Structural Durability & Health Monitoring

Tech Science Press

https://doi.org/10.32604/sdhm.2026.078870

ARTICLE

Metaheuristic-Based Optimum Design Methods for Multiple Tuned Mass Dampers under Seismic Excitations

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Received: 09 January 2026; Accepted: 24 March 2026; Published: 30 June 2026

ABSTRACT: This study presents a comparison study between a Tuned Mass Damper (TMD) and Multiple Tuned Mass Dampers (MTMD). The main novelty of this study lies in the unified optimization-based comparison of optimized and fixed-mass MTMD configurations, highlighting the trade-off between seismic performance and practical applicability compared to a single large TMD installed at the top of the building. The Jaya algorithm is used to determine the optimum solution of mass damper parameters. Three cases are examined. The first case is the building with the TMD system. The second case is the building with the MTMD system. Tuned mass dampers are connected to each floor; the masses of these dampers are optimized using the Jaya algorithm. In the third case, the building is fitted with an MTMD system whose damper mass is fixed at 50,000 kg rather than optimized. Comparisons are made between the cases in terms of vibration energy distribution and floor displacement reduction ratio. The results show that all systems are effective in reducing energy vibration and floor displacements. The optimized MTMD system provides superior performance compared to the TMD system, while the fixed-mass MTMD system exhibits lower control efficiency but offers a more practical alternative to a single large TMD installed at the top of the building.

KEYWORDS: Tuned mass damper; multiple-tuned mass damper; floor displacement; vibration energy; Jaya algorithm optimization

1 Introduction

One of the greatest challenges facing buildings is their exposure to forced vibrations caused by undesirable excitations such as earthquakes and winds. This situation becomes even more dangerous in high-rise buildings due to their thinness and flexibility [1]. In many cases, forced vibrations cause significant failure in the main structural elements, ultimately leading to the complete collapse of the structure [2]. Therefore, structural controls play a crucial role in reducing vibration generated by both earthquakes and winds [3]. To mitigate the impact of earthquakes and winds on structures, researchers have sought to find optimal solutions for reducing the structure's response. They have focused on adding elements with specific mass, stiffness, and damping properties to the floors of the building. The primary function of these elements is to control vibration by dissipating the energy. This type is called passive vibration control and is the most popular among other types. The first person to use this concept was the German scientist Frahm [4], who invented and patented a device that dissipates vibration energy. One of the most important conclusions of his research was that this device is only effective when the natural frequency of the structure is close to the excitation frequency. This device was later modified by using damping elements, creating the Tuned Mass Damper (TMD) [5].

Although the tuned mass damper offers significant advantages in damping the energy vibration in structures, it also has significant drawbacks that cannot be ignored. Among these is that this system only affects control of a specific mode and therefore may not be effective in controlling the other modes [6]. Furthermore, the mass required for production is very large, requiring adequate space, and the large mass makes maintenance difficult. All these factors have led to significant interest in developing a system similar to the tuned mass damper that is both effective in continuous damping across different modes and easy to install and maintain. This system is a multiple-tuned mass damper. It was first used in the John Hancock Building in the United States in the 1970s. Studies have demonstrated the significant contribution of the Multiple Tuned Mass Damper (MTMD) in damping the vibration of the buildings [7].

Many studies have described the effectiveness of multiple-tuned mass dampers in damping building vibration compared to tuned mass dampers. Igusa and Xu investigated the effectiveness of multiple-tuned mass dampers in damping the vibration of a building due to wind loads [8]. They designed the mass parameters using the optimum solution and then compared the results with the single-tuned mass damper case for the same building. From the results, it was found that multiple-tuned mass dampers were more effective and had higher efficiency than the single-tuned mass damper. Li and Liu compared tuned mass damper and multiple tuned mass damper systems by considering soil-structure interaction in irregular structures and found that the MTMD was more effective than the TMD [9]. Wang and Lin investigated the effect of multiple tuned mass dampers on an irregular torsional structure subjected to seismic-induced vibration [10]. The results showed that multiple tuned mass dampers were more efficient in damping energy than single tuned mass dampers when the effect of soil interaction was considered. Moon investigated the distribution of vertically tuned mass dampers along a 60-story high-rise building [11]. He discussed the effect of this distribution on vibration energy absorption. The results showed that the energy absorption efficiency was significantly higher than that of the conventional installation of a single tuned mass damper at the highest point of the high-rise building. Chen and Wu, in their study of tuned mass dampers and multiple tuned mass dampers, confirmed that the effectiveness of multiple tuned mass dampers in absorbing energy was higher than that of the tuned mass damper, even when the mass values were equal in both cases [12]. Brandao and Miguel investigated a steel building subjected to multiple seismic excitations [13]. They then examined the building in three different cases. In the first case, a tuned mass damper was installed on the top floor. In the second case, multiple tuned mass dampers were installed on all floors. In the third case, multiple tuned mass dampers were installed on the top floor. They calculated the mass damper parameters using a metaheuristic optimization algorithm. The results showed that all cases performed well in reducing floor displacements. Djerouni et al. [14] proposed MTMDs for adjacent structures and a hybrid metaheuristic method was used to find optimum design values. As a result of the study, MTMDs that are distributed along the height of the adjacent buildings perform better. Wang et al. proposed an Adaptive-Passive Variable-Mass Multiple Tuned Mass Damper (APVM-MTMD) system to mitigate detuning effects in MTMD applications for large-span floor structures. In their approach, each damper can retune its frequency by adjusting its mass based on dominant structural frequencies identified via the Short Time Fourier Transformation (STFT), resulting in improved vibration control compared to mistuned passive MTMD systems. Although effective, the system requires sensing, control hardware, and mechanical mass-adjustment mechanisms, which increase complexity relative to fully passive optimized MTMD solutions [15].

Tuned mass dampers and multiple-tuned mass dampers do not perform well without tuning. Determining the optimum values of these parameters is crucial for achieving good vibration energy absorption results. Numerous researchers have conducted extensive research on how to determine these values, using various equations, called algorithms, to calculate optimal solutions. Mathematical programming and algorithms have proven highly effective in complex optimization processes. These algorithms have been developed by researchers to find general solutions to solve complex problems. The algorithm was tested for 10-story shear frames as well as for controlling the response of a 76-story concrete building under wind excitation. The results showed great effectiveness in vibration damping due to the optimum calculation of the tuned mass damper. Singh et al. presented a study on the optimal design of tuned mass dampers using genetic algorithms to mitigate the seismic response of torsional building systems to seismic excitations [16]. The study provided important insights into the use of genetic algorithms in calculating the optimum parameters of tuned mass dampers. Frans and Arfiadi conducted a study to determine the parameters of multiple-tuned mass dampers using Hybrid Coded Genetic Algorithms (HCGAs) [17]. The study also focused on determining the optimal location of the mass dampers. For this purpose, they examined three different cases: the first is a three-story building, the second is a 10-story building, and the third is a 40-story building. The results showed great effectiveness in absorbing vibration energy after determining the optimal values and location of the mass dampers. Bekdaş et al. used the new Bat algorithm to calculate the mass, period, and damping ratio of a 10-story building [18]. They then calculated the previous parameters using analytical methods and other methods and compared the results between the previous methods. The results showed that the new Bat algorithm yielded good results, leading to good building behavior. Cetin and Ayden calculated the parameters of a tuned mass damper using the transfer function [19]. They calculated the mass, damping, and stiffness of the tuned mass damper using this function. Thus, they reduced the response of the structure in terms of displacement and acceleration caused by seismic excitations. The results showed that the transfer function was highly effective in reducing the response of the structure. Ozturk et al. discussed how to calculate the optimum parameters of tuned mass dampers for a 10-story building and how to determine the optimum location of the tuned mass dampers [20]. Kaveh et al. determined the optimum parameters of a tuned mass damper using a novel Chaotic Optimization Algorithm (COA) [21]. Wang et al. investigated a seismic multi-objective stochastic optimization strategy for MTMD systems applied to a large podium–twin towers structure. Using a Kanai–Tajimi-based stochastic earthquake model and the artificial fish swarm algorithm, they demonstrated that optimally distributed MTMDs can effectively mitigate coupled translational and torsional seismic responses under bi-directional excitations [22]. Ghojehbiglou et al. investigated the efficiency of a hybrid nature-inspired optimization algorithm for tuning multiple tuned mass dampers in buildings considering soil–structure interaction. By combining the Marine Predators Algorithm with Particle Swarm Optimization and applying it to a 10-story building under various earthquake spectra, they showed that optimally distributed TMDs can significantly reduce seismic responses, with performance strongly influenced by soil conditions [23].

Several recent studies have investigated the use of inerter-based multiple tuned mass damper systems for improving the seismic performance of building structures. For instance, Shahrouzi et al. proposed an optimization approach for multiple tuned mass damper inerter (MTMDI) systems using the escaping bird search algorithm to enhance seismic control of buildings [24]. Similarly, Djerouni et al. examined the effectiveness of MTMDI systems in mitigating earthquake-induced vibrations in structures [25]. A multi-objective optimization framework for inerter-based mass dampers was presented by Garrido et al. [26]. In another study, Wang et al. developed a root-locus-based optimal formulation for structures equipped with tuned mass damper inerters enhanced with negative stiffness while accounting for non-resonant modal contributions [27]. Additionally, Kiran et al. investigated the optimization of tuned mass damper inerter systems for both base-isolated and fixed-base structures under seismic excitations [28].

The aim of this study is to compare the effects of tuned mass dampers (TMD) and multiple tuned mass dampers (MTMD) in reducing floor displacements and vibration energy in a 10-story building subjected to seismic activity. A 10-story building was examined for this purpose. Three main cases were then examined.

The first case involved a tuned mass damper installed on the tenth floor. The second case involved multiple-tuned mass dampers installed on all floors. The third case involved multiple tuned mass dampers installed on all floors. However, the difference between this case and the previous one is that the damper mass was assumed to be 50,000 kg. The stiffness and damping ratio values of the masses connected to each floor were also calculated using the Jaya algorithm, and the optimum solution was achieved. The vibration energy and floor displacements resulting from the seismic effect were calculated for each case. Finally, a comparison was made between the previous three cases.

Despite extensive studies on TMD and MTMD systems, the practical implications of using optimized vs fixed damper mass distributions within MTMD configurations have not been sufficiently quantified under a unified optimization framework. The main contribution of this paper is a systematic comparison of TMD, optimized MTMD, and fixed-mass MTMD systems using the same Jaya-based optimization approach, providing clear insights into the trade-off between seismic performance, energy dissipation efficiency, and practical applicability.

