



REVIEW

Quantum Fuzzy Neural Networks: A Review of Foundations, Modeling Routes, and Open Problems

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ABSTRACT: Quantum fuzzy neural networks (QFNNs) integrate fuzzy systems, neural networks, and quantum models, aiming to leverage their complementary strengths in handling uncertainty, parameter learning, and feature representation. However, a unified framework for effectively combining these three components remains lacking, and the existing literature reflects diverse and sometimes inconsistent modeling strategies. This paper provides a comprehensive review of the fundamental theories underlying QFNNs, including the core design principles and mathematical formulations, as well as the major categories of network architectures. Representative training strategies and typical application scenarios are also systematically examined. Furthermore, persistent issues in the current literature are discussed in detail. These include blurred conceptual boundaries between fuzzy and quantum components, the absence of standardized experimental benchmarks, and the fact that most reported results remain limited to simulated environments without physical validation. Overall, QFNNs are still at an early stage of development and should be regarded as a promising direction for hybrid intelligent modeling rather than an established paradigm.

KEYWORDS: Quantum fuzzy neural networks; fuzzy neural networks; quantum machine learning; hybrid quantum-classical models; uncertainty modeling

1 Introduction

Fuzzy neural networks and quantum machine learning originally emerged from two distinct research traditions. The former emphasizes the use of membership functions, fuzzy rules, and parameter learning to handle uncertainty, and has developed a relatively stable set of basic structures within classical intelligent systems [1,2]. The latter revolves around quantum feature maps, parameterized quantum circuits, and variational optimization, and seeks to provide implementations for classification and representation learning that differ from those of classical models [3–5]. As trainable quantum circuits and hybrid optimization frameworks have gradually matured, researchers have begun to introduce quantum representational mechanisms into learning models equipped with fuzzy components. The force driving this convergence is not merely the appeal of interdisciplinarity. Conventional deep models often lack explicit means to represent fuzzy boundaries and uncertain semantics, whereas classical fuzzy neural networks remain constrained, to some extent, in complex representation learning and structural scalability.

The earliest studies in this direction were not quantum circuit models in the strict sense. Instead, they rewrote certain components of conventional fuzzy neural networks by drawing on quantum probability,

quantum entropy, or quantum learning rules. Early work had already attempted to incorporate quantum entropy and compensatory operations into fuzzy classification so as to improve decision behavior under uncertainty [6]. Subsequent studies embedded nonlinear quantum learning strategies into neuro-fuzzy networks and further extended quantization-inspired ideas to membership-function design and classification modeling [7,8]. In recent years, quantum-inspired optimization has also been used for self-organizing fuzzy neural modeling. This trajectory indicates that the field has not progressed along a single path; rather, the mode of quantum intervention has shifted across different stages [9].

The existing literature, however, has not arrived at a unified definition of quantum fuzzy neural networks. Similar names often conceal substantial differences in the internal division of labor. In some studies, the quantum module is placed at the back end of the model and is used for defuzzification or final decision mapping, with the aim of handling complex uncertainty in multimodal semantics [10]. In others, the quantum fuzzy network is embedded into a larger system framework and serves as a local inference unit in federated learning environments, where the concern is no longer limited to the classification performance of a single model [11]. Other studies move the quantum circuit forward to the stage of membership learning or hierarchical feature modeling, allowing the quantum component to participate directly in the formation of fuzzy representations [12]. More radical attempts even represent the fuzzy inference process itself as a variational quantum circuit, rather than treating the quantum module as a merely attached component [13]. Fig. 1 outlines this idea, showing the foundations, model families, functional roles, and application areas.

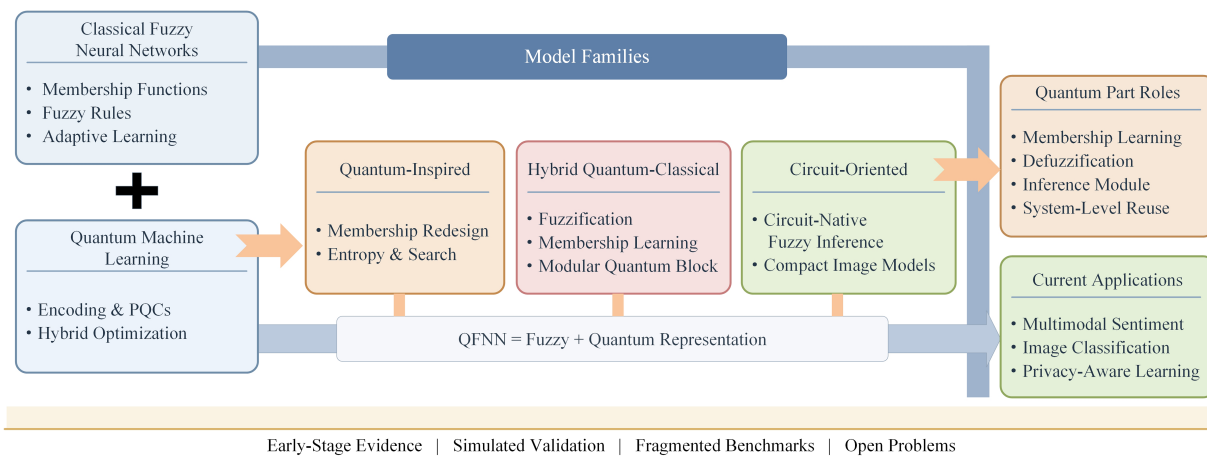


Figure 1: Conceptual overview of quantum fuzzy neural networks: foundations, model families, functional roles, and current application directions.

In this review, we use QFNN as a working umbrella term for models that combine fuzzy representation or fuzzy inference mechanisms, neural learning structures, and quantum-inspired or quantum-executable components within a unified learning framework. This working definition is used only to organize the current literature and should not be interpreted as a settled community-wide definition.

