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Shape Sensitivity Analysis of Bioheat Transfer in the System Blood Vessel - Surrounding Tissue

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Introduction

Thermal processes proceeding in the system blood vessel - biological tissue are described by the Pennes equation (Poisson-type partial differential equation) and the equation determining the change of blood temperature along the vessel [2, 4, 5, 12, 13]. The boundary condition given on the vessel wall constitutes the coupling element of the model presented.

The steady temperature field in the tissue domain (axially-symmetrical problem is considered) is described by the equation (Fig. 1)

$$\frac{\lambda}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T\left(r,z\right)}{\partial r}\right]+\lambda\frac{\partial^{2}T\left(r,z\right)}{\partial z^{2}}+c_{B}G_{B}\left[T_{a}\left(z\right)-T\left(r,z\right)\right]+Q_{m}=0 \qquad (1)$$

where $T(r,z)$ denotes a tissue temperature, λ is a tissue thermal conductivity, c_B is a volumetric specific heat of blood, G_B[m^{3}blood/m^{3}tissue] is a perfusion rate, Q_m is a metabolic heat source, $T_a(z)$ is an arterial blood temperature.

In the paper [9] the following equation determining the course of $T_B(z)$ is proposed

$$\frac{\mathrm{d}T_{B}\left(z\right)}{\mathrm{d}z}+\frac{2\alpha}{c_{B}wR_{1}}\left[T_{B}\left(z\right)-T\left(R_{1},z\right)\right]=0 \qquad (2)$$

where R_1= const is a vessel radius, α is a heat transfer coefficient on a vessel wall, w is a blood flow velocity. Additionally it is assumed that $T_B(r, z) = T_B(z)$ and for z = 0: $T_B(0) = T_{B0}$.

On the vessel wall the Robin condition is given

$$(r,z)\in\Gamma_{1}: \quad q\left(r,z\right)=-\lambda\frac{\partial T\left(r,z\right)}{\partial r}=\alpha\left[T\left(R_{1},z\right)-T_{B}\left(z\right)\right] \qquad (3)$$

while for $r = R$: T = 37^{o}C (Dirichlet condition). The part of boundary for which the no-flux condition is assumed is marked in Figure 1.

In literature [4, 11] the traversing and supplying vessel models are considered. In the first case
it is assumed that the temperature $T_a(z)$ in equation (1) is a constant value $T_a(z) = T_{B0}$, in the second case $T_a(z) = T_B(z)$. In the next chapters the both cases will be taken into account.

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Shape Sensitivity Analysis

For the needs of further considerations the definition of substantional derivative is introduced [3, 7]

$$\frac{\mathrm{D}T(r,z)}{\mathrm{D}b}=\frac{\partial T(r,z)}{\partial b}+\frac{\partial T(r,z)}{\partial r}v_r+\frac{\partial T(r,z)}{\partial z}v_z \tag{4}$$

where b is a shape parameter (here $b=R_1$), v_r, v_z is the velocity field associated with the shape parameter b. Now the basic equations constituting the mathematical description of the process will be differentiated with respect to the shape parameter (a direct approach). So

$$\begin{gathered}\lambda\frac{\mathrm{D}}{\mathrm{D}b}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T(r,z)}{\partial r}\right)\right]+\lambda\frac{\mathrm{D}}{\mathrm{D}b}\left(\frac{\partial^2 T(r,z)}{\partial z^2}\right)+\\ c_B G_B\left[\frac{\mathrm{D}T_a(z)}{\mathrm{D}b}-\frac{\mathrm{D}T(r,z)}{\mathrm{D}b}\right]=0\end{gathered} \tag{5}$$

or (after the rather complex mathematical manipulations [6])

$$\begin{gathered}\frac{\lambda}{r}\frac{\partial}{\partial r}\left[r\frac{\partial U(r,z)}{\partial r}\right]+\lambda\frac{\partial^2 U(r,z)}{\partial z^2}+G_B c_B\left[\frac{\mathrm{D}T_a(z)}{\mathrm{D}b}-U(r,z)\right]-\\ \frac{\lambda}{r}\left[\frac{\partial T(r,z)}{\partial r}\left(\frac{v_r}{r}+\frac{\partial v_r}{\partial r}\right)+\frac{\partial T(r,z)}{\partial z}\frac{\partial v_z}{\partial r}\right]-2\lambda\left(\frac{\partial^2 T(r,z)}{\partial r^2}\frac{\partial v_r}{\partial r}+\frac{\partial^2 T(r,z)}{\partial z^2}\frac{\partial v_z}{\partial z}\right)-\\ \lambda\frac{\partial T(r,z)}{\partial r}\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{\partial^2 v_r}{\partial z^2}\right)-2\lambda\frac{\partial^2 T(r,z)}{\partial r\partial z}\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)-\lambda\frac{\partial T(r,z)}{\partial z}\left(\frac{\partial^2 v_z}{\partial r^2}+\frac{\partial^2 v_z}{\partial z^2}\right)=0\end{gathered} \tag{6}$$

