Structural Durability &
Health Monitoring

Tech Science Press

https://doi.org/10.32604/sdhm.2026.079036

ARTICLE

Nonlinear Vibration and Resonance Behavior of Semi-Rigid Steel Frames with Second-Order Effects

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Received: 13 January 2026; Accepted: 02 March 2026; Published: 30 June 2026

ABSTRACT: The vibration and resonance behavior of steel frame structures are significantly affected by connection flexibility and geometric nonlinearity. Conventional vibration analyses often assume fully rigid beam-column connections and neglect second-order effects, which can lead to inaccurate predictions of natural frequencies and resonance conditions. This study proposes a finite element-based approach for the vibration analysis of planar steel frames with linear and nonlinear semi-rigid connections, explicitly incorporating geometric nonlinearity. Beam-column connections are modeled using nonlinear moment-rotation relationships, while elastic and geometric stiffness matrices, together with a consistent mass matrix, are integrated into the governing dynamic equations. The time-history response is computed using the Newmark integration scheme. The proposed formulation is validated through benchmark examples and comparisons with published results and commercial finite element software. Numerical results reveal that the combined effects of semi-rigid connections and geometric nonlinearity significantly alter the effective structural stiffness, leading to noticeable variations in natural vibration frequencies. In particular, the fundamental frequency evolves during dynamic response, resulting in resonance shifting and time-dependent resonance behavior. These findings suggest that resonance in steel frames with semi-rigid connections should be regarded as a state-dependent phenomenon rather than a fixed structural property, with important implications for vibration assessment and resonance control in structural design.

KEYWORDS: Semi-rigid connections; steel frames; second-order vibration; resonance behavior; finite element analysis

1 Introduction

Natural vibration characteristics are inherent properties of structures subjected to dynamic loading and play a crucial role in ensuring structural safety and serviceability. Resonance phenomena, which occur when the excitation frequency approaches the natural frequency of a structure, may significantly amplify structural responses and lead to severe damage or even collapse. According to classical structural dynamics theory [1], resonance has been recognized as one of the critical causes of damage in structures subjected to dynamic actions such as earthquakes and wind excitations. In engineering practice, dynamic analyses of steel frames are commonly performed assuming rigid or pinned connections [2,3]. However, such simplifications may lead to inaccurate estimations of structural dynamic behavior, particularly for slender frames where connection flexibility and geometric nonlinearity are non-negligible [4,5]. For a more realistic assessment of dynamic response, actual structural characteristics, including semi-rigid connection behavior, geometric nonlinearity, and second-order effects, should be explicitly considered.

Previous studies have demonstrated that the finite element method (FEM) provides an effective and versatile framework for vibration analysis of steel frames with semi-rigid connections [6–8]. Various beam and frame elements have been developed to account for connection flexibility, including moment-rotation relationships based on nonlinear spring models [9,10]. Chan and Ho [7] and Chan and Chui [8] proposed hybrid element formulations that incorporate beam elements and rotational springs to simulate the stiffness degradation of semi-rigid connections. Batelo et al. [9] investigated the vibration characteristics of steel frames with nonlinear semi-rigid connections while considering the P–Δ effect. However, most of these studies primarily focus on element formulation and numerical implementation, whereas the influence of connection placement on vibration characteristics and resonance behavior has rarely been addressed in a systematic manner.

In addition to connection flexibility, geometric nonlinearity play a significant role in the dynamic response of slender steel frames. Several researchers have investigated vibration and dynamic instability of semi-rigidly connected frames by incorporating geometric nonlinearity [4,5,11,12]. Gerstle [4] and Lui and Lopes [5] highlighted that neglecting second-order effects may result in unconservative predictions of structural response. Nevertheless, in many existing vibration analyses of steel frames with semi-rigid connections, the P–Delta effect is either neglected or only partially considered [13,14]. Neglecting the P–Delta effect may lead to unconservative estimations of natural frequencies and resonance amplitudes, particularly for slender frames subjected to significant axial forces.

More recent studies have extended vibration analysis of semi-rigid steel frames by incorporating advanced displacement functions and numerical techniques. Suarez et al. [15] and Sekulovic et al. [16] employed higher-order displacement interpolation functions to evaluate the dynamic response of frames with semi-rigid connections. Zlatkov [17] proposed a novel method for constructing shape functions of beam elements with semi-rigid end connections using the principle of minimum potential energy, which was later extended to dynamic and seismic analyses [18,19]. Studies on semi-rigid steel frames have always been a topic of interest for authors [20–24]. Nevertheless, these studies mainly focused on connection modeling and formulation aspects, and the combined influence of connection placement, geometric nonlinearity, and resonance behavior remains insufficiently explored.

In addition, classical finite element and structural dynamics formulations provide the theoretical foundation for analyzing vibration and stability of frame structures [25–29].

In recent years, several studies have further investigated vibration characteristics, dynamic stability, and damage identification of structural systems with semi-rigid connections using analytical, numerical, and experimental approaches [30–32].

Recent studies have devoted increasing attention to the dynamic behavior and numerical modeling of steel frames with semi-rigid connections. Advanced computational formulations, improved connection models, and experimental investigations have been proposed to better capture the nonlinear and dynamic responses of such structural systems. These studies include analytical frequency-domain solutions for frames with semi-rigid joints, component-based connection models, nonlinear dynamic behavior analysis of slender frames, and experimental–numerical investigations of semi-rigid beam-to-column connections [33–36].

Overall, although vibration analysis of steel frames with semi-rigid connections has attracted considerable research attention, existing studies mainly emphasize connection modeling, numerical formulation, or geometric nonlinearity effects separately. A comprehensive investigation into the combined effects of semi-rigid connection behavior, connection placement, and geometric nonlinearity on resonance phenomena is still lacking, particularly for planar steel frames.

To address these limitations, this paper proposes a finite element-based vibration analysis framework for planar steel frames with semi-rigid connections, explicitly incorporating connection nonlinearity and the P–Delta effect. The proposed formulation enables systematic investigation of the influence of semi-rigid connection placement on vibration characteristics and resonance behavior. The accuracy and robustness of the proposed model are verified through numerical examples and comparisons with established analytical solutions and previously published results.