For this very reason, although the number of publications continues to grow, their conclusions are not easily comparable. There is no agreement on whether the quantum component is meant to perform representation learning, inference computation, or system-level reuse, and neither the experimental settings nor the comparison targets are usually consistent. Some recent studies have attempted to improve model expressivity under limited qubit resources by using strategies such as data re-uploading [14]. Others have discussed, from the perspectives of circuit expressibility and entangling capability, what kind of structural

gain parameterized quantum circuits can actually provide [15]. These discussions suggest that quantum fuzzy neural networks are better understood as an evolving direction of hybrid modeling than as a model class with clear boundaries and a settled definition.

This review article aims not merely to list past research findings. Instead, it organizes the main research routes of quantum fuzzy neural networks by examining how quantum components are integrated into models. First, it reviews the fundamentals of fuzzy neural networks and the theory of quantum machine learning, which are most relevant to this field. Next, it discusses several representative research routes, including quantum-inspired models, hybrid quantum-classical models, and circuit-oriented models. Building on this foundation, the article further compares the differences among these models in terms of structural design, training strategies, application scenarios, and research limitations. This organization aims to more clearly elucidate the connections and boundaries between different types of research, while providing a more stable framework for subsequent concept clarification, model comparison, and problem definition.

This review is intended as a conceptual and structured narrative survey rather than a systematic review. The representative studies discussed here were selected according to whether they combine fuzzy representation or inference mechanisms, neural learning structures, and quantum-inspired or quantum-executable components within a unified modeling framework. Rather than organizing the literature chronologically, we group prior work according to how the quantum component enters the model and what functional role it plays in the overall architecture.

2 Background

2.1 Classical Fuzzy Neural Networks

Quantum fuzzy neural networks did not arise in isolation from prior research traditions. Their most direct structural source remains the classical fuzzy neural network. Conventional fuzzy neural networks place membership functions, fuzzy rules, and parameter learning within the same framework, allowing the model both to represent continuous uncertainty and to adjust internal parameters according to data [1,2]. Structurally, the input is typically first mapped to different membership degrees, then aggregated through a rule layer, and finally converted into outputs through trainable consequent parameters or a network output layer [1]. This distinguishes fuzzy neural networks from fuzzy systems that rely purely on handcrafted rules and, at the same time, provides a clear interface for later integration with quantum modules.

Early studies had already shown, from different directions, that fuzzy inference and neural learning can be incorporated into a single model. Research on self-learning fuzzy control and backpropagation-based fuzzy modeling showed that fuzzy systems are not incompatible with gradient-based updating. On the contrary, data-driven parameter learning can improve the adaptability of the model to complex inputs [16,17]. Another line of work emphasized rule acquisition and structural organization, showing that fuzzy rules need not be completely specified by hand but can emerge gradually during learning [18,19]. Although these classical routes do not involve quantum computing, they had already established the three core components later required by QFNNS: membership representation, rule aggregation, and adaptive parameter updating.

From the standpoint of later developments, the influence of classical fuzzy neural networks on QFNNS is manifested mainly in three respects. First, membership functions provide a direct mechanism for handling fuzzy boundaries and uncertain semantics, which is particularly important in language, multimodal settings, and noisy environments. Second, the rule layer preserves an interpretable intermediate inference stage, preventing the model from collapsing entirely into an uninterpretable end-to-end black box [2,19]. Third, the parameter-learning mechanism allows the fuzzy system to adjust automatically to data, rather than remaining trapped within the framework of a static knowledge base [1,16]. Later quantumization attempts

therefore rarely discard the fuzzy structure as a whole. Instead, they reassign functions around these core components: some retain fuzzy rules while placing the quantum module at the decision end; some retain the notion of membership and allow quantum circuits to participate in membership learning; others attempt to rewrite the fuzzy inference process more directly as a quantum computational structure.

2.2 Quantum Fuzzy Neural Networks

If classical fuzzy neural networks provide the structural foundation for QFNNs, quantum machine learning provides a set of computational tools that can be embedded into hybrid models. The most important of these is the parameterized quantum circuit. Compared with earlier and more idealized quantum algorithms, parameterized quantum circuits are more suitable for joint training with classical optimizers and have therefore become the most common mode of quantum implementation in current QFNN research [4,5]. Studies on quantum feature maps, circuit-based classifiers, and trainable quantum neural networks further indicate that the quantum component does not need to cover the entire model. More often, it is embedded as a local representation-learning module within a larger hybrid architecture [3,20].

Within the specific context of QFNNs, the quantum component usually revolves around four concepts. The first is feature encoding, namely how classical inputs are mapped to quantum states. Different encoding schemes lead to different input structures that the quantum circuit is able to represent [3,4]. The second is the parameterized circuit block, which serves as the trainable transformation and is the most direct learning unit of the quantum part [4,20]. The third is the measurement stage, that is, how quantum states are transformed back into outputs that can be used for classification, membership evaluation, or local inference [20,21]. The fourth is hybrid optimization, where parameter updating is carried out jointly through repeated quantum evaluations and a classical optimizer. This has become an almost universal premise of current hybrid quantum learning models [5,21].

Recent studies have introduced two additional concepts that are particularly relevant to QFNNs. One is data re-uploading, whose basic idea is to inject classical information repeatedly under limited qubit resources so as to enhance the expressivity of shallow quantum circuits [14]. The other concerns expressibility and entangling capability, which are used to analyze the representational richness and structural potential of parameterized quantum circuits [15]. Understanding these foundational concepts is therefore necessary not only for introducing the background of quantum machine learning, but also for providing criteria with which to judge the structural choices and experimental results of QFNNs in the following sections.