where $U=\mathrm{D}T/\mathrm{D}b$.

The equation concerning a blood vessel is also differentiated with respect to b

$$\frac{\mathrm{D}}{\mathrm{D}b}\left(\frac{\mathrm{d}T_B(z)}{\mathrm{d}z}\right)+\frac{\mathrm{D}}{\mathrm{D}b}\left[\frac{2\alpha}{c_B w R_1}\left(T_B(z)-T(R_1,z)\right)\right]=0 \tag{7}$$

this means

$$\begin{gathered}\frac{\mathrm{d}U_B(z)}{\mathrm{d}z}+\frac{2\alpha}{c_B w R_1}\left[U_B(z)-U(R_1,z)\right]-\frac{2\alpha}{c_B w R_1^2}\left[T_B(z)-T(R_1,z)\right]-\\ \frac{\partial T_B(z)}{\partial r}\frac{\partial v_r}{\partial z}-\frac{\partial T_B(z)}{\partial z}\frac{\partial v_z}{\partial z}=0\end{gathered} \tag{8}$$

where $U_B=\mathrm{D}T_B/\mathrm{D}b$.

The Robin condition (3) leads to the following formula

$$\begin{gathered}(r,z)\in\Gamma_1:\quad \lambda\frac{\partial U(r,z)}{\partial r}\ -\frac{\partial v_r}{\partial r}q(R_1,z)-\\ \lambda\left[2\frac{\partial v_r}{\partial r}\frac{\partial T(r,z)}{\partial r}+\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)\frac{\partial T(r,z)}{\partial z}\right]=\alpha\left[U(R_1,z)-U_B(z)\right]\end{gathered} \tag{9}$$

If one assumes that $b=R_1$ then the transformation velocities can be defined as follows

$$\begin{gathered}v_r=\begin{cases}\frac{r}{b}, & 0\le r<R_1\\ \frac{R-r}{R-b}, & R_1\le r\le R\end{cases}\\ v_z=0\end{gathered} \tag{10}$$

For above definition the equations (6), (8) take a form

$$\frac{\lambda}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)+\lambda\frac{\partial^2 U}{\partial z^2}+G_B c_B\left[\frac{\mathrm{D}T_a(z)}{\mathrm{D}b}-U\right]- \\ \frac{\lambda}{r}\frac{\partial T}{\partial r}\frac{R-2r}{r(R-b)}+\frac{2\lambda}{R-b}\frac{\partial^2 T}{\partial r^2}=0 \tag{11}$$

and

$$\frac{\mathrm{d}U_B(z)}{\mathrm{d}z}+\frac{2\alpha}{c_B w R_1}\left[U_B(z)-U(R_1,z)\right]-\frac{2\alpha}{c_B w R_1^2}\left[T_B(z)-T(R_1,z)\right]=0 \tag{12}$$

while the condition (9)

$$(r,z)\in\Gamma_1:\quad \lambda\frac{\partial U(r,z)}{\partial r} \;=\alpha\left[U(R_1,z)-U_B(z)\right]-\frac{q(R_1,z)}{R-b} \tag{13}$$

After differentiation of the others boundary conditions we obtain for $r = R$: $U(R, z) = 0$, while for $z = 0$ and $z = Z$: $-\lambda \quad \partial U(r, z)/\partial z = 0$.

Results of Computations

The basic problem and additional one connected with the sensitivity analysis have been solved using the hybrid algorithm basing on the boundary element method [1, 8] (tissue sub-domain) and finite differences method (blood vessel sub-domain). The details concerning the numerical solution of basic problem can be found in [9, 10], the sensitivity one is solved in a similar way.