The main contributions of this study are summarized as follows:

1. A consistent and efficient finite element formulation is developed for vibration analysis of planar steel frames with semi-rigid connections, explicitly accounting for connection nonlinearity and second-order geometric effects.
2. The evolution of effective structural stiffness during dynamic response is investigated, revealing the time-dependent nature of natural frequencies and resonance phenomena induced by semi-rigid connections and geometric nonlinearity.
3. The accuracy and robustness of the proposed formulation are demonstrated through comprehensive numerical examples and comparisons with analytical solutions, published results, and commercial finite element software.

It should be emphasized that the novelty of this study is computational and application-oriented rather than theoretical or algorithmic, as the proposed framework enables a unified and consistent investigation of state-dependent resonance behavior in semi-rigid steel frames by simultaneously incorporating connection nonlinearity, connection placement, and second-order geometric effects.

2 Structural Modeling of Steel Frames with Semi-Rigid Connections

In this study, a multi-storey planar steel frame is considered as the representative structural system, which is commonly encountered in civil and industrial engineering applications. The planar configuration is adopted due to its simple geometry, which facilitates numerical modeling and dynamic analysis and allows a clear investigation of the effects of semi-rigid connections and geometric nonlinearity on structural vibration. The frame is composed of columns and beams connected at the joints through semi-rigid connections. Both columns and beams are modeled using hybrid beam-column elements with two end nodes, denoted as A and B, where rotational springs are integrated at both ends, following the nonlinear moment-rotation formulation proposed by Vu et al. [6], as illustrated in Fig. 1. The quantities M_A, k_A, θ_{cA} and M_B, k_B, θ_{cB} denote the bending moment, instantaneous rotational stiffness, and rotation angle at the connections at ends A and B, respectively, at a given calculation time. The initial length of the element is denoted by L. The connections are assumed to be dimensionless and non-eccentric, the material behavior is linear elastic, viscous damping is neglected throughout the analysis, and the mass density of all members is assumed to be constant. Accordingly, the present formulation is restricted to planar steel frames with non-eccentric semi-rigid connections and is intended for undamped vibration analysis. Additional symbols and modeling assumptions are provided in [6].

The semi-rigid connection is modeled using spring connection elements. The behavior of the semi-rigid connection is represented by an instantaneous moment-rotation relationship at the connection, denoted as $M - \theta_c$, as proposed in [8]:

$$M = f\left(\theta_c\right), \tag{1}$$

where M and θ_c denote the bending moment and the corresponding rotation angle at the connection, respectively.

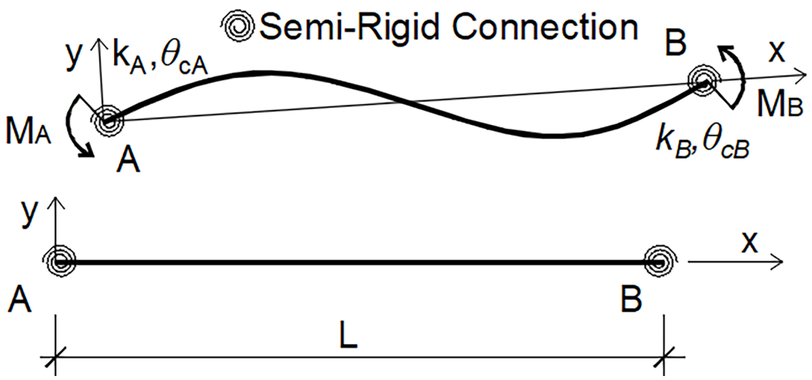


Figure 1: Configuration and deformation of hybrid elements.

For linear semi-rigid connections, the rotational stiffness remains constant throughout the loading process of the frame with respect to rotation. In contrast, for nonlinear semi-rigid connections, the moment-rotation backbone relationship under static loading exhibits a nonlinear, monotonic response, reflecting a progressive reduction in connection stiffness as the applied moment increases, as reported by Chan and Chui [8]. Among the available nonlinear connection models, the Chen-Lui exponential model proposed by Lui and Chen (1986, 1988), which provides a smooth and continuous representation of nonlinear moment-rotation behavior, is adopted in the present study and is expressed in the following form [8,14].

$$M = M_0 + \sum_{j=1}^{n} C_j \left[1 - \exp\left(\frac{-|\theta_c|}{2j\alpha} \right) \right] + k_p |\theta_c|, \tag{2}$$

the instantaneous stiffness of the connection, k_c, has the form:

$$k_c = \left. \frac{dM}{d\theta_c} \right|_{|\theta_c|=|\theta_c|} = \sum_{j=1}^{n} \frac{C_j}{2j\alpha} \exp\left(\frac{-|\theta_c|}{2j\alpha} \right) + k_p, \tag{3}$$

and the initial stiffness, k_0, can be obtained as:

$$k_0 = \left. \frac{dM}{d\theta_c} \right|_{|\theta_c|=0} = \sum_{j=1}^{n} \frac{C_j}{2j\alpha} + k_p, \tag{4}$$

in which M_0 is the initial moment, k_p is the strain-hardening stiffness of the connection, α is the scaling factor, C_j is the curve-fitting coefficient, and n is the number of terms considered. Based on past experimental results, Chan and Chui [8] determined the values of the curve-fitting parameters for the model. These parameters were identified for four types of connections: single web angle, top and bottom seated angle, flush end plate, and extended end plate joints, as summarized in Table 1.

The four-parameter model was originally proposed by Richard and Abbott (1975). The moment-rotation relationship is expressed by the following equation [6,8]:

$$M = \frac{\left(k_0 - k_p\right)|\theta_c|}{\left[1 + \left| \frac{\left(k_0 - k_p\right)|\theta_c|}{M_0} \right|^{n} \right]^{\frac{1}{n}}} + k_p |\theta_c|, \tag{5}$$

and the corresponding tangent stiffness by:

$$k = \left.\frac{dM}{d\theta}\right|_{|\theta_c|=|\theta_c|} = \frac{\left(k_0 - k_p\right)}{\left[1 + \left|\frac{\left(k_0 - k_p\right)|\theta_c|}{M_0}\right|^n\right]^{\frac{n+1}{n}}} + k_p. \tag{6}$$

In Eqs. (5) and (6), k_0 is the initial stiffness, k_p is the strain-hardening stiffness, M_0 is the reference moment, and n is a parameter defining the sharpness of the curve. Under dynamic loading, the moment-rotation behavior of the connection is modeled using the independent hardening method [8], in which the instantaneous stiffness is updated based on the current loading state.

