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Numerical Study of the Effect of Non-Uniform Porosity on Hybrid Nanofluid Flow inside Porous Microchannel Affected by Magnetic Field

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ABSTRACT: This study numerically investigated the flow of incompressible water-based hybrid nanofluid (Al_2O_3 –Cu/water) in a two-dimensional horizontal microchannel with non-uniform porosity in the transverse direction. The medial region of the microchannel received a steady and uniform heat flux and was simultaneously exposed to a constant transverse magnetic field. While the remaining sections had an insulated wall without a magnetic effect. The results showed that the non-uniform porous layer suppressed the axial velocity, while the magnetic field further damped momentum by approximately 6–12% as Hartmann number increased from 0 to 40. Thermal response strongly correlated with flow inertia: the maximum dimensionless temperature increased from 0.03 at Reynolds number = 500 to 0.14 at Reynolds number = 10, confirming the dominance of conduction at low Reynolds number and convection at high Reynolds number. Increasing the hybrid nanoparticle concentration enhanced heat transfer; the local Nusselt number at the heated upper wall increased from $\text{Nu}_s \approx 36$ for the base fluid to $\text{Nu}_s \approx 42$ –44 when using the Cu– Al_2O_3 hybrid mixture with $\varphi_1 = \varphi_2 = 0.04$, representing a 15–20% improvement. Comparative analysis revealed that copper nanoparticles provide strong conductivity enhancement, whereas aluminum oxide improves thermal stability; they generate a synergistic effect absent in single-nanoparticle suspensions. The contour fields demonstrated a clear transition from diffusion-dominated transport ($\text{Re} = 10$) to convection-dominated flow ($\text{Re} = 500$), with isotherms tilting downstream and velocity lines becoming fully parallel at high flow rates. The numerical results show very good agreement with available analytical and numerical data for both velocity and temperature distributions. The small deviations observed confirm the accuracy and reliability of the present numerical model. Overall, the results demonstrate that combining non-uniform porosity with hybrid nanofluids under a magnetic field provides an effective approach for controlling the flow behavior and improving heat transfer performance in microchannel cooling systems. In addition, the close agreement between the present results and previously published studies confirms the accuracy and reliability of the numerical model.

KEYWORDS: Hybrid nanofluid; non-uniform porosity; magnetic field

1 Introduction

Emerging artificial intelligence is leading to a new kind of renaissance for the future, a major part of this renaissance will be the role played by microelectromechanical systems (MEMS). These developments will further develop MEMS systems for related applications, but additionally encourage innovation through advancement within MEMS technology. Hybrid nanofluids such as Cu– Al_2O_3 /water have received much attention compared to conventional nanofluids in the last several years, demonstrating that they can enhance heat transfer rates and the performance of thermal systems [1]. The combination of copper (Cu) and aluminum oxide (Al_2O_3) nanoparticles in the presence of water exhibits greater effectiveness to

enhance thermal transfer and system performance [2]. The heat exchanger and cooling systems based on Cu-Al₂O₃/water hybrid nanofluids were demonstrated to remarkably enhance the heat transfer performance of the various applications compared to traditional nanofluids [3]. Hybrid and ternary hybrid nanofluids have gained more attraction recently owing to the better thermal transport properties for porous media and magnetohydrodynamic flows. For example, Priyadharshini et al. [4] studied the flow of a ternary hybrid nanofluid over a symmetrically stretched surface along with incorporating machine learning techniques for enhanced flow and thermal performance analysis. Similarly, Ali et al. [5] analyzed heat and mass transfer phenomena of hybrid nanofluid flow over a stretched surface utilizing the homotopic perturbation technique, with a considerable improvement in heat transfer efficiency, Lund et al. [6] also considered the unsteady Darcy-Forchheimer flow of a Cu-Al₂O₃/water hybrid nanofluid in a porous medium while taking into account Joule heating and viscous dissipation with an emphasis on the role of inertial resistance. Further, the studies of Baithalu et al. [7] and Lund et al. [8] have also shown the role of magnetic fields, porous media characteristics, and viscous dissipation in fluid flow and thermal transport phenomena.

In the past few years, researchers have been trying to find out the effect of the electromagnetic activity on fluid flow and heat transfer in microchannels. Khasawneh et al. [9] find that when the electromagnetic field is applied to a small channel, both the velocity distribution and the heat transfer can change significantly. As the orienting angle of the magnetic field moves up, fluid speed decreases too. Similarly, Kharouf et al. [10] showed that heat transfer rates are higher when a radial magnetic field is applied. Zhao et al. [11] reported that the magnetic field intensity and nanoparticle volume fraction strongly influence the friction coefficient, where simultaneous increases in both parameters lead to higher frictional resistance and consequently greater pressure drop. Moreover, further research on the transport in porous media has contributed to knowledge of heat transfer and mass transport and the thermodynamic irreversibility for a variety of applications. For optimization of such systems, parallel plate microreactors based on homogeneous porous media have been investigated. Porous media flow, utilization in solar thermal utilization as well as separation, e.g., filtering, and absorption of chemical reaction, are widely studied. These studies have provided profound insights into the evaluation of the optimum functioning of the efficient operation of solid matrix heat exchangers, solar thermal units, and chemical engineering techniques, and the optimization of the efficiency increase in removal and overall batch processing [12].

The non-uniform porosity and heat exchange behavior of microchannels is extensively studied. To explore non-uniform porosity in the heat transfer field, a new work was reported, which showed that the higher the porosity parameter, the lower the flow intensity [13]. Alibeigi and Farahani [14] also proved that (temperature-dependent) influence of porous film formation on thermal properties of porous medium position on thermal transfer on microchannels and postulated a significant influence of porous film location on heat transfer. Simultaneously, another contribution has been made by Meria et al. [15] It is also found that porous site has a significant influence on thermal properties and optimized thermal properties of porous media on microchannels, where porous surface thickness on fraction (the fraction of 0.8) with high Darcy number would be done with these porous media and could have high maximum optimum thermal property on microchannel. The investigation on the non-uniform porosity and heat transfer in the microchannels is expanding very rapidly. Cancilla et al. [16] suggest that heat in a microchannel is distributed based on the porous medium as well as the type of porous medium. Soltanipour and Pourfattah [17] achieved an increase in heat transfer rate when the porous medium occupied the mid-point and near the wall. According to an existing work, Reynolds number affects the internal heat transfer to the ordered porous structure, and the converse permeability affects Darcy velocity, which in turn depends on Reynolds number. Besides, the internal heat transfer of the non-ordered porous system is determined by Reynolds number, with

permeability affecting Darcy velocity, which affects Reynolds number. Another study by Polyaev and Mozhaev [18] found that the effect defined by the permeability of a non-ordered porous system at Darcy velocity is influenced by Reynolds number. Shangquan et al. [19] examined the distribution of porosity and contact angle, showing that the correlation between porosity gradient and contact angle at a high porosity gradient and water distribution is the influence of water saturation, but also the significance of lower saturation of water at higher contact angles and porosity gradients. Gorjaei and Shahidian [20] studied skin friction and convective heat transfer coefficients based on dimensionless eccentricity, boundary conditions, Richardson number, and discovered nanoparticle concentration and eccentricity to be significant parameters to the heat exchanger performance. When it comes to fluid dynamics and heat transfer, the heating or cooling of fluid in or under channels or ducts is one of the main factors determining the performance of thermal systems.

Non-homogeneous heating will cause a high loss of liquid flow in microchannels, and of heat transfer to porous media (the movement will be particularly sensitive to thermal transition and the wall). Aminossadati et al. [21] as a function of the fraction of nanoparticles, the average Nusselt value along the centrally heated wall increased, which is clearly higher for increasing concentration. Liu et al. [22]. This section deals with the application of infrared thermography and image segmentation to inspect non-uniform heat transfer characteristics of prefabricated wall systems. From the results, the methodology and thermal images processed by low Reynolds and Hartmann numbers were adopted to compute significant thermal parameters to determine non-uniform heat dissipation accurately. Automated forecasts and laboratory measurements showed trivial differences (~5% for the cumulative temperature-difference ratio, 10% for the equivalent heat-transfer coefficient). When synthesized, these observations illustrate that it will be geometrical or material elements, such as spatially variable porosity and/or the positioning of porous inserts, that will guarantee higher thermal performance of the microchannel system and therefore more effective heat transfer.

Some works have addressed this systematically in the form of analysis, numerical, and experimental [23,24]. For two-dimensional and three-dimensional space, first/second-order boundary conditions were defined as flow in microchannels [10,25–27]. The explanations for the flow and heat transfer of fluids in complex geometry using these analytical solutions conform, Algwauish [28] calculated convection and conduction heat transfer simulations numerically. Lamraoui et al. [29] investigated the hydrodynamic and thermal properties of Al_2O_3 -Cu/water hybrid nanofluids by numerical investigation and showed (for all) that the rates for heat transfer, Reynolds number, and solid volume fraction contribute significantly to higher rates. Jalili et al. [30], by using the finite element method and Akbari-Ganji method to analyze three fluids in semi-porous channels for solving nonlinear differential equations, confirmed that flow velocity decreases with increasing Hartmann number. Hoseinzadeh et al. [31] also reported the numerical result that with increasing Reynolds number, the velocity gradient increases, and overall heat transfer of Al_2O_3 /water nanofluid flow in a laminar two-dimensional channel increases. Toghraie et al. [32] explored turbulent flow under a nanofluid from the finite volume method and reported statistically significantly higher heat transfer with a higher volume fraction, while a fivefold greater increase in axial velocity was recorded with increasing Reynolds number. Ali et al. [33] studied Cu- Al_2O_3 hybrid nanofluid as part of a finite element-based photovoltaic thermal system. The Reynolds number, together with Cu concentration in the base fluid, increases the performance of the photovoltaic thermal system. Experimentally, Çolak et al. [34] It is all a bit of this and all related research that refer a lot to the importance of LTNE in maximizing the optimization of thermal system design and operations. These studies collectively highlight the importance of considering LTNE in designing and optimizing thermal systems. They demonstrate that both the positioning and

characteristics of porous media and the interaction between the solid and fluid phases play critical roles in enhancing heat transfer and improving the overall performance of thermal systems.