2.3 Why Fuzzy Models and Quantum Models Are Combined

The repeated placement of fuzzy mechanisms and quantum modules within the same framework in recent years does not follow from any natural belonging to a single theoretical system. It reflects the fact that they address different problems in learning models. The strength of fuzzy models lies in their ability to represent uncertainty more directly through membership degrees and rule structures. The appeal of quantum models lies in their providing implementations for representation learning and local mapping that differ from those of classical networks [3,5]. When the target problem simultaneously involves fuzzy boundaries, complex interactions, or multisource information, it is unsurprising that researchers attempt to place these two mechanisms within the same model and test whether they can complement each other [10,12].

Some studies introduce the quantum module after fuzzy processing, assigning the quantum part to defuzzification, decision mapping, or local fusion [10]. Others move the quantum module forward to the stage of membership learning or local representation formation, allowing the quantum circuit to participate directly in the generation of fuzzy representations [12]. Still other work embeds quantum fuzzy networks into larger system frameworks, such as federated learning or privacy-preserving environments, where the

concern is no longer limited to single-shot classification accuracy but extends to system-level inference and deployment [11].

Because the combination method is not unique, the structure of quantum fuzzy neural networks (QFNNs) shows obvious diversity. In different studies, the boundary between the fuzzy component, the quantum component and the surrounding classical architecture is not fixed. Quantum modules are sometimes used as back-end mappers, sometimes as front-end learners, and sometimes even as local components in larger systems. In other words, if the literature is only listed by year, it cannot truly reflect the actual development of this field. In order to understand more effectively, we need to pay attention to how the quantum part is integrated into the model and the specific functions it undertakes. Next, we will not introduce according to the timeline, but classify the existing QFNN studies according to the research route.

3 Research Routes of Quantum Fuzzy Neural Networks

3.1 Basis for Classification

Although many studies use the name “Quantum Fuzzy Neural Network” (QFNN) or similar terminology, the internal logic of models categorized into this group is inconsistent. The real difference lies not in the task they are designed for, but in how the quantum component enters the model. In some studies, the quantum component is more of a conceptual rewrite; in others, it is embedded as a trainable module; and still others go further, attempting to allow the quantum component to directly participate in the circuit-level inference process [6,10].

From this perspective, existing research can be broadly categorized into three main lines: quantum-inspired models, hybrid quantum-classical models, and circuit-oriented models. This classification has at least two direct advantages. First, it distinguishes early quantum-inspired research from recent hybrid models and quantum circuit models, avoiding the use of the same standards to evaluate fundamentally different models [6–8]. Second, this classification is closer to a core question: what role does the quantum component play in the model?

The answers to this question vary greatly among different studies. Some studies focus on redesigning membership functions or quantizing membership mechanisms [8,22]; others focus on defuzzification processes or local inference stages [10,12]; and still others attempt to transform fuzzy inference itself into quantum circuits [13]. In addition to these more explicit QFNN models, some studies also discuss fuzzy mechanisms in quantum environments from a broader “fuzzy-quantum” perspective, including content related to quantum annealing [23]. Table 1 summarizes these three research approaches from two aspects: the entry point of the quantum part into the model and the functions it undertakes.

Table 1: Main categories of models reviewed in this manuscript.

| Category | Representative Studies | Main Role of the Quantum Part | Role of the Fuzzy Part | Typical Tasks |
|------------------|------------------------|------------------------------------------------------|------------------------------|-----------------------------------------|
| Quantum-inspired | [6–8] | Membership redesign or quantum-inspired optimization | Classical inference backbone | Classification, modeling, path planning |

(Continued)

Table 1 (continued)

| Category | Representative Studies | Main Role of the Quantum Part | Role of the Fuzzy Part | Typical Tasks |
|--------------------------|------------------------|------------------------------------|-----------------------------------------------------|-----------------------------------------------------------------|
| Hybrid quantum-classical | [10–12] | Embedded trainable quantum modules | Fuzzification, rule aggregation, or local inference | Multimodal analysis, image classification, distributed learning |
| Circuit-oriented | [13,24,25] | Circuit-level fuzzy inference | Rule encoding or fuzzy-state representation | Image classification and structured-data classification |

3.2 Quantum-Inspired Models

The quantum-inspired route appeared first in QFNN-related work. Here, quantum is a design language, not an execution platform. Researchers borrow quantum probability, quantum entropy, probability amplitudes, or quantum learning rules. They then rewrite membership functions, classification steps, or parameter updates in classical fuzzy neural networks. The full model still runs on classical hardware [6,7].

The point of this route is therefore not real quantum circuits. It is the expansion of the design space of classical fuzzy neural networks through quantized descriptions.

Early studies already showed several directions. One line introduced quantum entropy and compensatory operations into fuzzy classification and tried to improve decisions under uncertainty [6]. Another embedded nonlinear quantum learning strategies into neuro-fuzzy networks and located the quantum element mainly in the learning rule [7]. Later work extended quantized ideas to membership-function design and classification modeling. Some studies replaced conventional memberships with quantum membership functions. Others used quantum-inspired mechanisms to strengthen classification [8,22]. More recently, quantum-inspired optimization has been used in self-organizing fuzzy neural modeling. That shift moved this route from membership rewriting toward structural search and optimization [9].

The main value of this route is conceptual. It shows that fuzzy neural networks need not rely only on classical probability or standard set-based interpretations. They can also adopt quantized parameter forms or quantum-inspired search [7,8].

Its limits are equally clear. Inference and training still depend entirely on classical execution. Reported gains therefore do not establish a genuine quantum advantage. They more likely come from altered parameterization, revised search, or structural reorganization [6,9]. From today's standpoint, quantum-inspired models are best treated as the starting point of QFNN research. They opened the question of how quantum ideas enter fuzzy neural networks, but did not yet turn the quantum part into a true computational module.