Table 1: Connection parameters of the Chen-Lui exponential model.

	Connection Type (Kip-in)			
	A Single Web Angle (Type A) [8]	**B Top and Seated Angle (Type B) [8]**	**C Flush-End Plate (Type C) [8]**	**D Extended End Plate (Type D) [8]**
M_0	0	0	0	0
k_p	0.47104×10^2	0.43169×10^3	0.96415×10^3	0.41193×10^3
α	0.51167×10^{-3}	0.31425×10^{-3}	0.31783×10^{-3}	0.67083×10^{-3}
C_1	-0.43300×10^2	-0.34515×10^3	-0.25038×10^3	-0.67824×10^3
C_2	0.12139×10^4	0.52345×10^4	0.50736×10^4	0.27084×10^4
C_3	-0.58583×10^4	-0.26762×10^5	-0.30396×10^5	-0.21389×10^5
C_4	0.12971×10^5	0.61920×10^5	0.75338×10^5	0.78563×10^5
C_5	-0.13374×10^5	-0.65114×10^5	-0.82873×10^5	-0.99740×10^5
C_6	0.52224×10^4	0.25506×10^5	0.33927×10^5	0.43042×10^5
k_0	0.48000×10^5	0.95219×10^5	0.11000×10^6	0.30800×10^6

3 Nonlinear Vibration Analysis Procedure

Building on the structural model introduced in Section 2, this section outlines the nonlinear vibration analysis procedure employed to investigate the time-dependent dynamic response and natural frequencies of steel frames with semi-rigid connections.

Fig. 2 summarizes the overall computational framework adopted in this study. The procedure consists of two main stages: (i) second-order static analysis to establish the initial stress state and geometric stiffness, and (ii) nonlinear dynamic time-history analysis with iterative updates of structural matrices. At each converged time step, the instantaneous natural frequencies are evaluated by solving the corresponding eigenvalue problem. This unified framework enables consistent investigation of state-dependent vibration and resonance behavior in semi-rigid steel frames.

For all nonlinear dynamic analyses, the Newmark average acceleration method with parameters $\gamma = 0.5$ and $\beta = 0.25$ was adopted to ensure unconditional numerical stability. The time-step size was taken as $\Delta t = 0.001$ s. The convergence criterion was defined based on the displacement norm with a tolerance of 1×10^{-6}, and the maximum number of iterations per time step was limited to 50. These numerical parameters were consistently applied throughout all numerical examples to ensure reproducibility and numerical stability.

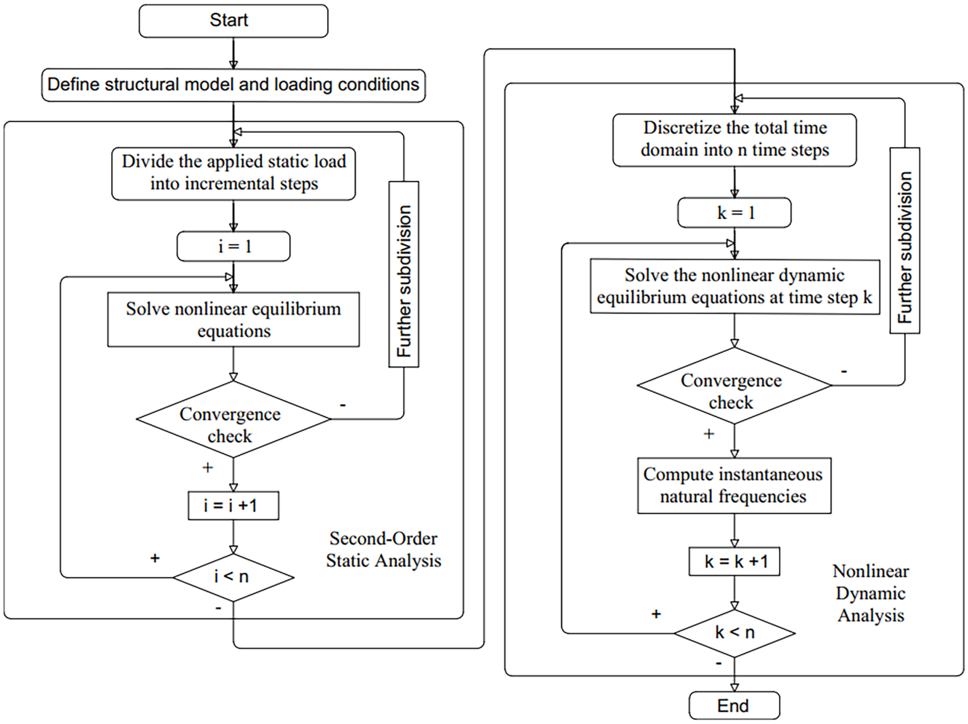


Figure 2: Flowchart of the proposed nonlinear vibration analysis procedure.

For steel frames with fully rigid or linear semi-rigid connections, the structural stiffness remains constant during vibration. In contrast, when nonlinear semi-rigid connections are subjected to dynamic loading, the combined effects of connection nonlinearity and the P–Delta phenomenon cause the connection stiffness to evolve with time, resulting in variations in the global stiffness and mass of the frame. As reported in the literature [8,14], vibration analysis of steel frames using the finite element method is inherently coupled with dynamic time-history analysis. Accordingly, the response is evaluated by discretizing the time domain into incremental steps, within which the system behavior is assumed to be linear. At a given time step i^{th}, the incremental equilibrium equations governing the motion of a multi-degree-of-freedom system can be expressed as [8,25]:

$$[M_s^*]\{\Delta\ddot{\delta}_s^*\} + [K_s^*]\{\Delta\delta_s^*\} = \{\Delta P_s^*\}. \tag{7}$$

here, in Eq. (7), $\{\Delta\ddot{\delta}_s^*\}$ and $\{\Delta\delta_s^*\}$ are the incremental acceleration and displacement vectors, respectively; $[K_s^*]$ denotes the stiffness matrix, and $\{\Delta P_s^*\}$ denotes the incremental applied external load vector, as defined in [6]. $[M_s^*]$ is the mass matrix, established from the consistent mass matrix $[m_s]_e$ in Eq. (8) and the lumped mass matrix. The matrix $[m_s]_e$ is defined as follows:

$$[m_s]_e = m \int_0^L [N(x)]^T [N(x)]\,\mathrm{d}x, \tag{8}$$

in which, m denotes the mass per unit length of the structural member, and *N(x)* are the shape functions of the beam-column element with semi-rigid connections [6].

