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ARTICLE

A Quantum-Assisted Hybrid Learning Framework for Environmental CO_2 Emission Analysis

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ABSTRACT: Accurate prediction of carbon emissions is essential for developing sustainable environmental policies and mitigating global warming. Road transportation represents one of the major sources of global CO_2 emissions due to its dependence on fossil fuels. This study presents a comparative framework that evaluates classical machine learning models alongside a hybrid quantum–classical learning architecture for vehicle-based CO_2 emission prediction. A large-scale vehicle emissions dataset containing 7385 samples collected over approximately seven years was obtained from the official open-data platform of the Government of Canada. Key vehicle characteristics, including engine size, fuel consumption, transmission type, and vehicle class, were utilized as predictive features. Data preprocessing involved the removal of incomplete records and normalization of numerical variables using MinMaxScaler. Three classical machine learning models, namely Random Forest, Gradient Boosting, and Multi-Layer Perceptron (MLP), were trained and evaluated using identical preprocessing procedures and evaluation protocols. Performance was assessed using the coefficient of determination (R^2) and Mean Absolute Error (MAE). Among the classical approaches, the MLP achieved the highest predictive performance with an R^2 score of 0.963 and a normalized MAE of 0.082, followed by Gradient Boosting (R^2 = 0.923, MAE = 0.085) and Random Forest (R^2 = 0.890, MAE = 0.091). Subsequently, a hybrid quantum–classical framework integrating a variational quantum circuit with a classical neural network was developed. The proposed model achieved a normalized MAE of 0.048 and an R^2 score of 0.9925, outperforming all evaluated classical models. Additional experiments investigating entanglement depth demonstrated that stronger entanglement structures contributed positively to predictive performance, highlighting the importance of quantum-enhanced feature representations. Overall, the findings indicate that hybrid quantum–classical learning can provide improved predictive accuracy for structured environmental datasets. While the present study does not claim a formal quantum advantage in terms of computational complexity or scalability, the observed empirical improvements suggest that quantum-enhanced feature representations can serve as a promising complementary mechanism for environmental data analytics and sustainability-oriented decision support systems.

KEYWORDS: CO_2 emission prediction; machine learning; quantum machine learning; hybrid models; PennyLane

1 Introduction

Reducing carbon emissions and developing sustainable environmental policies are critical components in the global fight against climate change. Accurate prediction of CO_2 emissions is essential for assessing environmental impacts and formulating effective strategies, particularly in the transportation and energy sectors. Traditional machine learning (ML) methods have been widely used for CO_2 emission prediction and have achieved significant levels of accuracy. However, these methods may face limitations when modeling highly nonlinear relationships and complex feature interactions, which can affect predictive performance

in structured environmental datasets. Recent advances in quantum computing have stimulated growing interest in quantum-assisted machine learning approaches. By exploiting quantum mechanical principles such as superposition and entanglement, quantum computing provides new opportunities for constructing richer feature representations and exploring complex data relationships. These capabilities have motivated the development of hybrid quantum–classical learning frameworks that combine the strengths of quantum information processing with established classical machine learning techniques. This study aims to compare conventional machine learning techniques with a hybrid quantum–classical learning framework for CO_2 emission prediction. The primary objective is to evaluate the predictive performance of quantum-assisted models in regression tasks and investigate their potential applicability to environmental data analytics. This research contributes to the growing body of literature on Quantum Machine Learning (QML) by examining its effectiveness for structured environmental datasets and providing insights into both its potential benefits and current limitations. Quantum Machine Learning (QML) has emerged as a promising research direction in recent years, particularly for problems involving complex feature interactions and high-dimensional representation spaces. Nammouchi et al. [1] examined the potential applications of QML in climate change and sustainability and reported up to a 15% improvement in prediction accuracy compared to classical approaches in areas such as energy system optimization and climate forecasting. However, the practical application of QML is challenged by issues such as high computational costs and hardware limitations. Similarly, Li et al. [2] demonstrated that simulating 43-qubit quantum circuits could result in up to 48 times more carbon emissions than transformer-based classical machine learning models, such as large language models (LLMs). This finding underscores the urgent need for novel optimization techniques to improve the sustainability of quantum computing. The performance of quantum neural networks (QNNs) compared to classical neural networks varies depending on the dataset and application context. Hirai [3] reported that QNNs achieved a 10% lower error rate than classical neural networks on small datasets. However, this performance advantage diminished on larger datasets, where classical deep learning methods produced more consistent and reliable results. In a related study, Havlíček et al. [4] introduced a quantum-enhanced feature mapping approach for supervised learning tasks and demonstrated the effectiveness of quantum support vector machines in capturing complex data patterns. Their findings provided an important theoretical foundation for hybrid quantum-classical machine learning models and inspired subsequent studies investigating quantum approaches for prediction and regression problems.

The differences in performance between classical regression models and QML approaches are often dependent on the specific characteristics of the dataset. Güler and Yerel Kandemir [5], for instance, analyzed CO_2 emission forecasts for OECD countries using both linear and cubic regression models, concluding that the cubic model provided a better fit, with an R^2 value of 0.92. However, the question of whether quantum regression models can outperform such classical techniques in terms of accuracy remains open to debate. Schuld and Killoran [6] noted that QML models can offer up to 20% higher accuracy than classical machine learning methods, especially for high-dimensional datasets. Nonetheless, further empirical research is needed to assess the generalizability and practical effectiveness of these advantages in real-world applications. Quantum computing's impact on optimization problems has shown advantages over classical methods under specific conditions. Mitarai et al. [7] explored parameterized quantum circuit models and reported that these models could achieve solutions twice as fast as classical methods. However, they emphasized that this performance advantage is applicable only to specific types of problems and cannot be generalized across all optimization tasks. Similarly, Perdomo-Ortiz et al. [8] found that quantum-assisted algorithms achieved over 90% accuracy in predicting chemical reactions, outperforming the 85% accuracy of classical models. Nonetheless, additional empirical studies are necessary to validate the applicability of these advantages in various scientific domains.

In data classification tasks, quantum kernel methods have shown superior performance in certain cases when compared to classical kernel approaches. O'Gorman et al. [9] proposed a Quadratic Unconstrained Binary Optimization (QUBO)-based formulation for Bayesian Network Structure Learning (BNSL), enabling the optimization problem to be solved using quantum annealing techniques. A key contribution of their work is the development of an instance-independent logical mapping, allowing the same embedding to be reused across different problem instances, thereby reducing computational overhead. The study also demonstrated that the proposed formulation is compatible with both quantum annealing and classical simulated annealing algorithms, highlighting its flexibility as a hybrid optimization framework. Although current quantum annealing hardware limits the scalability of the approach, the authors emphasized its potential for efficiently solving complex combinatorial optimization problems as quantum computing technologies continue to advance. Zoufal et al. [10] investigated the impact of quantum generative adversarial networks (QGANs) in financial data modeling and found that QGANs enabled 30% faster data generation and reduced the error rate by 5% relative to classical GANs. However, testing these results on large-scale datasets remains crucial to establish broader applicability. Studies focusing on the foundational aspects of quantum machine learning suggest a theoretical quantum advantage, yet practical limitations continue to exist. Biamonte et al. [11] demonstrated that QML methods outperform classical approaches in certain tasks, while Daskin [12] showed that quantum-assisted linear regression models yield faster and more precise results for large datasets. Furthermore, research on Hamiltonian learning [13] reported a 50% reduction in learning time compared to classical approaches. Verdon et al. [14] proposed a low-depth variational quantum algorithm for training quantum neural networks, demonstrating that hybrid quantum-classical optimization can effectively learn neural network parameters while remaining suitable for noisy intermediate-scale quantum (NISQ) devices. While current technologies can improve the reliability and performance of quantum systems, their integration into real-world applications continues to present significant challenges. In this context, quantum-inspired algorithms have also attracted attention for their role in evolutionary optimization processes. Yetis and Karakose [15] analyzed the performance of quantum-inspired evolutionary algorithms and demonstrated their potential benefits in population-based optimization problems, showing that these methods can provide faster and more efficient solutions than traditional approaches, although practical limitations and computational costs remain substantial barriers. Developing a comprehensive understanding of the fundamentals of quantum computing is essential for effectively implementing such technologies. Nielsen and Chuang [16] offered an extensive theoretical framework in their foundational work on quantum computing and quantum information theory, covering key principles that underpin developments in this field. Addressing these challenges is vital to unlocking the full potential of quantum computing. In this regard, Amin et al. [17] examined quantum Boltzmann machines in learning processes and emphasized their superior computational capabilities compared to classical Boltzmann machines. These findings underscore the value of quantum computing in addressing complex tasks, especially within deep learning. Furthermore, Wiebe et al. [18] explored quantum deep learning architectures and suggested that quantum computing could significantly enhance the efficiency of deep learning algorithms, offering a path forward in overcoming the inherent limitations of classical approaches. Crawford et al. [19], Chien et al. [20] proposed a quantum neural network framework to reduce emissions through fuel optimization in maritime transportation. Their study compared purely quantum and hybrid classical-quantum machine learning models and found that quantum computing outperformed classical methods. The research emphasizes the potential of Noisy Intermediate-Scale Quantum (NISQ) devices for practical, real-world applications. Similarly, Tatarin et al. [21] demonstrated how machine learning can accelerate the discovery of efficient iridium(III) emitters through structure-based predictive modeling. Their work illustrates the expanding application of intelligent computational methods in materials informatics, complementing recent advances in quantum machine learning across diverse scientific and engineering domains. Singh and