Non-uniform porosity in the Y -direction adds a higher level of complexity to microchannel analysis because the porous structure varies across the height of the channel rather than remaining constant. In this case, the pores are not uniformly arranged or aligned parallel to the flow direction, and their distribution changes spatially along the Y -axis. Such configurations are common in practical applications, including advanced filtration systems, layered composite materials, biomedical devices, and engineered thermal management systems. The Table 1 below provides comparative information among my work and other researchers' studies on fluid type, porosity model, magnetic field application, geometry, thermal boundary conditions, and numerical technique.

Table 1: Comparison of recently performed numerical analyses of MHD hybrid nanofluid flow through porous channels with this study.

Reference	Fluid Type	Porosity Model	Magnetic Field	Geometry	Thermal Boundary Condition	Numerical/Solution Method
Sheikhpour 2024 [35]	Hybrid nanofluid	Porous sinusoidal channel	Yes (MHD)	Sinusoidal channel	Non-uniform heat flux	Numerical simulation
Abbas 2024 [13]	Williamson hybrid nanofluid	Uniform Porous medium	Yes (MHD)	Moving thin needle	Non-uniform heat rise/fall	Numerical method
Kannaiah 2022 [36]	Third-grade hybrid nanofluid	Vertical porous channel	Yes (MHD)	Vertical channel	Non-uniform heat source/sink	Numerical study
Laureti [37]	Hybrid nanofluid	Uniform Porous medium	Yes (MHD)	Diverging channel	Heat transfer with stretchable walls	Numerical approach
Ndelwa 2024 [38]	Cu-Al ₂ O ₃ /water hybrid nanofluid	Porous rotating channel	Yes (MHD)	Rotating channel	Entropy-based thermal analysis	MATLAB
Ali Raza 2024 [39]	Jeffery hybrid nanofluid	Uniform Porous medium	Yes (MHD)	Inclined asymmetric microchannel	Coupled heat and mass transfer	Fractional Numerical techniques
Present Study	Hybrid nanofluid	Non-Uniform porous medium	Yes (MHD)	Microchannel	Specified thermal boundary condition	CFD/Numerical Method

Based on this review and those published in the literature, it is evident that the non-uniform porosity and the combined influence of magnetic fields and hybrid nanofluids in microchannels have not yet been thoroughly investigated. Most prior studies have focused on either uniform porous media, single-component nanofluids, or magnetic effects considered independently. To the best of the authors' knowledge, this work represents one of the first comprehensive numerical studies to simultaneously couple magnetic field effects, non-uniform porosity, and hybrid nanofluid transport in microchannel flows, thereby overlooking the complex coupled mechanisms that govern thermal and hydrodynamic behavior in realistic microchannel systems. The present research aims to bridge this gap by developing a detailed CFD model to analyze the resulting thermo-hydrodynamic behavior and to validate the numerical predictions through systematic comparison with available analytical and numerical benchmark solutions, thereby establishing both the novelty and reliability of the proposed approach. This integrated approach not only fills an important

gap in the current body of knowledge but also provides new physical insights and design guidelines for advanced microchannel thermal management systems.

The structural framework of this study is delineated as follows: Section 2 describes the physical model and the associated mathematical formulation. Section 3 details the numerical method employed to solve the governing equations. Grid independence and validation of the numerical code are presented in Section 4 to ensure the accuracy of the results. Section 5 discusses the numerical results in detail, and Section 6 concludes the study by highlighting the main findings.

2 Problem Description

In this study, incompressible hybrid nanofluid (Cu-Al₂O₃/water) flows in non-uniform, porous horizontal two-dimensional microchannel subjected to a transverse magnetic field B_0 with uniform strength. The temperature of the middle portion of the microchannel is constant and homogeneous for heating, while the other blocks are insulated without the presence of a magnetic field, as illustrated in Fig. 1. The length and height of the microchannel are 2.5 mm and 25 μ m, respectively. L_1 and L_3 are symmetric with a length of 750 μ m and thermally insulated, L_2 is affected by heat flux $q'' = 25$ kW/m² and transverse magnetic field B_0 . The origin is set at the lower left edge of the microchannel inlet. Hybrid nanofluid (Cu-Al₂O₃/water) flows with y -direction from left to right in a non-uniform porosity, at inlet velocity u_0 and inlet temperature 298 K. The hybrid nanofluid is composed of copper, water, and alumina nanoparticles, which are in thermal equilibrium. The hypothesis of LTE was made due to the assumption that the nanofluid was considered as a homogeneous one phase system with small volume fraction of nanoparticles and thus having insignificant temperature difference between the porous medium and fluid. It is therefore appropriate to use only one energy equation. The hybrid nanofluid is assumed to be incompressible and Newtonian. The physical characteristics of copper, pure water, and aluminum nanoparticles are related in Table 2.

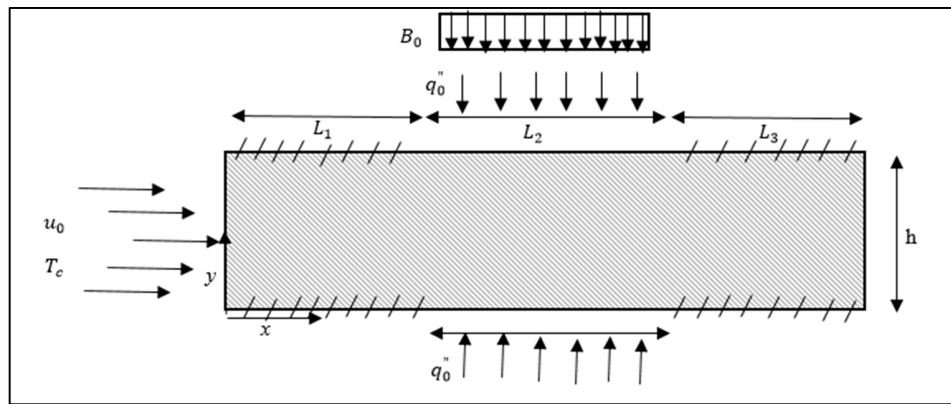


Figure 1: Schematic diagram of the problem.

This study will consider Darcy damping term only, which is given as follows: $R = \frac{\mu}{\beta} V$, where R is the viscous resistance of the porous medium, β is the permeability of the porous medium, and μ is the dynamic viscosity of the fluid [4].

The linear variation of porosity used in the current work was chosen according to previous studies on the functional grading of porous media that have been conducted. Specifically, the porosity variation followed the work done by Chen et al. [40] on functionally graded porous media, where porosity variations were considered to analyze the flow behavior in porous media. (Dynamic behavior and energy absorption in functionally graded porous media) Although the value of 0.99 is close to free-flow conditions, porosities

of such high values have been observed in other studies on metal foams, fibrous porous media, and light porous media. The linear equation of non-uniform porosity in the microchannel is as follows:

$$\phi_Y = \phi_0 + (\phi_{\text{Top}} - \phi_0)h_Y \quad (1)$$

where ϕ_Y is the porosity in the Y direction. ϕ_0 is the mean porosity at the lower wall (0.5). ϕ_{top} is the porosity at the upper wall (0.99), and h_Y is the height of the microchannel, as shown in Fig. 2.

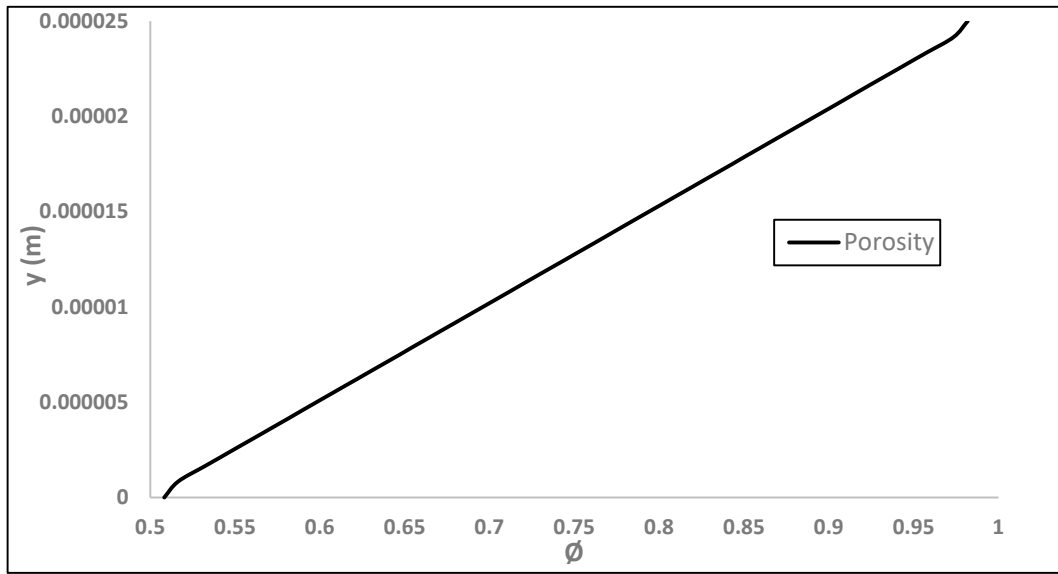


Figure 2: Linear porosity profile.

The following assumptions are used to model and analyze this problem mathematically:

1. The flow is steady, laminar, and two-dimensional.
2. The working fluid is an incompressible and Newtonian fluid.
3. The thermophysical properties of the fluid are assumed to be constant.
4. Radiation heat transfer and internal sources are neglected.
5. At all solid boundaries, no-slip conditions are applied.
6. At the inlet, a uniform flow is specified, and at the outlet, the condition of a fully developed flow is specified.
7. The fluid and the porous medium are assumed to be in local thermal equilibrium.