The consistent mass matrix of a beam element is fundamentally derived from the mass density of the structural member. For convenience of formulation, the mass per unit length is represented through an equivalent uniformly distributed mass, which may be expressed in a form analogous to a distributed load without implying any physical loading effect.

After each transient response analysis step, the stiffness of the connections, the axial forces in the elements and the geometrical changes of the structure are calculated, and updated in the linear elastic stiffness matrices, geometric stiffness matrices, mass matrices, and nodal load vectors. When analyzing the vibration, also at the calculation time step i^{th}, these quantities continue to be used to determine the natural vibration frequencies, through the frequency equation of the system [26,27], which has the following form:

$$\left|\left[K_s^*\right] - \omega^2 \left[M_s^*\right]\right| = 0. \tag{9}$$

In Eq. (9), $\left[K_s^*\right]$ and $\left[M_s^*\right]$ are defined as in Eq. (8), ω is the vibration circular frequency, also known as the natural vibration frequency. The smallest and non-zero value of ω is the fundamental vibration frequency. In case the steel frame is not loaded, the natural vibration frequencies are determined from Eq. (9). In case the steel frame is only statically loaded, the second-order static analysis procedure described in [6] is used, and then Eq. (9) is employed to determine the natural vibration frequencies.

It should be noted that although the total mass of the structure remains constant, the consistent mass matrix is updated during the nonlinear analysis due to the evolution of the interpolation (shape) functions associated with the semi-rigid connections. This update reflects numerical consistency of the finite element formulation rather than any physical redistribution of mass.

4 Numerical Verification and Discussion

4.1 Example 1—Vibration of L-Shaped Steel Frame

The first example aims to validate the proposed nonlinear vibration formulation by comparing the natural vibration results of an L-shaped steel frame with available experimental data reported by Kawashima and Fujimoto [28], analytical results by Chan and Chui [8], and numerical results obtained using SAP2000 V14 software [29]. The frame consists of two elements with lengths of 150 and 100 cm, respectively, the flexural rigidity is 42,300 kgf cm^2, illustrated in Fig. 3. The two elements in the frame are connected to each other by fully rigid or linear semi-rigid connections. The stiffness of the connection in the above studies is given by the formula:

$$k = \frac{3EI}{L}\frac{v}{1-v}. \tag{10}$$

The parameter v varies from 0.0 for hinged connections to 1.0 for fully rigid connections.

After programming using Matlab software, the analysis results are obtained and presented in Tables 2 and 3. Since no external axial load is applied to the frame, second-order (P–Delta) effects do not develop in this case. As shown in Tables 2 and 3, the natural vibration frequencies obtained using the proposed formulation and those calculated using SAP2000 are in very close agreement with the experimental data reported in [28]. For example, in the first vibration mode, the natural frequency obtained from SAP2000 is 16.27 Hz, while the present formulation yields 16.2746 Hz, corresponding to a relative difference of approximately 0.03%, which can be considered negligible.

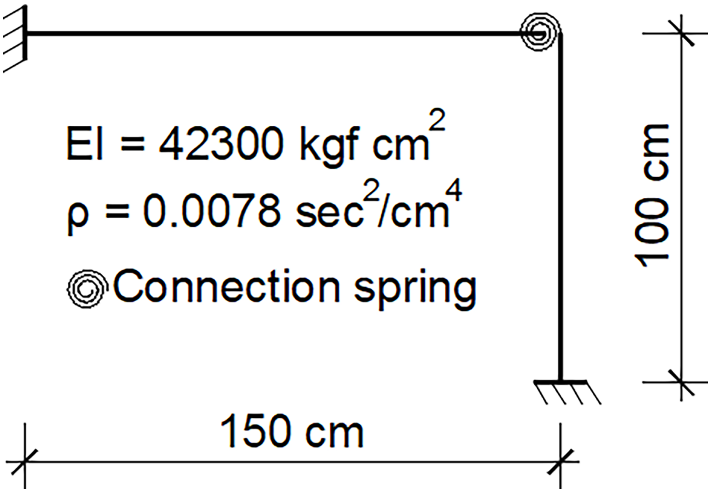


Figure 3: L-shaped steel frame diagram.

Table 2: Natural frequencies of L-shaped frame with fully rigid connection.

Vibration Mode	Natural Frequency (Hz)			
	SAP 2000	Experiment (Kawashima and Fujimoto [28])	Present Theory	Error (%)
1	16.27	16.3	16.2746	0.16
2	35.827	35.9	35.8487	0.14
3	51.582	52.0	51.6454	0.68
4	94.466	96.2	94.7003	1.56

Table 3: Natural frequencies of L-shaped frame with linear connection.

Vibration Mode	Connection Stiffness Parameter, *v*	Natural Frequency (Hz)				
		Experiment (Kawashima and Fujimoto [28])	Chan and Chui [8]	SAP2000	Present theory	Error (%)
1	0.47	15.5	15.22	15.733	15.7365	1.53
	0.307	14.1	14.76	15.373	15.3762	9.05
	0.25	13.9	13.76	13.895	13.8965	0.03
2	0.47	30.8	33.52	34.674	34.6833	12.61
	0.307	30.2	32.59	33.888	33.8976	12.25
	0.25	29.9	30.86	31.078	31.0853	3.96
3	0.47	45.0	46.69	48.588	48.6681	8.15
	0.307	44.1	45.64	47.242	47.3151	7.29
	0.25	43.9	44.27	44.422	44.4728	1.31
4	0.47	95.8	92.48	93.855	94.3272	1.54
	0.307	89.1	92.07	93.546	94.034	5.54
	0.25	83.4	91.19	92.133	92.6204	11.05

The relative errors between the present results and experimental data are also reported for quantitative comparison.

In Table 3, when changing the stiffness of the connection through the parameter v, it can be clearly seen that the semi-rigid connection has a significant influence on the natural vibration frequency. For the same vibration mode, it can be seen that when the semi-rigid connection is more flexible, the natural vibration frequency decreases. With the same value of connection stiffness, the frequency increases rapidly with higher order vibration modes. This shows that the semi-rigid connection reduces the overall stiffness of the frame, resulting in lower vibration resistance.