2 Conceptual Framework

2.1 Tuned Mass Damper System (TMD)

A tuned mass damper is a device that reduces the amplitude of vibrations resulting from forced movements of structures. A tuned mass damper operates by adjusting its frequency to match the frequency of the structure. In this case, the tuned mass damper absorbs the vibration energy, creating a force equal and opposite to the vibration force, which damps the vibration. The dynamic behavior of a tuned mass damper is expressed by the equation of motion for a system with two degrees of freedom. Fig. 1 shows the tuned mass damper system.

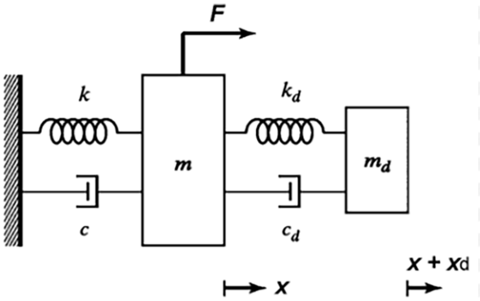

Figure 1: Tuned mass damper system (TMD).

The equation includes the displacement, velocity, and acceleration of the tuned mass damper. The equation of motion of the Tuned Mass Damper (TMD) system associated with a Single Degree of Freedom (SDOF) structure under seismic excitation is as follows:

$$[\boldsymbol{M}]\,\ddot{X} + [\boldsymbol{C}]\,\dot{X} + [\boldsymbol{K}]\,X = -[M]\,1\ddot{X}_g \tag{1}$$

here:

$\ddot{X}$: Acceleration vector, $\dot{X}$: Velocity vector, X: Displacement vector, $\ddot{X}_g$: Ground acceleration.

The mass matrix is given by the following equation:

$$[\boldsymbol{M}] = \begin{bmatrix} m_1 & 0 \\ 0 & md \end{bmatrix} \tag{2}$$

here:

m_1: Mass of the structure, m_d: Mass of the tuned mass damper.

The damping matrix is given by the following equation:

$$[\boldsymbol{C}] = \begin{bmatrix} c_1 & 0 \\ 0 & c_d \end{bmatrix} \tag{3}$$

here:

c_1: Damping of the structure, c_d: Damping of the tuned mass damper.

The stiffness matrix is given by the following equation:

$$[\boldsymbol{K}] = \begin{bmatrix} k_1 & 0 \\ 0 & k_d \end{bmatrix} \tag{4}$$

here:

k_1: Stiffness of the structure, k_d: Stiffness of the tuned mass damper.

Eq. (7) can be written as follows:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_d \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_d \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_d \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_d \end{Bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_d \end{bmatrix} \begin{Bmatrix} x_1 \\ x_d \end{Bmatrix} = - \begin{bmatrix} m_1 \\ m_d \end{bmatrix} \ddot{X}_g \tag{5}$$

2.2 Motion Equation of a Multiple-Storey Building with Tuned Mass Damper

In this system, the tuned mass damper is placed at the highest point of the building. Furthermore, each floor is assumed to have one degree of freedom. The tuned mass damper also has one degree of freedom. Therefore, this system consists of $(n + 1)$ degrees of freedom. Fig. 2 shows the tuned mass damper system:

The equation of motion of the previous system is similar to Eq. (1). The mass matrix of the system is diagonal matrix. This matrix is written as follows:

$$\boldsymbol{M} = \begin{bmatrix} m_1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & m_2 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & m_3 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & m_{n-1} & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & m_n & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & m_d \end{bmatrix} \tag{6}$$

here:

$m_1, m_2 \dots . m_{n-1}, m_n$: Masses of the floors of the structure.

m_d: Mass of the tuned mass damper. The system's stiffness matrix is written as follows:

$$\boldsymbol{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & \dots & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \dots & 0 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & k_{n-1} + k_n & -k_n & 0 \\ 0 & 0 & 0 & \dots & -k_n & k_n + k_d & -k_d \\ 0 & 0 & 0 & \dots & 0 & -k_d & k_d \end{bmatrix} \tag{7}$$

here:

$k_1, k_2 \ldots .k_{n-1}, k_n$: Stiffnesses of the floors of the structure.

k_d: Stiffness of the tuned mass damper. The damping matrix of the system is written as follows:

$$\boldsymbol{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & \ldots & 0 & 0 & 0 \\ -c_2 & c_2 + c_3 & -c_3 & \ldots & 0 & 0 & 0 \\ 0 & -c_3 & c_3 + c_4 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & c_{n-1} + c_n & -c_n & 0 \\ 0 & 0 & 0 & \ldots & -c_n & c_n + c_d & -c_d \\ 0 & 0 & 0 & \ldots & 0 & -c_d & c_d \end{bmatrix} \tag{8}$$

here:

$c_1, c_2 \ldots, c_{n-1}, c_n$: Damping of the floors of the structure.

c_d: Damping of the tuned mass damper.

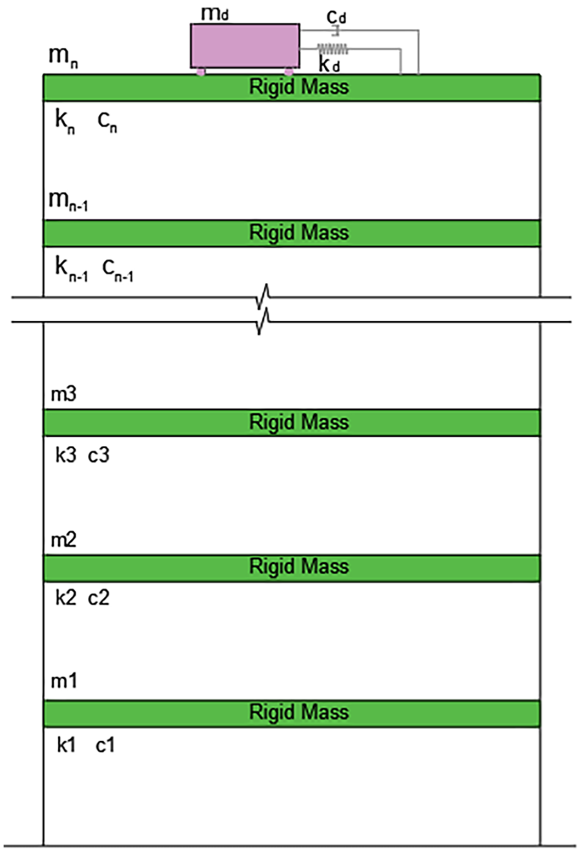

Figure 2: System of multiple-storey building with TMD.

2.3 Motion Equation of a Multiple-Storey Building with a Multiple-Tuned Mass Damper

A multiple-tuned mass damper (MTMD) is more effective than a single-tuned mass damper because each layer is connected to a small mass damper. This system contributes to the effective damping of vibrations induced by seismic excitations. Fig. 3 shows a multiple-tuned mass damper system:

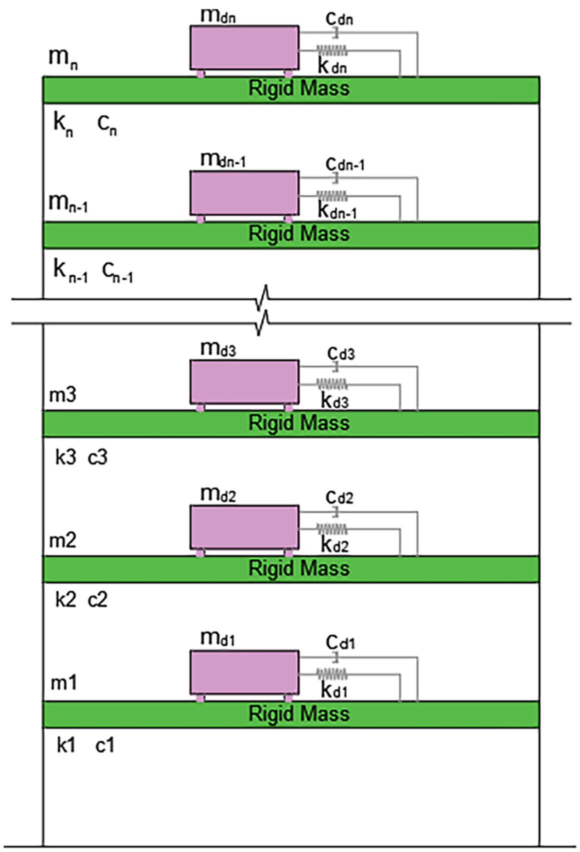

Figure 3: System of multiple-storey building with MTMD.

The equation of motion of the previous system is similar to Eq. (1). The mass matrix of the system is diagonal. This matrix is written as follows:

$$\boldsymbol{M}=\begin{bmatrix} m_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & m_2 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & m_3 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & m_{n-1} & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & m_n & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & m_{d1} & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & m_{d2} & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & m_{d3} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & m_{dn-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & m_{dn} \end{bmatrix} \tag{9}$$

here:

$m_1, m_2, \ldots, m_{n-1}, m_n$: Masses of the floors of the structure.

$m_{d1}, m_{d2}, \ldots, m_{dn-1}, m_{dn}$: Mass of the tuned mass damper.

The system's stiffness matrix is written as follows:

$$\boldsymbol{K}=\begin{bmatrix} k_1+k_2+k_{d1} & -k_2 & \cdots & 0 & -k_{d1} & 0 & \cdots & 0 & 0 \\ -k_2 & k_2+k_3+k_{d2} & \cdots & 0 & 0 & -k_{d2} & \cdots & 0 & 0 \\ 0 & -k_3 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -k_{dn-1} & 0 \\ 0 & 0 & \cdots & k_{n-1}+k_n+k_d & -k_n & 0 & \cdots & 0 & 0 \\ -k_{d1} & 0 & \cdots & -k_n & k_{d1} & 0 & \cdots & 0 & -k_{dn} \\ 0 & -k_{d2} & \cdots & 0 & 0 & k_{d2} & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & -k_{dn-1} & 0 & 0 & \cdots & k_{dn-1} & 0 \\ 0 & 0 & 0 & 0 & -k_{dn} & 0 & \cdots & 0 & k_{dn} \end{bmatrix} \tag{10}$$

here:

$k_1, k_2, \ldots, k_{n-1}, k_n$: Stiffnesses of the floors of the structure.