The governing equations in dimensional form were converted into dimensionless form using characteristic length scale depending upon hydraulic diameter, characteristic velocity scale depending on inlet velocity, and characteristic heat flux scale. Following substitution of dimensionless terms and division by suitable reference values, the final non-dimensional equations were arrived at. The non-dimensional formulation involves the inclusion of Reynolds number, Hartmann number, Prandtl number, Eckert number, and Darcy numbers describing the transport phenomena. The dimensionless forms of continuity, momentum, and energy equations can be given as follows [4,16]:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (2)$$

$$\phi_Y \left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = -\phi_Y \frac{\partial P}{\partial X} + \frac{1}{\text{Re}} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{\text{Ha}^2}{\text{Re}} \phi_Y U - \frac{1}{\text{ReDa}} U \quad (3)$$

$$\phi_Y \left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = -\phi_Y \frac{\partial P}{\partial Y} + \frac{1}{\text{Re}} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{\text{Ha}^2}{\text{Re}} \phi_Y V - \frac{1}{\text{ReDa}} V \quad (4)$$

$$\phi_Y \left(U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right) = \frac{1}{\text{RePr}} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) + \frac{\text{Ec}}{\text{Re}} \phi + \frac{\text{Ha}^2}{\text{Re}} \text{Ec} \phi_Y U^2 \quad (5)$$

where ϕ is viscous dissipation $(2(\frac{\partial U}{\partial X})^2 + 2(\frac{\partial V}{\partial Y})^2 + (\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y})^2)$, Ec is Eckert number, Pr is Prandtl number and Da is Darcy number. In these formulations, the governing expressions incorporate the following non-dimensional parameters:

$$X = \frac{x}{h}, Y = \frac{y}{h}, U = \frac{u}{u_0}, V = \frac{v}{u_0}, \theta = \frac{T-T_c}{\Delta T}, \rho = \frac{\rho}{\rho_{\text{hnf}}}, \text{Re} = \frac{u_0 h}{\nu_f}, \text{Pr} = \frac{\nu_f}{\alpha_f}, \text{Ha} = (B_0 h \sqrt{\frac{\sigma_{\text{hnf}}}{\rho_{\text{hnf}} \nu_f}}), \text{Ec} = \frac{u_0^2}{C_{p(T)}}, P = \frac{P}{\rho^2 u C_p}.$$

The corresponding boundary conditions in non-dimensional form are given as follows:

$$U = 1, V = 0 \text{ and } \theta = 0 \text{ for } X = 0 \text{ and } 0 \leq Y \leq 1$$

$$V = 0 \text{ and } \frac{\partial U}{\partial X} = \frac{\partial \theta}{\partial X} = 0 \text{ for } X = 100 \text{ and } 0 \leq Y \leq 1$$

$$U = 0, V = 0 \text{ and } \frac{\partial \theta}{\partial Y} = 0 \text{ or } \frac{\partial \theta}{\partial Y} = -\frac{K_f}{K_{\text{hnf}}} \text{ for } Y = 0 \text{ and } 0 \leq X \leq 100$$

$$V = 0, U = 0 \text{ and } \frac{\partial \theta}{\partial Y} = 0 \text{ or } \frac{\partial \theta}{\partial Y} = +\frac{K_f}{K_{\text{hnf}}} \text{ for } Y = 1 \text{ and } 0 \leq X \leq 100$$

These nondimensional boundary conditions represent the physics of the flow and the heat transfer within the computational domain. The horizontal velocity component at the inlet $X = 0$ for $0 \leq Y \leq 1$ is given by $U = 1$, and the vertical component by $V = 0$, with the nondimensional temperature $\theta = 0$, representing a uniform inflow with a reference temperature. In the outlet at $X = 100$, a zero vertical velocity condition $V = 0$ is imposed along with zero gradients of velocity and temperature in the streamwise direction, which signifies fully developed flow. On the lower wall ($Y = 0$) and upper wall ($Y = 1$), the no-slip condition is imposed as $U = 0$ and $V = 0$. The thermal boundary conditions at these walls are either adiabatic $\frac{\partial \theta}{\partial Y} = 0$ or conjugate heat transfer conditions that involve the ratio of thermal conductivities $\frac{K_f}{K_{\text{hnf}}}$.

Table 2: Thermophysical properties of Al_2O_3 , Cu, and water.

	Pr	ρ (kg/m ³)	Cp (J/kg K)	K (W/m K)
Pure water	6.2	997.1	4179	0.613
Al_2O_3		3970	765	40
Cu		8933	385	400

(Cu- Al_2O_3)/water Hybrid nanofluids thermophysical properties:

Density [41].

$$\rho_{\text{hnf}} = (1 - \phi_1 - \phi_2) \rho_f + (\phi_1 \rho_{S_1}) + \phi_2 \rho_{S_2} \quad (6)$$

Heat capacity [41].

$$(\rho C_p)_{\text{hnf}} = (1 - \phi_1 - \phi_2) (\rho C_p)_f + \phi_1 (\rho C_p)_{S_1} + \phi_2 (\rho C_p)_{S_2} \quad (7)$$

Viscosity [42].

$$\mu_{hnf} = \mu_f / (1 - \varphi_1 - \varphi_2)^{2.5} \quad (8)$$

Electrical conductivity [43].

$$\sigma_{hnf} = (1 - \varphi_1 - \varphi_2)\sigma_f + \varphi_1\sigma_{Cu} + \varphi_2\sigma_{Al_2O_3} \quad (9)$$

Thermal conductivity [44].

$$k_{hnf} = k_f \left[1 + \left(\frac{k_{Cu}}{k_f} \frac{A_{s1}}{A_f} + C_{Patel} Pe_1 \frac{A_{s1}}{A_f} \right) + \left(\frac{k_{Al_2O_3}}{k_f} \frac{A_{s2}}{A_f} + C_{Patel} Pe_2 \frac{A_{s2}}{A_f} \right) \right] \quad (10)$$

$$\text{where } \frac{A_{s1}}{A_f} = \frac{D_f}{D_{s1}} \cdot \frac{\varphi_1}{1 - \varphi_1 - \varphi_2}, Pe_1 = \frac{u_{s1} D_{s1}}{\alpha_f}, u_{s1} = \frac{2k_B T}{\pi \mu_f D_{s1}^2}.$$

μ_f , ρ_f , k_f , and $(\rho C_p)_f$ are viscosity, density, thermal conductivity, and specific heat for base fluid (water). s_1 for Cu, s_2 for Al_2O_3 nanoparticles, and u_{s1} , Pe are the Brownian motion and Peclet number terms.

In the above equation, k_B is the Boltzmann constant (1.3807×10^{-23} J/K), the solid nano partial diameter $D_s = 40$ nm, and the molecular diameter of the base fluid is $D_f = 2^\circ A$.

3 Numerical Approach

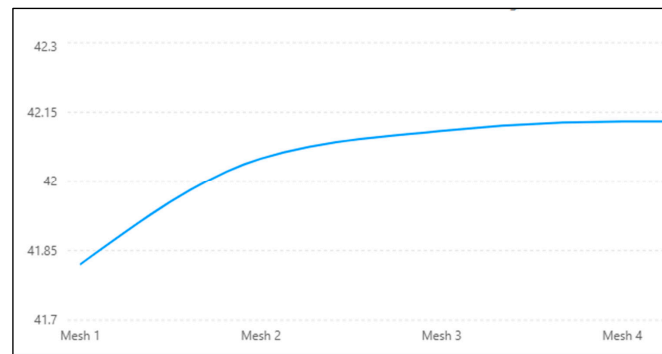
The dimensionless governing Eqs. (2)–(5) were solved using the finite volume method based on a control-volume discretization scheme in conjunction with SIMPLE algorithm for pressure velocity coupling. The numerical simulations were carried out by employing the commercial CFD software ANSYS Fluent, which offers appropriate and validated solvers for the coupled fluid flow and heat transfer problems. A second-order upwind scheme was used to discretize the convective terms to enhance the accuracy of the solutions, while central differencing was used for diffusive terms. Convergence was considered achieved when the maximum mass-imbalance within any control volume decreased below 10^{-8} . A grid resolution of 30×300 cells was employed, and each case was advanced for 1000 iterations to ensure stability of the solution. Once the velocity (U) and temperature (Θ) fields were obtained, post-processing allowed the extraction of performance metrics, including the Nusselt number. The local Nusselt number at the heated upper wall, Nu_s , is defined as $= \frac{\lambda h}{k_f}$.

4 Grid Independence and Code Validation

After simulating this study code using Ansys and obtaining many results about velocity and temperature distribution with different parameters, Table 3 summarizes the mesh-independence assessment carried out using three grid resolutions. For each mesh configuration, the non-dimensional outlet centerline temperature and velocity were computed at three different Reynolds numbers, while keeping the Hartmann number and nanoparticle concentration fixed at $Ha = 20$ and $\phi = 0.02$. The comparison shows that the 30×300 grid yields results that are sufficiently stable and closely aligned with the finer meshes, offering an appropriate compromise between numerical accuracy and computational cost. A study on grid independence, as shown in Fig. 3, was carried out further using the Grid Convergence Index (GCI) approach, which is aimed at determining the numerical uncertainty due to spatial discretization. Four systematic refinement grids were considered, and the average Nusselt number was chosen as a representative variable to analyze the problem. It was observed that the values obtained from the computation of GCI are below 1–2%, which proves that numerical uncertainty due to grid resolution is insignificant.

Table 3: Grid independency study ($Ha = 20$, volume fraction = 0.02, and $Da = 0.2$).

Grid Points	10×100	30×300	50×500
Re = 10			
θ_{out} at Y = 0.5	1.35709	1.32565	1.32565
U_{out} at Y = 0.5	1.03309	1.04069	1.04296
Re = 100			
θ_{out} at Y = 0.5	0.132568	0.132568	0.132568
U_{out} at Y = 0.5	0.828188	0.837317	0.842438
Re = 500			
θ_{out} at Y = 0.5	0.0265198	0.0265198	0.0265198
U_{out} at Y = 0.5	0.743564	0.763594	0.772742

**Figure 3:** Variation of the average Nusselt number with increasing mesh density.

The present numerical code for hybrid nanofluid has been validated by comparing the results with a previous analytical published study [30] and a numerical published study [16]. The present findings exhibit close correspondence with the solution for flow between parallel plates subjected to a magnetic field, thereby validating the accuracy and robustness of the numerical scheme. The fully developed, dimensionless velocity distribution shown in Fig. 4 further reinforces the reliability of the results generated.