The relative errors between the present results and experimental data are also reported for quantitative comparison. It is also observed that the present results show very close agreement with those obtained from SAP2000, confirming the numerical consistency of the proposed formulation.

Compared with the experimental results reported in [28] and the analytical results in [8,29], the present formulation demonstrates very good agreement across all vibration modes. The results in Tables 2 and 3 clearly indicate that the influence of semi-rigid connections on the natural vibration characteristics of the frame is significant and mode-dependent. A reduction in connection stiffness leads to a decrease in natural frequencies, reflecting the reduction in the overall structural stiffness of the frame. These comparisons confirm the accuracy and reliability of the proposed formulation in predicting the vibration behavior of steel frames with semi-rigid connections.

4.2 Example 2—Vibration of Single-Span Two-Story Portal Frame under Static Loading

The second example analyzes and verifies the natural vibration calculation results of a single-span two-story portal frame with the analysis results of Chan and Chui [8] and the SAP2000 V14 program [29]. The frame is 7.32 m (288 in) high and has a span of 6.1 m (240 in), shown in Fig. 4. To activate the semi-rigid connection, concentrated forces P and 0.5 P are applied at the mid-span of the beam and at the frame nodes, respectively, P = 1601 kN. This loading arrangement induces bending moments at the beam ends. Combined with geometric nonlinear effects, these moments modify the effective connection stiffness and, consequently, the natural vibration frequencies of the frame. Six types of connections are considered, namely: the hinged connection, the single web angle connection (A), the top and seated angle connection with double web cleats (B), the flush end plate connection (C), extended end plate connection (D), and fully rigid connection. The nonlinear semi-rigid connections A, B, C, and D follow the exponential model of Lui and Chen (1988), the corresponding curve fitting data are given in Table 1.

In Table 4, all semi-rigid connection types (A–D) are modeled as linear connections using their initial rotational stiffness values. In contrast, Table 5 presents results obtained using the full nonlinear exponential moment-rotation model.

The second-order static analysis of steel frames with semi-rigid connections was performed according to Vu et al. [6]. After static analysis, the frequency equation was established and solved using Matlab software to obtain natural vibration frequencies. Table 4 presents the analysis results when the connection follows a linear model with initial stiffness. Table 5 presents the analysis results when the frame has connections following a nonlinear model.

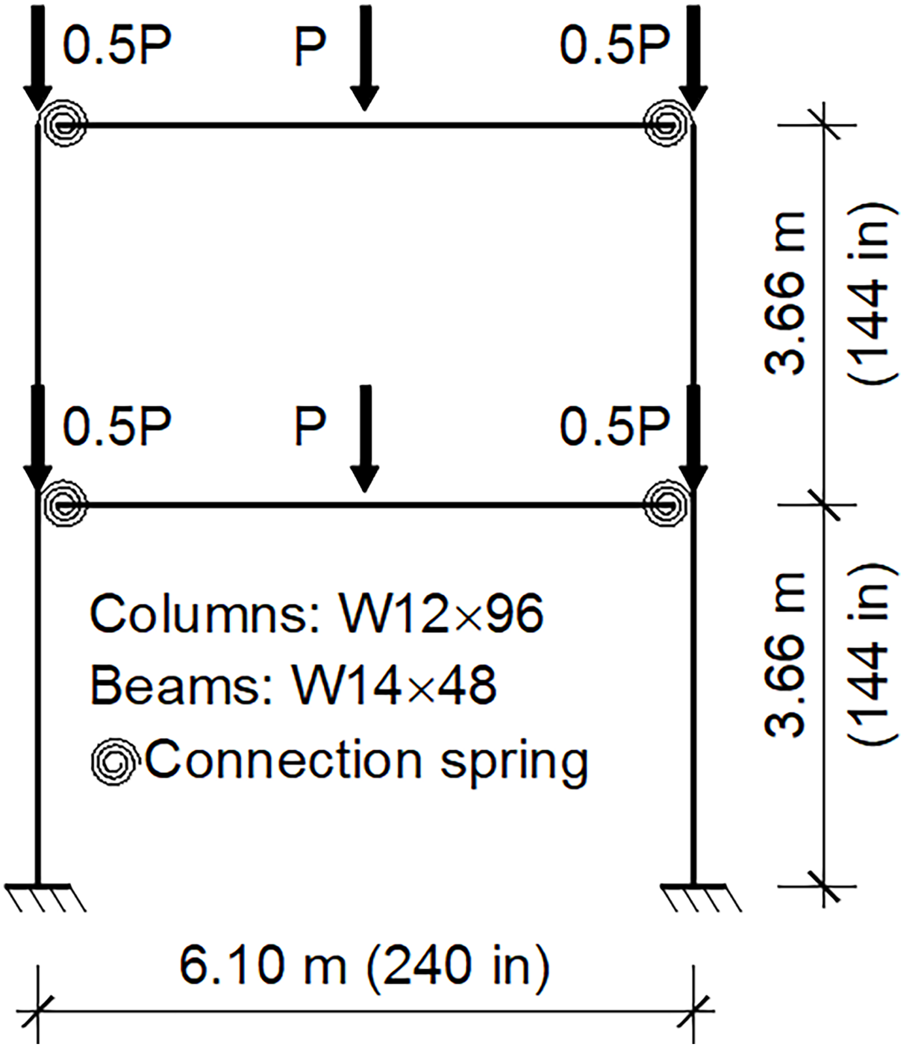


Figure 4: Geometry and loading of the two-story portal steel frame (dimensions in meters with inches in parentheses).

Table 4: Natural frequencies of two-story portal frame with linear connections.

Connection Type	**Natural Frequency (Hz)**			
	Chan and Chui [8]	**SAP2000**	**Present Theory**	**Error (%)**
Hinge		4.9232	5.23	
A	6.44	6.0885	6.4224	0.27
B	7.13	6.7647	7.106	0.34
C	7.30	6.9174	7.2595	0.55
D	8.53	8.1113	8.4524	0.91
Fully rigid	10.35	9.8463	10.1695	1.74

Note: The relative errors between the present results and the reference solutions reported by Chan and Chui [8] are also provided for quantitative comparison.

Table 5: Natural frequencies of two-story portal frame with nonlinear connections.

Connection Type	**Natural Frequency (Hz)**		
	Chan and Chui [8]	**Present Theory**	**Error (%)**
A	5.26	5.2316	0.54
B	5.27	5.2448	0.48
C	5.29	5.2629	0.51
D	5.29	5.2653	0.47

Note: The relative errors between the present results and the reference solutions reported by Chan and Chui [8] are also provided for quantitative comparison.

