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Theory and Comments on Standard Dilatometric Back Analysis

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Summary

The robust analytical solution was carried out to describe the stress state in the massive round boreholes. It gives the chance for complex back analysis of dilatometric in situ measurements. The main goal of this presentation is to present the incorporating phenomenon of influence zone around the boreholes. The analytical solution gives the chance to describe the progress of plastic zone around the hole.

Introduction

The paper deals with dilatometer measurements in deep boreholes. The dilatometer probe is used for in-situ measurements in boreholes to determine deformation properties of rock massif. We describe the Solexperts dilatometer system due to our practical experience with measurements, evaluation and interpretation of more than 120 in-situ tests executed during 2004 and 2006. The dilatometer measurements are frequently used as a part of geotechnical investigation for tunneling or dam foundations. Further information can be found in [1] and [2].

The dilatometer probe developed by Solexperts AG (Switzerland) has been successfully used for more than 30 years. Determination of the mechanical properties of the rock mass is based on real-time measurement of the applied pressure and the resulting deformation – the change of the borehole diameter. The dilatometer probe consists of a metal core, which holds three potentiometeric displacement transducers placed in the centre of the core, Figure 1. The axes of the transducers are oriented 120° to each other and vertical distance between the transducers is 75 mm. The borehole wall is loaded by means of a reinforced one meter long packer sleeve fixed to the core of the probe at its both ends. The packer can be expanded by compressed nitrogen or air via a high-pressure hose connected to either nitrogen bottles or a compressor. The deformation of the borehole wall is directly measured by the transducers, which contact the borehole wall through steel pins with spherical heads. The installation of the probe into the borehole is usually done with a drilling rig.

A dilatometer test has usually two to four loading cycles, as described in the example from the investigation for the Brenner Base Tunnel, Figure 2. At first,

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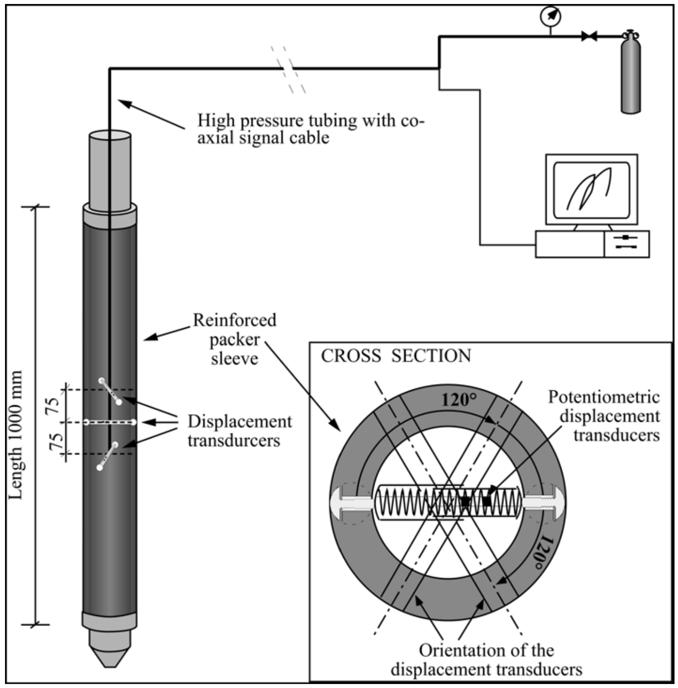

Figure 1: Scheme of Solexperts dilatometer, after [2]

the probe is inflated to a base pressure, about 0.5 MPa higher than the hydrostatic pressure at the test position in order to establish a good contact between the packer and the borehole wall. The first cycle consists of initial loading and subsequent unloading to the base pressure. The following cycles consists of reloading up to the maximum pressure of the previous load cycle, initial loading to a higher level and then unloading to the base pressure level.