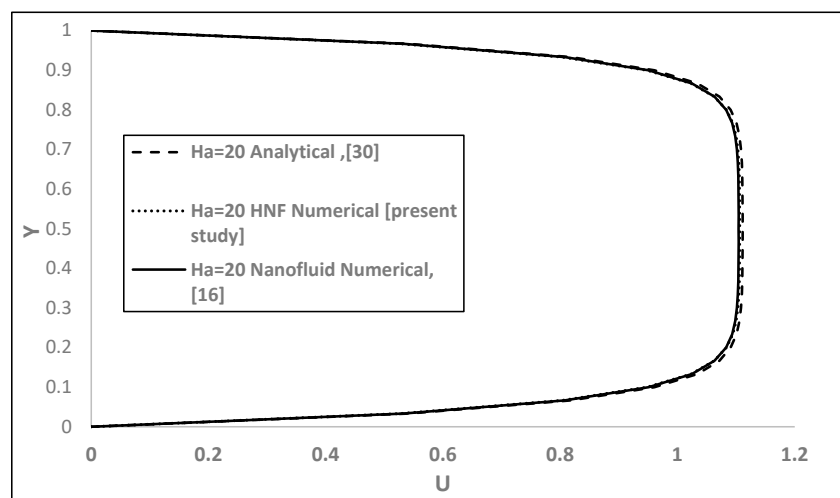
**Figure 4:** Comparison of the present result of the dimensionless velocity at $X = 50$, $Re = 100$, $\phi = 0$, $Da = 0.2$, $Ha = 20$, and $\phi = 0.02$ for different Ha numbers with the analytical solution [30], HNF numerical solution [21], and nanofluid numerical solution [16].

Fig. 5 compares the dimensionless temperature of two cases: one with a conventional nanofluid ($\text{Al}_2\text{O}_3/\text{water}$) [16] and another with a hybrid nanofluid ($\text{Al}_2\text{O}_3\text{-Cu}/\text{water}$). Both cases are without a porous medium. The results show good agreement.

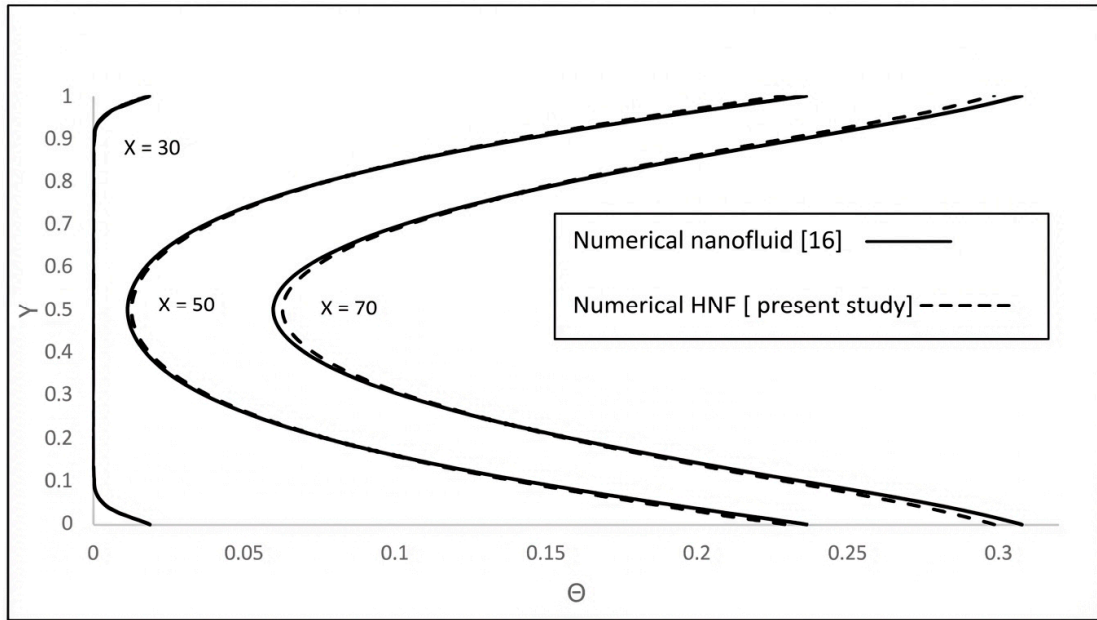


Figure 5: Comparison of the dimensionless temperature at $\text{Re} = 100$, $\phi = 0$, $\text{Da} = 0.2$, and $\phi = 0.02$ with different X , numerical solution [21], and nanofluid numerical solution [16].

Table 4 presents the percentage errors between the present numerical results and those provided in [16], which demonstrate acceptable percentage errors.

Table 4: Percentage errors between the present HNF numerical results and numerical results [16].

Case	Parameter	Reference Value [16]	Present Study Value	Error (%)
Temperature profile	$X = 50$	0.0114136	0.0126648	0.109
Temperature profile	$X = 70$	0.0597534	0.0627746	0.0505
Velocity profile	$\text{Ha} = 20$	1.10609	1.10422	0.00169

5 Results and Discussion

The velocity distribution along the microchannel centerline is shown in Fig. 6 for two cases: a clear channel containing hybrid nanofluid, and a non-uniform porous configuration that incorporated both viscous and inertial resistance. Adding porosity considerably reduces velocity, primarily induced by the enhanced flow resistance and the damping by the applied magnetic field ($\text{Ha} = 20$). Of these two scenarios, the non-uniform porous structure is characterized by the greatest velocity suppression due to the increasing drag on the fluid, due to the gradual change in porosity. This behavior demonstrates the combined effects of the Lorentz force and spatial variations in porous resistance.

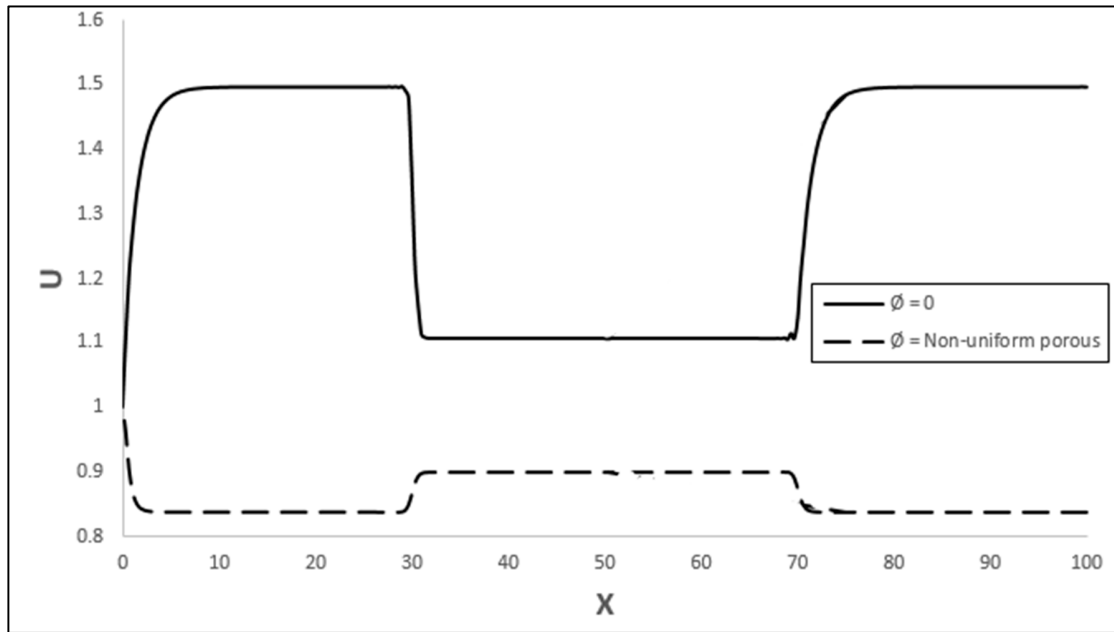


Figure 6: Dimensionless velocity along the center line of the microchannel, $Re = 100$, $Ha = 20$, $\phi = 0.02$, and $Da = 0.2$.

Dimensionless temperature characteristics along the microchannel centerline with both porosity configurations are shown in Fig. 7. In the clear channel, the temperature starts to rise gradually when the fluid has entered the heated region near $X \approx 30$, and that is characteristic of the thermal development process. The non-uniform porous case exhibits the highest temperature increase, as shown by the values. This increased heating is attributed to the spatial variation in porosity, which increases local heat absorption and slows down the fluid at the same time, allowing additional time for heat to diffuse into the flow.

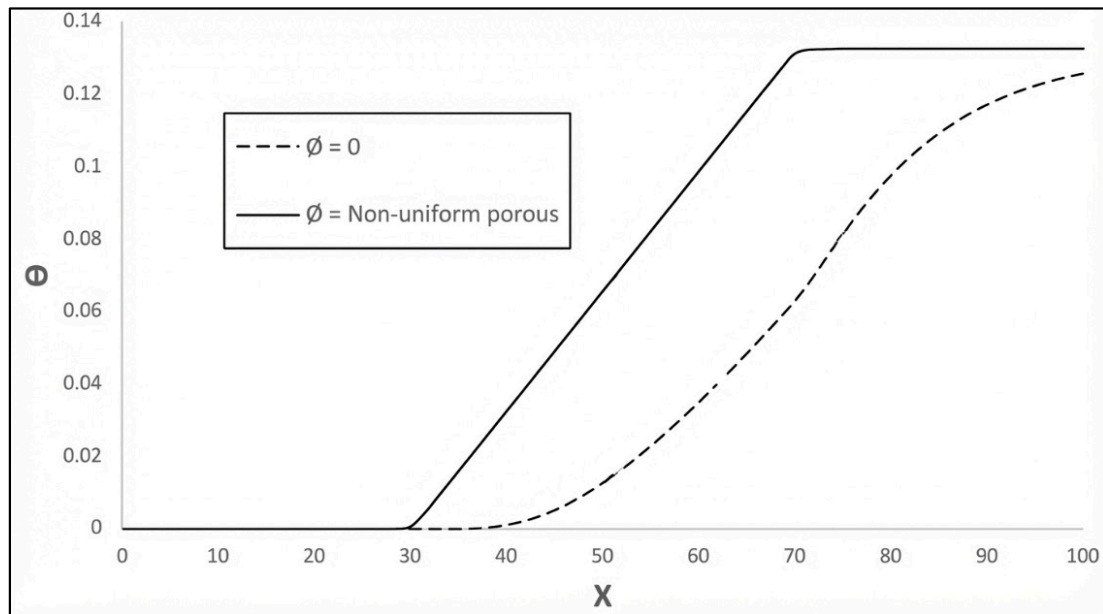


Figure 7: Dimensionless temperature along the center line of the microchannel, $Re = 100$, $Ha = 20$, $\phi = 0.02$, and $Da = 0.2$.

