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Laplace Distribution for A Priori Information for Damage Detection

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Summary

A priori information is discussed in order to overcome the ill-posedness of damage detection. We compared Gauss and Laplace distributions to express the uncertainties of a priori information. Uncertainty level of a priori and observation information is related to the balance of a priori and observation terms in the objective function. Maximum likelihood method is used to determine the balance adaptively. The method is examined through numerical simulations of identification problem to detect damage of a bridge based on coupling vibration with moving vehicles.

Introduction

An effective maintenance of infra-structures is indispensable in infra-management in Japan. A basic study was performed to examine the possibility of detecting damage of a bridge based on vibration induced by a moving vehicle (Yoshida et al. 2006). Damage detection problem can be interpreted as an inverse problem, of which ill-posedness is a known difficulty. It is basically caused by lack of information compared with the size of the solution space. Many regularization methods, which are techniques to overcome the problem of ill-posedness, can be interpreted as adding information, which is known as a priori information. It corresponds to a regularization or penalty term of the objective function of the inverse problem. It is common practice to use residual sum of squares of the empirical value of the unknown parameter. From a probabilistic interpretation, the square means that the error of the empirical value is assumed to follow a Gaussian distribution. If we assume Laplace distribution instead, then a residual sum of absolute differences instead of square of the differences should be used.

In this paper, a regularization method is introduced for the damage detection problem of a bridge with vibration induced by a moving vehicle. A formulation is proposed to estimate uncertainty level of the a priori and a posterioi information, in addition to unknown parameters, using maximum likelihood method. Gaussian or Laplace distribution is used for the uncertainty of a priori information. The approach is examined through numerical simulations of the damage detection with coupled vibration of a bridge with a moving vehicle.

Numerical simulation of damage detection of a bridge

Inverse problems can be interpreted from two parts, 1)definition of objective function, 2)minimization of the objective function. Most important issue is how to

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define the objective function. Once objective function is defined, well-known existing optimization algorithms can be applied. In this study, DFP method is applied to minimize the objective function.

The objective function with Gaussian type a priori information is introduced briefly, because details of the derivation are stated in several papers, for example, Jackson & Matsu'ura (1985), Hoshiya & Yoshida (1996). We assume that the mean vector and the variance-covariance matrix of unknown parameters $\boldsymbol{x}$ are given as a priori information.

$$\bar{\boldsymbol{x}} = E[\boldsymbol{x}], \quad \boldsymbol{M} = E[(\boldsymbol{x}-\bar{\boldsymbol{x}})(\boldsymbol{x}-\bar{\boldsymbol{x}})^T] \tag{1}$$

Observation vector $\boldsymbol{z}$ is given as,

$$\boldsymbol{z} = \boldsymbol{H}(\boldsymbol{x}) + \boldsymbol{v} \tag{2}$$

The vector $\boldsymbol{v}$ denotes observation error, $\boldsymbol{R} = E[\boldsymbol{v}\boldsymbol{v}^T]$ is variance-covariance matrix of the error. The following objective function is derived by Maximum Likelihood method, assuming the a priori information is a part of the observation.

$$J = (\boldsymbol{x}-\bar{\boldsymbol{x}})^T\boldsymbol{M}^{-1}(\boldsymbol{x}-\bar{\boldsymbol{x}}) + (\boldsymbol{z}-\boldsymbol{H}(\boldsymbol{x}))^T\boldsymbol{R}^{-1}(\boldsymbol{z}-\boldsymbol{H}(\boldsymbol{x})) + \ln|\boldsymbol{M}| + \ln|\boldsymbol{R}| + (n+m)\ln(2\pi) \tag{3}$$

where n = number of unknown parameter; m= number of observation data. When uncertainty of unknown parameter and observation error are independent and identical as shown in Equation (4), the objective function is expressed in Equation (5).