$k_{d1}, k_{d2}, \ldots, k_{dn-1}, k_{dn}$: Stiffness of the tuned mass damper.

The damping matrix of the system is written as follows:

$$\boldsymbol{C}=\begin{bmatrix} c_1+c_2+c_{d1} & -c & \cdots & 0 & -c_{d1} & 0 & \cdots & 0 & 0 \\ -c & c_2+c_3+c_{d2} & \cdots & 0 & 0 & -c_{d2} & \cdots & 0 & 0 \\ 0 & -c_3 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -c_{dn-1} & 0 \\ 0 & 0 & \cdots & c_{n-1}+c_n+c_d & -c_n & 0 & \cdots & 0 & 0 \\ -c_{d1} & 0 & \cdots & -c_n & c_{d1} & 0 & \cdots & 0 & -c_{dn} \\ 0 & -c_{d2} & \cdots & 0 & 0 & c_{d2} & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & -c_{dn-1} & 0 & 0 & \cdots & c_{dn-1} & 0 \\ 0 & 0 & 0 & 0 & -c_{dn} & 0 & \cdots & 0 & c_{dn} \end{bmatrix} \tag{11}$$

here:

$c_1, c_2, \ldots, c_{n-1}, c_n$: Damping of the floors of the structure.

$c_{d1}, c_{d2}, \ldots, c_{dn-1}, c_{dn}$: Damping of the tuned mass damper.

3 Method

Metaheuristics are regarded as a higher-level procedure to find the optimum of a problem or tune a system or algorithm. These algorithms use iterative and random methods that can be used with problems that cannot be fully mathematically formalized. Generally, an inspiration is used in the metaheuristics to reach optimum solutions.

The Jaya algorithm was proposed by Rao [29]. It is considered simple yet powerful because it converges to the best solutions and avoids the worst ones. The Jaya equation is an optimization equation designed for both constrained and unconstrained solutions that depend on the population. In the Jaya equation phase, the design constants, design variables, and population size must be defined. Then, the design variables of the initial solution matrix must be randomly determined according to their lower and upper limits. The following equations are used for this purpose:

$$m_d = m_{d\,min} + rand(m_{d\,max} - m_{d\,min}) \tag{12}$$

$$T_d = T_{d\,min} + rand(T_{d\,max} - T_{d\,min}) \tag{13}$$

$$C_r = C_{r\,min} + rand(C_{r\,max} - C_{r\,min}) \tag{14}$$

here: m_d: Mass of the tuned mass damper. T_d: Period of the tuned mass damper. C_r: Damping ratio. The basic equation of the Jaya algorithm is as follows:

$$X_{i,new} = X_{i,j} + r_1\left(X_{i,g\,the\,best} - \left|X_{i,j}\right|\right) - r_2\left(X_{i,g\,the\,worst} - \left|X_{i,j}\right|\right) \tag{15}$$

here: $X_{i,new}$: The updated value of the design variable. $X_{i,j}$: The current value of the design variable. $X_{i,g\,the\,best}$: The value of the variable for the best candidate solution. $X_{i,g\,the\,worst}$: The value of the variable for the worst candidate solution. r_1, r_2: Random numbers between 0 and 1.

To ensure reproducibility, the implementation details of the Jaya algorithm are explicitly defined in this study. The optimization procedure was coded in MATLAB, and the design variables (damper mass, period, and damping ratio) were bounded according to Eqs. (16)–(19). A population size of 20 was adopted and the algorithm was terminated after a predefined number of iterations once convergence of the objective function was observed. At each iteration, candidate solutions were updated using Eq. (15), where the current solution is moved toward the best solution and away from the worst solution in the population. Random numbers uniformly distributed between 0 and 1 were generated at each step to maintain exploration capability.

The Jaya algorithm, proposed by Rao, was selected because it is a parameter-less metaheuristic algorithm that does not require algorithm-specific control parameters such as crossover rate, mutation rate, inertia weight, or acceleration coefficients, which are required in algorithms like Genetic Algorithms or Particle Swarm Optimization. This simplicity improves transparency and reproducibility while reducing the risk of biased tuning of algorithmic parameters. Furthermore, Jaya simultaneously promotes convergence toward the best solution and divergence from the worst solution, enhancing both exploitation and exploration capabilities. Since the MTMD optimization problem involves multiple continuous design variables and nonlinear dynamic structural responses, a robust and computationally efficient population-based optimizer without additional calibration requirements is particularly suitable. Therefore, the Jaya algorithm provides a reliable and unbiased framework for the optimal tuning of MTMD systems.

3.1 Definition of Structural Building

This study focuses on improving the behavior of a 10-story building subjected to seismic excitation using a tuned mass damper for the first case and a multiple-tuned damper for the second. The acceleration, velocity, and displacement of the floors were obtained using MATLAB and Simulink. The Jaya algorithm was also used to tune the mass damper parameters. Each floor in the building has the same properties for mass (50,000 kg), stiffness (900 MN/m), and damping coefficient (7 MNs/m). The building is subjected to seismic excitation and it is the El-Centro earthquake.

Figs. 4 and 5 show the TMD & MTMD system for the two basic cases of 10-story structure.

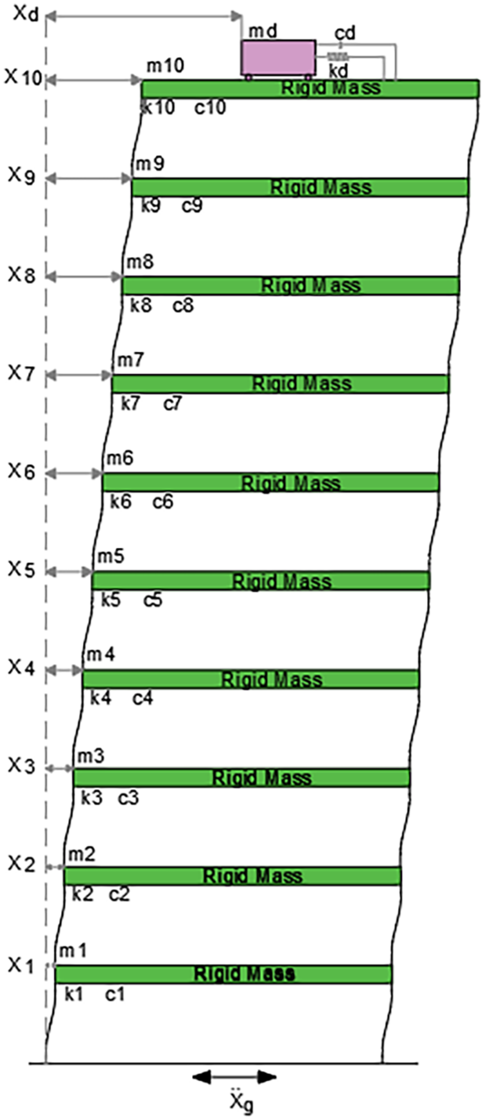

Figure 4: TMD system on 10-story structure.

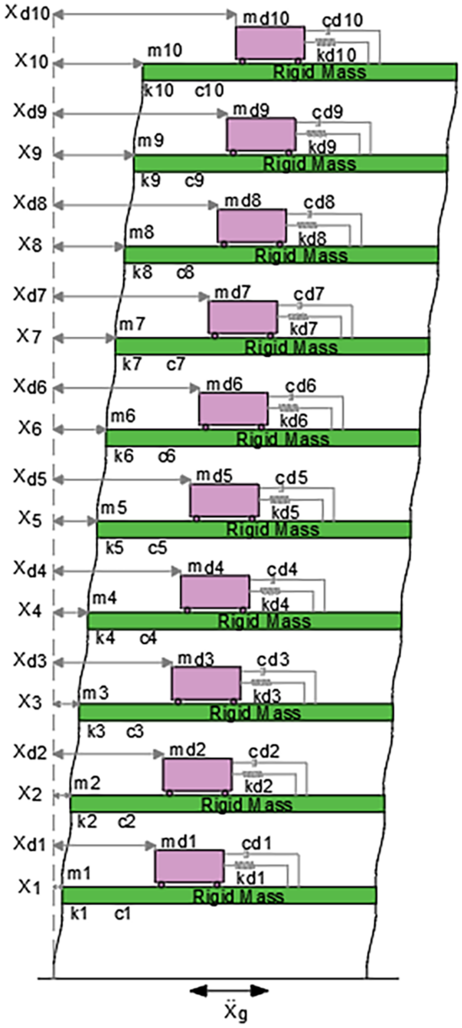

Figure 5: MTMD system on 10-story structure.

where:

$m_1, m_2, m_3, \ldots, m_9, m_{10}$: Floors masses.

$c_1, c_2, c_3, \ldots, c_9, c_{10}$: Floors damping.

$k_1, k_2, k_3, \ldots, k_9, k_{10}$: Floors stiffness.

m_d: The mass of tuned mass damper (TMD case).

c_d: The damping of tuned mass damper (TMD case).

k_d: The stiffness of tuned mass damper (TMD case).

$m_{d1}, m_{d2}, m_{d3}, \ldots, m_{d9}, m_{d10}$: The mass of tuned mass dampers (MTMD case).

$c_{d1}, c_{d2}, c_{d3}, \ldots, c_{d9}, c_{d10}$: The damping of tuned mass dampers (MTMD case).

$k_{d1}, k_{d2}, k_{d3}, \ldots, k_{d9}, k_{d10}$: The stiffness of tuned mass dampers (MTMD case).

$x_1, x_2, x_3, \ldots, x_9, x_{10}$: Floors displacements.

$x_{d1}, x_{d2}, x_{d3}, \ldots, x_{d9}, x_{d10}$: Diplacements of tuned mass dampers (MTMD case).

3.2 1st Case: Tuned Mass Damper (TMD)

In this case, a tuned mass damper is connected to the 10th floor. Because of the TMD connection, the number of degrees of freedom in this system is 11. The mass, stiffness, and damping matrix must have rows and columns of 11 × 11.