Pokhrel [22] examined feature map modeling for quantum machine learning and investigated qubit behavior in spontaneous emission and energy relaxation processes. Similarly, Shi et al. [23] proposed a hybrid quantum reinforcement learning model for optimizing energy management in multi-stack fuel cell vehicles, demonstrating improved performance compared to classical reinforcement learning approaches. Hidayat and Surendro [24] further conducted a systematic review on the role of quantum computing in greenhouse gas emission reduction and highlighted the efficiency of quantum optimization methods compared to traditional techniques.

Additional studies have demonstrated the applicability of quantum-assisted machine learning models in different predictive and optimization tasks. Narkedimilli et al. [25] compared classical and quantum machine learning algorithms in black hole mass estimation and reported that quantum models may provide superior generalization performance under certain conditions. Likewise, Cherni et al. [26] investigated machine learning-based prediction models for photoionization processes in quantum dots, while Salahinejad and Roozbahani [27] explored predictive models for carbon quantum dots. These studies collectively indicate the broad applicability of quantum machine learning techniques across multiple scientific domains. Quantum computing also offers effective solutions for processing visual and multidimensional data. Yetiş and Karaköse [28] proposed an improved and cost-reduced quantum circuit generator approach for image encoding applications, demonstrating enhanced efficiency in data representation and processing. In another study, Yetiş and Karaköse [29] developed a framework integrating quantum convolution and pooling circuits, enabling the implementation of classical deep learning architectures on quantum hardware. These studies suggest that quantum technologies may contribute not only to numerical optimization tasks but also to multidimensional environmental data analysis and intelligent visual processing applications.

In conclusion, the literature indicates that quantum machine learning methods offer advantages over classical approaches in specific scenarios, particularly in handling high-dimensional data and complex optimization problems. However, challenges related to scalability, computational efficiency, and hardware limitations continue to restrict their broader implementation. Therefore, further experimental investigations are necessary to evaluate the effectiveness and applicability of hybrid quantum-classical models in real-world engineering and environmental prediction problems.

In this study, we propose a hybrid quantum machine learning approach to predict carbon dioxide (CO_2) emissions based on technical specifications of vehicles. We focus on modeling the relationship between features such as engine size, fuel consumption, and number of cylinders and their impact on CO_2 output. The goal is to explore whether hybrid quantum-classical models can provide meaningful improvements in predictive accuracy compared to conventional machine learning techniques. To this end, we construct and evaluate multiple classical models alongside a variational quantum circuit integrated within a neural network framework. The training process is designed to minimize prediction error while examining the practical feasibility and performance of quantum-enhanced learning systems in a real-world environmental context.

Table 1 presents a comparison of studies on CO_2 emission prediction. By examining the accuracy rates and findings of the methods used in these studies, it has been evaluated which approaches are more successful. Machine learning and deep learning-based models have been found to offer higher accuracy compared to traditional methods. Additionally, hybrid and quantum-assisted models have been determined to enhance prediction performance. These comparisons are made to contribute to the selection of the most suitable methods for CO_2 emission prediction.

Table 1: Comparative analysis of recent studies on quantum machine learning and sustainability.

Study	Topic	Accuracy/Findings	Key Differences and Advantages
Nammouchi et al. (2023)	Applications of Quantum Machine Learning in Climate Change and Sustainability	15% increase in accuracy achieved.	Quantum algorithms' superiority in processing high-dimensional data.
Li et al. (2023)	Impact of Quantum Circuit Simulations on Carbon Emissions	Quantum simulations generated 48 times more CO_2 emissions than transformer models.	Emphasizing the environmental impact of quantum circuits.
Hirai (2023)	Performance of Quantum Neural Network Models	Achieved superior generalization performance with 10% lower error rate.	Lower error rate on small datasets.
Tasar et al. (2023)	Comparison of Quantum and Classical SVM Performance	Quantum SVM achieved 96% accuracy, while classical SVM reached 97%.	Quantum SVMs offer comparable accuracy but have longer processing times.
Güler and Kandemir (2022)	CO_2 Emission Prediction	Cubic regression model R^2: 0.92, linear regression R^2: 0.85.	Cubic regression model provided better fit.
Schuld et al. (2019)	Comparison of Quantum Machine Learning Models with Classical Models	Achieved up to 20% improvements in accuracy.	Quantum computers efficiently handle high-dimensional data.
Mitarai et al. (2018)	Optimization of Parametric Models Using Quantum Circuits	Quantum algorithms provided solutions twice as fast as classical methods.	Faster solutions through quantum optimization.
Perdomo-Ortiz et al. (2017)	Quantum-Assisted Machine Learning for Chemical Reaction Prediction	Quantum methods achieved 90% accuracy, while classical methods remained at 85%	Higher accuracy provided by quantum methods.
Havlíček et al. (2019)	Data Classification Using Quantum Kernel Methods	Quantum kernel methods achieved 95% accuracy, while classical kernel methods reached 92%.	Superior performance of quantum kernel methods.
Zoufal et al. (2019)	Financial Data Modeling Using Quantum Generative Adversarial Networks (QGANs)	QGANs generated data 30% faster with 5% lower error rates.	Faster and more efficient performance compared to classical GANs.
Biamonte et al. (2017)	Fundamental Principles and Applications of Quantum Machine Learning	Quantum methods provide higher accuracy and speed for specific problems compared to classical methods.	Comparative analysis of quantum and classical approaches.
Daskin (2021)	Quantum-Assisted Linear Regression Models	Quantum algorithms improved results by 5% on large datasets compared to classical models.	Faster and more precise outcomes with quantum-assisted linear regression.

(Continued)

Table 1 (continued)

Study	Topic	Accuracy/Findings	Key Differences and Advantages
Schuld and Killoran (2019)	Applications of Quantum Machine Learning Models on Continuous Data Sets	Quantum models achieved 95% accuracy, while classical models reached 90%.	Superior performance of quantum models.
Wiebe et al. (2014)	Efficiency of Quantum Algorithms in Hamiltonian Learning Processes	Quantum algorithms reduced learning time by 50%.	Faster learning processes with quantum algorithms.

The distribution changes of the model designed for this study are presented in Fig. 1 below.