3.3 Hybrid Quantum-Classical Models

Compared with quantum-inspired models, a prominent line of recent research has tended to embed small-scale quantum modules into classical models. This is also the most representative technical route in the current QFNN research. In this kind of design, the classical part usually undertakes tasks such as high-dimensional feature extraction, rule organization and large-scale parameter update; the quantum part mainly

plays a local role, such as defuzzification, membership learning, local nonlinear mapping, or multimodal information fusion [10–12]. In other words, this route is no longer just borrowing quantum ideas to rewrite the original model, but embeds trainable quantum circuits into the system as actual computing units.

Representative studies have shown that this kind of hybrid model is not single in terms of structural layout. In multimodal emotional analysis and irony recognition tasks, quantum modules are often located at the back end of the model and are responsible for mapping fuzzy representations to the final decision-making space. In this case, the quantum part is closer to the role of defuzzer and classifier [10]. In the federated learning scenario, the local QFNN is embedded in a larger distributed system. At this time, the quantum fuzzy network is no longer just an independent model, but participates in node-level inference and system-level aggregation at the same time [11]. In the image classification task, the quantum module may also move forward to the membership learning stage, generate fuzzy representations through quantum circuits, and work together with the classical vision branch [12]. These examples show that even if it belongs to the same quantum-classical hybrid route, the specific position and function of quantum modules may be significantly different.

An important reason why this line has become increasingly visible is that it achieves a relative balance between feasibility and structural innovation. The quantum part does not have to cover the entire network, so it is usually within the controllable range in terms of the number of quantum bits, training difficulty and experimental cost [5,10]. At the same time, the fuzzy mechanism has not been abandoned. It is still responsible for tasks such as uncertainty characterization, rule aggregation and intermediate state interpretation [11,12].

However, there are also obvious limitations on this route. The performance of the model often depends highly on the position of the insertion of the quantum module and the way it is coordinated with the surrounding classical structure. That is to say, even if both studies are classified as hybrid models, they may still be inconsistent in structural logic. Therefore, in the comparison in Section 4, they cannot be discussed side by side just because they are all mixed models. A more reasonable approach is to analyze where the quantum part is placed and what function it assumes.

3.4 *Circuit-Oriented and Recent Architectures*

Although hybrid quantum-classical models introduce quantum modules, they still retain a clear overall boundary, namely the combination of a classical trunk and a quantum module. In contrast, the architecture for quantum circuits goes further. This kind of research no longer regards the quantum part as an add-on component, but tries to represent fuzzification, rule computation, inference composition, and even defuzzification directly as quantum circuits or gate sequences [13]. Under this idea, fuzzy inference is no longer just an auxiliary mechanism around quantum modules, but is reorganized into a calculation process at the circuit level.

Judging from the existing literature, this route is still in the early stage. There have been studies to build quantum convolutional networks around variable quantum fuzzy inference and use them for small-scale classification tasks [13]. Since then, some works have further proposed a more compact quantum-fuzzy structure in an attempt to continue to compress the circuit scale in image classification [24]. Recent studies have also begun to discuss the hierarchical structure of dual quantum bits and the problem of relational learning. In these models, entanglement is no longer just an accessory design, but gradually becomes an important mechanism that affects the expression ability of the model [25]. At the application level, there are also studies that try to extend such structures to image classification tasks with stronger structure. This shows that fuzzy inference based on quantum circuits is gradually moving from early concept verification to more specific application scenarios [26].

Even so, this route is still a clear distance from the mature method framework. First of all, the existing evidence mainly comes from the research on the nature of conference papers, preprints or concept verification, and the experimental scale and control settings are still limited [13,24]. Secondly, the consistency of different studies in terms of data sets, baseline models and evaluation indicators is weak, so it is still difficult to accurately judge how much additional benefits the native design of the circuit can bring [25,26]. Thirdly, once the model further increases the depth, entanglement structure or relationship coding, its trainability and realization cost will rise rapidly, which cannot be ignored. Based on these circumstances, a more sound expression should be that this route cannot yet be regarded as an established mainstream route, but it remains one of the more promising recent directions in the further evolution of QFNN.

4 Representative Architectures and Comparative Analysis

To compare how representative architectures divide labor among fuzzy, quantum, and classical components, Table 2 summarizes their structural design, task setting, and main architectural contribution. The following sections discuss the multimodal QFNN, the local QFNN in QFFL, QA-HFNN, and VQFI-QCNN on that basis.

Table 2: Representative QFNN architectures and their division of labor among fuzzy, quantum, and classical components.

| Model | Fuzzy Mechanism | Quantum Mechanism | Classical Support | Main Task Setting | Main Architectural Contribution |
|-------------------------|--------------------------------------|---------------------------------|----------------------------|--------------------------------------------------|-------------------------------------------|
| QFNN [10] | Task-aware fuzzification | Quantum defuzzifier | Encoder-decoder backbone | Sentiment and sarcasm detection | Staged fuzzy-quantum decision pipeline |
| Local QFNN in QFFL [11] | Fuzzification and t-norm aggregation | Local QNN with global inference | Distributed node structure | Privacy-aware intelligent information processing | Distributed reuse in federated inference |
| QA-HFNN [12] | Quantum memberships and fuzzy rules | PQC with re-uploading | Parallel DNN branch | Image classification | Compact hybrid membership learning |
| VQFI-QCNN [13] | Variational fuzzy-rule inference | Variational QCNN block | Limited classical baseline | Structured-data classification | Proof-of-concept circuit-native inference |

In addition to architectural differences, an important comparative issue is whether the reported gains can be attributed to the quantum component itself, to the fuzzy mechanism, or to stronger surrounding classical support.