Numerical verification shows that the analytical results of the present theory are in good agreement with the comparative results. And in general, the flexibility of a connection affects the vibration characteristics of a structure through the variation of the stiffness of the connection. The natural vibration frequency increases as the stiffness of the connection increases. For the same type of connection in a nonlinear model, the natural vibration frequency will have the highest value when calculated with the initial stiffness.

4.3 Example 3—Vibration of Two-Span Six-Story Frame under Dynamic Loading

The third example aims to verify the calculation results as well as study the influence of resonance on the performance of a Vogel two-span six-story planar steel frame with different types of connections, including the P–Delta effect. This example is specifically designed to study the time-dependent resonance behavior and the evolution of natural frequencies under dynamic loading conditions. According to Chan and Chui [8], frames with rigid connections were studied by Vogel (1985) and many other authors, frames with semi-rigid connections were studied by Yau and Chan (1994). The Vogel frame configuration with dimensions and profiles of steel columns and beams is shown in Fig. 5. To focus on the research content, the material is assumed to be elastic and viscous damping is neglected. The beams are subjected to uniformly distributed static loads of 31.7 kN/m at the top floor and 49.1 kN/m at the remaining floors. These loads are added as additional mass together with the self-weight density of 7.8 kN s^2/m^4 for all frame members. The horizontal concentrated harmonic forces $F_1(t) = 10.23\sin(\omega t)$ kN and $F_2(t) = 20.44\sin(\omega t)$ kN are applied to the six boundary frame nodes. These harmonic excitations are introduced to activate the fundamental vibration mode and to investigate resonance and resonance shifting phenomena. Young's modulus is 205 GPa. An initial geometric imperfection of $\mu = 1/450$ is applied to columns. The semi-rigid connection is a flush end plate type, following the Richard-Abbott (1975), considered for three cases: fully rigid, linear semi-rigid (with the stiffness of the connection equal to the initial stiffness of the connection), and nonlinear semi-rigid. The fundamental vibration frequencies for the fully rigid and the linear semi-rigid connection cases are 2.41 rad/s and 1.66 rad/s, respectively, corresponding to the fundamental circular frequencies [8,14].

The analysis results performed using Matlab software are given in Figs. 6 and 7.

Fig. 6 illustrates the time-history dynamic displacements of the Vogel frame subjected to harmonic loading with different excitation frequencies for various connection models. It can be clearly observed that the dynamic response of the frame is highly sensitive to both the excitation frequency and the connection stiffness.

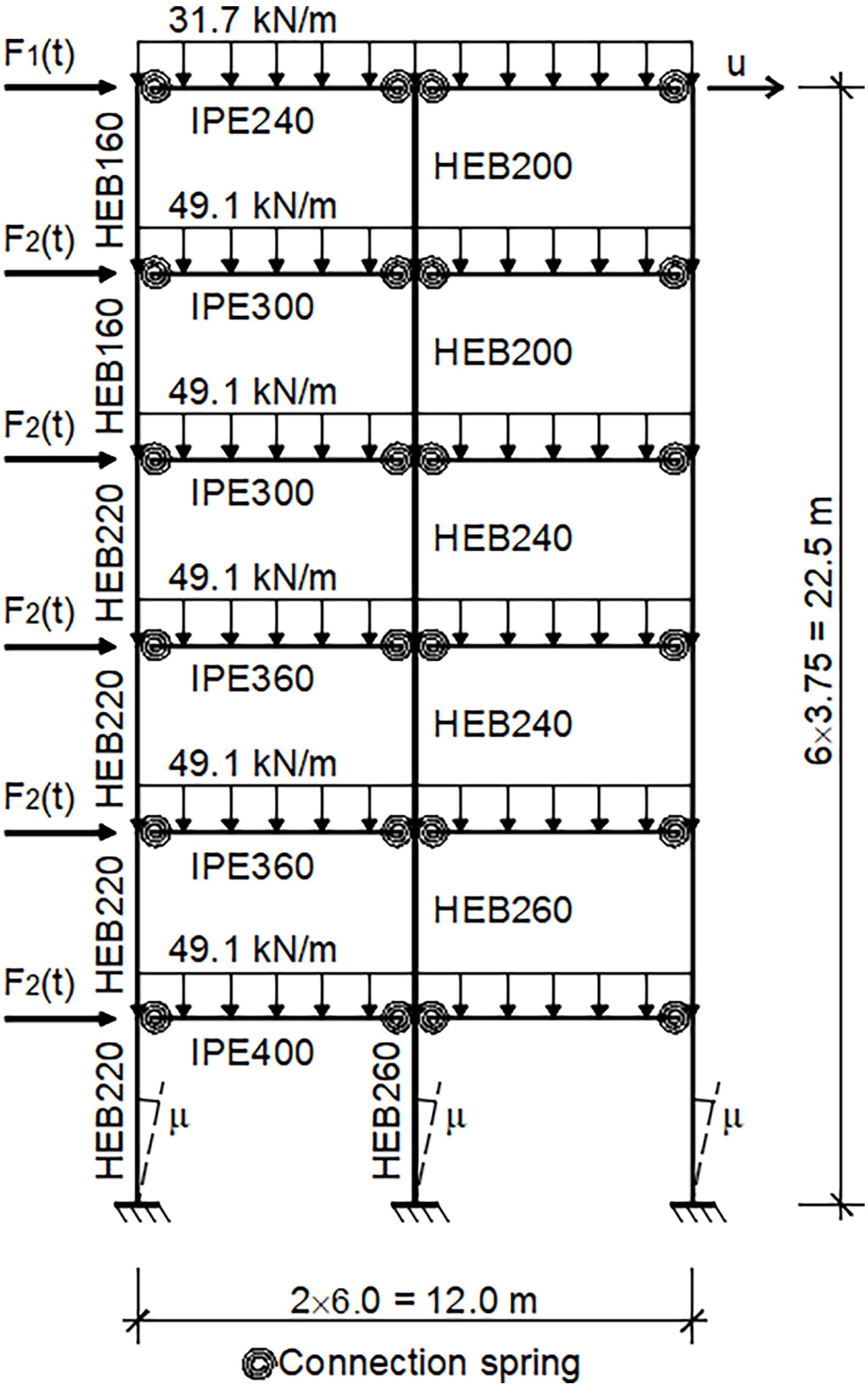
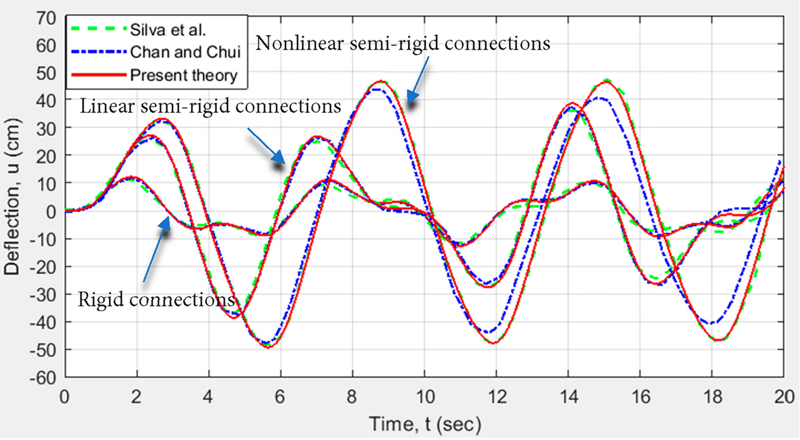
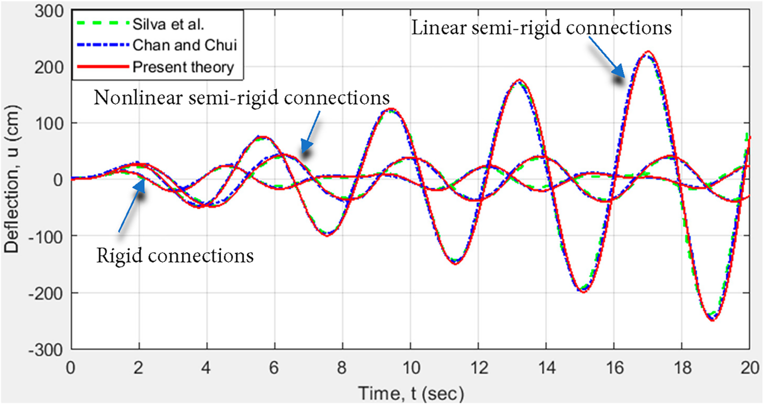