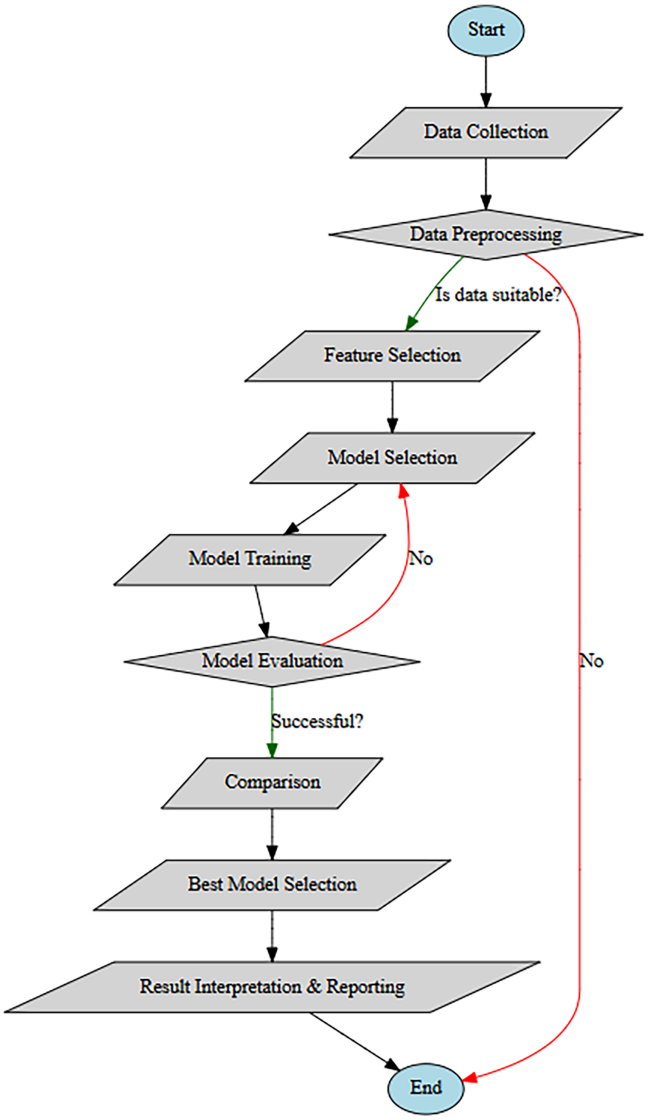

Figure 1: Algorithm flowchart of the designed system.

According to the algorithm flowchart shown in Fig. 1, the proposed method for CO_2 emission prediction is modeled using a systematic flowchart consisting of eight fundamental steps. The process begins with the collection of CO_2 emission data. Next, a data preprocessing phase is conducted, including tasks such as missing data cleaning, normalization, and scaling. Following this, feature selection is performed to identify the most significant variables in order to enhance model accuracy. Based on the selected features, a choice is made between traditional machine learning models and quantum-assisted models, and the training phase is carried out. After training, the models are evaluated in terms of accuracy and performance metrics, allowing for a comparison between classical machine learning and quantum-based models. As a result of this comparison, the model with the highest performance is identified. The process concludes with the interpretation and reporting of the obtained results. Through this systematic approach, the effectiveness of both classical and quantum-based methods is analyzed, aiming to select the most optimal model.

2 Proposed Method

The emission rates produced by the automobile models specified in Fig. 2 are shown below.

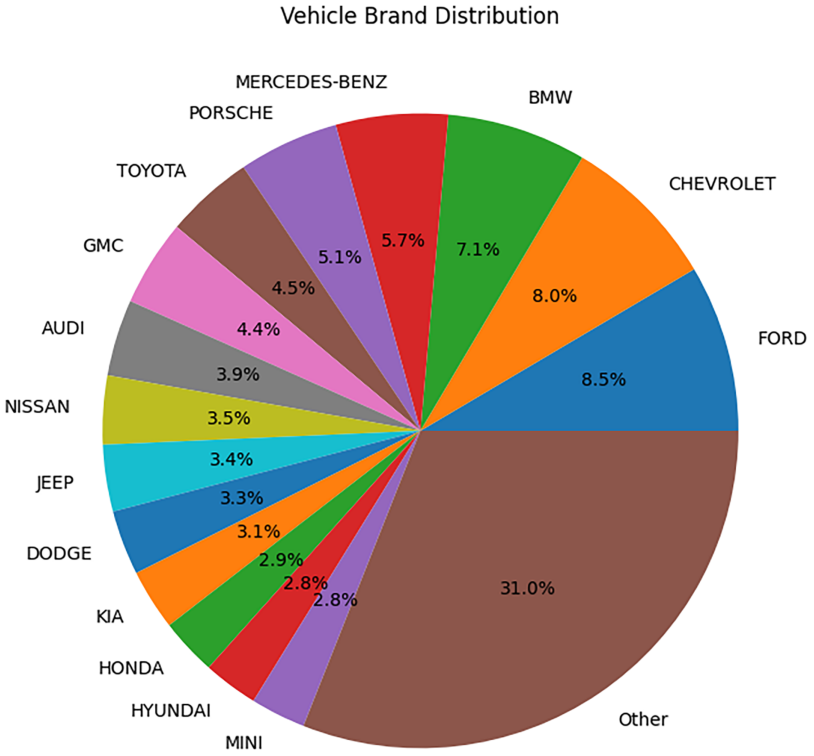

Figure 2: Car bands and emission rates.

As seen in Fig. 2, Ford has the highest emission rate (8.5%), while Mini has the lowest emission rate (2.8%) among the car brands in the dataset. The data is sorted by the number of vehicles for the top 15 brands and displayed separately. All remaining brands are grouped under the "Other" category.

As seen in Fig. 3, the top 10 car brands with the highest average CO_2 emissions are visualized using a bar chart. This graph highlights which brands have higher carbon emissions. According to the graph, Bugatti has the highest average CO_2 emissions, while Land Rover has the lowest.

Below, in Fig. 4, a graph comparing engine displacement (L) and CO_2 emissions (g/km) is presented, followed by its analysis and interpretation.

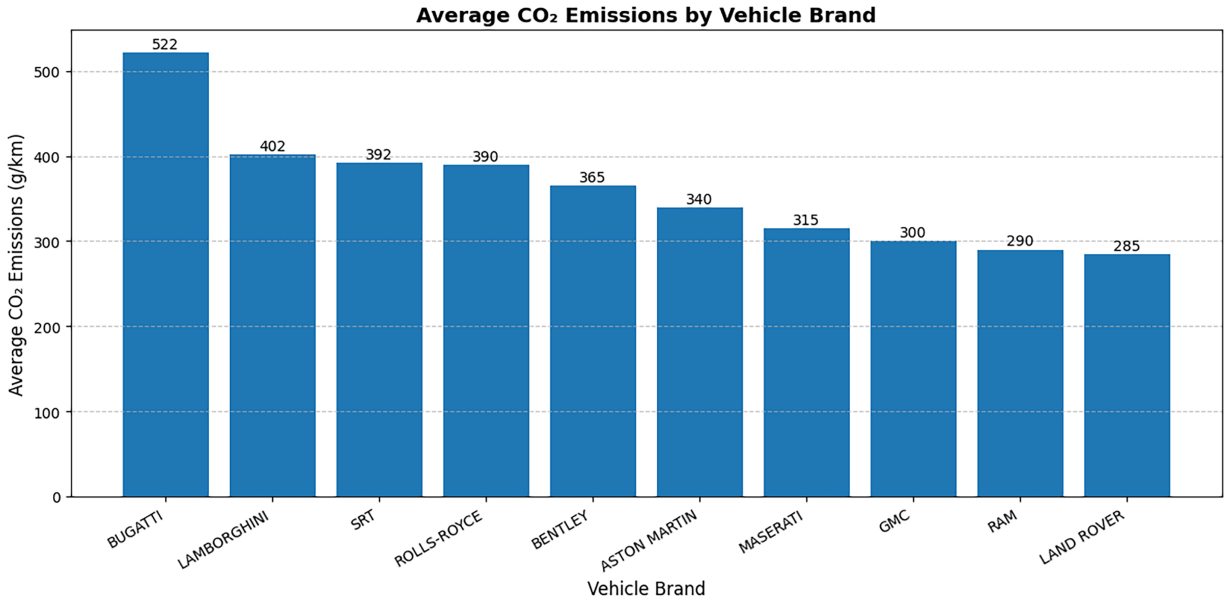

Figure 3: Top 10 brands with the highest average CO_2 emissions (g/km).