4.1 QFNN for Multimodal Sentiment and Sarcasm Detection

In existing QFNN research, models for multimodal sentiment analysis and irony detection generally present a clear hierarchical structure. Their complete process typically includes three interconnected stages: first, classical neural networks extract textual and multimodal contextual features; then, complex-valued fuzzy representations characterize ambiguity and uncertainty in language; finally, parameterized quantum circuits are used to complete defuzzification and classification tasks [10]. The most noteworthy aspect of this design is the position of the quantum module. It does not participate in front-end feature formation but is placed at the end of the decision chain.

From a structural design perspective, this arrangement of “classical first, fuzzy in the middle, quantum last” has strong engineering controllability. The quantum circuit is only responsible for the mapping in the final stage, so the model does not need to maintain large-scale quantum representations at the front end, making implementation relatively more stable. Meanwhile, the fuzzy part plays a role in intermediate semantic organization, and what enters the quantum module is not the original high-dimensional features but a fuzzy state shaped by uncertainty [10]. Existing experiments have also compared various encoding methods, and the results show that angle encoding is more suitable for this task. This, to some extent, illustrates that the quantum module is not arbitrarily added to the model, but rather assumes a relatively defined function in the backend decision-making stage.

However, the limitations of this structure are also quite apparent. First, it is highly dependent on the specific task, especially on the degree of matching between fuzzy representations and phenomena such as linguistic ambiguity, semantic conflict, and sentiment reversal. Second, and more importantly, when the quantum part mainly performs the defuzzification function, it is not easy to determine where the model performance improvement comes from. The performance improvement may come from the quantum circuit itself, or it may mainly benefit from the preceding fuzzy representations, because a large amount of information has actually been pre-screened and organized before entering the quantum module.

4.2 Local QFNN in Quantum Fuzzy Federated Learning

The value of the local QFNN in QFFL lies in its shift from single-model prediction to system-level inference. In this framework, each local node maintains a small quantum fuzzy network. Inside the node, the model still includes fuzzification, quantum t-norm aggregation, and a parameterized quantum neural layer. The system-level focus, however, moves away from one model’s internal structure. It turns instead to how multiple local models coordinate, when they aggregate, and how the system reduces communication and privacy risk.

Conventional federated learning usually aggregates model parameters. QFFL instead stresses prediction aggregation based on quantum-assisted density estimation. This turns local QFNN plus global inference strategy into one integrated architecture [11]. The importance of this work therefore does not lie only in the local QFNN itself. It also shows that quantum fuzzy processing can exist as a dedicated component inside a larger distributed system.

From a comparative perspective, the most important feature of QFFL is the relocation of the architectural question. The emphasis shifts from internal division of labor within one network to organization across modules. That means the model cannot be judged only by the classification ability of the local QFNN. The evaluation must also ask whether the global aggregation strategy is the main source of benefit. Reviews and method studies on QFL repeatedly show that communication cost, aggregation rules, and privacy constraints can substantially shape system performance [27,28]. Part of QFFL’s contribution comes from the design of

the local QFNN. Another part clearly comes from the federated framework around it. Because these two contributions coexist, the model requires careful interpretation.

4.3 QA-HFNN for Image Classification

If the multimodal QFNN places the quantum module at the back end, and QFFL places the QFNN inside a larger system, QA-HFNN makes a third choice: it moves the quantum circuit directly into the membership-learning stage [12]. In this model, one branch uses single-qubit parameterized circuits to learn membership functions from the input features. These quantum-generated memberships then enter a fuzzy rule layer for aggregation. At the same time, a classical deep branch extracts spatial image features. The two branches meet in a fusion layer and then feed a classifier. Compared with the previous two models, this architecture places the quantum module more explicitly in representation formation rather than in back-end mapping or system-level reuse.

The key feature of this design is not the size of the quantum part but its position. The module does not merely refine an existing result at the end. It intervenes directly in how the input is interpreted as a fuzzy representation [12]. Because the circuit is small, the authors can compare circuit depth, gate type, parameter count, and noise conditions in a more systematic way. This makes QA-HFNN one of the few QFNN architectures that examines the actual contribution of the quantum module in some detail. Its lightweight design, combined with data re-uploading, also preserves some expressivity under limited quantum resources.

Even so, a familiar problem remains. The reported improvement may come from the representational properties of the quantum circuit; it may also come from the simple addition of another trainable nonlinear module. Stronger controls are still needed to separate these effects. QA-HFNN remains important nonetheless. It represents a clear design logic within the hybrid route: let the quantum module handle membership learning, while the classical branch continues to handle mature visual representation learning.

5 Training Strategies and Implementation Platforms

5.1 Membership Modeling and Feature Encoding

From a training perspective, the key issue for quantum fuzzy neural networks (QFNNs) is not simply choosing an optimizer. The more fundamental question is: at what stage does fuzzification occur, on what objects does it act, and where does the quantum encoding enter the model? Early quantum-inspired research has shown that membership modeling is not a dispensable preprocessing step; it often directly affects the organization of subsequent classifiers [6–9]. In these studies, quantum factors are usually reflected in the rewriting of membership functions, or in the reconstruction of entropy terms, compensation operations, and learning rules. Taking a neural fuzzy classifier based on quantum entropy and compensation mechanisms as an example, it is clear that fuzzy representation and decision structure are coupled together from the beginning, rather than being two independent modules [29].

As QFNNs have gradually developed to the quantum-classical hybrid stage, this issue has become more specific and prominent. Different models show significant disagreement on “when to perform fuzzification.” Some approaches first convert task-related features into fuzzy representations before feeding them into the quantum module; others allow parameterized quantum circuits to directly participate in membership learning before connecting them to the fuzzy rule layer; still others place the quantum circuits in a later position, making them mainly responsible for defuzzification or decision mapping [10–12]. In this context, the encoding strategy is no longer just a detail choice at the implementation level. For example, methods such as data re-uploading can affect how information is injected into the model with limited quantum bit resources, thereby changing the expressive power of shallow quantum modules [14]. Therefore, in QFNNs,

membership modeling and feature encoding actually constitute two aspects of the same interface: the former is responsible for organizing uncertainty, and the latter determines how this uncertainty enters the quantum computing process.

