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Homogenized response of jointed rock masses with periodic fields

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Summary

Rock masses with relatively high concentration of discontinuities or joints are considered. Being aware of limitations of various averaging techniques such as the self consistent or Mori-Tanaka methods in providing reliable estimates of generally nonlinear macroscopic response of jointed rock masses, the paper introduces a notion of statistically equivalent periodic unit cell (SEPUC). Such a unit cell contains, in order to reduce the problem complexity, of the orders of magnitude less number of joints in comparison with the actual material system. In analogy with two-phase composites, the SEPUC is expected to be found in a statistical sense by matching suitable microstructure descriptors of both the actual microstructure and periodic one. A possibility of using the second order intensity function as a informative descriptor of the cracks distribution is investigated and possible improvements, although without computational support, are proposed. These will be described in the following paper.

Introduction

Modeling of highly jointed rock masses presents a formidable challenge owing to the large complexity of the problem. It has been recognized long ago that discrete modeling of each joint is in such a case not only impractical, but computationally infeasible. Instead, elements of homogenization rooted in analyses of composites have been employed. Application of micromechanics based averaging techniques essentially demanding separation of scales is admissible, since rock joints, although of the order of meters, are still considerably smaller in size when compared with the dimensions of the analyzed problem (large underground caverns, deep seated tunnels, etc). Introduction of homogenization then transforms the original discontinuous body into a continuum with certain equivalent material properties as schematically depicted in Fig. 1.

In this context the jointed rocks can be viewed as a special class of solids weakened by cracks. Estimates of overall elastic moduli of such material systems can be obtained by several well known methods including the self-consistent [1] and Mori-Tanaka methods [2]. A variant of the dilute approximation has been introduced in [3] to seek macroscopic nonlinear response of highly jointed rocks exploiting the dilation constitutive model presented in [6]. A particular application of the Mori-Tanaka method to the analysis of cracked rocks can be found in [4]. A

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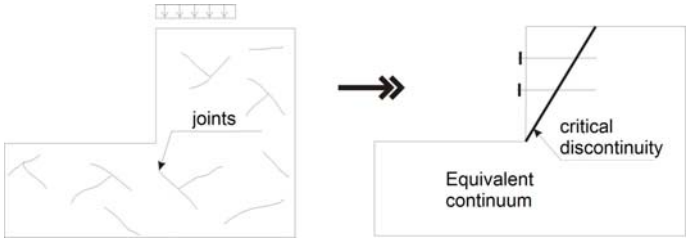


Figure 1: Jointed rock mass and equivalent continuum model

comprehensive survey of various averaging techniques available for a number crack geometries also suggesting their potential drawbacks and essentially promoting the most simple ones, the dilute approximation and the Mori-Tanaka method, is given in [5]. Although computationally very attractive, the observed deficiencies of simple averaging techniques, further reviewed in the next section, may open the way to a more accurate approach presented in the framework of statistically equivalent periodic unit cells [9].

Evaluation of homogenized response of cracked solids - reasons for unit cell models

A key point in the derivation of macroscopic or homogenized response is a proper definition of macroscopic volume averages of stress and strain fields. To that end, consider a generally heterogeneous material weakened by cracks and loaded by remote displacements or remote tractions consistent with macroscopically uniform strain E or stress Σ fields. Further denote the displacement jump along the two crack faces

$$[\![u]\!]_i = u_i^+ - u_i^-, \qquad m_i^+ = -m_i^-, \tag{1}$$

where m_i^+, m_i^- represent the outer unit normals along the opposite crack faces. Standard volume averaging then provides

$$\langle \varepsilon_{ij}(\mathbf{x}) \rangle = \frac{1}{V}\int_{V_M} \varepsilon_{ij}(\mathbf{x})\,\mathrm{d}V + \frac{1}{2V}\int_{\Gamma^+} ([\![u_i]\!]\, m_j^+ + [\![u_j]\!]\, m_i^+)\,\mathrm{d}\Gamma = \varepsilon_{ij,M} + J_{ij} \tag{2}$$

The above equation receives a slightly different format when applied to material systems that assume geometrical periodicity. In such a case, the local displacement field $u_i(\mathbf{x})$ and the corresponding local strain $\varepsilon_{ij}(\mathbf{x})$ admit the following decomposition [7]

$$u_i(\mathbf{x}) = E_{ij}x_j + u_i^*(\mathbf{x}), \qquad \varepsilon_{ij}(\mathbf{x}) = E_{ij} + \varepsilon_{ij}^*(\mathbf{x}), \tag{3}$$

where $u_i(\mathbf{x})$ is a fluctuation part of the total displacement being periodic. Thus for periodic fields, taking into account the fact that $[\![E_{ij}x_j]\!] = 0$ and limiting our