The dimensionless velocity distribution along the length of the microchannel is shown in Fig. 8 at three transverse positions ($Y = 0.1, 0.2$, and 0.3). The velocity magnitude decreases progressively as the measurement line moves closer to the lower wall, where viscous and porous resistances are strongest. At $Y = 0.1$, the profile exhibits the lowest velocities, reflecting the substantial shear and damping effects within the near-wall region. At $Y = 0.2$, the velocity increases but remains significantly lower than the centerline value, indicating the continued influence of the non-uniform porosity and magnetic field. At $Y = 0.3$, closer to the channel core, the velocity reaches its highest levels between the three cases, displaying a flatter plateau within the porous region and sharper transitions at the entry and exit of the heated zone.

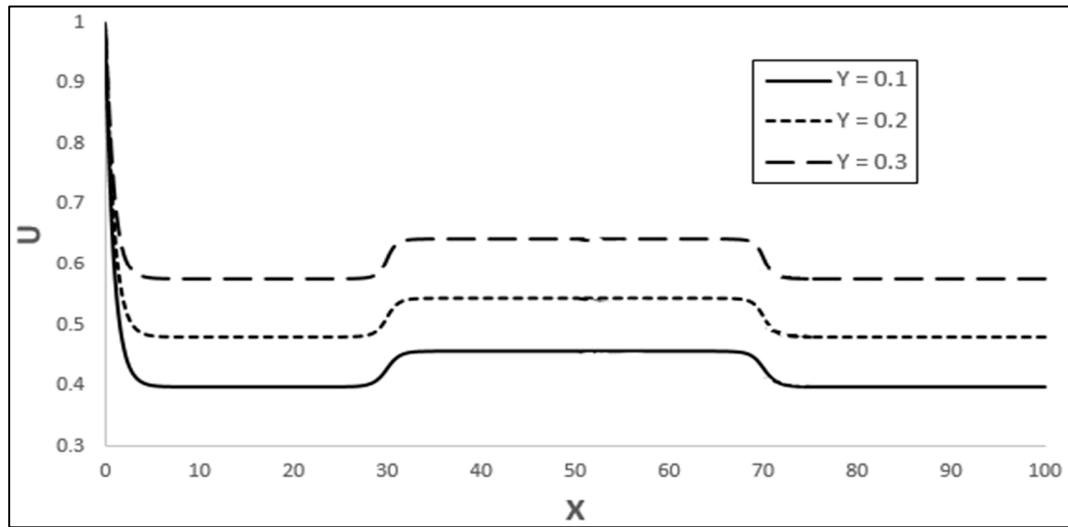


Figure 8: Dimensionless velocity contour line along the microchannel, $Re = 100$, $Ha = 20$, $Da = 0.2$, and $\phi = 0.02$ at different Y .

The dimensionless velocity patterns in the microchannel height, at different points ($X = 30, 50, 70$, and 90), are given in Fig. 9. The velocity profile exhibits strong damping at the lower wall, the most direct result of the combined effects of porous resistance and magnetic retardation. Immediately after the flow enters the heated porous region, the velocity gradually increases in the core region of the porous medium after moving on to $X = 50$ and $X = 70$, indicating the developing flow behavior within the porous medium. The shift in the peak of velocity toward higher values at these locations illustrates that the role of local resistance diminishes as the flow progresses more efficiently. At $X = 90$, after exiting the porous and heated zone, the velocity profile recovers even more, and the peak is near the upper part of the channel.

Fig. 10 presents the dimensionless pressure variation with non-uniform porosity at various streamwise points on the microchannel. With downstream transmission of the liquid, the porosity gradient and the magnetic field both affect the flow resistance and cause a progressive drop in pressure along the channel. This effect is exaggerated in the downstream regions, where the fluid encounters a longer distance with the porous structure and becomes persistently magnetically damped. Thus, the implication of the findings is that non-uniform porosity leads directly to the pressure development, and this determines the hydrodynamic behavior of the microchannel very effectively.

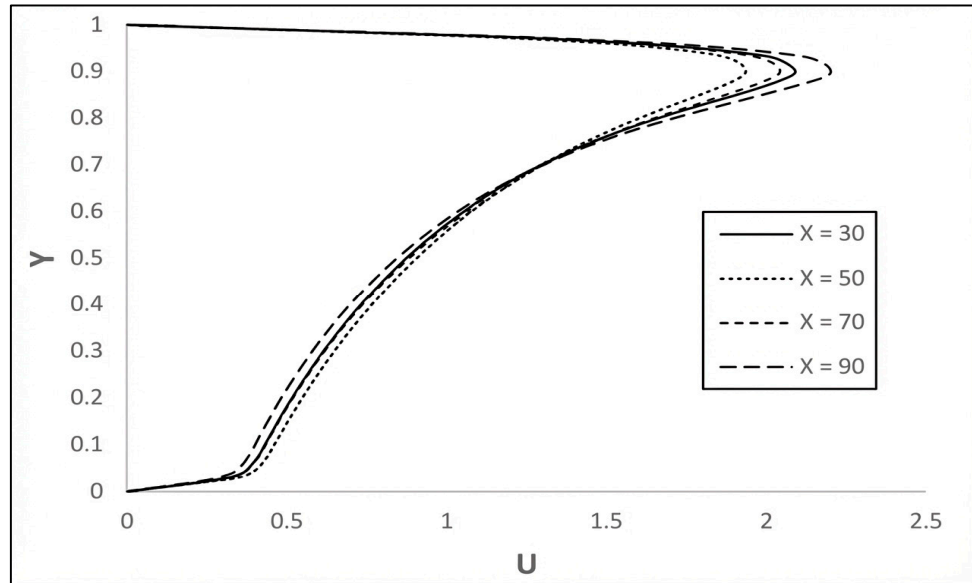


Figure 9: Dimensionless velocity along the center line, $Ha = 20$, non-uniform porosity, and $\varphi = 0.02$ for different X .

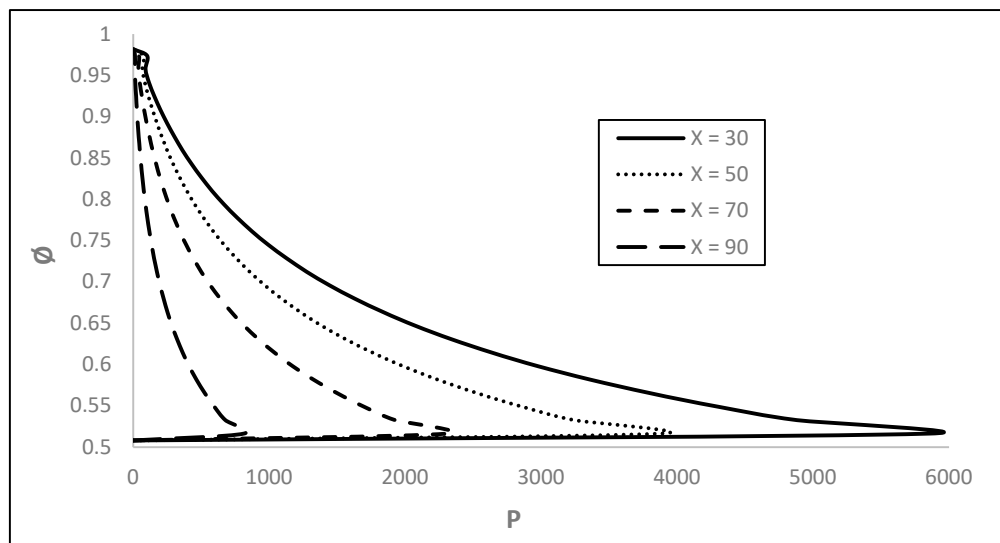


Figure 10: The dimensionless pressure (P) and the non-uniform porosity (φ) at different axial locations along the microchannel, $Re = 100$, $Ha = 20$, $Da = 0.2$, and $\varphi = 0.02$.

Fig. 11 depicts the dimensionless velocity profiles at various Darcy numbers ($Da = 0.2, 0.5$, and 0.8) in the microchannel height, at two different points ($X = 50$ and $X = 90$). With a smaller Darcy number $Da = 0.2$, flow resistance is more pronounced, and near both walls a markedly damped velocity profile is apparent. With the Darcy number increased to 0.5 and 0.8 , the porous medium becomes more permeable, reducing viscous resistance and allowing higher velocities in the core region. At $X = 90$, the flow has finally adjusted after leaving the magnetic and heated zone, and the trend becomes even stronger. Here, the profiles separate quite distinctly, with the highest velocities observed for $Da = 0.8$ because there is less drag and more penetration of flow.