5.2 Parameter Optimization

Judging from the existing research, the parameter optimization of QFNNs almost always adopts a hybrid scheme. Even those models that are closer to native quantum circuit inference still rely on the combination of classical optimizers and repeated evaluation of quantum circuits during training, and there is still no genuine all-quantum training framework [10–13]. This is not surprising. Under current hardware and algorithmic conditions, large-scale, stable, and end-to-end quantum optimization remains difficult to achieve.

In earlier quantum-inspired models, classical training methods were still commonly used, such as backpropagation, cluster updating, least squares, and quantum heuristic search [6,7,9]. After entering the hybrid-model stage, research has become more inclined to adopt parameter-shift rules, gradient approximation methods, and classical optimizers wrapped around quantum forward computation [10–12].

However, what is really worth paying attention to is not whether this optimization method is “pure,” but whether it can support the model from concept verification to relatively stable structural design. Although representative QFNNs vary greatly in model form, they show similar characteristics in their training mode: the overall process is still dominated by classical optimization, while the quantum part participates in the computation cycle [10–12]. In this regard, QFNNs have not opened up a training path that is fundamentally different from general hybrid quantum learning. The optimization strategy is by no means an accessory technical detail. It largely determines whether the model remains trainable, comparable, and interpretable after the introduction of quantum modules.

5.3 Robustness, Expressibility, and Complexity

The evaluation of QFNN research is no longer limited to the single indicator of accuracy. More and more studies have begun to pay attention to issues such as noise robustness, circuit complexity and training stability. In other words, the focus of the study is gradually shifting from “whether the model has achieved higher scores” to “whether the result is reliable”.

In some representative models, noise tests usually introduce disturbances such as amplitude attenuation, depolarization, bit flipping or phase flipping, and observe the subsequent changes in model performance [10,12]. QFFL also gives a very inspiring phenomenon: the increase in the number of quantum bits does not necessarily lead to performance improvement, but may make the training process more unstable [11]. This shows that in QFNN-related models, more quantum resources are not automatically equivalent to stronger modeling capabilities.

Of course, by increasing the circuit depth and entanglement structure, the expression ability of QFNN can be further improved. But at the same time, with each layer of complexity, the cost of training and deployment will also rise. For the time being, the more realistic idea may not be to build a large and complete quantum network, but to limit the quantum part to a position with clear functions and controllable input scale. In this regard, QFNNs face the same realistic boundaries as general hybrid quantum learning: structural innovation can continue to be promoted, but training stability and realization cost are always unavoidable constraints. Recently, there has been a study to estimate the expression ability of parametric quantum circuits by using graph neural networks, which provides a more specific tool for analyzing the representation potential of quantum modules [30].

5.4 Platform Choice and the Comparison Problem

At present, research on QFNNs still relies heavily on simulation platforms rather than real quantum hardware at the implementation level. Whether it is multimodal QFNN, QFFL, or QA-HFNN, although quantum modules are introduced, the main experimental results still come from the simulation environment. Although some studies have added noise models to test robustness, there is still little work for real deployment to physical quantum devices [10–12]. This does not mean that existing studies are worthless, but it does limit the range of conclusions that they can support. At this stage, the more core question is still whether the model architecture is reasonable, not whether it has mature deployment conditions.

In broader quantum-classical hybrid learning research, platform selection is also closely related to data boundaries and privacy constraints. Studies have begun to focus on hybrid visual models for privacy protection, which shows that the implementation limitations are not only hardware problems, but also involve how data flows, which calculations can be shared, and which modules must be kept locally [31,32]. From another perspective, the comparative study of quantum-classical hybrid neural networks also points out that model conclusions are often highly dependent on task setting, parameter scale and contrast design [33]. Therefore, the platform selection and the comparison framework are actually inseparable. If most of the results still come from the simulation platform, then the judgment of the quality of the model relies more on rigorous and reproducible control experiments. In general, the current promotion of QFNNs at the implementation level is still relatively cautious, and the focus is still on testing whether the structural concept is valid, rather than proving that it has the ability to deploy on a large scale.

6 Application Scenarios

6.1 Why Current Applications Concentrate in a Few Scenarios

This is not just a matter of researcher preference. It reveals where QFNNs currently make structural sense. The tasks most often used share three features. First, the inputs or labels contain clear uncertainty or fuzzy boundaries. Second, the task allows the quantum part to remain a small local unit with a clear role. Third, the setup is suitable for prototype validation in simulation [10–13].

The present application pattern is therefore not accidental. Tasks that require explicit uncertainty modeling and also allow local quantum modules are natural testbeds for QFNNs. By contrast, if a task has little need for fuzzy representation, or demands large-scale, end-to-end, hardware-deployable quantum structures, QFNNs currently show no clear advantage. The task distribution already marks the limits of the field. At this stage, QFNNs fit some structurally plausible problems; they are not ready to serve as a general-purpose method.

6.2 Multimodal Sentiment and Semantic Tasks

Multimodal sentiment analysis and related semantic tasks are among the earliest settings in which QFNNs show a strong structural fit. Beyond representative models for sentiment and sarcasm detection, quantum multimodal fusion systems and their extensions to social-media misinformation detection suggest that this line is not confined to one task. It recurs across heterogeneous semantic modeling more broadly [10,34,35]. Recent work has further explored quantum-neuro-fuzzy approaches specifically designed for sentiment analysis, demonstrating the continued interest in this application domain [36]. These tasks share a common pattern: text, vision, and context all contribute to the judgment, while sentiment polarity, semantic reversal, and local conflict naturally exhibit fuzzy boundaries.