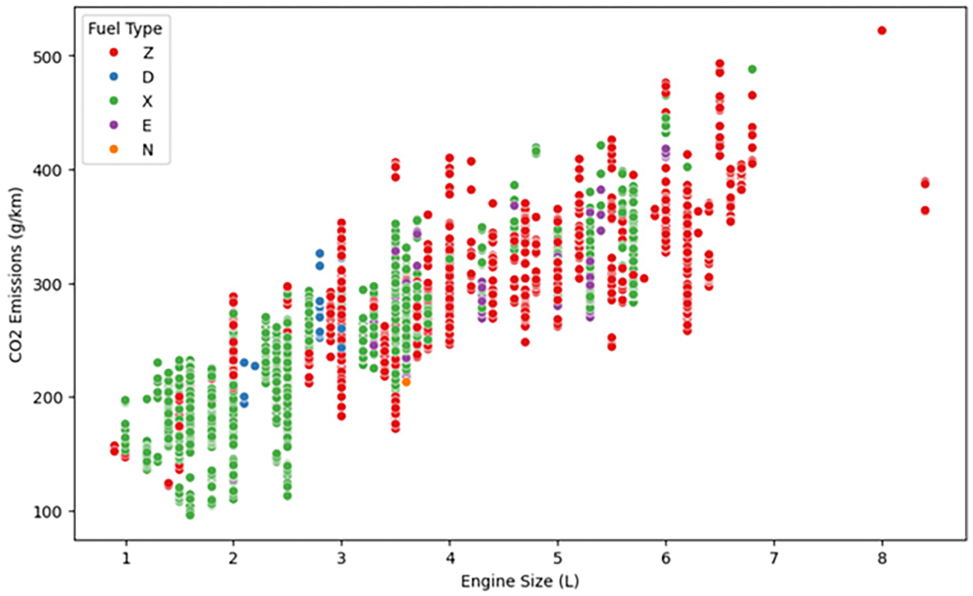

Figure 4: Engine displacement and CO_2 emission (g/km) comparison.

When analyzing Fig. 4, it is observed that vehicles with an engine displacement between 1.0 and 2.0 L tend to have CO_2 emissions concentrated between 100–250 g/km. Additionally, as engine displacement increases, emission levels rise significantly, with vehicles having an engine displacement of 4.0 L or more exceeding 300 g/km in CO_2 emissions. When evaluating the graph in terms of fuel types, it is notable that vehicles using E-type fuel (marked in red) generally exhibit higher emission values. While emissions for these vehicles can reach up to 500 g/km, those using X- and D-type fuels tend to show lower emission levels. Specifically, vehicles with a 2.0 L engine displacement using X-type fuel exhibit emissions primarily concentrated between 150–250 g/km. Overall, the analysis suggests that as engine displacement increases, CO_2 emissions also rise considerably. However, fuel type plays a critical role in this relationship. This finding

underscores the importance of emission control policies that focus not only on engine displacement but also on the type of fuel utilized.

All experiments were conducted using the PennyLane and PyTorch libraries. The study was carried out in a classical simulation environment rather than on real quantum hardware. To ensure reproducibility, fixed random seed values were used throughout the training process. The experimental setup was implemented in a Python 3.10 environment using PennyLane (version 0.35.1) and PyTorch (version 2.2.0). All computations were performed on Google Colab, utilizing a system equipped with an NVIDIA Tesla T4 GPU. The source code will be made publicly available after publication to support transparency and reproducibility.

3 Simulation Result

Below, Fig. 5 presents a correlation matrix designed to illustrate the relationships between engine displacement, cylinder count, fuel consumption, and CO_2 emissions, followed by an in-depth interpretation.

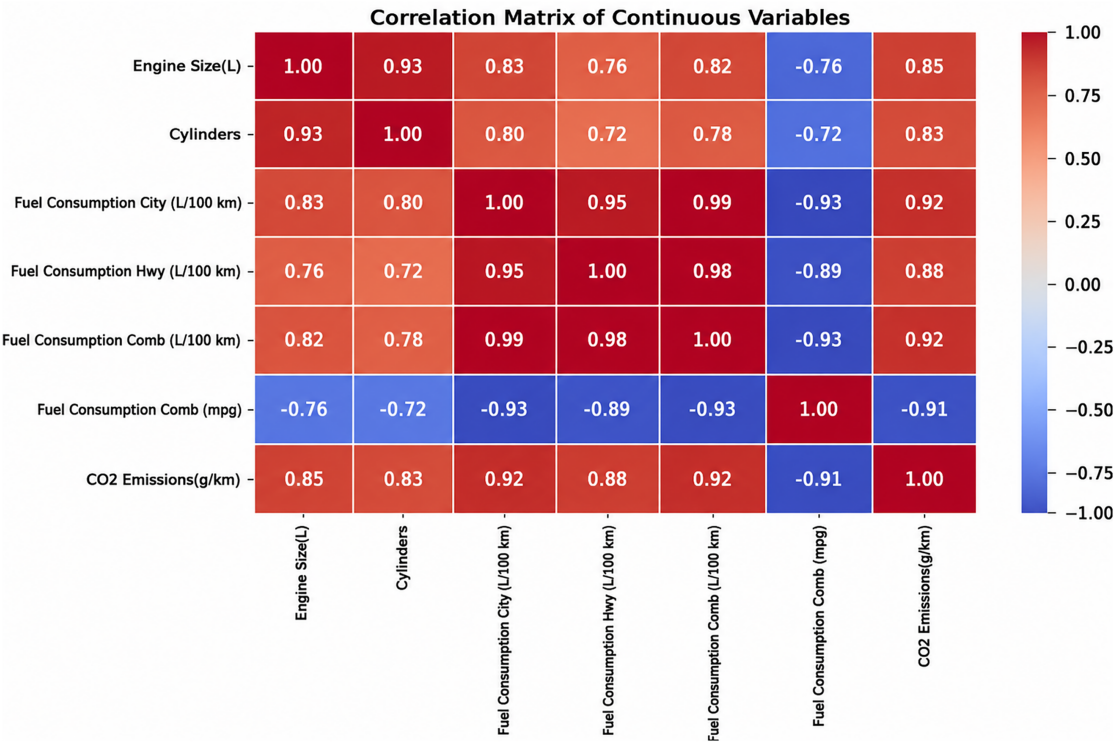

Figure 5: Correlation matrix of continuous variables.

As shown in Fig. 5, there is a strong positive correlation between engine displacement (r = 0.85), cylinder count (r = 0.83), and fuel consumption under different driving conditions (r ≈ 0.88–0.92) with CO_2 emissions. In contrast, fuel efficiency, represented by combined miles per gallon (mpg), exhibits a significant negative correlation (r = −0.91) with CO_2 emissions. These findings confirm that vehicles with larger engine displacements and higher fuel consumption tend to produce higher CO_2 emissions. Therefore, strategies aimed at reducing CO_2 emissions should also focus on optimizing fuel consumption. In this context, adopting electric and hybrid engine technologies, promoting low-emission engine designs, and implementing aerodynamic improvements are crucial steps to enhance environmental sustainability.

Below, Fig. 6 presents a comparative analysis of model prediction accuracy and error rates, along with their respective interpretations.

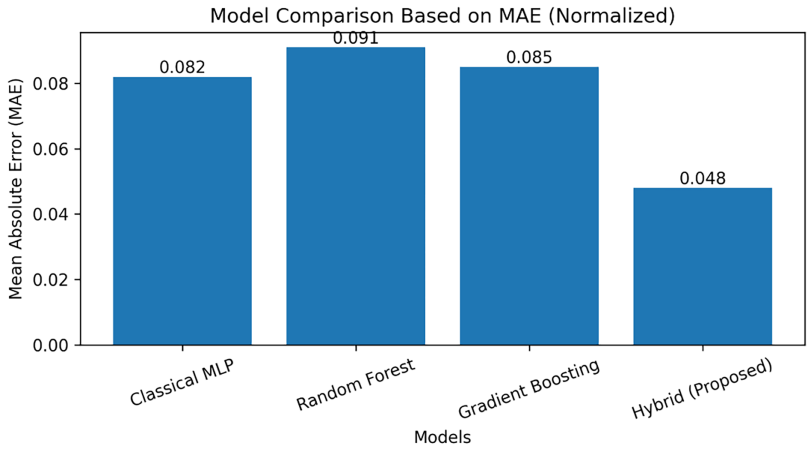

Figure 6: Model MAE comparison.

Fig. 6 presents the comparison of different prediction models based on the normalized Mean Absolute Error (MAE) metric. MAE evaluates the average magnitude of prediction errors, where lower values indicate better predictive performance. Among the evaluated models, the proposed Hybrid model achieved the lowest MAE value of 0.048, demonstrating the highest prediction accuracy and the smallest deviation from the actual CO_2 emission values. In contrast, the Random Forest model produced the highest MAE value of 0.091, indicating comparatively larger prediction errors. The Classical MLP and Gradient Boosting models achieved MAE values of 0.082 and 0.085, respectively, showing moderate predictive performance. Overall, the proposed Hybrid model significantly outperformed the conventional machine learning models by reducing the prediction error and providing more accurate CO_2 emission forecasts.