$$\boldsymbol{M} = \sigma_M^2\boldsymbol{I}, \quad \boldsymbol{R} = \sigma_R^2\boldsymbol{I} \tag{4}$$

$$J = \frac{1}{\sigma_M^2}\sum_{i=1}^{n}(\bar{x}_i - x_i)^2 + \frac{1}{\sigma_R^2}\sum_{i=1}^{m}(z_i - h_i(x))^2 + 2n\ln\sigma_M + 2m\ln\sigma_R + (n+m)\ln(2\pi) \tag{5}$$

In the above objective function, independent identical Gaussian distributions given in Equation (6) are assumed for the a priori information.

$$f(\boldsymbol{x}) = \prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma_M}\exp\left(-\frac{(x_i-\bar{x}_i)^2}{2\sigma_M^2}\right) \tag{6}$$

where $x_i, \bar{x}_i = i$-th component of vector ($\boldsymbol{x}, \bar{\boldsymbol{x}}$ = unknown parameter vector and expectation of the vector). When Laplace distribution is assumed instead of Gaussian distribution for a priori information, probability density function of the a priori information is,

$$f(\boldsymbol{x}) = \prod_{i=1}^{n}\frac{1}{2b}\exp\left(-\frac{|x_i-\bar{x}_i|}{b}\right) \tag{7}$$

The following objective function is derived in the same way with Laplace type a priori information.

$$J = 2n\ln(2b) + \frac{2}{b}\sum_{i=1}^{n}|x_i - \bar{x}_i| + m\ln(2\pi) + 2m\ln\sigma_R + \frac{1}{\sigma_R^2}\sum_{i=1}^{m}(z_i - h_i(\boldsymbol{x}))^2 \quad (8)$$

Numerical simulation of damage detection of a bridge

A bridge vibrates due to a moving vehicle on the bridge. A method to simulate the coupled vibration is proposed and discussed in Kawatani et al. 2000, Kim et al. 2005. The acceleration and/or displacement time histories due to the moving vehicle are supposed to be recorded at several points of a bridge. The stiffness distribution of the bridge is identified from observed data. The part where the estimated stiffness is small, is considered as damaged. The regularization technique is studied with this damage detection problem. The technique discussed in this paper is, however, applicable to general inverse problems.

The formulation of the coupling vibration is briefly introduced. The motion of a bridge and vehicle is described by the following equation.

$$\begin{bmatrix} \boldsymbol{M}_{bb} & 0 \\ 0 & \boldsymbol{M}_{vv} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{D}}(t) \\ \ddot{\delta}(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{bb}(t) & \mathbf{C}_{bv}(t) \\ \mathbf{C}_{vb}(t) & \mathbf{C}_{vv} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{D}}(t) \\ \dot{\delta}(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{bb}(t) & \mathbf{K}_{bv}(t) \\ \mathbf{K}_{vb}(t) & \mathbf{K}_{vv} \end{bmatrix} \begin{Bmatrix} \mathbf{D}(t) \\ \delta(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_{bb}(t) \\ \mathbf{f}_{vv}(t) \end{Bmatrix} \quad (9)$$

Subscripts b, v and bv denote terms related to the bridge, vehicle and bridge-vehicle interaction respectively. $\boldsymbol{C}_{bv}(t)$ and $\boldsymbol{K}_{bv}(t)$ are coupled damping and stiffness matrices between bridge and vehicle systems. $\boldsymbol{M}_{vv}$, $\boldsymbol{C}_{vv}$ and $\boldsymbol{K}_{vv}$ are mass, damping and stiffness matrices for the vehicle. $\boldsymbol{f}_{bb}(t)$ and $\boldsymbol{f}_{vv}(t)$ indicates external moving vehicular loadings on the bridge and dynamic wheel loads of the vehicle respectively. The simultaneous differential equation is solved by Newmark's β method in the following numerical examples.