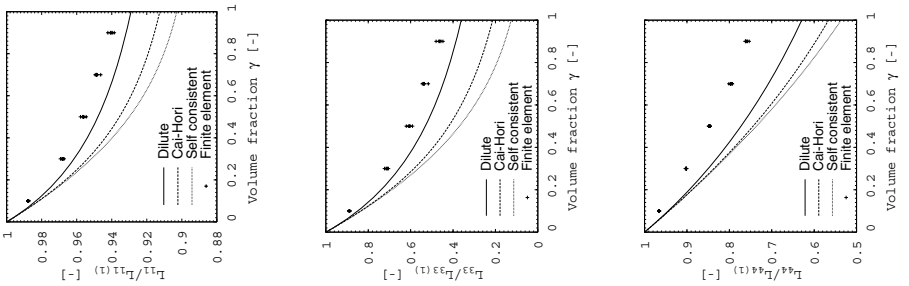
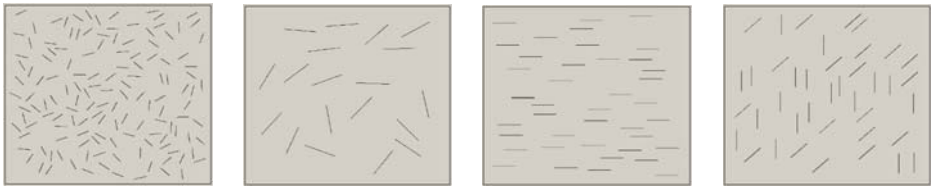


Figure 2: Parallel distribution of cracks - homogenized coefficient of the effective stiffness matrix which change as a function of crack density γ



(a) (b) (c) (d)

Figure 3: Two possible arrays of randomly distributed cracks of various sizes but the same crack density: (a), (b); random distribution of parallel cracks: (d) one group of cracks, (c) two groups of cracks

attention to a homogeneous matrix with cracks, Eq. (2) modifies as

$$\langle \varepsilon_{ij}(\mathbf{x}) \rangle = \frac{1}{V} \int_{V_M} \varepsilon_{ij}(\mathbf{x})\, \mathrm{d}V + \frac{1}{2V} \int_{\Gamma^+} (\llbracket u_i^* \rrbracket\, m_j^+ + \left[\!\left[u_j^* \right]\!\right] m_i^+)\, \mathrm{d}\Gamma, \tag{4}$$

$$\langle \varepsilon \rangle = \langle \varepsilon_M \rangle + J^*. \tag{5}$$

To derive the macroscopic response it is advantageous, particularly with reference to averaging techniques, to proceed in the stress control regime. Then, the available analytical solutions of a single crack in an infinite medium loaded by a remote uniform stress Σ, the stepping stone of all averaging schemes, are readily applicable. Introducing a volume average of a certain crack influence function H such that

$$\langle \varepsilon \rangle = \mathsf{M}_M \Sigma + J = (\mathsf{M}_M + \mathsf{H})\Sigma, \tag{6}$$

provides together with Eq. (2) the macroscopic compliance matrix M in the form

$$\mathsf{M} = \mathsf{M}_M + \mathsf{H}. \tag{7}$$

Specific estimates of matrix H are evaluated here by the dilute approximation and the self-consistent, Mori-Tanaka and Cai-Horii methods. The influence of individual methods on the prediction of overall response appears in Fig. 2 immediately

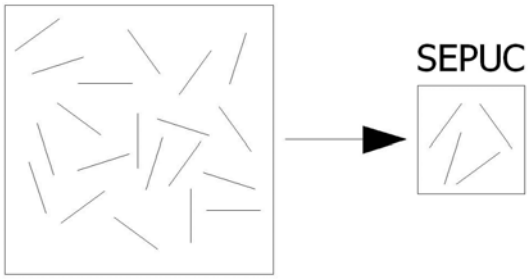


Figure 4: Concept of statistically equivalent periodic unit cell

disclosing their major drawback - severe overestimation of the interaction effect. In view of these results, the dilute approximation appears as the most reliable method when compared with the finite element solutions, see also [5] for detailed discussion on this subject. Note that the finite element results were derived with the help of 1st order homogenization method, e.g. [7], [9], assuming periodic distribution of cracks within a certain unit cell such as the one plotted in Fig. 3(c).

Another drawback arises when implementing averaging techniques in the framework of multiscale analysis [3]. The specific format of local stresses (3×1 vector of surface tractions) developed within joints does not allow for straightforward up-scaling to the macroscopic stress tensor. Incremental solutions in the form of the forward Euler integration scheme are therefore required, which may lead to erroneous results especially for larger load increments since equilibrium on the macroscale is not specifically enforced. The detailed finite element analysis on the other hand presents no such complications. Additional weighty reason for using more complicated approaches is general inability of averaging techniques to rigorously account for actual microstructural details. Being dependent solely on a given crack density these methods cannot clearly distinguish between the two crack patterns depicted in Fig. 3(a)(b).