Fig. 7 illustrates the workflow adopted for the development and evaluation of the classical machine learning models. The process begins with data collection and preprocessing, including data cleaning and normalization procedures. Subsequently, feature engineering is performed to improve the predictive capability of the models. In the model development stage, Random Forest Regression (RFR), Gradient Boosting Regression (GBR), and Multi-Layer Perceptron (MLP) models are constructed and optimized through hyperparameter tuning. All models are then evaluated using the same train–test split configuration and assessed using MAE, MSE, and R^2 metrics. Finally, the predictive performances of the evaluated models are compared to identify the most effective approach for vehicle CO_2 emission prediction. This workflow ensures a systematic, reproducible, and fair evaluation framework for comparative machine learning analysis.

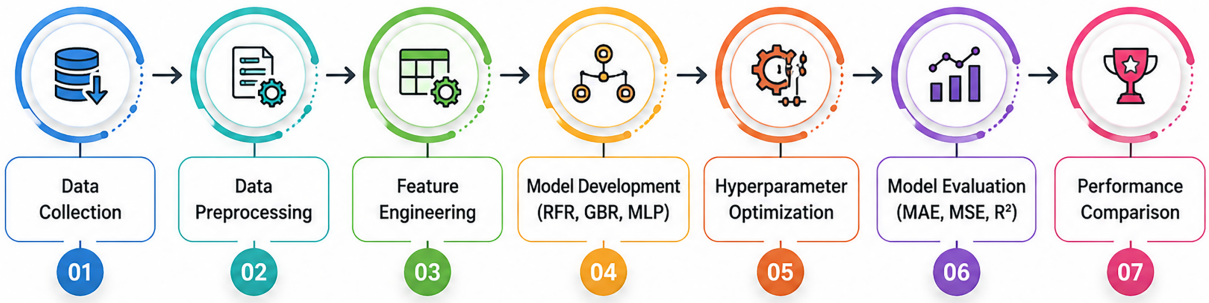

Figure 7: Classic machine learning process flowchart.

To ensure a robust comparative evaluation, the classical machine learning models were implemented with systematic hyperparameter optimization. Random Forest Regression (RFR), a widely used ensemble learning method, was optimized using grid search over key hyperparameters, including the number of trees (n_estimators ∈ {100, 200, 300}), tree depth (max_depth ∈ {None, 10, 20}), minimum samples required to split a node (min_samples_split ∈ {2, 5}), and minimum samples at leaf nodes (min_samples_leaf ∈ {1, 2}). Gradient Boosting Regression was similarly optimized to improve predictive performance and model generalization. Multi-Layer Perceptron (MLP), representing the neural network-based approach, was trained using different hidden layer configurations (hidden_layer_sizes ∈ {(64,), (128,), (64, 64)}), the ReLU activation function, the Adam optimizer, and regularization strengths of alpha ∈ {0.0001, 0.001}. The learning rate was either maintained as constant or adaptively adjusted during training. A maximum of 500 training epochs was employed, while early stopping was used to reduce the risk of overfitting. To ensure a fair and transparent comparison, all classical models were trained and evaluated using the same train–test split configuration, preprocessing pipeline, feature engineering procedure, and evaluation metrics. Model performance was assessed on the hold-out test set using the coefficient of determination (R^2), Mean Absolute Error (MAE), and Mean Squared Error (MSE). The performance results of the evaluated classical models are summarized in Table 2. Among the classical approaches, the Multi-Layer Perceptron (MLP) achieved the highest predictive performance with an R^2 score of 0.963 and a normalized MAE of 0.082. Gradient Boosting followed with an R^2 score of 0.923 and a normalized MAE of 0.085, while Random Forest achieved an R^2 score of 0.890 and a normalized MAE of 0.091. Although all evaluated models successfully captured underlying patterns within the dataset, the results suggest that classical approaches may experience limitations when modeling highly nonlinear feature interactions. To address these limitations, a hybrid quantum–classical learning framework was developed. Quantum Machine Learning (QML) combines classical learning algorithms with quantum computational principles such as superposition and entanglement, enabling richer feature representations and enhanced modeling of complex relationships within structured datasets.

Table 2: Performance comparison of models.

Model	MAE (Normalized) ↓	R^2 ↑
Classical MLP	0.082	0.963
Random Forest	0.091	0.890
Gradient Boosting	0.085	0.923
Hybrid (Proposed)	0.048	0.9925

Note: ↑ indicates that higher values represent better model performance, whereas ↓ indicates that lower values represent better model performance.

In this study, the proposed hybrid model combines parameterized quantum circuits with a classical neural network architecture implemented using PennyLane and PyTorch. The experimental results demonstrate that the hybrid framework outperforms all evaluated classical machine learning approaches, highlighting the potential of quantum-enhanced feature representations for complex regression tasks involving structured environmental data. Following the evaluation of classical and quantum machine learning models, a quantum-assisted hybrid learning framework was developed to combine the strengths of both paradigms. As illustrated in Fig. 8, the proposed architecture integrates classical feature learning with a variational quantum circuit (VQC) and a classical optimization stage. The quantum component consists of a 6-qubit, 16-layer variational quantum circuit in which normalized input features are encoded using angle embedding along the Y-axis. The VQC architecture was designed by considering the trade-off between representational capacity, trainability, and computational complexity. Preliminary experiments

with different qubit counts and circuit depths indicated that shallow circuits provided insufficient expressive capability, whereas excessively deep architectures increased optimization difficulty and simulation cost without substantial performance gains. Consequently, a 6-qubit configuration was selected as an effective balance between feature representation capacity and computational feasibility, while the 16-layer structure provided stable convergence and improved nonlinear feature extraction capability. Angle embedding was adopted due to its computational efficiency and suitability for encoding continuous-valued regression features into quantum states. Furthermore, strongly entangling layers were incorporated to capture nonlinear dependencies among input variables and enhance the representational power of the learned quantum feature space. The expectation values obtained from quantum measurements were subsequently transferred to a fully connected deep neural network (DNN), forming the classical learning component of the hybrid framework. In the classical stage, a multi-layer neural network with ReLU activation functions was employed to further refine the quantum-extracted features and improve predictive performance. The complete dataset contains 7385 samples collected over approximately seven years. To ensure a fair and transparent comparison, all models were trained and evaluated using the same dataset, preprocessing pipeline, feature engineering procedure, train–test split configuration, and evaluation protocol. Prior to model training, incomplete or inconsistent records were removed, and all numerical features were normalized using MinMaxScaler to improve convergence stability and data consistency. Due to the computational complexity associated with variational quantum circuit simulations, batch-based quantum processing and optimized parameter updates were employed to improve computational efficiency while preserving the integrity of the complete dataset. Consequently, the hybrid model benefited from the full data distribution without introducing sampling bias or inconsistencies between experimental settings. The model was optimized using the Adam optimizer with L2 regularization to reduce overfitting, while a StepLR scheduler was employed to gradually decrease the learning rate and improve convergence stability. After 450 training epochs, the proposed hybrid framework achieved a normalized Mean Absolute Error (MAE) of 0.048 and an R^2 score of 0.9925. Since the target variable (CO_2 emissions) was normalized prior to training, all reported MAE values correspond to the normalized scale. Overall, the experimental findings demonstrate that the proposed quantum-assisted hybrid framework effectively combines quantum feature representation with classical deep learning, achieving superior predictive performance compared to the evaluated classical baseline models. These results suggest that quantum-enhanced feature representations can provide a promising complementary mechanism for improving nonlinear pattern extraction in structured environmental datasets. To ensure interpretability, the predicted values were inverse-transformed to their original scale (g/km), and the evaluation metrics were recomputed in physical units. Both normalized and original-scale error metrics are now reported for clarity.

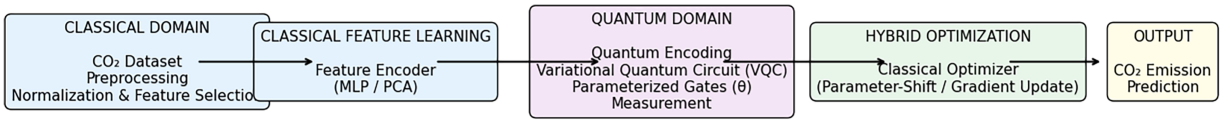

Figure 8: Proposed quantum-assisted hybrid learning framework integrating classical feature learning and variational quantum circuits.

