177.9536	735.4745	142.3369	68.1839	454.8826	88.0337	42.1709
	2	2	2176.8379	420.0751	200.4822	822	158.6254	75.7045	502.5	96.9699	46.2792
	2	3	2437.0926	469.4491	223.5046	906.0448	174.5284	83.0929	546.6376	105.297	50.1319
	2	4	2669.1606	513.5699	244.1503	979.3688	188.4392	89.5837	584.731	112.5074	53.4858
	3	1	1922.5442	372.0651	178.2279	735.4929	142.3381	68.1833	455.228	88.0991	42.2015
	3	2	2185.5238	421.7381	201.2738	821.4049	158.5056	75.6465	502.2818	96.9248	46.2572
	3	3	2437.2153	469.4444	223.5069	904.5335	174.2268	82.951	546.6651	105.296	50.1324
	3	4	2682.9453	516.2422	245.4219	978.396	188.2593	89.4986	585.0375	112.5707	53.5162
	4	1	1925.4783	372.6398	178.5031	734.6019	142.1682	68.1019	455.2159	88.0984	42.2012
	4	2	2185.5079	421.746	201.2778	822.0119	158.6269	75.7045	502.5073	96.9708	46.2792
	4	3	2437.0926	469.4444	223.5093	904.488	174.2268	82.9519	547.2058	105.4054	50.185
	4	4	2683.0521	516.25	245.4271	979.365	188.4411	89.5856	585.3932	112.6364	53.5477
2	2	1	281.8182	111.5152	71.124	111.9256	44.2888	28.2473	71.5885	28.3275	18.0672
	2	2	331.0106	129.3732	81.6479	130.2036	50.8892	32.1163	82.39	32.2016	20.3225
	2	3	379.1736	147.3017	92.4256	147.0513	57.1266	35.8446	92.2211	35.8261	22.4794
	2	4	422.8884	163.8093	102.4744	162.0695	62.779	39.2727	100.4652	38.916	24.3448
	3	1	281.8182	111.5165	71.124	111.9869	44.3136	28.2627	71.5635	28.3179	18.0609
	3	2	331.5926	129.5979	81.7884	130.0228	50.8174	32.0705	82.3535	32.1866	20.3127
	3	3	379.9565	147.6087	92.6149	147.0505	57.1274	35.8438	92.267	35.8446	22.4902
	3	4	423.8741	164.1888	102.7133	162.0695	62.7781	39.2727	100.5207	38.937	24.3582
	4	1	281.0385	111.2088	70.9286	111.9234	44.2888	28.2473	71.5371	28.3077	18.0545
	4	2	331	129.3732	81.6479	130.1994	50.8892	32.1163	82.3152	32.1734	20.3047
	4	3	379.1736	147.2975	92.4298	147.0513	57.125	35.8462	92.1285	35.7892	22.4578
	4	4	420.9352	163.0463	102	161.7829	62.6655	39.2028	100.5774	38.958	24.3717
3	2	1	156.3648	76.0866	52.7454	62.9757	30.6438	21.2431	40.7769	19.8419	13.755
	2	2	187.4094	89.4597	61.104	74.8633	35.7359	24.4088	47.9794	22.9029	15.6435
	2	3	217.5968	103.0672	69.8893	85.6174	40.5537	27.4992	54.3992	25.7668	17.4723
	2	4	243.6	115.0267	77.7422	94.9913	44.8544	30.3154	59.771	28.2236	19.0752
	3	1	156.3661	76.0866	52.7441	62.9429	30.6276	21.2314	40.7772	19.8419	13.7546
	3	2	188.0404	89.7626	61.3081	74.7631	35.6888	24.3755	47.9794	22.9034	15.643
	3	3	217.1716	102.8639	69.7515	85.5524	40.5221	27.4779	54.454	25.7923	17.4896
	3	4	243.6	115.0267	77.74	94.9091	44.8156	30.2883	59.8036	28.239	19.0851
	4	1	155.9581	75.8848	52.6073	62.9767	30.6427	21.2431	40.7497	19.8276	13.7456
	4	2	187.4094	89.4564	61.1007	74.8633	35.7346	24.4075	47.9794	22.9021	15.6426
	4	3	216.7402	102.6614	69.6142	85.7508	40.6168	27.5421	54.3992	25.7668	17.4723
	4	4	242.5221	114.5221	77.3982	94.827	44.7785	30.263	59.7059	28.1939	19.0545

3.2.2 Indifference Quality Region (IQR)

In IQR, the product is accepted with maximum and minimum probabilities 0.95 and 0.50, that correspond to (AQL, $1-\alpha$) and (LQL, β) respectively. Hence, $R_0 = \mu_0 - \mu_1$ is the range

of IQR and lies in the interval $\mu_1 < R_0 < \mu_0$. This is obtained from the average probability of acceptance Eq. (11).