3.2.1 Definition of Design Variables and Population Numbers

The design variables are related to the parameters of the tuned mass damper, including the mass, period, and damping ratio. The values of these parameters are adopted according to the following:

$$m_{dmax} = 0.1 \times \left(\sum_{n=10}^{n=1} m_n \right) \tag{16}$$

$$m_{dmin} = 0.005 \times \left(\sum_{n=10}^{n=1} m_n \right) \tag{17}$$

$$T_{dmax} = \frac{3\pi}{\omega_{min}} \tag{18}$$

$$T_{dmin} = \frac{0.25\pi}{\omega_{min}} \tag{19}$$

$\mu_{max} = 0.5$, $\mu_{min} = 0.01$.

where:

m_d: Mass of the tuned mass damper (kg).

m_n: Mass of the floors (kg), $n = 1, 2, 3, \ldots, 10$.

T_d: Period of the tuned mass damper (s).

μ: Damping ratio.

ω: Building frequency (rad/s).

$P_n = 20$: Population number.

3.2.2 Definition of the Objective Function

The objective function is to reduce the displacement of the floors. It can be expressed as follows:

$$f\left(X\right) = \max(|x_1\, x_2\, x_3\, \ldots\, x_9\, x_{10}|) \tag{20}$$

here:

$x_1, x_2, x_3, \ldots, x_9, x_{10}$: Floors displacements.

3.2.3 Definition of Design Constraints

The adopted design constraints can be defined as follows:

$$g\left(x\right) = \frac{\max(|x_d - x_{10}|)_{with\,TMD}}{|x_{10}|_{without\,TMD}} \leq st_{max} \tag{21}$$

where:

x_d: Displacement of the tuned mass damper,

x_{10}: Displacement of the tenth floor,

st_{max}: Maximum limit of the design constraint ($st_{max} = 2$).

3.3 2nd Case: Multiple-Tuned Mass Dampers (MTMD)

This case involves attaching a small tuned mass damper to each floor. The tuned mass dampers have a specific stiffness, damping, and mass, resulting in 20 degrees of freedom. The mass, stiffness, and damping matrix should have a 20 × 20 row and column size.

3.3.1 Definition of Design Variables and Population Numbers

$$T_{dmax} = \frac{3\pi}{\omega_{min}} \tag{22}$$

$$T_{dmin} = \frac{0.25\pi}{\omega_{min}} \tag{23}$$

$\mu_{max} = 0.5$, $\mu_{min} = 0.01$

$P_n = 20$: Population number.

The lower and upper limit values of the tuned mass dampers were investigated for two cases:

- Case A: The variables of the masses are as follows:

$$m_{dmax} = 0.1 \times \left(\sum_{n=10}^{n=1} m_n\right), m_{dmin} = 0.005 \times \left(\sum_{n=10}^{n=1} m_n\right)$$

- Case B: Using a constant value of mass:

$$m_d = 50,000\ kg$$

3.3.2 Definition of the Objective Function

The equation of the objective function for this case is the same equation mentioned in Section 3.2.2.

3.3.3 Definition of Design Constraints

$$g\left(x\right) = \frac{\max(|x_{dn} - x_n|)_{with\ TMD}}{|x_n|_{without\ TMD}} \leq st_{max} \tag{24}$$

4 Results

In this study, the effectiveness of a tuned mass damper (TMD) and a multiple-tuned mass damper (MTMD) system in dissipating vibration energy and reducing floor displacements was investigated for a 10-story building under the seismic excitation, which is the El-Centro earthquake. Each floor in the building has a specific stiffness, mass, and damping coefficient. The building has been analyzed using MATLAB and Simulink. Three different cases have been examined as follows:

1. The first case is the use of the TMD system connected to the 10th floor. The mass, stiffness, and damping parameters of the TMD are calculated using the Jaya algorithm.
2. The second case is the use of the MTMD system connected to all floors. The mass, stiffness, and damping parameters of the MTMD are calculated using the Jaya algorithm. This case is called MTMD-A.

3. The third case is the use of the MTMD system. However, in this case, the difference from the previous case is that the MTMD masses are set at 50,000 kg. The remaining MTMD parameters are calculated using the Jaya algorithm. This case is called MTMD-B.

4.1 The First Case: The Building with TMD

4.1.1 Stiffness, Mass and Damping Matrices of the System

The ideal properties of the tuned mass damper can be obtained from the above matrices as shown in Table 1.

Table 1: The properties of the tuned mass damper.

Mass (m_d)	Stiffness (k_d)	Damping (c_d)	Period	Damping Ratio
kg	N/m	N s/m	s	
5×10^5	1.2145×10^7	7×10^6	1.2718	0.2955

4.1.2 The Floors Displacements

The results show a significant improvement in floor displacements compared to the floor displacements in buildings without any damping system. The following tables (Tables 2 and 3) show the floor displacement values for all floors every 5 s:

Table 2: Floor displacement values for the case of the building without TMD (m).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	0.0273	0.0111	0.0027	0.0032	0.0018	0.0026
2nd Floor	0.0545	0.0212	0.0046	0.0061	0.0034	0.0054
3rd Floor	0.0806	0.0288	0.0107	0.0100	0.0059	0.0073
4th Floor	0.1046	0.0376	0.0130	0.0134	0.0078	0.0103
5th Floor	0.1253	0.0322	0.0169	0.0168	0.0101	0.0123
6th Floor	0.1434	0.0487	0.0187	0.0197	0.0109	0.0144
7th Floor	0.1594	0.0525	0.0212	0.0205	0.0127	0.0151
8th Floor	0.1722	0.0580	0.0238	0.0234	0.0137	0.0172
9th Floor	0.1816	0.0579	0.0250	0.0252	0.0162	0.0172
10th Floor	0.1836	0.0600	0.0260	0.0257	0.0163	0.0178

Table 3: Floor displacement values for the case of the building with TMD (m).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	0.0104	0.0018	0.0025	0.0018	0.0009	0.0008
2nd Floor	0.0216	0.0031	0.0041	0.0037	0.0019	0.0014
3rd Floor	0.0326	0.0043	0.0057	0.0064	0.0034	0.0027

(Continued)

Table 3 (continued)

Floor No.	Time (s)					
	5	10	15	20	25	30
4th Floor	0.0421	0.0049	0.0082	0.0083	0.0043	0.0052
5th Floor	0.0503	0.0031	0.0095	0.0089	0.0058	0.0032
6th Floor	0.0565	0.0021	0.0138	0.0101	0.0064	0.0051
7th Floor	0.0661	0.0023	0.0146	0.0124	0.0075	0.0056
8th Floor	0.0721	0.0024	0.0161	0.0148	0.0099	0.0062
9th Floor	0.0759	0.0034	0.0186	0.0149	0.0086	0.0062
10th Floor	0.0776	0.0040	0.0190	0.0171	0.0089	0.0069

Table 4 shows the improvement in displacement resulting from adding the TMD system.

Table 4: Percentage improvement in displacement (TMD case).

Floor No.	Time (s)						
	5	10	15	20	25	30	Average
1st Floor	62%	84%	7%	44%	50%	69%	53%
2nd Floor	60%	85%	11%	39%	44%	74%	52%
3rd Floor	60%	85%	47%	36%	42%	63%	56%
4th Floor	60%	87%	37%	38%	45%	50%	53%
5th Floor	60%	90%	44%	47%	43%	74%	60%
6th Floor	61%	96%	26%	49%	41%	65%	56%
7th Floor	59%	96%	31%	40%	41%	63%	55%
8th Floor	58%	96%	32%	37%	28%	64%	53%
9th Floor	58%	94%	26%	41%	47%	64%	55%
10th Floor	58%	93%	27%	33%	45%	61%	53%
Average improvement of displacement in the building							53%

Fig. 6 shows that the TMD system provides an average displacement reduction of about 53%–54% compared to the uncontrolled building.

4.1.3 Dissipation of Vibrational Energy

Kinetic and potential energy were calculated for all floors of the building, with and without TMD. The resulting energy values are shown in the tables below (Tables 5–11).

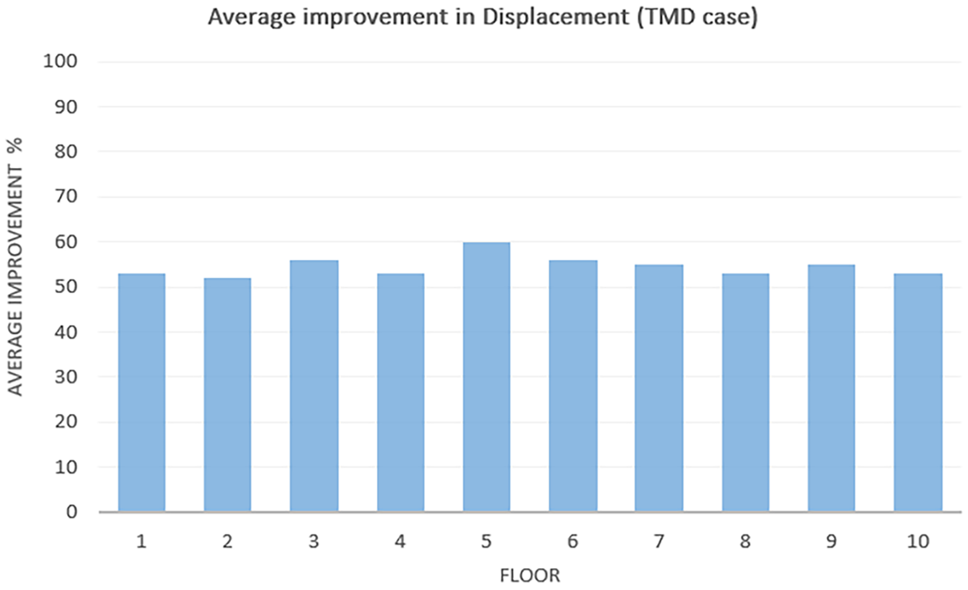

Figure 6: Average improvement in displacement for TMD case (%).