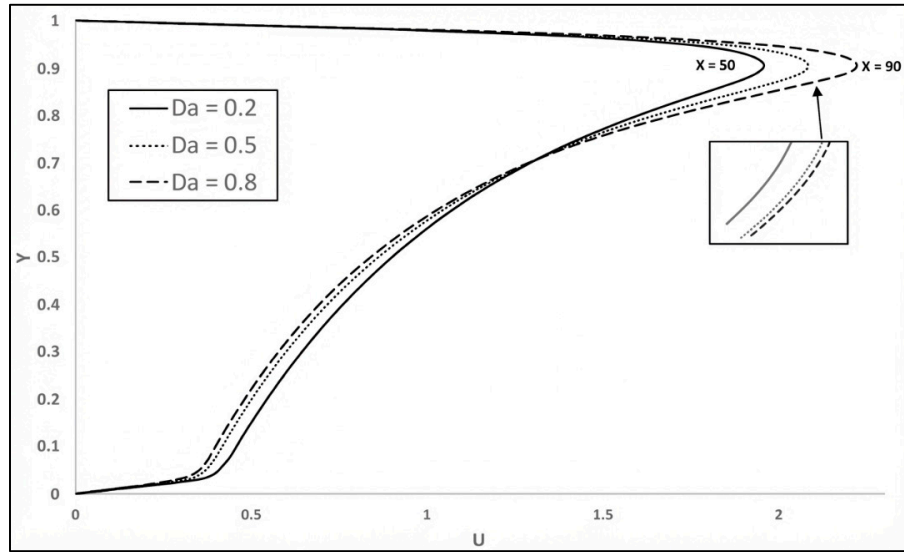


Figure 11: Dimensionless velocity profile, $Ha = 20$, $Re = 100$, non-uniform porosity, and $\phi = 0.02$ for different values of Da and X .

At different Reynolds numbers, the pumping power for the clear microchannel and the non-uniform porous microchannel has been observed to vary greatly, as shown in Fig. 12. For every additional Reynolds number, there is a very striking increase in pumping power for both the clear microchannel and the non-uniform porous microchannel. This increase in energy demand for the flowing fluid follows a cubic law that is proportional to the velocity of the flowing fluid. Due to the combined effects of inertial resistance and porous drag, there is a higher pumping power requirement for the non-uniform porous microchannel compared to the clear microchannel.

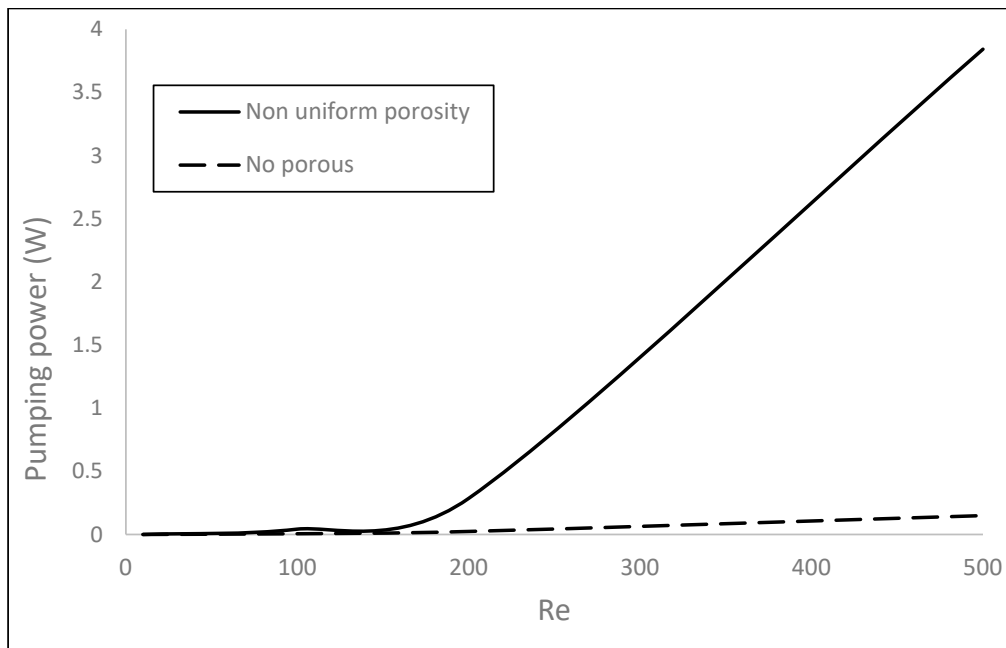


Figure 12: Pumping power for the clear microchannel and the non-uniform porous microchannel at different Reynolds numbers.

Fig. 13 illustrates the dimensionless centerline velocity distribution of the microchannel for varying Reynolds numbers ($Re = 10, 100, 500$) and non-uniform porosity. The trend is that the centerline velocity increases with the increased Reynolds number. The lower Reynolds number ($Re = 10$) is characterized by velocity suppression due to magnetic damping and porous resistance, translating to an average velocity of 0.7636 at the outlet of the microchannel. At a higher Reynolds number of 100, the fluid momentum is stronger, the flow centerline velocity is up to 0.8373. However, the porous region with non-uniformity still shows a large flat plateau in the middle of the microchannel. For $Re = 500$, fluid momentum and resistance create the maximum outlet centerline velocity of approximately 1.0407; the velocity profile rapidly adjusts to achieve higher velocities, and sharp accelerations are evident at the entrance and exit of the microchannel. It is shown that increasing the Reynolds number reduces the effect of the porous effect and Lorentz force, so that the flow can obtain higher velocity within the microchannel.

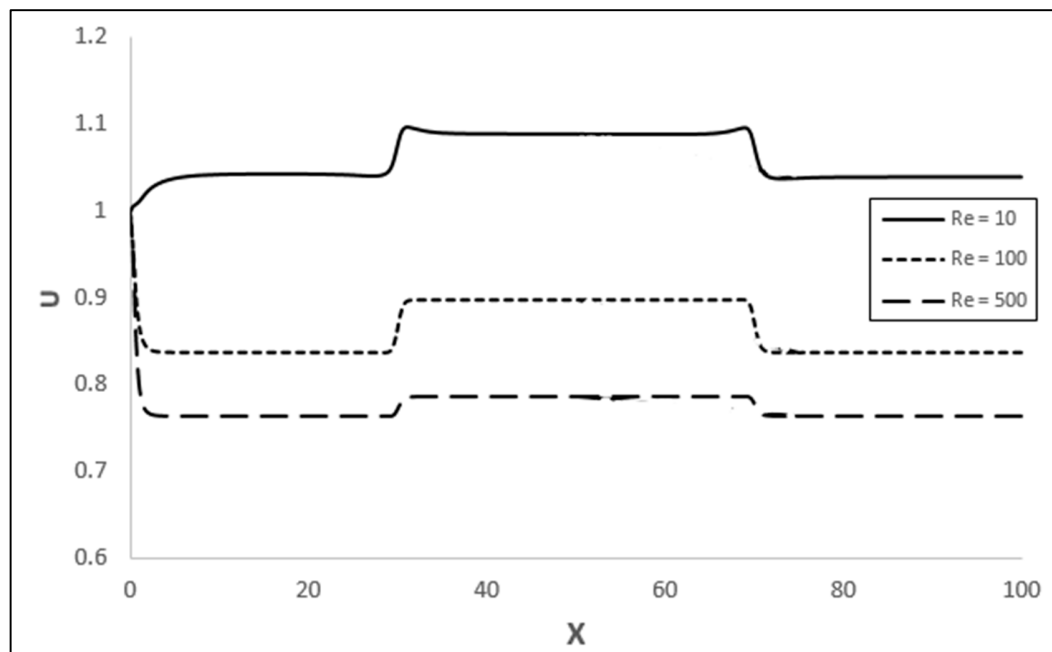


Figure 13: Dimensionless velocity along the center line of the microchannel, $Ha = 20$, non-uniform porosity, and $\varphi = 0.02$, for different values of Re .

Fig. 14 illustrates the dimensionless temperature distribution along the microchannel centerline for non-uniform porosity at three Reynolds numbers ($Re = 10, 100$, and 500). The results clearly show that the thermal response is strongly influenced by the flow inertia. At the lowest Reynolds number, $Re = 10$, the fluid moves slowly through the porous region, allowing significant heat absorption. As a result, the temperature rises sharply once the flow enters the heated zone at $X \approx 30$ and reaches the highest dimensionless temperature among the three cases. For $Re = 100$, the temperature increase is more moderate; the fluid absorbs heat at a slower rate due to its higher momentum, producing a smoother and less steep thermal profile. At $Re = 500$, the strong inertial forces dominate, causing the fluid to spend less time in the heated zone. This greatly limits heat diffusion, resulting in the lowest centerline temperature rise. In this current investigation, the Darcy model was chosen considering the fact that the viscous effect is the dominant force responsible for controlling the flow behavior inside the porous medium whereas the effect of inertia is negligible. Despite the Reynolds number based on the channel dimension being high ($Re = 500$) the Reynolds number within the porous medium based on the pore dimension is low because of the smaller

permeability of the flow. In addition to this, the main goal of this investigation is to analyze the effect of varying porosity on the fluid flow and heat transfer through the simplified form of the porous medium formulation. Overall, the figure demonstrates that decreasing the Reynolds number enhances thermal penetration within the porous region, while higher Reynolds numbers delay temperature development because the flow transfers heat more rapidly through the channel.

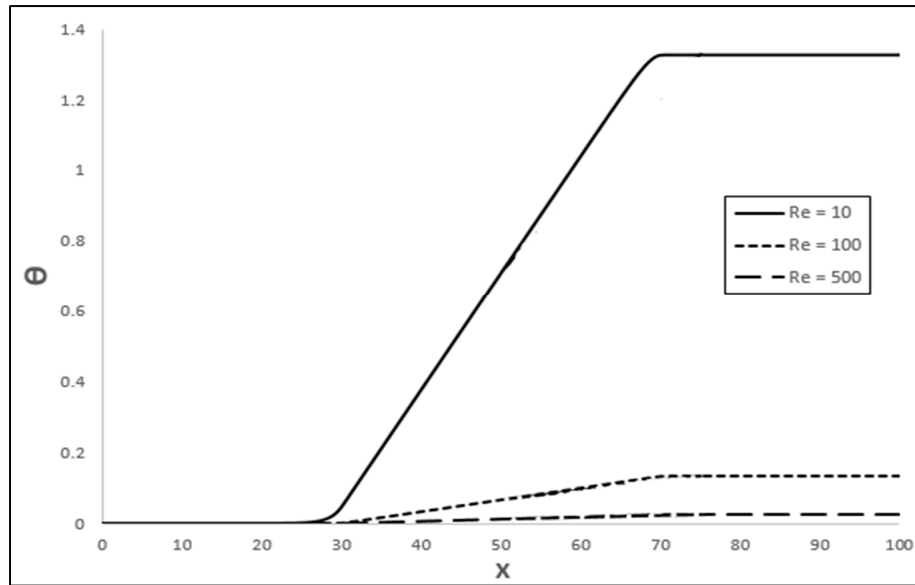


Figure 14: Dimensionless temperature along the center line of the microchannel, $Ha = 20$, non-uniform porosity, and $\varphi = 0.02$, for different values of Re .