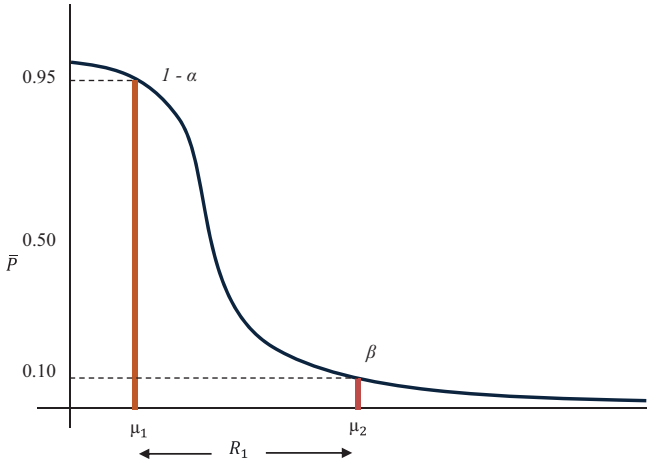

Figure 3: OC curve with pair of coordinates for PQR

Example 2: Let $s = 1$ at $\alpha = 0.10$ and $\beta = 0.05$, $\mu_1 = 0.008$ and $\mu_2 = 0.90$, then the operating ratio $\frac{\mu_2}{\mu_1} = 112.5$. From Tab. 2 the value 112.6364 with design parameters $s = 1, r = 4, i = 4$ is approximately equal to this calculated operating ratio. With the same design parameters $s = 1, r = 4$ and $i = 4$ from Tab. 1, the corresponding value of $g\mu_1 = 0.044$. The number of groups can be obtained by $\frac{g\mu_1}{\mu_1} = 5.5$-the required minimum number of groups is approximately 6 in order to get the required operating ratio. For this operating ratio, the parametric values are $s = 1, g = 6$, $r = 4, i = 4$ at $\alpha = 0.10$ and $\beta = 0.05$. Also, the corresponding value of AQL is $\mu_1 = 0.008$ and LQL is $\mu_2 = 0.9$.

3.3 Selection of Sampling Plans

For different values of s and r in Tab. 3 the operating ratio T is given, i.e., $T = \frac{\mu_2 - \mu_1}{\mu_0 - \mu_1} = \frac{R_1}{R_0}$, where R_1 and R_0 are used to characterize the sampling plan. Operating ratio $T = \frac{R_1}{R_0}$ can be found for any given values of PQR (R_1) and IQR (R_0). First we find the value in Tab. 3 under the column of T, that is equal to or just less than the specified ratio. Then for this operating ratio, the corresponding parameters values can be determined for s, r and i in BGChSP.