The common test evaluation is derived from Lamé's equation:

$$E = \frac{\Delta p * d * (1+\nu)}{\Delta d}, \tag{1}$$

where Δp = pressure difference; d = borehole diameter; Δd = change of borehole diameter; ν = Poisson's ratio. In case where no results of laboratory tests are available for the tested rock formation, we assume the Poisson's ratio of 0.33 with respect to rock quality. The averaged displacement measurements of the all three transducers are mainly used for the calculation of the Young's moduli. The deformation modulus (V) is calculated based on data of initial loading from each load cycle using linear regression along the corresponding section of the curve. The deformation modulus from reloading phase of each cycle is calculated in a similar manner.

The modulus of elasticity E_1 is calculated from the slope of a secant connecting the first and the last point of the unloading phase, which corresponds to the initial loading range for each cycle. However in cases of very step slopes which correspond to very small elastic deformation of the borehole wall and thus to very high moduli, it appears to be more appropriate to calculate E_2 from the entire unloading curve for each load cycle. An example of a dilatometer test is depicted in Figure 2.

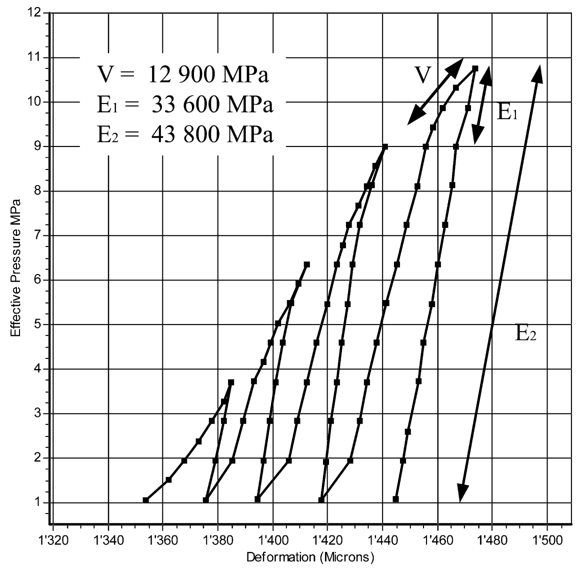

Figure 2: Dilatometer test in borehole Va-B-03/04s, 986.5 m depth, chlorite schists. Graphical representation from the DilatoII software program: average from deformation measurements of all three extensometers, after [2]

Governing theory of the back analysis

Theoretical base of the back analysis is as follows. Let us assume displacement field in the form

$$u(r;\varphi;x) = u(r;\varphi), \quad v(r;\varphi;x) = v(r;\varphi), \quad w(r;\varphi;x) = w(r;\varphi) + ax + b. \tag{2}$$

a, b are unknown parameters. Referring to the Kantorovich method (details in Rektorys (1969)) the functions of the displacement field can be in cylindrical coordinative system expressed in series

$$u(r;\varphi) = u_0(r)\psi_0 + \sum_{j=1}^{\infty} u_j(r)\psi_j + \sum_{j=1}^{\infty} \bar{u}_j(r)\bar{\psi}_j,$$

$$v(r;\varphi) = v_0(r)\psi_0 + \sum_{j=1}^{\infty} v_j(r)\psi_j + \sum_{j=1}^{\infty} \bar{v}_j(r)\bar{\psi}_j,$$

$$
w(r;\varphi) = w_0(r)\psi_0 + \sum_{j=1}^{\infty} w_j(r)\psi_j + \sum_{j=1}^{\infty} \bar{w}_j(r)\bar{\psi}_j;
$$

$$
\psi_0 = 1, \quad \psi_j = \cos j\varphi, \quad \bar{\psi}_j = \sin j\varphi; j = 1,2,\ldots
$$