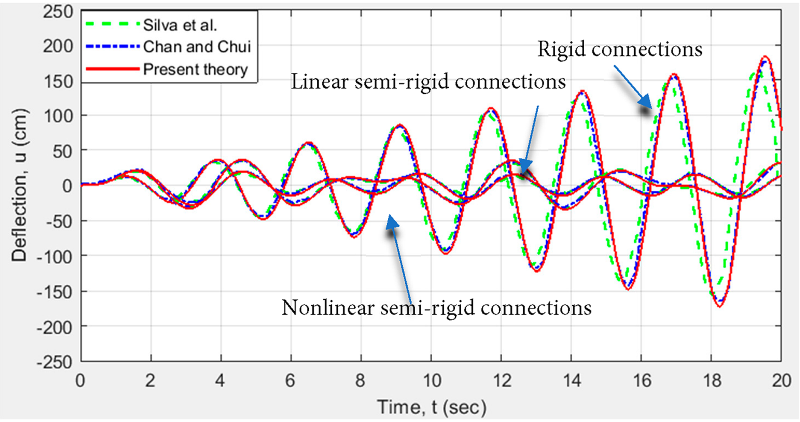
Figure 5: Two-span six-story Vogel steel frame diagram.



(**a**) $\omega = 1.00\ rad/s$

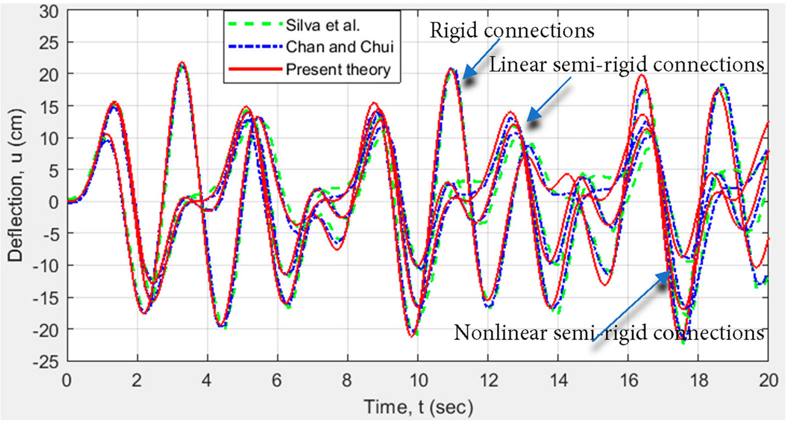


(**b**) $\omega = 1.66\ rad/s$



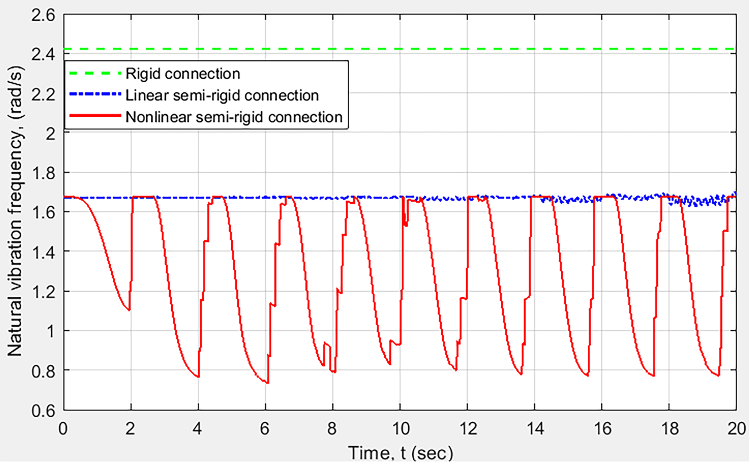
(**c**) $\omega = 2.41\ rad/s$

Figure 6: (Continued)