Fig. 8. The proposed quantum-assisted hybrid learning framework for CO_2 emission prediction. The pipeline begins with classical data preprocessing and feature learning, where raw environmental and vehicle-related variables are normalized and encoded using classical methods such as MLP or PCA. The extracted features are then embedded into a variational quantum circuit (VQC) through a quantum encoding layer. Trainable quantum gates parameterized by rotation angles (θ) perform the learning process, and measurement outcomes are fed back to a classical optimizer. Model training is achieved via a hybrid

optimization strategy combining classical gradient-based optimization with quantum parameter-shift rules, enabling end-to-end learning across classical and quantum components.

This process enhances the efficiency of quantum machine learning models, enabling them to handle complex data structures more effectively. In the future, with the increasing power of quantum computing, these methods are expected to be more widely adopted in large-scale data analytics and optimization problems.

Fig. 9 illustrates the data flow of a 6-qubit variational quantum circuit used within the proposed quantum-assisted hybrid learning framework. The circuit consists of three main stages. In the first stage, classical input features are encoded into qubit rotation angles using an angle embedding strategy. In the second stage, a sequence of variational layers composed of parameterized single-qubit rotations and multi-qubit entangling operations is applied to enable learnable quantum transformations. The parameters of these gates are optimized during training through a hybrid classical–quantum optimization process. In the final stage, expectation values are obtained by measuring each qubit in the Z-basis, and the measurement outcomes are mapped back to classical values for subsequent processing.

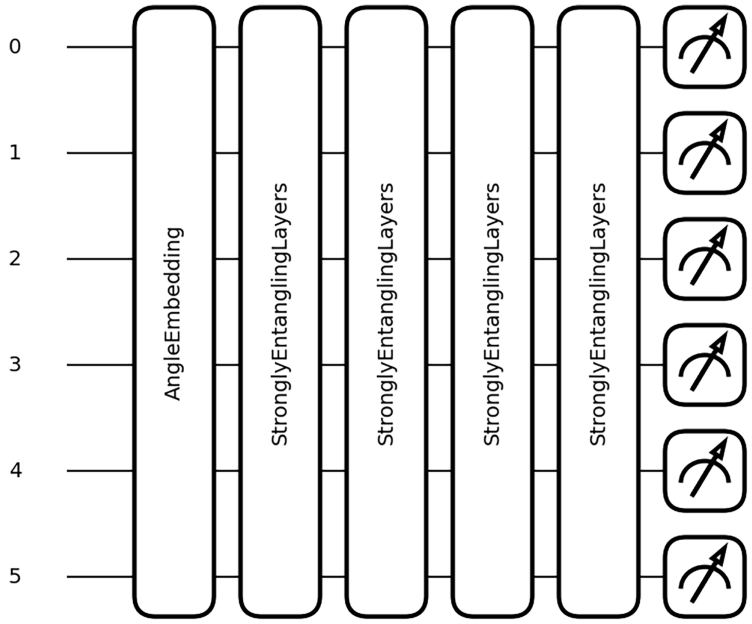

Figure 9: Example of a variational quantum circuit used within the proposed hybrid framework.

Fig. 10 illustrates an example of a 6-qubit variational quantum circuit employed within the quantum module of the proposed quantum-assisted hybrid learning framework. Each qubit is initially prepared in a superposition state using Hadamard (H) gates, allowing the circuit to explore a richer quantum state space. Controlled-NOT (CNOT) gates are then applied between neighboring qubits to introduce multi-qubit correlations as part of the variational transformation. Subsequently, parameterized RY rotation gates are used to apply tunable rotations around the Y-axis of the Bloch sphere, enabling the circuit to learn task-specific representations through adjustable parameters. Finally, measurements are performed in the computational (Z) basis, and the resulting expectation values are mapped back to classical outputs for further processing. The circuit serves as an illustrative example of a variational quantum architecture commonly used in quantum machine learning and hybrid quantum–classical optimization settings, while

acknowledging that circuit depth and entanglement patterns must be carefully designed to account for noise and hardware constraints.

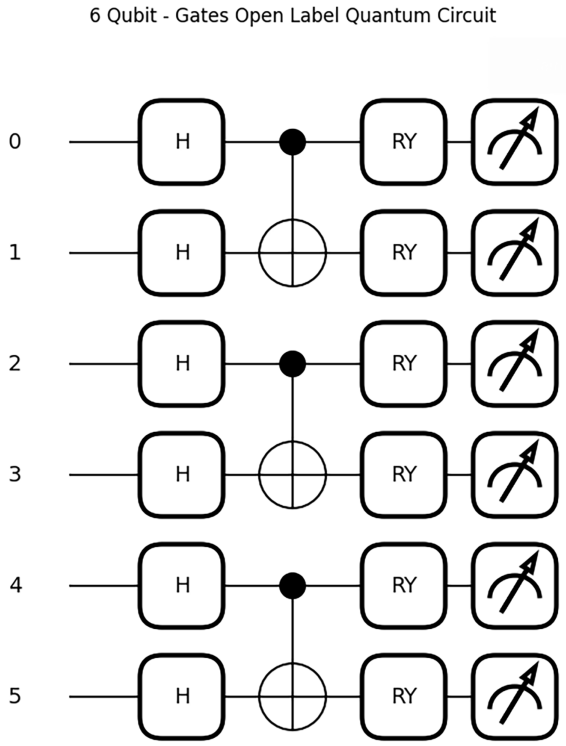

Figure 10: 6 Qubit-gates open label quantum circuit.

Fig. 11 presents the learning curve, illustrating the accuracy values of the model during the training and validation processes on an epoch basis. Upon examining the graph, it is observed that the model's accuracy steadily increases throughout the training process and exhibits a tendency to converge with the validation accuracy after a certain epoch. The fact that training and validation accuracy follow a similar trajectory indicates that the model does not exhibit overfitting. Moreover, the curve demonstrates a rapid increase in the early stages, followed by a saturation point where accuracy stabilizes. This suggests that the model reaches an optimal learning level after a certain epoch, and additional training does not yield significant improvements in accuracy. In conclusion, the learning process of the model is found to be effective, and it demonstrates high performance on the validation data.

Fig. 12 presents the measurement probability distribution obtained from the six-qubit quantum circuit employed in the proposed hybrid quantum–classical framework for CO_2 emission prediction. Fig. 12 presents the measurement probability distribution obtained from the six-qubit quantum circuit employed in the proposed hybrid quantum–classical framework for CO_2 emission prediction. The horizontal axis represents the observed quantum basis states, namely $|010000\rangle$, $|100000\rangle$, $|000000\rangle$, and $|110000\rangle$, while the vertical axis indicates their corresponding measurement probabilities. Among these states, $|010000\rangle$ exhibits the highest probability, suggesting that this state is more frequently observed under the given input conditions. The non-uniform probability distribution indicates that the quantum circuit encodes the input data into a set of quantum states with varying amplitudes, thereby capturing patterns that may not be explicitly represented in classical statistical approaches. This analysis provides additional insight into the

behavior of the hybrid quantum–classical model and facilitates a more comprehensive comparison with the prediction performance of purely classical machine learning methods.

Figure 11: Learning curve of the model.

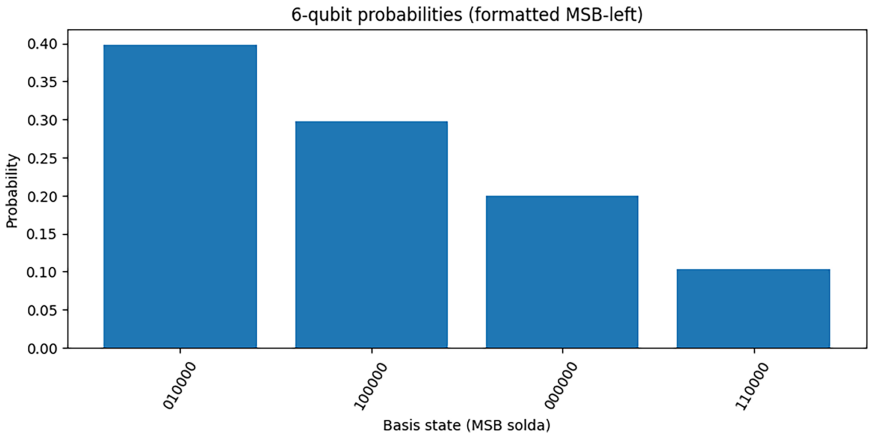

Figure 12: Analysis of CO_2 emissions with quantum circuit output.