System of the ordinary differential equation can be derived for elastic isotropic material using standard steps of deformation variant. Their solution is the displacement field vector

$$
\begin{aligned}
u =& C_{10}\ln r + C_{20} + \sum_{j=1}^{\infty}\left(C_{1j}r^{j} + C_{2j}r^{-j}\right)\cos j\varphi + \sum_{j=1}^{\infty}\left(\bar{C}_{1j}r^{j} + \bar{C}_{2j}r^{-j}\right)\sin j\varphi,\\
v =& A_{10}r + A_{20}r^{-1} + \left[A_{11}r^{2} + A_{21}r^{-2} + A_{31} + A_{41}\ln r\right]\cos\varphi +\\
&\left[B_{11}r^{2} + B_{21}r^{-2} + B_{31} + B_{41}\ln r\right]\sin\varphi +\\
&\sum_{j=2}^{\infty}\left(A_{1j}r^{j+1} + A_{2j}r^{-(j+1)} + A_{3j}r^{j-1} + A_{4j}r^{-(j-1)}\right)\cos j\varphi +\\
&\sum_{j=2}^{\infty}\left(B_{1j}r^{j+1} + B_{2j}r^{-(j+1)} + B_{3j}r^{j-1} + B_{4j}r^{-(j-1)}\right)\sin j\varphi,\\
w =& B_{10}r + B_{20}r^{-1} + \left[B_{11}\frac{5-4\nu}{-1+4\nu}r^{2} - B_{21}r^{-2} + B_{31} + B_{41}\left(\frac{1}{3-4\nu} + \ln r\right)\right]\cos\varphi +\\
&\left[A_{11}\frac{-5+4\nu}{-1+4\nu}r^{2} + A_{21}r^{-2} - A_{31} - A_{41}\left(\frac{1}{3-4\nu} + \ln r\right)\right]\sin\varphi +\\
&\sum_{j=2}^{\infty}\left(B_{1j}\frac{j+4-4\nu}{j-2+4\nu}r^{j+1} - B_{2j}r^{-(j+1)} + B_{3j}r^{j-1} + B_{4j}\frac{-j+4-4\nu}{j+2-4\nu}r^{-(j-1)}\right)\cos j\varphi +\\
&\sum_{j=2}^{\infty}\left(A_{1j}\frac{-j-4+4\nu}{j-2+4\nu}r^{j+1} + A_{2j}r^{-(j+1)} - A_{3j}r^{j-1} - A_{4j}\frac{j-4+4\nu}{j+2-4\nu}r^{-(j-1)}\right)\sin j\varphi,
\end{aligned}
\tag{3}
$$

and corresponding isotropic elastic stress state

$$
\begin{aligned}
\frac{\sigma_r}{2G} =& \frac{A_{10}}{1-2\nu} - A_{20}r^{-2} + \left[\frac{-2}{-1+4\nu}A_{11}r - 2A_{21}r^{-3} + \frac{3-2\nu}{3-4\nu}A_{41}r^{-1}\right]\cos\varphi +\\
&\left[\frac{-2}{-1+4\nu}B_{11}r - 2B_{21}r^{-3} + \frac{3-2\nu}{3-4\nu}B_{41}r^{-1}\right]\sin\varphi +\\
&+\sum_{j=2}^{\infty}\left(A_{1j}\frac{j^{2}-j-2}{j-2+4\nu}r^{j} - A_{2j}(j+1)r^{-(j+2)} + A_{3j}(j-1)r^{j-2}\right.\\
&\left. - A_{4j}\frac{j^{2}+j-2}{j+2-4\nu}r^{-j}\right)\cos j\varphi +
\end{aligned}
$$

$$+\sum_{j=2}^{\infty}\left(B_{1j}\frac{j^2-j-2}{j-2+4\nu}r^j-B_{2j}(j+1)r^{-(j+2)}+B_{3j}(j-1)r^{j-2}\right.$$
$$\left.-B_{4j}\frac{j^2+j-2}{j+2-4\nu}r^{-j}\right)\sin j\varphi,$$