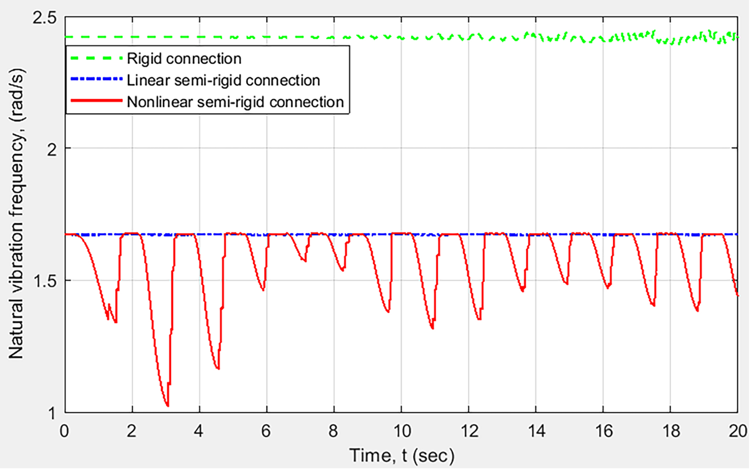


(d) $\omega = 3.30\ rad/s$

Figure 6: Dynamic displacements of the Vogel frame under various harmonic load frequencies: corresponding comparison with Silva et al., and Chan and Chui [8,14].



(a) ω = 1.66 rad/s



(b) ω = 2.41 rad/s

Figure 7: Fundamental vibration frequency of Vogel frame at resonance.

These observations clearly indicate that, for frames with nonlinear semi-rigid connections, resonance should be regarded as a time-dependent phenomenon rather than a fixed condition associated with a single natural frequency.

For the fully rigid connection case, resonance occurs when the excitation frequency approaches the initial fundamental frequency of the structure, resulting in a rapid amplification of displacement amplitude. In contrast, for frames with linear and nonlinear semi-rigid connections, the peak dynamic responses are shifted to lower excitation frequencies due to the reduction in global stiffness induced by connection flexibility.

Moreover, in the nonlinear semi-rigid connection case, the displacement amplitude does not grow monotonically with time even near resonance conditions. This behavior indicates that the effective stiffness of the structure varies during vibration as a result of connection nonlinearity and P–Delta effects, leading to a time-dependent modification of the resonance characteristics.

Fig. 7 presents the evolution of the fundamental natural frequency of the Vogel frame during dynamic loading under resonance conditions. The results demonstrate that the natural frequency is no longer a constant structural property when semi-rigid connections and geometric nonlinearity are considered.

In the case of linear semi-rigid connections, the fundamental frequency remains approximately constant and lower than that of the fully rigid frame, reflecting the reduced but stable stiffness of the system. However, for nonlinear semi-rigid connections, the fundamental frequency exhibits noticeable variation during the dynamic response.

This phenomenon can be attributed to the continuous degradation and recovery of connection stiffness associated with nonlinear moment-rotation behavior, combined with the influence of P–Delta effects. Consequently, resonance in semi-rigid frames should be regarded as a time-varying phenomenon rather than a single fixed condition, which has important implications for resonance prediction and vibration control in practical design.

Overall, the numerical results obtained from the Vogel frame example highlight the crucial role of semi-rigid connections and geometric nonlinearity in the dynamic behavior of multi-storey steel frames. The interaction between connection flexibility and P–Delta effects not only alters the natural vibration frequencies but also leads to resonance shifting and time-dependent resonance behavior. These findings indicate that resonance assessment based on linear assumptions or fixed modal properties may significantly underestimate the dynamic demand of steel frames with semi-rigid connections.

Therefore, the Vogel frame example serves as a representative benchmark to demonstrate the capability of the proposed formulation in capturing state-dependent resonance phenomena in planar steel frames.

5 Limitations and Future Work

The present study is conducted under several simplifying assumptions in order to clearly investigate the fundamental influence of semi-rigid connections and geometric nonlinearity on the vibration and resonance behavior of planar steel frames. The material behavior of all structural members is assumed to be linear elastic, and viscous damping is neglected throughout the analysis, which may influence the predicted resonance amplitudes but does not alter the observed time-dependent resonance trends. These assumptions allow the essential mechanisms of resonance shifting and time-dependent vibration characteristics induced by connection flexibility and P–Delta effects to be identified without additional complexity.

Furthermore, the semi-rigid connections considered in this study are modeled using commonly adopted moment-rotation relationships and do not account for possible degradation, strength deterioration, or damage accumulation under long-term cyclic or seismic loading. The proposed formulation is currently applied to two-dimensional frame systems, and the validation is mainly based on benchmark comparisons with previously published experimental and numerical results.

Future research will focus on extending the proposed framework to include material nonlinearity, damping effects, and degradation mechanisms of semi-rigid connections under repeated and cyclic loading conditions. In addition, the methodology will be further developed for three-dimensional steel frame systems and supported by comprehensive experimental validation, with the aim of enhancing its applicability to practical structural design and vibration control problems.

6 Conclusions and Recommendations

This paper has presented a consistent finite element-based approach for the vibration analysis of planar steel frames with semi-rigid connections, explicitly accounting for geometric nonlinearity through the P–Delta effect. By incorporating nonlinear angular moment relationships at beam-column connections and updating structural characteristics during dynamic response, the proposed method provides an efficient framework for determining and evaluating time-dependent vibration characteristics and resonance behavior in semi-rigid steel frames.

The numerical investigations demonstrate that semi-rigid connections have a pronounced influence on the vibration characteristics of steel frames by reducing the effective global stiffness and modifying the natural vibration frequencies. In particular, the results indicate that resonance phenomena in frames with nonlinear semi-rigid connections are not governed by a single fixed natural frequency. Instead, the combined effects of connection nonlinearity and geometric nonlinearity lead to resonance shifting and time-dependent variations of the fundamental natural frequency, especially under dynamic loading conditions.

Comparisons with experimental data, established analytical solutions, and commercial finite element software confirm the accuracy and robustness of the proposed formulation. The results further highlight that neglecting the flexibility and nonlinear behavior of beam-column connections may result in an incomplete or inaccurate assessment of resonance conditions and dynamic demand in steel frame structures.

Acknowledgement: Not applicable.

Funding Statement: The authors received no specific funding for this study.

Author Contributions: Dung Bao Trung Le contributed to the conceptualization and methodology. Quoc Anh Vu contributed to the validation and literature review. Hai Quang Nguyen developed the numerical model, performed the analysis, and wrote the manuscript. All authors reviewed and approved the final version of the manuscript.

Availability of Data and Materials: The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

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