Table 5: Potential energy of floors for the building without TMD (J).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	335,381	55,445	3281	4608	1458	3042
2nd Floor	1,336,613	202,248	9522	16,745	5202	13,122
3rd Floor	2,923,362	373,248	51,521	45,000	15,665	23,981
4th Floor	4,923,522	636,192	76,050	80,802	27,378	47,741
5th Floor	7,065,041	466,578	128,525	127,008	45,905	68,081
6th Floor	9,253,602	1,067,261	157,361	174,641	53,465	93,312
7th Floor	1,1433,762	1,240,313	202,248	189,113	72,581	102,605
8th Floor	13,343,778	1,513,800	254,898	246,402	84,461	133,128
9th Floor	14,840,352	1,508,585	281,250	285,768	118,098	133,128
10th Floor	15,169,032	1,620,000	304,200	297,221	119,561	142,578

Table 6: Kinetic energy of floors for the building without TMD (J).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	6249	467	101	250	56	85
2nd Floor	23,948	5134	726	853	344	315
3rd Floor	48,378	11,385	1471	2107	726	541
4th Floor	76,259	17,305	2209	3221	1336	1116
5th Floor	110,058	22,171	2601	4914	2338	1332
6th Floor	148,726	27,308	2873	6989	2647	1798

(Continued)

Table 6 (continued)

Floor No.	Time (s)					
	5	10	15	20	25	30
7th Floor	193,292	30,276	3528	9419	3770	2098
8th Floor	237,803	36,960	3709	10,609	3988	2167
9th Floor	273,948	43,660	3147	12,410	4830	2673
10th Floor	295,121	53,430	3534	11,621	4907	3003

Table 7: Total vibration energy of floors for the building without TMD (J).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	341,629	55,911	3382	4858	1514	3127
2nd Floor	1,360,560	207,382	10,248	17,597	5546	13,437
3rd Floor	2,971,740	384,633	52,991	47,107	16,391	24,521
4th Floor	4,999,781	653,497	78,259	84,023	28,714	48,856
5th Floor	7,175,099	488,749	131,126	131,922	48,242	69,413
6th Floor	9,402,328	1,094,568	160,233	181,629	56,112	95,110
7th Floor	11,627,054	1,270,589	205,776	198,531	76,350	104,702
8th Floor	13,581,581	1,550,760	258,607	257,011	88,448	135,295
9th Floor	15,114,300	1,552,245	284,397	298,178	122,928	135,801
10th Floor	15,464,153	1,673,430	307,734	308,841	124,468	145,581

Table 8: Potential energy of floors for the building with TMD (J).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	48,672	1458	2813	1458	365	288
2nd Floor	209,952	4325	7565	6161	1625	882
3rd Floor	478,242	8321	14,621	18,432	5202	3281
4th Floor	797,585	10,805	30,258	31,001	8321	12,168
5th Floor	1,138,541	4325	40,613	35,645	15,138	4608
6th Floor	1,436,513	1985	85,698	45,905	18,432	11,705
7th Floor	1,966,145	2381	95,922	69,192	25,313	14,112
8th Floor	2,339,285	2592	116,645	98,568	44,105	17,298
9th Floor	2,592,365	5202	155,682	99,905	33,282	17,298
10th Floor	2,709,792	7200	162,450	131,585	35,645	21,425

Table 9: Kinetic energy of floors for the building with TMD (J).

Floor No.	Time (s)					
	5	**10**	**15**	**20**	**25**	**30**
1st Floor	1509	61	85	27	4	5
2nd Floor	5944	306	230	125	298	4
3rd Floor	127	531	745	174	49	6
4th Floor	20,022	628	1112	361	224	16
5th Floor	28,798	402	1600	1246	437	53
6th Floor	36,538	426	1661	2252	469	88
7th Floor	42,560	467	2328	2139	732	77
8th Floor	45,008	543	2762	2798	729	42
9th Floor	42,333	740	2525	3919	524	94
10th Floor	44,479	853	2642	2900	1050	259

Table 10: Total vibration energy of floors for the building with TMD (J).

Floor No.	Time (s)					
	5	**10**	**15**	**20**	**25**	**30**
1st Floor	50,181	1519	2897	1485	369	293
2nd Floor	215,896	4631	7794	6286	1922	886
3rd Floor	478,369	8852	15,366	18,606	5251	3287
4th Floor	817,607	11,432	31,370	31,362	8544	12,184
5th Floor	1,167,339	4727	42,213	36,891	15,575	4661
6th Floor	1,473,051	2411	87,359	48,156	18,901	11,793
7th Floor	2,008,704	2847	98,250	71,331	26,044	14,189
8th Floor	2,384,292	3135	119,406	101,366	44,834	17,340
9th Floor	2,634,698	5942	158,207	103,823	33,806	17,392
10th Floor	2,754,271	8053	165,092	134,484	36,694	21,684

Table 11: Percentage improvement in energy dissipation (TMD case).

Floor No.	Time (s)						
	5	**10**	**15**	**20**	**25**	**30**	**Average**
1st Floor	85%	97%	14%	68%	75%	91%	72%
2nd Floor	84%	98%	21%	63%	68%	93%	71%
3rd Floor	84%	98%	72%	59%	67%	86%	78%
4th Floor	84%	98%	60%	62%	70%	75%	75%
5th Floor	84%	99%	68%	72%	67%	93%	81%
6th Floor	84%	100%	46%	74%	66%	87%	76%
7th Floor	83%	100%	53%	63%	65%	86%	75%
8th Floor	82%	100%	54%	60%	48%	87%	72%
9th Floor	83%	100%	45%	65%	72%	87%	75%

(Continued)

Table 11 (continued)

Floor No.	Time (s)						
	5	10	15	20	25	30	Average
10th Floor	82%	100%	47%	56%	70%	85%	73%
Average improvement of energy dissipation in the building							75%

The energy dissipation rate of the building with the TMD system reached 75% compared to the same building without the damping system. Fig. 7 presents the average improvement in energy dissipation for the TMD case. The results indicate a significant increase in the total dissipated energy compared to the uncontrolled structure. This confirms that TMD not only reduces displacements but also enhances the overall damping capacity of the system by absorbing and dissipating a portion of the input seismic energy.

4.2 Second Case: The Building with MTMD System (MTMD-A)

4.2.1 Stiffness, Mass and Damping Matrices of the System

The ideal properties of multiple-tuned mass dampers can be obtained from the above matrices as shown in Table 12.

Table 12: The properties of tuned mass dampers.

Floor No.	Mass (m_d) kg	Stiffness (k_d) N/m	Damping (c_d) N s/m	Period s	Damping Ratio
1st Floor	217,501	3.887×10^6	8.675×10^5	1.4863	0.4717
2nd Floor	207,850	3.714×10^6	8.437×10^5	1.4863	0.4801
3rd Floor	319,012	5.701×10^6	9.625×10^5	1.4863	0.3568
4th Floor	320,553	5.729×10^6	9.112×10^5	1.4863	0.3362
5th Floor	327,648	5.855×10^6	9.001×10^5	1.4863	0.3249
6th Floor	337,246	6.027×10^6	9.271×10^5	1.4863	0.3251
7th Floor	347,669	6.213×10^6	9.609×10^5	1.4863	0.3269
8th Floor	382,387	6.834×10^6	1.153×10^6	1.4863	0.3565
9th Floor	374,616	6.695×10^6	1.054×10^6	1.4863	0.3328
10th Floor	394,576	7.051×10^6	1.187×10^6	1.4863	0.3557

4.2.2 The Floors Displacements

The floor displacements for the building with MTMD-A is shown in Table 13 for different times of simulation.

By comparing the previous table with Table 3, the improvement rate of floor displacements can be determined according to Table 14.

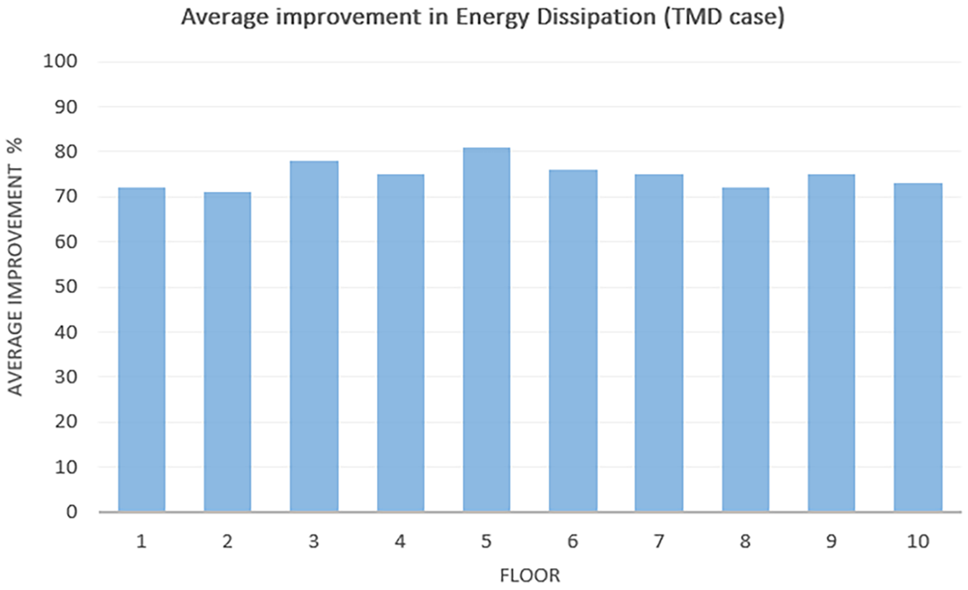

Figure 7: Average improvement in energy dissipation for TMD case (%).

Table 13: Floor displacement values for the building with MTMD-A (m).

Floor No.	**Time (s)**					
	5	**10**	**15**	**20**	**25**	**30**
1st Floor	0.0046	0.0012	0.0006	0.0005	0.0023	0.0007
2nd Floor	0.0101	0.0013	0.0009	0.0015	0.0016	0.0015
3rd Floor	0.0146	0.002	0.0014	0.0022	0.0017	0.0023
4th Floor	0.0194	0.0019	0.0016	0.0022	0.0027	0.0028
5th Floor	0.0231	0.0013	0.0026	0.0043	0.0038	0.0036
6th Floor	0.028	0.0016	0.0033	0.0046	0.0064	0.0041
7th Floor	0.032	0.0024	0.0038	0.0044	0.0072	0.0046
8th Floor	0.035	0.0029	0.0048	0.0042	0.0062	0.0051
9th Floor	0.0362	0.0033	0.0044	0.0047	0.0081	0.0048
10th Floor	0.0354	0.0032	0.0053	0.0053	0.0096	0.0053

Table 14: Percentage improvement in displacement (MTMD-A case).