The effect of Reynolds number on the local Nusselt number Nu_s along the heated wall of the microchannel is presented in Fig. 14. The local Nusselt number is defined as $Nu_s = \left[\frac{q'' D_h}{K_f (T_w - T_b)} \right]$ where (q'') is the imposed wall heat flux, (D_h) is the hydraulic diameter of the microchannel, (K_f) is the thermal conductivity of the reference fluid, (T_w) is the local wall temperature, and (T_b) is the bulk fluid temperature. In the present configuration, a positive heat flux is applied only to the heated middle section of the upper wall, while the inlet and outlet sections are adiabatic.

Consistent with expectation, there is no heat transfer within the adiabatic inlet and exit zones; hence, (Nu_s) equals zero in these parts. When entering the heating zone, the local Nusselt number takes on its maximum value since the difference in temperature between the heated surface and the fluid entering is at its maximum, implying strong heat transfer due to high thermal gradients. While the hybrid nanofluid passes through the heating zone, it keeps receiving more heat, thus making the bulk temperature of the fluid rise. As a result, the temperature difference and, accordingly, the wall temperature gradient become smaller, making the local Nusselt number lower.

Furthermore, increasing the Reynolds number significantly enhances the local Nusselt number throughout the heated region. Higher Reynolds numbers correspond to larger flow velocities and stronger convective transport, which reduce the thermal boundary-layer thickness adjacent to the heated wall. As a result, heat is removed more efficiently from the surface, producing larger Nusselt numbers and improved heat-transfer performance. This behavior confirms the increasing dominance of convection over thermal diffusion as the Reynolds number increases.

Figs. 15–17 collectively demonstrate the coupled influence of the magnetic field on both the temperature and velocity development in the microchannel. As shown in Fig. 15, increasing the Hartmann number (Ha) enhances the fluid temperature within the heated region because the applied magnetic field suppresses the fluid motion, thereby increasing the thermal residence time and allowing the hybrid nanofluid to absorb more heat. The corresponding velocity response is presented in Fig. 16, where the peak centerline velocity decreases progressively from $Ha = 0$ to $Ha = 40$ within the porous and heated region due to the Lorentz force acting opposite to the flow direction. This reduction in velocity directly contributes to the temperature increase observed in Fig. 15. Furthermore, Fig. 17 confirms the coupled thermal and hydrodynamic effects of the magnetic field, demonstrating that stronger magnetic fields simultaneously suppress the flow and promote heat accumulation within the heated section, highlighting the strong interaction between momentum damping and heat transfer in the porous microchannel.

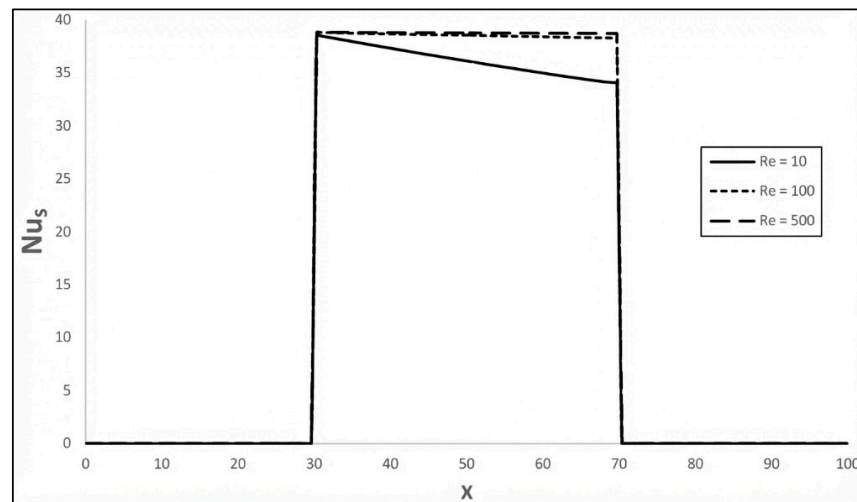


Figure 15: Dimensionless local Nusselt number along the upper wall, $Ha = 20$, non-uniform porosity, and $\phi = 0.02$ for different Re .

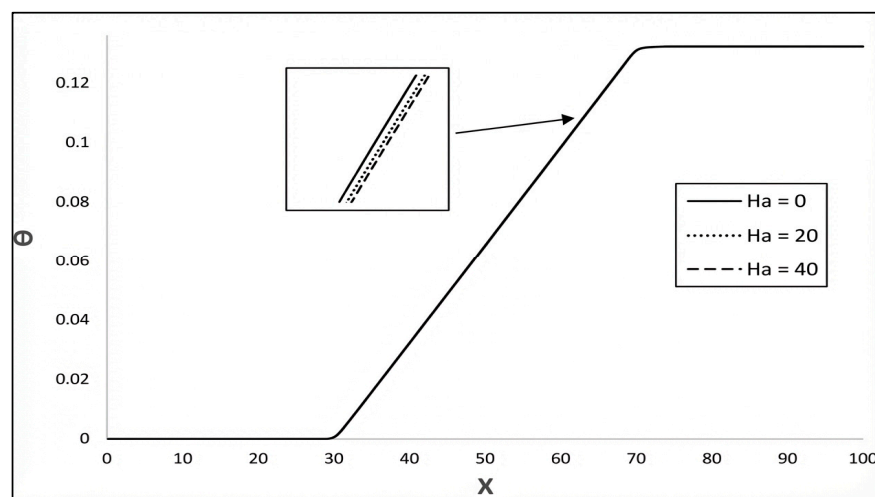


Figure 16: Dimensionless temperature along the center line of the microchannel, $Re = 100$, non-uniform porosity, and $\phi = 0.02$, with different values of Ha .

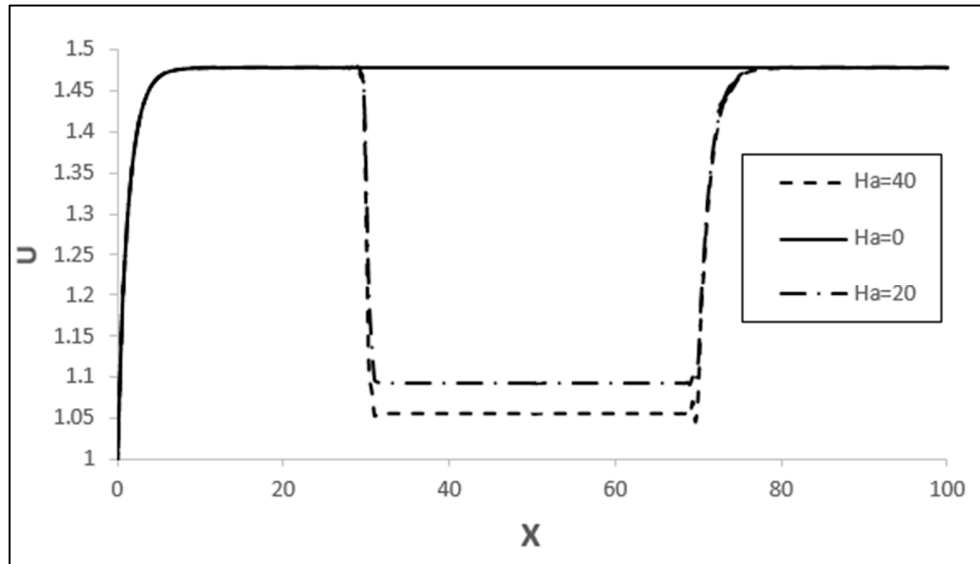


Figure 17: Dimensionless velocity along the center line of the microchannel, non-uniform porous, $Re = 100$, and $\varphi = 0.02$ for different values of Ha .

Figs. 18–20 collectively illustrate the influence of hybrid nanoparticle volume fractions on the hydrodynamic and thermal behavior of the hybrid nanofluid flowing through the non-uniform porous microchannel. As shown in Fig. 18, increasing the total nanoparticle concentration slightly elevates the centerline velocity within the porous region. This enhancement is most noticeable when both Cu and Al_2O_3 nanoparticles are added together ($\varphi_1 = 0.04$, $\varphi_2 = 0.04$), which increases the effective density and thermal conductivity while reducing viscous suppression caused by the porous medium. This occurs because Cu provides exceptionally high thermal conductivity and momentum enhancement, whereas Al_2O_3 contributes to flow stability and uniform heat diffusion; together, they create a synergistic thermophysical effect that neither nanoparticle can achieve alone. In contrast, cases with single component nanofluid (only Cu or only Al_2O_3) exhibit lower velocities due to weaker momentum transport compared with the hybrid mixture. Fig. 19 shows that the centerline temperature rises more prominently as the nanoparticle loading increases, especially in hybrid compositions. The improved thermal conductivity of the hybrid nanofluid allows more efficient heat absorption along the heated section, resulting in slightly higher dimensionless temperature values near the outlet for larger φ_1 and φ_2 . This thermal enhancement is further reflected in Fig. 20, where the local Nusselt number along the upper wall increases with higher nanoparticle concentrations. The hybrid mixtures consistently produce the highest Nu_s values, indicating stronger convective heat transfer at the wall due to improved thermal conductivity and enhanced energy transport mechanisms.

Fig. 21 presents the velocity and temperature contour distributions inside the microchannel for three Reynolds numbers ($Re = 10$, 100, and 500). At $Re = 10$, the velocity contours show strong recirculation and slow movement near the inlet, indicating that viscous and porous resistances dominate the flow. The corresponding temperature field is highly conductive, with nearly vertical isotherms that reveal limited convective transport. As the Reynolds number increases to 100, the flow becomes more uniform, the recirculation zones diminish, and the velocity contours flatten signifying the transition toward convection dominated transport. The temperature contours at this Reynolds number begin to tilt, showing the combined influence of forced convection and the applied heat flux on the thermal field. At $Re = 500$, the velocity distribution becomes fully developed and significantly faster, with straight, parallel contours that reflect

strong inertial forces overwhelming both viscous and porous resistance. The temperature field at this high Reynolds number displays strongly inclined isotherms, indicating intense convection that sweeps heat downstream and reduces thermal penetration toward the lower wall.