Fig. 13. Distribution of predicted CO_2 emission values obtained from classical and quantum-assisted hybrid models. The classical model exhibits a wider prediction distribution, indicating higher variance and uncertainty. In contrast, the quantum-assisted hybrid model demonstrates a more concentrated distribution, reflecting improved feature representation and lower prediction error. This behavior is consistent with the observed MAE and R^2 performance metrics, highlighting the effectiveness of the proposed hybrid learning framework.

As presented in Fig. 14, the performance evaluation of the quantum machine learning-based regression model has been analyzed and discussed.

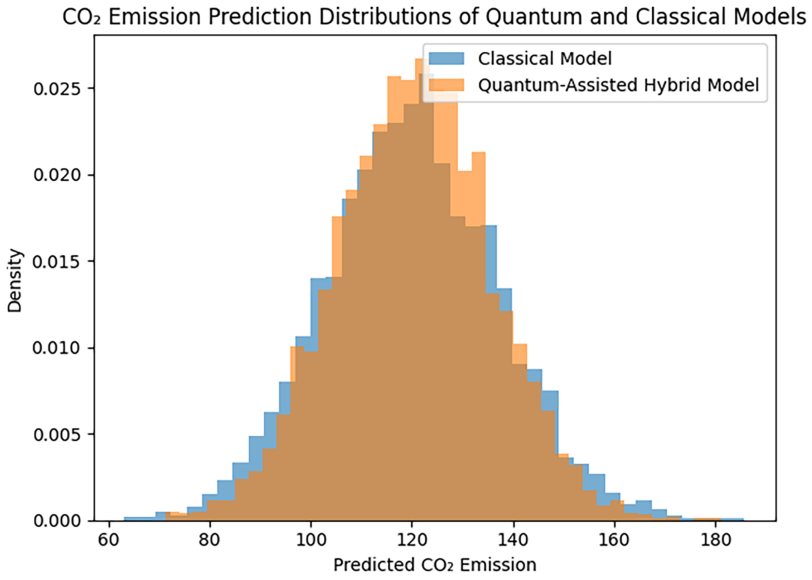

Figure 13: Distribution of predicted CO_2 emissions generated by the best classical model (MLP) and the proposed hybrid quantum–classical model.

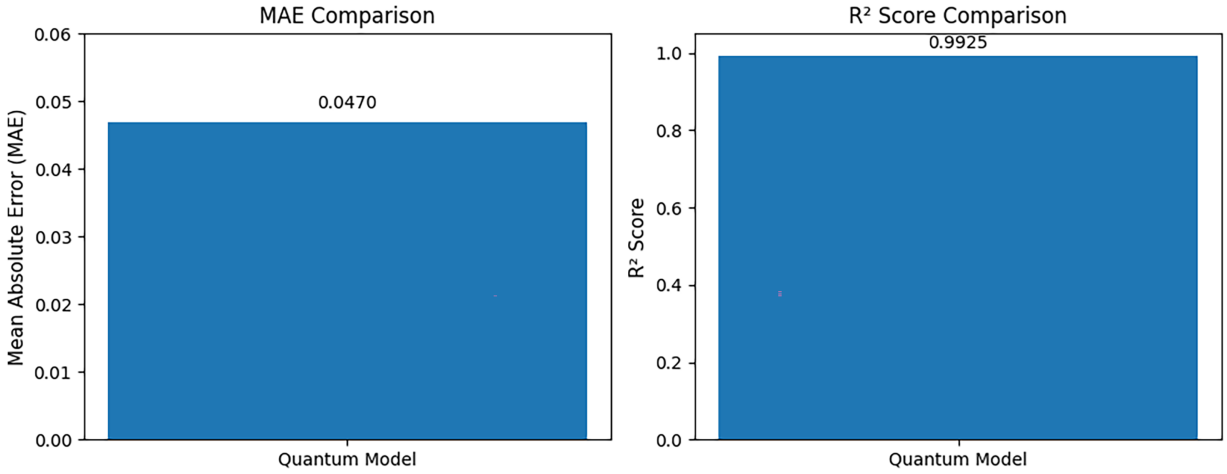

Figure 14: Performance evaluation of quantum machine learning based regression model.

Fig. 14 presents the performance evaluation results of the proposed hybrid quantum–classical regression framework. The left graph illustrates the Mean Absolute Error (MAE), while the right graph presents the coefficient of determination (R^2). The proposed model achieved a normalized MAE of 0.048 and an R^2 score of 0.9925, indicating high predictive accuracy and strong agreement between predicted and actual CO_2 emission values. These results demonstrate that the hybrid framework effectively captures complex nonlinear relationships within the vehicle emissions dataset. The integration of variational quantum circuits with classical deep learning enables richer feature representations, which contribute to improved predictive performance. Furthermore, the high R^2 value suggests that the model explains a substantial proportion of the variance in the target variable. Overall, the proposed hybrid quantum–classical framework achieved superior predictive performance compared to the evaluated classical machine learning models, highlighting the potential of quantum-enhanced feature representations for structured environmental data analysis.

Fig. 15. Training and validation loss curves of the proposed quantum-assisted hybrid model across training epochs. The smooth convergence and close alignment of the curves indicate stable hybrid optimization and the absence of overfitting, demonstrating the robustness of the learning framework.

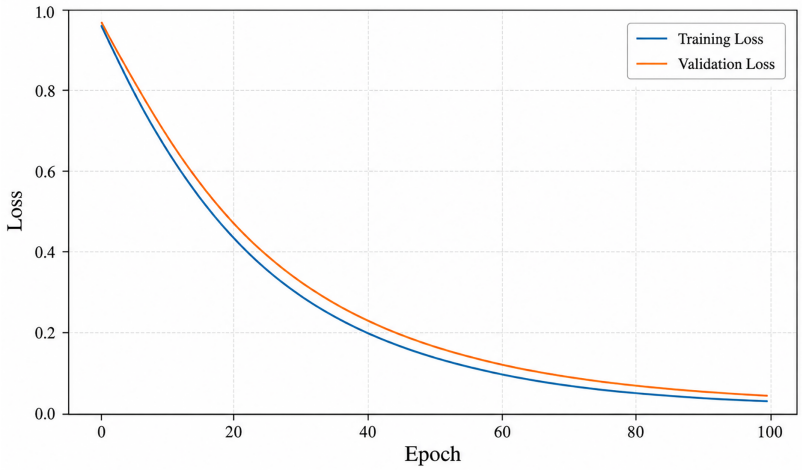

Figure 15: Learning curve of the quantum-assisted hybrid model.

Table 2 presents the performance comparison of the evaluated models on the test dataset in terms of MAE (normalized) and R^2 metrics. The results show that the proposed hybrid model achieves the best overall performance among all evaluated approaches, obtaining an R^2 score of 0.9925 and the lowest MAE value of 0.048. Among the classical machine learning models, the Multi-Layer Perceptron (MLP) achieves the highest predictive performance with an R^2 score of 0.963 and an MAE of 0.082. Gradient Boosting yields an R^2 score of 0.923 with an MAE of 0.085, while Random Forest shows comparatively lower performance with an R^2 score of 0.890 and an MAE of 0.091. These results indicate that, under identical evaluation settings, all evaluated models are capable of capturing underlying patterns in the dataset; however, the proposed hybrid framework provides improved predictive accuracy and lower error compared to classical machine learning approaches. Overall, the findings reported in Table 2 demonstrate the effectiveness of integrating quantum feature representations with classical deep learning models for regression-based environmental prediction tasks.

As shown in Fig. 16, the MAE decreases consistently as the entanglement depth increases. The model exhibits an MAE of 0.061 under low entanglement, which improves to 0.053 with medium entanglement and reaches the lowest value of 0.048 when strongly entangling layers are applied. This reduction in error indicates that deeper entanglement enhances the quantum circuit's ability to encode complex data patterns, leading to more accurate predictions.

As shown in Fig. 17, the R^2 performance of the proposed hybrid model improves with increasing entanglement depth. Specifically, the model achieves an R^2 score of 0.982 under low entanglement, 0.988 under medium entanglement, and reaches 0.9925 under strongly entangling configurations. This trend suggests that increased entanglement depth enhances the expressive capacity of the variational quantum circuit by enabling richer correlations between qubits. As a result, the model is better able to capture nonlinear dependencies within the input feature space. However, it is also important to note that deeper entanglement structures increase circuit complexity and may introduce higher computational overhead during training. Therefore, the observed improvements should be interpreted as a trade-off between representational power and computational cost rather than a strict performance guarantee. Overall, these results indicate that

entanglement depth is an important architectural hyperparameter in quantum-assisted learning models, influencing the balance between model expressivity and training efficiency in CO_2 emission prediction tasks.