A bridge model for the numerical simulation is shown in Figure 1. When a vehicle moves on a lane of the bridge, the dynamic response of the bridge is observed at several points. The detailed data of the model such as roadway surface roughness data is shown in Kim & Kawatani (2007). The bending stiffness of an element of the bridge is assumed to decrease at the damaged point. Damage index is defined to be the ratio of the damaged stiffness to undamaged stiffness. For example, when stiffness of an element decreases to 70%, the damage index of the element is 0.7.

Unknown parameter vector $\boldsymbol{x}$ in Equation (1) to (9) is the damage index vector. There are 16 elements in the model, of which damage indices are unknown parameters $\boldsymbol{x}$. $\boldsymbol{z}$ and $\boldsymbol{H}(\boldsymbol{x})$ are respectively observation data and response data calculated by Equation (9). The observation points are node number 4, 8 and 12. $\boldsymbol{R}$

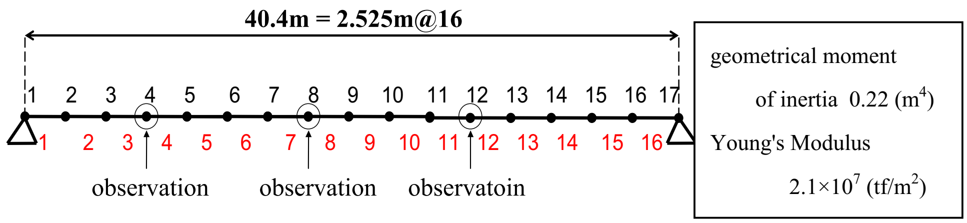

Figure 1: A bridge model for numerical simulation

is assumed to be an identity matrix, which means that observation error is identical and independent of each other. In this study, DFP method is applied to minimize the objective function.

Identification of initial model

It is important to estimate stiffness distribution of initial model without damage because the damage index is defined as the ratio of the damaged stiffness to the undamaged stiffness or initial stiffness. It is also important for good damage detection to estimate initial stiffness distribution since there are several uncertainties such as material randomness and construction error that are present.

Assumed initial stiffness distribution is shown by the line denoted "true" in Figure 2. The vertical axis expresses the ratio to nominal stiffness. The axis title should be called correction coefficient rather than damage index, however, same axis title is used to be consistent with the figures below. Response induced by a moving vehicle is calculated using the stiffness distribution and then response data at the observation points are obtained, which are subsequently used as observation data in damage detection simulations. Three cases are performed, without a priori information, with Gaussian type and Laplace type a priori information. When Gaussian type a priori information is used, the objective function of Equation (5) is minimized with respect to damage indices $\boldsymbol{x}$, standard deviation of a priori information and observation error. In the case of Laplace type a priori information, the objective function of Equation (8) is used. Parameter b is estimated instead of standard deviation. Mean value of damage index in a priori information is assumed to be 1.0.

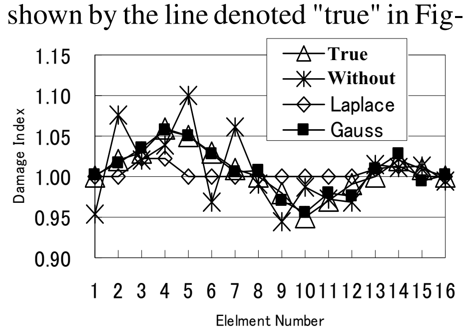

Figure 2: Estimation of initial model

The estimated distributions are also shown in Figure 2. The distribution estimated without a priori information is fluctuating and different from that of the true distribution. The damage indices estimated with Laplace type a priori information is close to 1.0, and also different from that of the true distribution. The distribution estimated with Gaussian a priori information is almost the same as that of true model. It is suggested that when the distribution is smooth, Gaussian type a priori information has an advantage in the identification problem.