In such settings, the appeal of QFNNs is not that they have already surpassed all mature language models. It is that they organize the problem differently. Standard deep models usually compress ambiguity

into vector representations. QFNNs more often organize uncertainty explicitly as fuzzy states and then assign local fusion or decision-related computation to the quantum module [10,34,35]. This design fits scenarios in which conflicting signals coexist, because both fuzzy representation and quantum mapping preserve more flexibility in intermediate states. Multimodal sentiment and semantic tasks therefore function less as proof of universal effectiveness than as high-fit testbeds.

6.3 Privacy Protection and Distributed Learning

Unlike the previous type of applications, the focus of privacy protection and distributed scenarios is not how a single model interprets input information, but what role quantum-fuzzy modules play in larger systems. In addition to QFFL, relevant studies have also extended similar ideas to dynamic quantum federated learning for intelligent diagnosis, a review of quantum federated learning, and multimodal task scenarios [11,27,28,37,38]. On this research line, the value of QFNNs lies not mainly in their performance as independent models, but in their ability to participate in system operation as local inference units in a distributed framework.

The importance of this kind of application is that it changes the way we understand QFNNs. In such a system, the model benefits can no longer be simply reduced to how much accuracy the quantum module has improved. Factors such as communication overhead, aggregation strategy, local density estimation and privacy constraints will jointly affect the performance of the whole system [27,28,37,38]. Because of this, this direction often reveals the true state of QFNNs better than many single-model studies. At least for now, QFNN is not a mature module that can be evaluated separately from the system environment. It still largely depends on the embedded task framework. The distributed scenario does open up a new application space for QFNNs, but at the same time, it also further amplifies the problems of difficult performance attribution and insufficient fairness comparison.

6.4 Image Classification and Vision Tasks

Image classification is another gradually expanding direction in research on QFNNs, but its significance is not exactly the same as that of multimodal semantic tasks. The key here is not the fuzzy semantics itself, but the visual task provides a clearer test environment for the position selection of the quantum module, the depth design of the circuit and the structural compression effect. From quantum-classic hybrid convolutional networks, expandible quantum convolutional structures, to hybrid models in remote sensing scenarios and more general image classification networks, this route has formed a relatively continuous visual research vein [39–42]. Similar development trends can also be seen in remote sensing tasks and shallow hybrid models [43,44]. These studies show that the reason why visual tasks appear frequently is not because it is naturally more suitable for fuzzy semantic modeling, but because it is more suitable for testing the specific role of local quantum modules in hierarchical representation.

In the visual scenario, the evidence on model robustness and structural selection has also accumulated relatively more. Some studies on adversarial robustness and multi-channel supervised learning show that there is no unified template for quantum-classical hybrid convolution models, and there may be large differences in input organization, channel design and noise sensitivity [45,46]. In addition, the relevant models for classification and medical diagnosis also show that the current object of visual research is no longer limited to standard image data sets, but gradually extends to more specific and targeted tasks [30,47]. However, at this stage, most of the results are still based on small-scale data sets, shallow circuits and simulation platforms. Therefore, the increase in the number of visual direction papers shows that QFNN is easier to carry out tests in this scenario, but does not directly prove that it has formed a stable and transferable engineering advantage.

7 Advantages, Limitations, and Open Problems

7.1 Main Advantages

From the current research accumulation, the most prominent advantage of QFNNs lies not in their proven performance superiority, but in providing a reasonable structural coordination mechanism. In this framework, the fuzzy mechanism handles uncertainty and fuzzy boundaries, the neural network undertakes parameter learning and adaptive adjustment, and the quantum component is embedded in the model as a local representation, mapping, or transformation unit [10,12]. For tasks with complex input sources, unclear boundaries, or difficult-to-characterize local structures, this division of labor is not a simple superposition, but rather has a certain task adaptability.

According to existing representative studies, the position of the quantum module in QFNN is not fixed. It can be placed at the back end of the model for defuzzification or decision mapping; it can be moved forward to the membership learning stage; or it can be embedded in a larger system structure, functioning as a local inference unit [10–13]. Therefore, QFNN is not suitable to be viewed as a highly unified set of models, but rather as an unclosed design space. This openness itself is an advantage, as it allows researchers to continuously explore more meaningful structural combinations regarding the placement, functional positioning, and integration with the fuzzy mechanism of the quantum module. Of course, this advantage, more accurately described as compatibility and flexibility in modeling, is not a universally proven superiority. To date, a more conservative assessment remains that fuzzy mechanisms and trainable quantum modules have demonstrated feasible synergistic potential in some tasks. Whether this potential can be further translated into stable, generalizable methodological advantages still depends on more rigorous theoretical analysis and experimental verification.

7.2 Main Limitations

The first outstanding problem facing QFNN at present is conceptual instability. Although many studies use “quantum fuzzy neural network” or similar names, they vary greatly on many key issues, such as whether the quantum part is really executed, whether the fuzzy mechanism constitutes the core inference structure, and whether the classical trunk remains structurally dominant in the model [10–13]. At present, QFNN is more like a collective label than a strictly defined method category. As long as this layer of ambiguity is not clarified, it is difficult for many subsequent comparisons to build on a solid foundation.

The second problem is that the performance attribution is weak. In many studies, the improvement of model performance is often accompanied by the adjustment of fuzzy mechanisms, the addition of quantum modules, stronger classical support structures, and changes in parameter scale. When the result is better, it is difficult to judge what is the factors that really play a decisive role. The previous comparison of representative models has shown that some performance improvements may come from the placement of quantum modules, and some may mainly come from the enhancement of peripheral classical structures; and in system-level scenarios, the benefits may even come from the overall inference framework rather than the local QFNN itself [10–13]. Therefore, what QFNN lacks at present is not only more experimental results, but more importantly, more rigorous attribution design.