Concept of statistically equivalent periodic unit cell

Images of real rock masses generated by Global positioning system receivers combined with geologic mapping, digital photography, remote sensing and tomography that show rock formation including distribution of fractures (joints) will soon become commonly available. It is therefore imperative to develop a general modeling concept that would take detailed information about real "microstructure" into account and if possible in an efficient way. A concept of statistically equivalent periodic unit cell (SEPUC) developed in our previous works for various classes of two-phase composites, see e.g. [9], presents one particular option.

The leading idea of this approach, evident from Fig. 4, is to replace a complex non-periodic microstructure by a certain periodic unit cell (PUC), which still optimally resembles the original microstructure in a proper sense. To reduce the

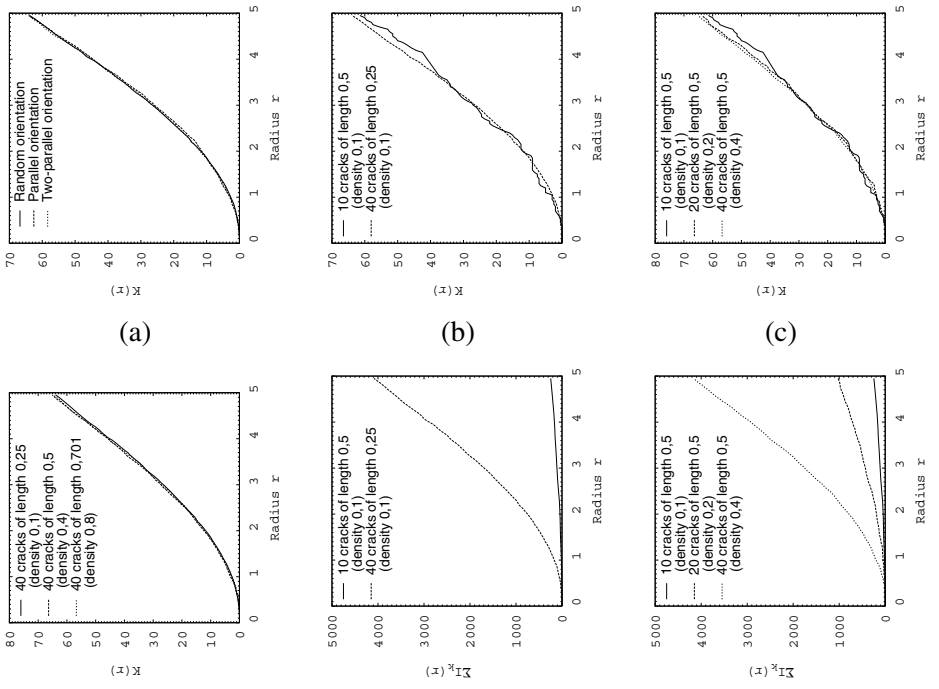


Figure 5: $K(r) - r$ plot: (a) Various orientations of cracks - the same density and the same crack length, (b) two crack systems with variable crack length but the same density, (c) variable density - different number of cracks of the same length, (d) variable intensity - the same number of cracks of different length; $\sum I_k - r$ plot: (e) - as (b), (f) - as (c)

problem complexity the periodic unit cell is described by a substantially smaller number of parameters. It has been found [9] that the predictive capabilities of the resulting SEPUCs strongly depend on the microstructure quantification by suitable geometrical descriptors.

Being inspired by our previous work, see also [8] and references therein, we offer as a possible descriptor of the microstructure morphology the second order intensity function $K(r)$ given by

$$K(r) = \frac{A}{N^2} \sum_{k=1}^{N} I_k(r), \tag{8}$$

where $I_k(r)$ is the number of points (centers of joints) within a circle with radius r centered at the k-th joint, N is the total number of joints in the sample and A is the sample area. Periodicity of the analyzed sample is often assumed to account

for points outside the sampling area. Examples of this function for various crack patterns are shown in Fig. 5.

It becomes evident from these plots that this function, as a descriptor of the points distribution, is not capable of capturing the essential differences between various crack patters. It cannot discriminate between different crack orientations, Fig. 5(a), different crack lengths, Fig. 5(a), and even different densities, Figs. 5(c)(d). Certain differences arise, by no surprise, when removing the scaling factor from Eq. 8, Figs. 5(e)(f). It appears that applicability of this function is thus limited to artificial or well defined crack patterns such as those displayed in Fig. 3 providing the same crack pattern is assumed for both the real microstructure representative and statistically equivalent periodic unit cell.

More sophisticated geometrical descriptors are therefore needed if we wish to address in situ observed crack patterns. Intuitively, a variant of the above technique that incorporates certain elements of the lineal path function can be adopted. In such a case, a segment of a given length and orientation is thrown into a medium and the number of cracks crossed by this segment is calculated. Such an approach would not only account for different crack patterns both with respect to the crack length and orientation, but would also reflect the possible statistical homogeneity, unlike the function $K(r)$ a-priori assuming a statistically isotropic medium.

Acknowledgement

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