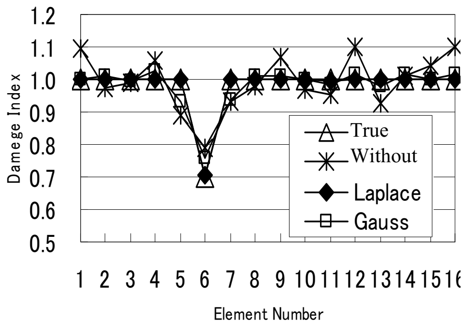

Figure 3: Damage detection of one damaged points model

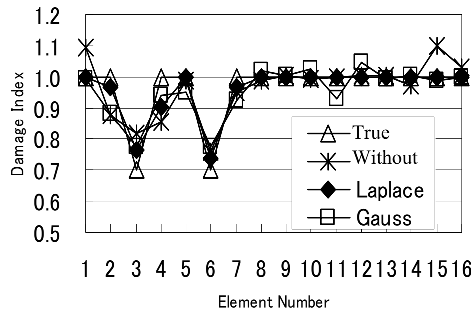

Figure 4: Damage detection of two damage Points model

Damage detection

The position and level of the damage are estimated in damage detection. The damaged point is assumed to be Element 6, and its damage index is 0.7. Three cases of simulation, without a priori information, with Gaussian type and Laplace type a priori information, are also performed. The estimated distributions are shown in Figure 3. Though all cases indicate small damage index around Element 6, the distribution estimated without a priori information has some fluctuations. The case with Gauss type a priori information shows better agreement with the true distribution, however still some estimation errors are observed. On the other hand, the case with Laplace type shows very good agreement. It is suggested that Laplace type a priori information has advantage in identification problem when only a specific part is damaged.

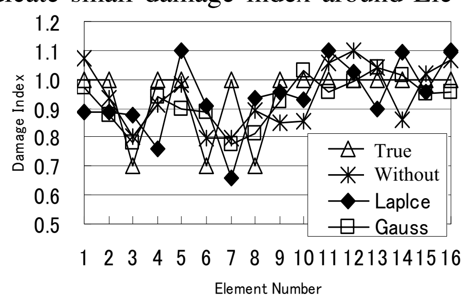

Figure 5: Damage detection of three damage points model

Damage detection simulation is performed in same way for the case with two damaged points, element number 3 and 6. The estimated distributions are shown in Figure 4. Gauss and Laplace type simulation shows almost similar agreement level. Figure 5 shows the estimated distribution when damages are assumed at element 3, 6 and 8. The agreement is not good in all cases for the model with three damaged points.

Conclusion

A regularization method with a priori information is introduced for the damage detection problem of a bridge with vibration induced by a moving vehicle. We compared Gauss and Laplace distributions to express the uncertainties of a priori information. Gaussian type a priori information gives us more stable result than Laplace type when the distribution of unknown parameters is smooth. Laplace type is, however, better when limited parts are damaged.

References

1. Hoshiya, M. & Yoshida, I. 1996. Identification of Conditional Stochastic Gaussian Field, *Jour. of EM*, *ASCE*, Vol.122, No.2, pp.101-108.
2. Jackson, D. & Matsu'ura, M. 1985. A Baysian Approach to Nonlinear Inversion, *Journal of Geophysical Research*, Vol.90, No.131, pp.581-591
3. Kawatani, M., Kobayashi, Y. and Kawaki, H. 2000. Influence of Elastomeric Bearings on Traffic-Induced Vibration of Highway Bridges, *Trasportation Reseach Record*, No.1696, Vol.2, pp.76-82.
4. Kim, C.W., Kawatani, M and Kim, K.B. 2005. Three-dimensional dynamic analysis for bridge-vehicle interaction with roadway roughness, *Computers and Structures*, Vol.83/19-20, pp.1627-1645.
5. Kim, C.W. & Kawatani, M. 2007. Pseudo-static approach for damage identification of bridges based on coupling vibration with a moving vehicle, *Structure and Infrastructure Engineering*, pp.1-9.
6. Yoshida, I., Kim, C.W. and Kawatani,M. 2006. Simulation of Damage Identification for Bridges, *Proceedings of 4th International Symposium on Steel Structures*, Korea, pp.237-242.