$$\frac{\sigma_\varphi}{2G}=\frac{A_{10}}{1+2\nu}+A_{20}r^{-2}+\left[\frac{-6}{-1+4\nu}A_{11}r+2A_{21}r^{-3}+\frac{-1+2\nu}{3-4\nu}A_{41}r^{-1}\right]\cos\varphi+$$
$$\left[\frac{-6}{-1+4\nu}B_{11}r+2B_{21}r^{-3}+\frac{-1+2\nu}{3-4\nu}B_{41}r^{-1}\right]\sin\varphi+$$
$$+\sum_{j=2}^{\infty}\left(A_{1j}\frac{-j^2-3j-2}{j-2+4\nu}r^j+A_{2j}(j+1)r^{-(j+2)}-A_{3j}(j-1)r^{j-2}\right.$$
$$\left.+A_{4j}\frac{j^2-3j+2}{j+2-4\nu}r^{-j}\right)\cos j\varphi+$$
$$+\sum_{j=2}^{\infty}\left(B_{1j}\frac{-j^2-3j-2}{j-2+4\nu}r^j+B_{2j}(j+1)r^{-(j+2)}-B_{3j}(j-1)r^{j-2}\right.$$
$$\left.-B_{4j}\frac{j^2-3j+2}{j+2-4\nu}r^{-j}\right)\sin j\varphi, \tag{4}$$

$$\sigma_x=\nu\left(\sigma_r+\sigma_\varphi\right).$$

The unknown integral constants are calculated from the known boundary conditions.

Improvement of the standard back analysis formula

The standard back analysis formula is derived from three boundary conditions, namely the zero displacement in radial direction in infinity, the measured displacement on the probe hole interface and the known pressure in the same place. We would like to enhance the formula from two aspects. At first, we would like to incorporate developing of the plastic zone around the hole. The thickness is described by radius $\bar{R}_1 > R_1$, where R_1 is the radius of the hole. The second aspect is the phenomenon of the influence zone around the hole described by R_2, we assume zero radial displacement in the finite distance R_2. Using equations (3) and (4) the three boundary conditions mentioned above yields

$$E=\frac{\Delta p*(1+\nu)}{\frac{1}{2}\Delta d*\frac{\bar{R}_1}{R_2^2-\bar{R}_1^2}\left(\frac{R_2^2}{\bar{R}_1^2}+\frac{1}{1-2\nu}\right)}. \tag{5}$$

There is no problem verified by the limit process thru formula (5) formula (1).

To introduce formula (5) we carried out back analysis on the results presented in Fig. 2. The input data for calculation are $\Delta p = 10.8$(MPa), $\Delta d = 30(\mu$m), $\nu = 0.33$, $R_1 = 46$(mm). Standard back analysis gives presented value of Young's modulus $E = 43.800$ (GPa). Let us assume the radius of influence zone in the range $R_2 \in \langle 3\bar{R}_1; 5\bar{R}_1 \rangle$ and let us estimate the thickness of plastic zone in the range $\delta_{pl} \in \langle 0,1\bar{R}_1; 0,9\bar{R}_1 \rangle$. We calculated the values of the Young's modulus on the boundary of estimated intervals. The values are listed in following tables. Table 1 for influence zone $R_2 = 3\bar{R}_1$, table 2 for $R_2 = 5\bar{R}_1$

Table 1

$\bar{R}_1$	$1.1R_1$	$1.2R_1$	$1.3R_1$	$1.4R_1$	$1.5R_1$	$1.6R_1$	$1.7R_1$	$1.8R_1$	$1.9R_1$
E(GPa)	32.46	35.41	38.36	41.32	44.27	47.22	50.17	53.12	56.07

Table 2

$\bar{R}_1$	$1.1R_1$	$1.2R_1$	$1.3R_1$	$1.4R_1$	$1.5R_1$	$1.6R_1$	$1.7R_1$	$1.8R_1$	$1.9R_1$
E(GPa)	41.62	45.41	49.19	52.97	56.75	60.54	64.32	68.11	71.89

Conclusion

Several ideas were introduced for back analysis of the dilatometric measurement in the deep boreholes. In this case geostatic stress state has a significant effect on the compaction of the rock massive. Due to this fact we started to incorporate the idea of influence zone for the back analysis. Neglecting this fact causes overestimation of the material parameters as a Young's modulus. On the other hand, the higher Young's modulus can be explained by progression of the plastic zone around boreholes.

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