Floor No.	**Time (s)**						
	5	**10**	**15**	**20**	**25**	**30**	**Average**
1st Floor	83%	89%	78%	84%	28%	73%	63%
2nd Floor	81%	94%	80%	75%	53%	72%	76%
3rd Floor	82%	93%	87%	78%	71%	68%	80%
4th Floor	81%	95%	88%	84%	65%	73%	81%
5th Floor	82%	96%	85%	74%	62%	71%	78%
6th Floor	80%	97%	82%	77%	41%	72%	75%

(Continued)

Table 14 (continued)

Floor No.	Time (s)						
	5	10	15	20	25	30	Average
7th Floor	80%	95%	82%	79%	43%	70%	75%
8th Floor	80%	95%	80%	82%	55%	70%	77%
9th Floor	80%	94%	82%	81%	50%	72%	77%
10th Floor	81%	95%	80%	79%	41%	70%	74%
Average improvement of displacement in the building							76%

According to the previous table, the improvement rate in floor displacements has reached 76%. Fig. 8 presents the average displacement improvement for the MTMD-A case and allows direct comparison with the single TMD results (Fig. 6). Compared to the TMD, MTMD-A provides higher displacement reduction.

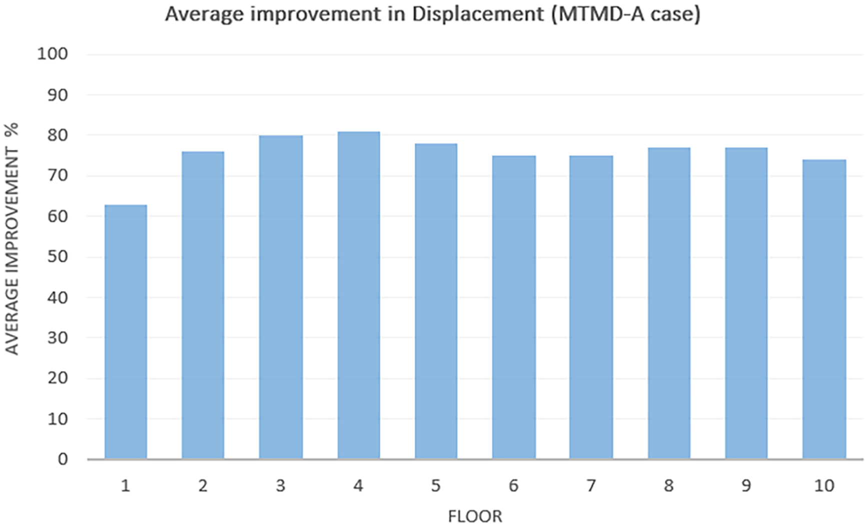

Figure 8: Average improvement in displacement for MTMD-A case (%).

4.2.3 Dissipation of Vibrational Energy

Kinetic and potential energy (Tables 15–17) with MTMD have been calculated for all floors of the building.

Table 15: Potential energy of floors for the building with MTMD-A (J).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	9522	221	162	113	2381	221
2nd Floor	45,905	23,981	365	1013	1152	1013
3rd Floor	95,922	51,521	882	2178	1301	2381
4th Floor	169,362	65,885	1152	2178	3281	3528

(Continued)

Table 15 (continued)

Floor No.	Time (s)					
	5	10	15	20	25	30
5th Floor	240,125	76,050	3042	8321	6498	5832
6th Floor	352,800	105,341	4901	9522	18,432	7565
7th Floor	460,800	33,282	6498	8712	23,328	9522
8th Floor	551,250	1013	10,368	7938	17,298	11,705
9th Floor	589,698	21,425	8712	9941	29,525	10,368
10th Floor	563,922	31,001	12,641	12,641	41,472	12,641

Table 16: Kinetic energy of floors for the building with MTMD-A (J).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	178	95	22	4	81	2
2nd Floor	467	289	19	176	32	5
3rd Floor	1296	443	298	55	93	39
4th Floor	2061	486	210	58	359	19
5th Floor	2490	571	374	155	1250	20
6th Floor	1513	737	504	315	1828	17
7th Floor	3097	949	305	970	1985	1
8th Floor	3306	1303	477	1225	2252	53
9th Floor	3364	1292	281	1296	37	110
10th Floor	3813	1580	600	1351	3600	10

Table 17: Total vibration energy of floors for the building with MTMD-A (J).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	9700	743	184	117	2462	222
2nd Floor	46,371	1050	383	1188	1184	1017
3rd Floor	97,218	2243	1180	2233	1394	2420
4th Floor	171,423	2111	1362	2236	3640	3547
5th Floor	242,615	1332	3416	8476	7748	5852
6th Floor	354,313	1889	5405	9837	20,260	7582
7th Floor	463,897	3541	6803	9682	25,313	9523
8th Floor	554,556	5088	10,845	9163	19,550	11,757
9th Floor	593,062	6193	8993	11,237	29,562	10,478
10th Floor	567,735	6188	13,241	13,991	45,072	12,650

By comparing the previous table with Table 8, the improvement rate of dispersion vibration energy can be determined according to Table 18.

Table 18: Percentage improvement in energy dissipation (MTMD-A case).

Floor No.	Time (s)						
	5	10	15	20	25	30	Average
1st Floor	97%	99%	95%	98%	63%	93%	91%
2nd Floor	97%	99%	96%	94%	78%	92%	93%
3rd Floor	97%	99%	98%	95%	92%	90%	95%
4th Floor	97%	100%	98%	97%	88%	93%	95%
5th Floor	97%	100%	98%	93%	86%	91%	94%
6th Floor	96%	100%	97%	95%	65%	92%	91%
7th Floor	96%	100%	97%	95%	68%	91%	91%
8th Floor	96%	100%	96%	97%	79%	91%	93%
9th Floor	96%	100%	97%	96%	75%	92%	93%
10th Floor	96%	100%	96%	96%	65%	91%	90%
Average improvement of energy dissipation in the building							93%

The energy dissipation rate of the building with the MTMD-A system reached 93% compared to the same building without the MTMD system. The effect of reduction is clearly seen for all stories as seen in Fig. 9.

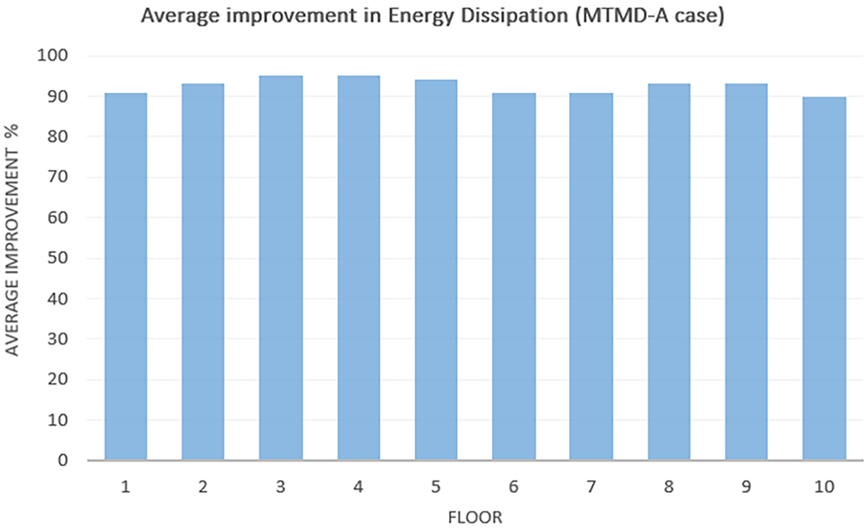

Figure 9: Average improvement in energy dissipation for MTMD-A case (%).

4.3 Third Case: The Building with MTMD System (MTMD-B)

4.3.1 Stiffness, Mass and Damping Matrices of the System

The ideal properties of multiple-tuned mass dampers can be obtained from the above matrices as shown in Table 19.

Table 19: The properties of tuned mass dampers.

Floor No.	Mass (m_d) kg	Stiffness (k_d) N/m	Damping (c_d) N s/m	Period s	Damping Ratio
1st Floor	5×10^4	1.641×10^6	1.113×10^5	1.0966	0.1942
2nd Floor	5×10^4	1.580×10^6	9.261×10^4	1.1177	0.1647
3rd Floor	5×10^4	1.644×10^6	9.650×10^4	1.0958	0.1683
4th Floor	5×10^4	1.867×10^6	9.727×10^4	1.0282	0.1591
5th Floor	5×10^4	2.101×10^6	8.619×10^4	0.9694	0.1329
6th Floor	5×10^4	2.069×10^6	8.803×10^4	0.9768	0.1368
7th Floor	5×10^4	1.866×10^6	9.621×10^4	1.0285	0.1574
8th Floor	5×10^4	1.924×10^6	9.561×10^4	1.0129	0.1541
9th Floor	5×10^4	1.906×10^6	9.665×10^4	1.0177	0.1565
10th Floor	5×10^4	1.869×10^6	9.804×10^4	1.0278	0.1603

4.3.2 The Floors Displacements

The floor displacements for the building with MTMD-B are shown in Table 20 for different times of simulation.

Table 20: Floor displacement values for the building with MTMD-B (m).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	0.0084	0.0023	0.0040	0.0007	0.0019	0.0010
2nd Floor	0.0179	0.0048	0.0085	0.0016	0.0041	0.0025
3rd Floor	0.0266	0.006	0.0126	0.0024	0.0057	0.004
4th Floor	0.034	0.0089	0.0166	0.0036	0.0087	0.005
5th Floor	0.0397	0.0091	0.0196	0.0026	0.0109	0.0055
6th Floor	0.0463	0.0119	0.0235	0.0043	0.0188	0.0065
7th Floor	0.051	0.0132	0.0262	0.0063	0.0219	0.0073
8th Floor	0.0565	0.0157	0.0272	0.0067	0.0239	0.0084
9th Floor	0.059	0.0183	0.0286	0.0063	0.0248	0.0081
10th Floor	0.0589	0.0178	0.0300	0.0070	0.0162	0.0082

By comparing the previous table with Table 3, the improvement rate of floor displacements can be determined according to Table 21.

Table 21: Percentage improvement in displacement (MTMD-B case).

Floor No.	Time (s)						
	5	10	15	20	25	30	Average
1st Floor	69%	79%	−48%	78%	−6%	62%	39%
2nd Floor	67%	77%	−85%	74%	−21%	54%	28%

(Continued)

Table 21 (continued)

Floor No.	Time (s)						
	5	10	15	20	25	30	Average
3rd Floor	67%	79%	−18%	76%	3%	45%	42%
4th Floor	67%	76%	−28%	73%	−12%	51%	38%
5th Floor	68%	72%	−16%	85%	−8%	55%	43%
6th Floor	68%	76%	−26%	78%	−72%	55%	30%
7th Floor	68%	75%	−24%	69%	−72%	52%	28%
8th Floor	67%	73%	−14%	71%	−74%	51%	29%
9th Floor	68%	68%	−14%	75%	−53%	53%	33%
10th Floor	68%	70%	−15%	73%	1%	54%	42%
Average improvement of displacement in the building							35%

The improvement rate in floor displacements has reached 35% and Fig. 10 shows the average improvement in displacement for the MTMD-B case (%).