The third problem is the lack of a unified experimental framework. The existing research is scattered on different tasks such as image classification, remote sensing, multi-channel supervised learning, etc., and the data sets, coding methods and circuit depths used are also inconsistent [39–42]. Similar dispersion phenomena also exist in a wider range of mixed visual models and specific application tasks [42–47]. In this case, even if different studies report positive results, it is difficult for them to be included in the same comparable evaluation framework. Recent comparative studies on quantum-classical hybrid neural networks

also point out that experimental conclusions are often highly dependent on task setting, parameter scale and comparison protocols [33]. This discreteness is not surprising for a field that is still developing, but it does weaken the judgment that “QFNN already has stable methodological value”.

The fourth problem is the lack of evidence at the level of real deployment. As mentioned above, most representative QFNN models are still mainly verified on the simulation platform [10–13]. This also exists in a wider range of hybrid visual models and convolutional models [39–42]. Even in studies with richer image tasks and supervised learning tasks, evidence from physical quantum devices is still quite limited [43–46]. Many structural options that are feasible in the simulation environment have not actually faced constraints such as noise, calibration drift, delay and cost in real hardware. Therefore, at this stage, QFNN can only show that such designs are feasible at the prototype level, but cannot be concluded that they have the conditions for stable deployment.

Finally, as the quantum structure becomes more complex, the difficulty of training will also increase rapidly. This is not an individual defect of a certain model, but a hard constraint commonly faced by hybrid quantum learning. With the deepening of the circuit, the increase of parameters and the complexity of the entangled structure, the modeling process tends to become more unstable. Even from a theoretical point of view, the variable quantum algorithm itself may face quite serious training difficulties [48]. Therefore, for QFNN, the real key question is not whether more quantum modules can continue to be added, but whether the model remains trainable, comparable and explainable after the introduction of these modules.

7.3 Open Problems

First of all, it is still a problem at the theoretical level. At present, there is still a lack of clear explanation of what kind of correspondence should be established between fuzzy membership, fuzzy rules and quantum states in this field. It is not clear whether membership should be understood as probability amplitude, measurement results, or quantum statistical objects in a more general sense. At the same time, when the fuzzy rule is put into the quantum circuit, whether it still retains the original inference meaning or just retains a certain semantic label has not been fully clarified [49,50].

The second open question belongs to the method level. If QFNN wants to go beyond the accumulation of results that “look effective but underexplained”, future research must establish a stricter framework for comparison and ablation. The version without fuzzy mechanism, the version without quantum module and the complete hybrid version should be compared under the controlled parameter budget, training budget and task conditions. Without this control, QFNN is likely to fall into a common dilemma: the model structure is becoming more and more complex, but the research conclusions are becoming more and more difficult to explain.

The third open question is related to the scale of the model and the design focus. The existing evidence has shown to a certain extent that the future development direction of QFNN is not necessarily to build a large-scale, general-purpose quantum network, but more likely to design small-scale, task-oriented and clearly functional quantum modules. This seems to be a biased judgment, but it actually leads to a sharper question: which local functions are really worth “quantizing” and which parts are still more suitable for maintaining the classical implementation?

The last open question involves the path from prototype verification to real deployment. Given that a large number of results still rely on simulators and small-scale experiments, whether QFNN can be further developed in the future may not mainly depend on whether to continue to propose new model names, but more on whether training stability, representation effectiveness and system implementability can be maintained at the same time under limited hardware conditions. If no substantial progress has been made

in this layer, QFNN may remain in a state of conceptually attractive, experimentally demonstrable, but still difficult to implement in practice.

8 Conclusions

In general, quantum fuzzy neural network (QFNN) is not a mature system of methods with clear boundaries. Instead of treating it as a fixed model class, it is better to understand it as a hybrid modeling idea that is still evolving. At this stage, the relevant research can be broadly understood along three paths: quantum-inspired models, hybrid quantum-classical models, and circuit-oriented models. However, whether in terms of model structure or research paradigm, this field has not yet formed a unified framework. In the end, the current research is still repeatedly exploring two basic issues: which link the quantum part should be embedded in the model, and what role it plays in the whole system [10–13].

The conclusion that the existing evidence can support should actually be restrained. The most noteworthy thing about QFNN at present is not that it is generally superior to classical methods in various tasks, but that it shows a certain structural rationality in a few representative scenarios. Especially in tasks that require explicit expression of uncertainty and are suitable for the introduction of small-scale quantum modules, fuzzy mechanisms and quantum modules have been able to combine operational prototype models. However, most of these results are still based on scattered task settings, limited experimental scale and simulation platforms, so they show that this direction has the value of further research, rather than determining that it has formed a mature methodology.

Judging from the subsequent development, what QFNN really needs to promote is not to continue to add model names or structural variants, but to solve several more basic problems. First, it is necessary to further clarify the theoretical relationship between the fuzzy mechanism and quantum representation; secondly, it is necessary to establish a stricter comparison and ablation framework to improve the interpretability and comparability between different models; thirdly, it is necessary to develop small-scale quantum modules that remain trainable, interpretable, and deployable under limited hardware conditions. Only by making more solid progress in these aspects can QFNN gradually move from an attractive mixed idea to a relatively stable method framework.

Overall, the present evidence suggests that the main value of QFNN lies in its role as a promising hybrid modeling direction rather than as a mature and standardized method class. Future progress will depend less on introducing additional model variants and more on clarifying conceptual boundaries, establishing fair benchmarking protocols, and identifying which local functions genuinely benefit from quantization under realistic resource constraints.

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