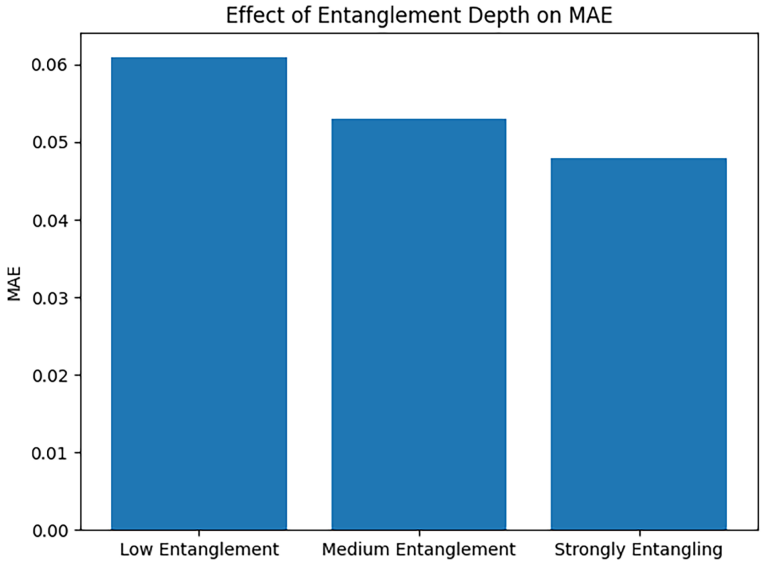

Figure 16: Effect of entanglement depth on MAE of the quantum-assisted hybrid model.

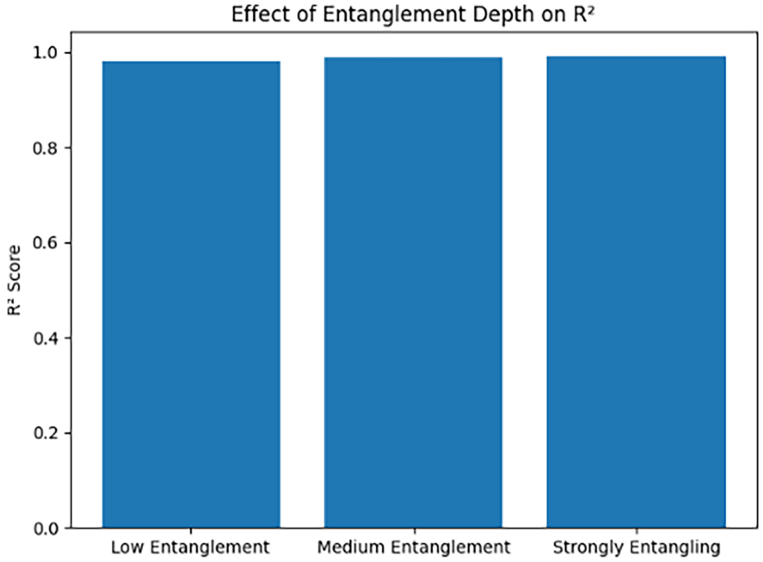

Figure 17: Effect of entanglement depth on R^2 performance of the quantum-assisted hybrid model.

As shown as Fig. 18, impact of simulated quantum noise levels on the prediction performance of the proposed quantum-assisted hybrid model. As noise intensity increases, the MAE gradually rises, indicating sensitivity to hardware imperfections while maintaining stable performance under moderate noise conditions.

The proposed quantum-assisted hybrid framework is designed with practical deployability in mind and can, in principle, be implemented on current quantum hardware platforms such as IBM Quantum Experience.

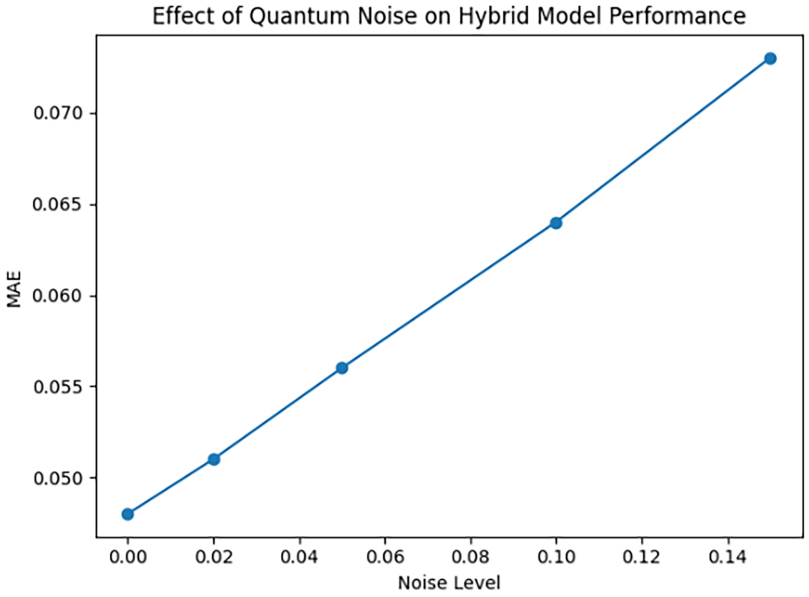

Figure 18: Effect of quantum noise on the performance of the hybrid model.

Google Quantum AI and Rigetti Forest are among the major platforms in the quantum computing ecosystem, while variational quantum circuits can be implemented using widely adopted quantum software frameworks such as PennyLane, Qiskit, and Cirq. The proposed variational quantum circuit is constructed using standard quantum gates that are natively supported within these frameworks, ensuring compatibility with existing quantum simulation and hardware environments. All experiments in this study are conducted using quantum circuit simulations rather than real quantum hardware. Therefore, it should be noted that factors such as noise, decoherence, and limited qubit connectivity in physical quantum devices may influence model performance in practical implementations. In such cases, error mitigation strategies and hardware-aware optimization techniques would be required for real-world deployment. Despite these limitations, the proposed hybrid framework demonstrates strong predictive performance in the simulation environment. In particular, the hybrid model achieves higher R^2 scores compared to classical baselines while maintaining a comparable level of model complexity in terms of trainable parameters. This suggests that the observed performance improvements are associated with more effective feature representation within the hybrid quantum–classical architecture rather than an increase in model size.

Overall, these findings indicate that variational quantum circuits can serve as a promising component in hybrid machine learning models by enhancing representation capability for structured regression problems such as CO_2 emission prediction.

4 Conclusions

This study explored the use of a quantum-assisted hybrid learning framework for vehicle CO_2 emission prediction by integrating classical feature learning with variational quantum circuits. The experimental results indicate that while classical machine learning models achieve strong performance, they may face limitations in modeling complex nonlinear feature interactions within structured environmental datasets. In contrast, the proposed hybrid quantum–classical approach, implemented using PennyLane and PyTorch, demonstrated improved representation capability for capturing such nonlinear relationships within a structured learning framework. To evaluate model performance, Mean Absolute Error (MAE) and the coefficient of determination (R^2) were employed. The hybrid model achieved a normalized MAE of 0.048 and an R^2

score of 0.9925, indicating high predictive accuracy and strong explanatory power. These findings suggest that the proposed hybrid framework can provide competitive and, in several cases, improved predictive performance compared to the evaluated classical machine learning models. Despite these promising outcomes, several challenges remain, including quantum hardware limitations, noise sensitivity, and the optimization complexity associated with hybrid quantum architectures. Overall, this work provides a systematic and reproducible framework for applying quantum-assisted learning to environmental data analysis. As quantum technologies continue to evolve, such hybrid models may play an increasingly important role in sustainability-focused data analytics and environmental decision-making. The proposed hybrid framework demonstrates an empirical performance improvement over the evaluated classical baselines in terms of prediction accuracy. However, the present study does not claim a formal quantum advantage in terms of computational complexity, scalability, or execution speed. Instead, the observed empirical improvements suggest that quantum-enhanced feature representations can serve as complementary learning mechanisms capable of improving nonlinear pattern extraction in structured tabular datasets.

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Availability of Data and Materials: The data used in this study are available upon request, though some may be restricted due to privacy, confidentiality, or ethical concerns.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

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