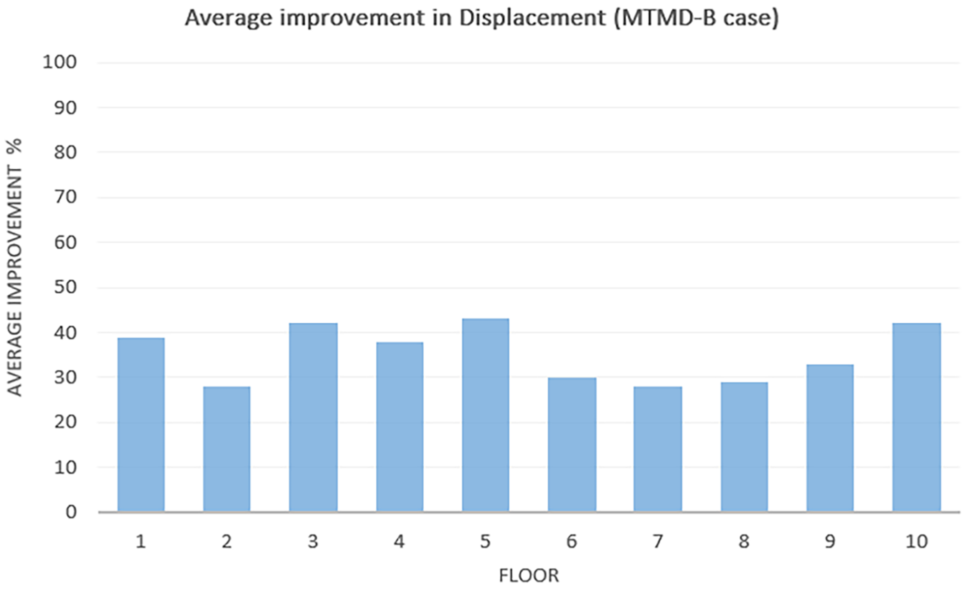

Figure 10: Average improvement in displacement for MTMD-B case (%).

4.3.3 Dissipation of Vibrational Energy

Kinetic and potential energy (Tables 22–24) with MTMD have been calculated for all floors of the building.

Table 22: Potential energy of floors for the building with MTMD-B (J).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	31,752	2381	7200	221	1625	450
2nd Floor	144,185	10,368	32,513	1152	7565	2813
3rd Floor	318,402	16,200	71,442	2592	14,621	7200
4th Floor	520,200	35,645	124,002	5832	34,061	11,250
5th Floor	709,241	37,265	172,872	3042	53,465	13,613
6th Floor	964,661	63,725	248,513	8321	159,048	19,013
7th Floor	1,170,450	78,408	308,898	17,861	215,825	23,981
8th Floor	1,436,513	110,921	332,928	20,201	257,045	31,752
9th Floor	1,566,450	150,701	368,082	17,861	276,768	29,525
10th Floor	1,561,145	142,578	405,000	22,050	118,098	30,258

Table 23: Kinetic energy of floors for the building with MTMD-B (J).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	38	7	261	33	138	2
2nd Floor	112	1444	1089	132	650	38
3rd Floor	202	2465	2435	355	1395	119
4th Floor	281	2804	4376	89	2581	142
5th Floor	536	3863	6257	225	4225	221
6th Floor	767	3900	7534	1656	5373	277
7th Floor	1056	4631	9168	710	7319	261
8th Floor	2435	4754	10,537	2688	7098	396
9th Floor	1218	1343	11,892	3053	10,171	447
10th Floor	1560	2280	12,466	2632	106,50	511

Table 24: Total vibration energy of floors for the building with MTMD-B (J).

Floor No.	Time (s)					
	5	10	15	20	25	30
1st Floor	31,790	2388	7461	254	1763	452
2nd Floor	144,297	11,812	33,602	1284	8215	2850
3rd Floor	318,604	18,665	73,877	2947	16,016	7319
4th Floor	520,481	38,448	128,378	5921	36,641	11,392
5th Floor	709,776	41,127	179,129	3267	57,690	13,833
6th Floor	965,428	67,625	256,047	9977	164,421	19,290
7th Floor	1,171,506	83,039	318,066	18,571	223,143	24,241
8th Floor	1,438,948	115,675	343,465	22,889	264,143	32,148
9th Floor	1,567,668	152,044	379,974	20,913	286,939	29,972
10th Floor	1,562,705	144,858	417,466	24,682	128,748	30,769

By comparing the previous table with Table 7, the improvement rate of dispersion vibration energy can be determined according to Table 25. The average improvement in energy dissipation for the MTMD-B case is plotted in Fig. 11.

Table 25: Percentage improvement in energy dissipation (MTMD-B case).

Floor No.	**Time (s)**						**Average**
	5	**10**	**15**	**20**	**25**	**30**	
1st Floor	91%	96%	−121%	95%	−16%	86%	38%
2nd Floor	89%	94%	−228%	93%	−48%	79%	13%
3rd Floor	89%	95%	−39%	94%	2%	70%	52%
4th Floor	90%	94%	−64%	93%	−28%	77%	44%
5th Floor	90%	92%	−37%	98%	−20%	80%	51%
6th Floor	90%	94%	−60%	95%	−193%	80%	17%
7th Floor	90%	93%	−55%	91%	−192%	77%	17%
8th Floor	89%	93%	−33%	91%	−199%	76%	20%
9th Floor	90%	90%	−34%	93%	−133%	78%	31%
10th Floor	90%	91%	−36%	92%	−3%	79%	52%
Average improvement of energy dissipation in the building							33%

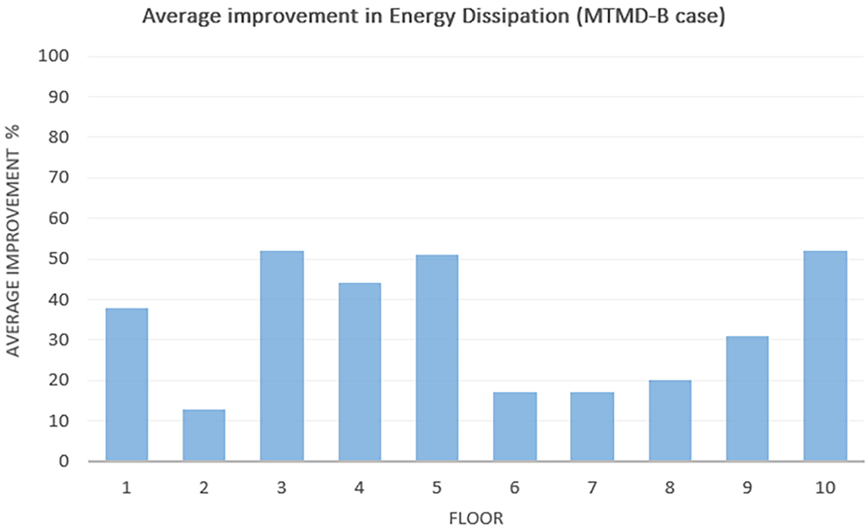

Figure 11: Average improvement in energy dissipation for MTMD-B case (%).

The minus sign in the previous table represents the increase in vibration energy in the MTMD-B case compared to the building without the MTMD system. According to the previous table, the energy dissipation ratio of the building with the MTMD-B system reached 33% compared to the same building without the MTMD system.

4.4 Comparison between TMD System and MTMD Case-A System

4.4.1 The Floors Displacements

The difference in floor displacements between the two cases can be calculated from Tables 3 and 13 considering that the minus sign indicates that the improvement rate of the floor displacements in the TMD case is greater than that in the MTMD case.

According to Table 26, the MTMD-A system provides 60% more improvement in floor displacements than the TMD system.

Table 26: Percentage improvement in displacement between TMD & MTMD-A.

Floor No.	Time (s)						
	5	10	15	20	25	30	Average
1st Floor	77%	40%	123%	113%	−88%	13%	46%
2nd Floor	73%	82%	128%	85%	17%	−7%	63%
3rd Floor	76%	73%	121%	98%	67%	16%	75%
4th Floor	74%	88%	135%	116%	46%	60%	86%
5th Floor	74%	82%	114%	70%	42%	−12%	62%
6th Floor	67%	27%	123%	75%	0%	22%	52%
7th Floor	70%	−4%	117%	95%	4%	20%	50%
8th Floor	69%	−19%	108%	112%	46%	19%	56%
9th Floor	71%	3%	123%	104%	6%	25%	55%
10th Floor	75%	22%	113%	105%	−8%	26%	56%
	Average improvement of displacement in the building						60%

4.4.2 Comparison of Maximum Displacement Values

This comparison provides important information about the percentage improvement in maximum floor displacements between the two cases. Table 27 considers the maximum values at each floor for both cases, regardless of the time at which the displacements occurred.

Table 27: Percentage improvement in maximum displacement between TMD and MTMD-A.

Floor No.	Maximum Displacement		Average
	TMD	MTMD	
1st Floor	0.0131	0.0116	12%
2nd Floor	0.0246	0.0225	8%
3rd Floor	0.0367	0.0326	11%
4th Floor	0.0465	0.0414	11%
5th Floor	0.0533	0.0489	8%
6th Floor	0.0615	0.0551	10%
7th Floor	0.0676	0.0603	17%
8th Floor	0.0718	0.0641	11%
9th Floor	0.0752	0.0661	12%

(Continued)

Table 27 (continued)

Floor No.	Maximum Displacement		Average
	TMD	MTMD	
10th Floor	0.0776	0.0701	9%
	Average improvement		11%

According to Table 27, the MTMD-A system provides 11% more improvement in maximum floor displacements than the TMD system.

4.4.3 Dissipation of Vibrational Energy

The difference in floor vibration energy distribution between the two cases can be calculated from Tables 10 and 17. The minus sign indicates that the improvement rate of the vibration energy distribution at a floor in the TMD case is greater than that in the MTMD-A case. According to Table 28, the MTMD-A system provides 95% more improvement in floor vibration energy distribution than the TMD system.