The blood vessel of radius $R_1 = 0.0002$ [m] is considered. The external radius of domain is assumed as $R = 10R_1$, while $Z = 0.18$ [m]. The following input data are taken into account [6, 11]: $\lambda = 0.5$ [W/(mK)], $Q_m = 25000$ [W/m^3], $G_B = 0.002$ [1/s], $c_B = 4.134\cdot 10^6$ [J/(m^{3}K)], $w = 0.01$ [m/s], $P/F = 2/R_1$ [1/m], $\alpha = 500$ [W/(m^{2}K]. The blood temperature T_{B0} equals $T_{B0} = 37^{\circ}$C, the tissue temperature $T(R, z)$ equals $T(R, z) = 37^{\circ}$C. The value of metabolic heat source assumed corresponds to the exercise conditions (in the case of rest conditions it is the essentially less value). In this way we obtain the effect of clear-cut changes of tissue temperature.

Figures 2 and 3 show the temperature profiles the radial direction at different co-ordinates z for the traversing and supplying vessel cases, respectively. The next Figures (4 and 5) illustrate the distribution of sensitivity function in a radial direction and different z.

One can see that the solution concerning the supplying vessel gives more visible changes of temperature distribution and this fact is confirmed in literature (see: [4]), the maximum temperature both in the case of supplying and traversing vessels is located in the same place corresponding to $r = 5$[mm].

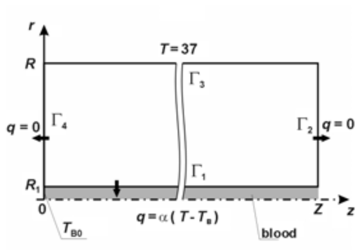

Figure 1: Domain considered

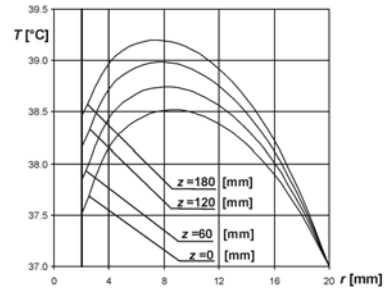

Figure 2: Temperature field (supplying vessel)

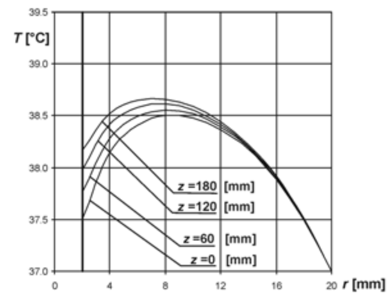

Figure 3: Temperature field (traversing vessel)

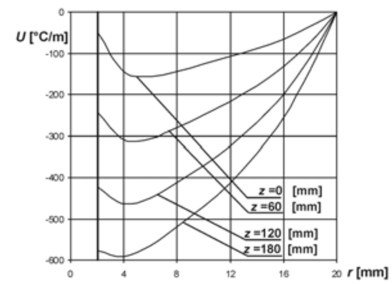

Figure 4: Sensitivity function (supplying vessel)

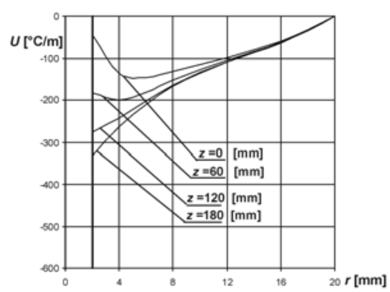

Figure 5: Sensitivity function (traversing vessel)

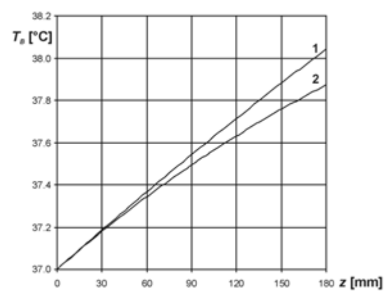

Figure 6: Changes of blood temperature 1 - supplying, 2 - traversing

The sensitivity distribution shows that the increase of vessel radius causes the debasement of temperature. It results from the introduction of negative values U to the Taylor formula. From the physical point of view such effect results from the better cooling conditions of tissue sub-domain. The zero value of sensitivity function of the external surface of the system results from the assumed boundary condition (core temperature). The model of supplying vessel is more sensitive than a traversing one. Additionally the maximum changes of temperature resulting from the increase of vessel radius take place close to the vessel wall (supplying vessel) or almost directly on the vessel wall surface (traversing vessel).

In Figure 6 the changes of blood temperature along the vessels are shown.

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