Table 3: For specified s, r and i values of PQR, IQR and operating ratio μ_2/μ_1

s	r	i	$g\mu_1$	$g\mu_0$	$g\mu_2$	gR_1	gR_0	T	μ_2/μ_1
1	2	1	0.0843	0.7339	5.7479	5.6636	0.6496	8.7186	68.1839
	2	2	0.0670	0.6247	5.0722	5.0052	0.5577	8.9747	75.7045
	2	3	0.0581	0.5788	4.8277	4.7696	0.5207	9.1599	83.0929
	2	4	0.0526	0.5546	4.7121	4.6595	0.5020	9.2819	89.5837
	3	1	0.0562	0.4892	3.8319	3.7757	0.4330	8.7199	68.1833
	3	2	0.0447	0.4164	3.3814	3.3367	0.3717	8.9768	75.6465
	3	3	0.0388	0.3859	3.2185	3.1797	0.3471	9.1607	82.9510
	3	4	0.0351	0.3697	3.1414	3.1063	0.3346	9.2836	89.4986
	4	1	0.0422	0.3669	2.8739	2.8317	0.3247	8.7209	68.1019
	4	2	0.0335	0.3123	2.5361	2.5026	0.2788	8.9763	75.7045
	4	3	0.0291	0.2894	2.4139	2.3848	0.2603	9.1617	82.9519
	4	4	0.0263	0.2773	2.3561	2.3298	0.2510	9.2820	89.5856
2	2	1	0.0914	0.6055	2.5818	2.4904	0.5141	4.8442	28.2473
	2	2	0.0722	0.5123	2.3188	2.2466	0.4401	5.1048	32.1163
	2	3	0.0624	0.473	2.2367	2.1743	0.4106	5.2954	35.8446
	2	4	0.0561	0.4526	2.2032	2.1471	0.3965	5.4151	39.2727
	3	1	0.0609	0.4037	1.7212	1.6603	0.3428	4.8434	28.2627
	3	2	0.0482	0.3415	1.5458	1.4976	0.2933	5.1060	32.0705
	3	3	0.0416	0.3153	1.4911	1.4495	0.2737	5.2959	35.8438
	3	4	0.0374	0.3017	1.4688	1.4314	0.2643	5.4159	39.2727
	4	1	0.0457	0.3027	1.2909	1.2452	0.2570	4.8451	28.2473
	4	2	0.0361	0.2561	1.1594	1.1233	0.2200	5.1059	32.1163
	4	3	0.0312	0.2365	1.1184	1.0872	0.2053	5.2957	35.8462
	4	4	0.0281	0.2263	1.1016	1.0735	0.1982	5.4162	39.2029
3	2	1	0.0946	0.5690	2.0096	1.915	0.4744	4.0367	21.2431
	2	2	0.0746	0.4798	1.8209	1.7463	0.4052	4.3097	24.4089
	2	3	0.0643	0.4421	1.7682	1.7039	0.3778	4.5101	27.4992
	2	4	0.0577	0.4226	1.7492	1.6915	0.3649	4.6355	30.3154
	3	1	0.0631	0.3794	1.3397	1.2766	0.3163	4.0360	21.2314
	3	2	0.0498	0.3199	1.2139	1.1641	0.2701	4.3099	24.3760
	3	3	0.0429	0.2947	1.1788	1.1359	0.2518	4.5111	27.4779
	3	4	0.0385	0.2817	1.1661	1.1276	0.2432	4.6365	30.2883
	4	1	0.0473	0.2845	1.0048	0.9575	0.2372	4.0367	21.2431
	4	2	0.0373	0.2399	0.9104	0.8731	0.2026	4.3095	24.4075
	4	3	0.0321	0.2210	0.8841	0.8520	0.1889	4.5103	27.5421
	4	4	0.0289	0.2113	0.8746	0.8457	0.1824	4.6365	30.2629

Example 3: At $\alpha = 0.05$ and $\beta = 0.1$ let $\mu_1 = 0.01$ and $\mu_2 = 0.25$, then the operating ratio will be $\mu_2/\mu_1 = 25$. From Tab. 3 for this operating ratio, the nearer values are 24.4089, 24.376 and 24.4075. The nearest values of PQR and IQR corresponding to $s = 3$ and $i = 2$ will be found based on the minimum number of groups g.

Hence from Tab. 4, through quality region the required plan has parameters $s = 3$, $g = 5$ and $r = 3$. The nearest values of PQR and IQR are $R_1 = 0.2328$, $R_0 = 0.0540$ and $T = 4.3099$ corresponding to $s = 3$, $g = 5$, $r = 3$, $i = 2$ and $\mu_1 = 0.0107$.

Table 4: For $s = 3$ and $i = 2$ and specified r the values of PQR, IQR, μ_2/μ_1 and T

r	μ_2/μ_1	$g\mu_1$	g	μ_1	μ_2	R_1	R_0	T
2	24.4089	0.0746	8	0.0093	0.2276	0.2183	0.0507	4.3097
3	24.3760	0.0498	5	0.0100	0.2428	0.2328	0.0540	4.3099
4	24.4075	0.0373	4	0.0093	0.2276	0.2183	0.0507	4.3095

4 Conclusion

In this acceptance sampling plan, both risks are used to balance the expected losses for the consumers and producers. The presented work is limited to BGChSP and estimate PQR and IQR for the specified producer's and consumer's risks. This research gives the idea to estimate an interval of average number of defectives for prespecified design parameters. Quality interval sampling plans have wider potential applications in the industry to ensure that the product or process complies with a higher quality standard. Thus, quality interval sampling could be useful for outlining product quality, planning, and quality control arrangements that are ready for electronic industrial applications. Many electronic components such as transport electronics systems, wireless systems, global positioning systems, and computer-supported and integrated manufacturing systems can be evaluated by using the proposed plan. Many other distributions and other quality and reliability characteristics can be explored in the future.

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