Table 28: Percentage improvement in vibration energy distribution between TMD and MTMD-A.

Floor No.	Time (s)						
	5	10	15	20	25	30	Average
1st Floor	135%	69%	176%	171%	−148%	27%	72%
2nd Floor	129%	126%	181%	136%	47%	−14%	101%
3rd Floor	132%	119%	171%	157%	116%	30%	121%
4th Floor	131%	138%	183%	173%	81%	110%	136%
5th Floor	131%	112%	170%	125%	67%	−23%	97%
6th Floor	122%	24%	177%	132%	−7%	43%	82%
7th Floor	125%	−22%	174%	152%	3%	39%	79%
8th Floor	125%	−47%	167%	167%	79%	38%	88%
9th Floor	127%	−4%	178%	161%	13%	50%	87%
10th Floor	132%	26%	170%	162%	−20%	53%	87%
Average improvement of energy dissipation in the building							95%

4.5 Comparison between TMD System and MTMD Case-B System

4.5.1 The Floors Displacements

Similarly, the difference in floor displacements between the two cases can be calculated from Tables 3 and 20. The minus sign indicates that the improvement rate of the floor displacements in the TMD case is greater than that in the MTMD case. According to Table 29, the TMD system provides 17% more improvement in floor displacements than the MTMD-B system.

Table 29: Percentage improvement in displacement between TMD & MTMD-B.

Floor No.	Time (s)						
	5	10	15	20	25	30	Average
1st Floor	24%	−22%	−38%	61%	−53%	−20%	−8%
2nd Floor	21%	−35%	−52%	57%	−54%	−44%	−18%
3rd Floor	23%	−28%	−55%	63%	−40%	−33%	−12%
4th Floor	24%	−45%	−51%	57%	−51%	4%	−10%
5th Floor	27%	−66%	−52%	71%	−47%	−42%	−18%
6th Floor	22%	−82%	−41%	57%	−66%	−22%	−22%
7th Floor	30%	−83%	−44%	49%	−66%	−23%	−23%
8th Floor	28%	−85%	−41%	55%	−59%	−26%	−21%
9th Floor	29%	−81%	−35%	58%	−65%	−23%	−20%
10th Floor	32%	−78%	−37%	59%	−45%	−16%	−14%
Average improvement of displacement in the building							−17%

4.5.2 Comparison of Maximum Displacement Values

In Table 30, the maximum values at each floor are considered for both cases, regardless of the time at which the displacements occur.

Table 30: Percentage improvement in maximum displacement between TMD and MTMD-B.

Floor No.	Maximum Displacement		Average
	TMD	MTMD	
1st Floor	0.0131	0.0143	−9%
2nd Floor	0.0246	0.0276	−12%
3rd Floor	0.0367	0.0386	−5%
4th Floor	0.0465	0.0493	−6%
5th Floor	0.0533	0.0586	−10%
6th Floor	0.0615	0.0683	−11%
7th Floor	0.0676	0.0749	−11%
8th Floor	0.0718	0.0805	−12%
9th Floor	0.0752	0.0855	−14%
10th Floor	0.0776	0.0862	−11%
Average improvement			−10%

According to Table 30, the TMD system provides 10% more improvement in maximum floor displacements than the MTMD-B system.

4.5.3 Dissipation of Vibrational Energy

From Tables 10 and 24, the difference in floor vibration energy distribution between the two cases can be calculated. The minus sign indicates that the improvement rate of the vibration energy distribution at a floor in the TMD case is greater than that in the MTMD-B case.

According to Table 31, the TMD system provides 21% more improvement in floor vibration energy distribution than the MTMD-B system.

Table 31: Percentage improvement in vibration energy distribution between TMD and MTMD-B.

Floor No.	Time (s)						
	5	10	15	20	25	30	Average
1st Floor	58%	−36%	−61%	83%	−79%	−35%	−12%
2nd Floor	50%	−61%	−77%	80%	−77%	−69%	−26%
3rd Floor	50%	−53%	−79%	84%	−67%	−55%	−20%
4th Floor	57%	−70%	−76%	81%	−77%	7%	−13%
5th Floor	64%	−89%	−76%	91%	−73%	−66%	−25%
6th Floor	53%	−96%	−66%	79%	−89%	−39%	−26%
7th Floor	71%	−97%	−69%	74%	−88%	−41%	−25%
8th Floor	66%	−97%	−65%	77%	−83%	−46%	−25%
9th Floor	68%	−96%	−58%	80%	−88%	−42%	−23%
10th Floor	76%	−94%	−60%	82%	−71%	−30%	−16%
Average improvement of energy dissipation in the building							−21%

4.6 The Maximum Displacement & Story Drift Values

In this section, a comparison is presented between the three cases studied in terms of the maximum values of displacement and story drift. Fig. 12 shows the maximum values of story displacements for all cases.

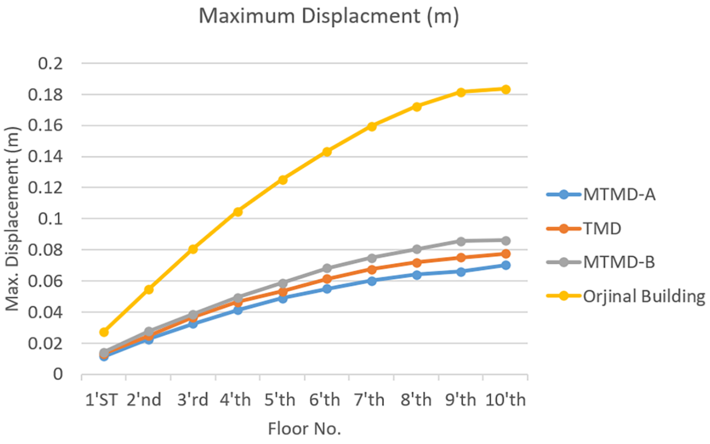

Figure 12: Maximum displacment values (m).

Fig. 13 shows the story drift for all cases (assuming a story height of 3.5 m):

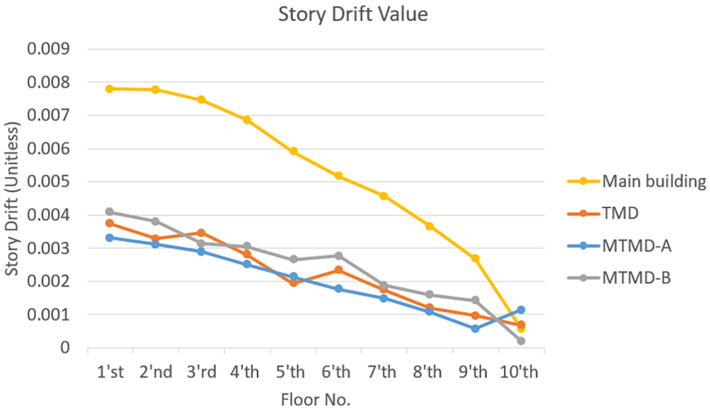

Figure 13: Story drift (Unitless).

5 Discussion

The results of all cases have been compared with the building without a damping system. The results have then been compared with the TMD and MTMD systems separately for each case. The results demonstrate the following key points:

1. The building with the TMD system demonstrated significant effectiveness compared to the building without the damping system. It reduced floor displacements by 54% and reduced the building's vibration energy by 75%.
2. The building with the MTMD-A system demonstrated a very high effectiveness in reducing floor displacements, reaching a percentage equal to 76%, compared to the same building without the damping system, and the percentage of vibration energy dissipation reached 93%.
3. The building with the MTMD-B system was found to be less effective in reducing floor displacements and dissipating building vibration energy compared to the previous two cases. Compared to a building without a damping system, the displacement reduction reached 35%, and the vibration energy dissipation rate reached 33%.
4. Comparing the TMD with MTMD-A cases, the MTMD-A system was found to be more effective than the TMD system in reducing displacements and energy dissipation by 60% and 95%, respectively. Conversely, the TMD system was found to be more effective than the MTMD-B system in reducing displacements and energy dissipation by 17% and 21%, respectively.
5. Although the MTMD-A system yielded better results than other systems, it has the disadvantage that the damper masses obtained using the Jaya algorithm are much larger than those used in other systems. The total mass used across all floors in the MTMD-A system is 3,229,058 kg. The total mass across all floors in the MTMD-B system is 500,000 kg. This value is the same as the value used in the TMD system; that is, there is a single mass weighing 500,000 kg on the tenth floor. Therefore, the mass weight of the dampers in the MTMD-A system is approximately 85% greater than in the TMD and MTMD-B systems. This result is considered a shortcoming of the MTMD-A system. This makes the MTMD-B system more practical and applicable than the MTMD-A system in terms of cost, installation, maintenance, and the cross-sectional area of the structural elements supporting the mass dampers.
6. This study emphasizes the importance of the MTMD system's overall effect on the behavior of buildings under seismic excitation compared to the TMD system. However, it should be noted that the results obtained in this study are limited to the cases examined.

6 Conclusion

This study focuses on the building's behavior when a tuned mass damper is added to the tenth floor. The second case focuses on the behavior when a tuned mass damper is added to each floor. Acceleration and displacement graphs for the floors are plotted using MATLAB. The accelerations and displacements of the tuned mass damper are also shown in this section. The cases are then compared. This information will help understand the effect of the tuned mass damper in damping vibration energy.

The significance of this study is focused on understanding the effectiveness of using tuned mass dampers and multiple-tuned mass dampers to improve the structural behavior of buildings. This provides designers and engineers with the flexibility to choose the best option based on the project and its specifications. Furthermore, the optimization results presented in this study provide the characteristics of the tuned mass damper that should be adopted for similar situations. Furthermore, the study focuses on optimizing the parameters of the tuned mass damper system using the Jaya algorithm within the scope of MATLAB simulation.

Acknowledgement: Not applicable.

Funding Statement: The authors received no specific funding.

Author Contributions: The authors confirm contribution to the paper as follows: study conception and design: Sinan Melih Nigdeli, Gebrail Bekdaş; data collection: Omer Elselumi; analysis and interpretation of results: Omer Elselumi, Sinan Melih Nigdeli, Gebrail Bekdaş; draft manuscript preparation: Omer Elselumi. All authors reviewed and approved the final version of the manuscript.

Availability of Data and Materials: The datasets generated and analyzed during the current study are available from the corresponding author on reasonable request.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

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