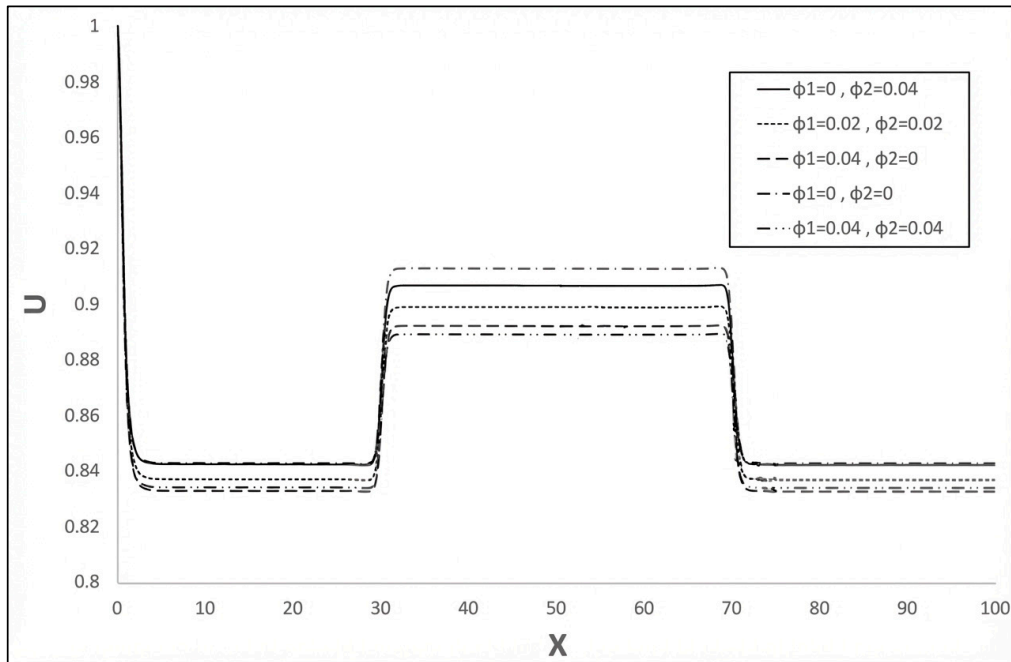


Figure 18: Dimensionless velocity along the center line of the microchannel, non-uniform porous, $Re = 100$, and $Ha = 20$ for different values of volume fraction.

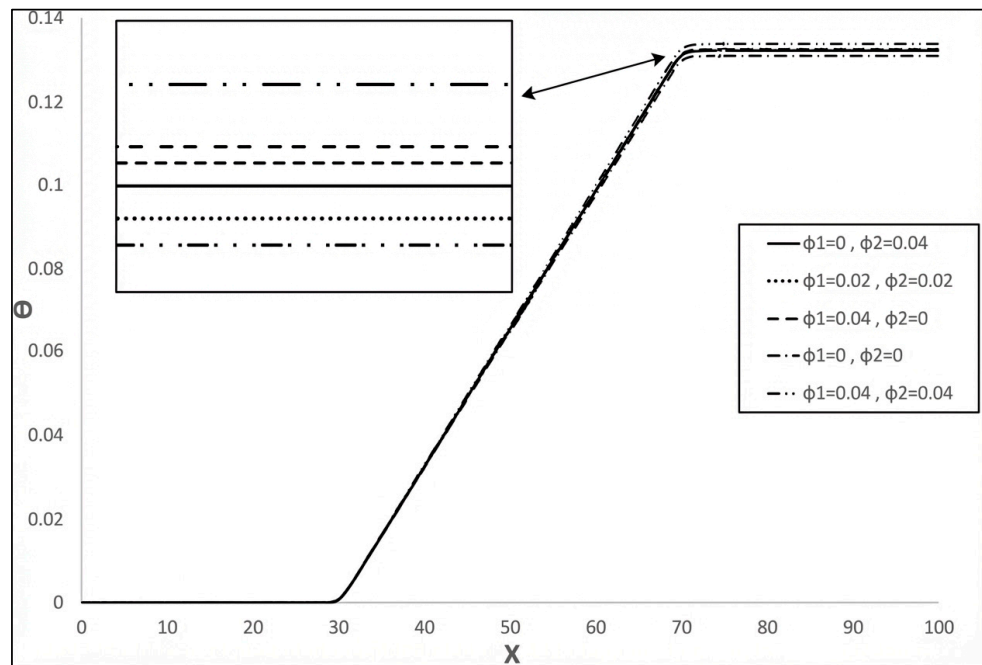


Figure 19: Dimensionless temperature along the center line of the microchannel, non-uniform porous, $Re = 100$, and $Ha = 20$ for different values of volume fraction.

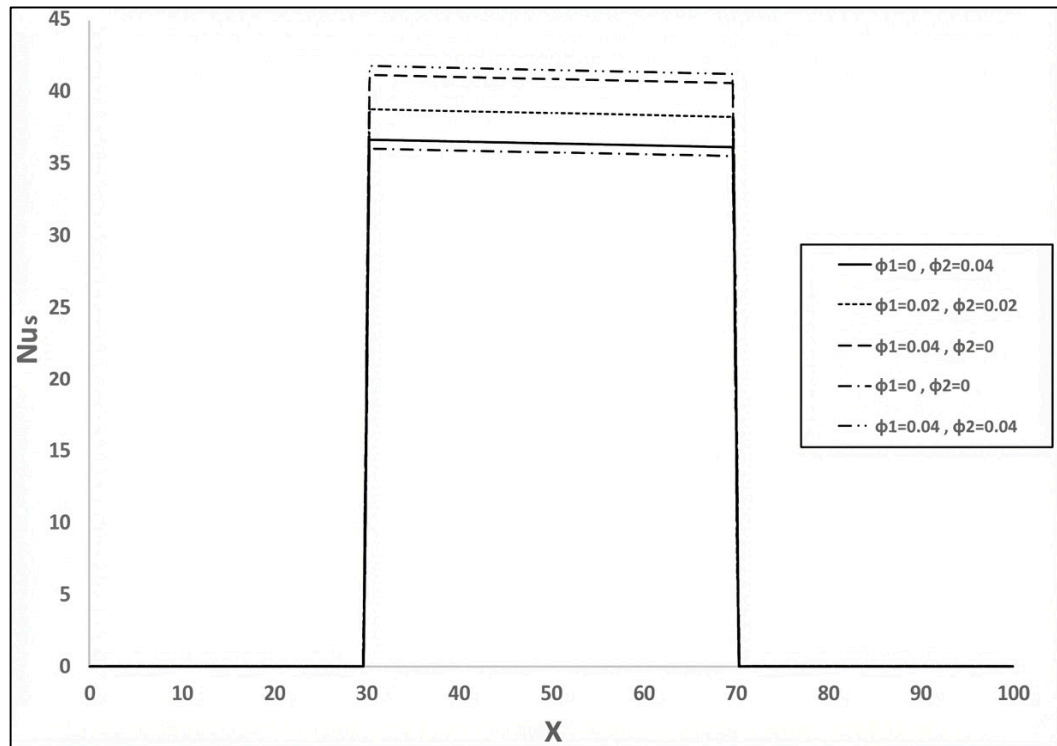


Figure 20: Dimensionless local Nusselt number along the upper wall of the microchannel, non-uniform porous, $Re = 100$, and $Ha = 20$ for different values of volume fraction.

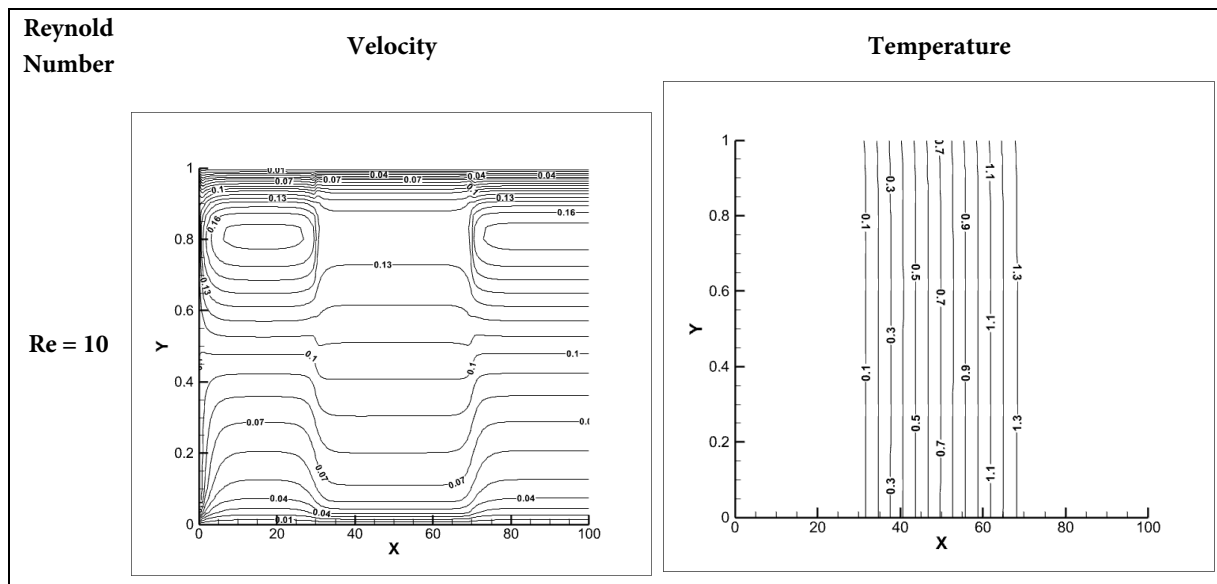


Figure 21: Cont.

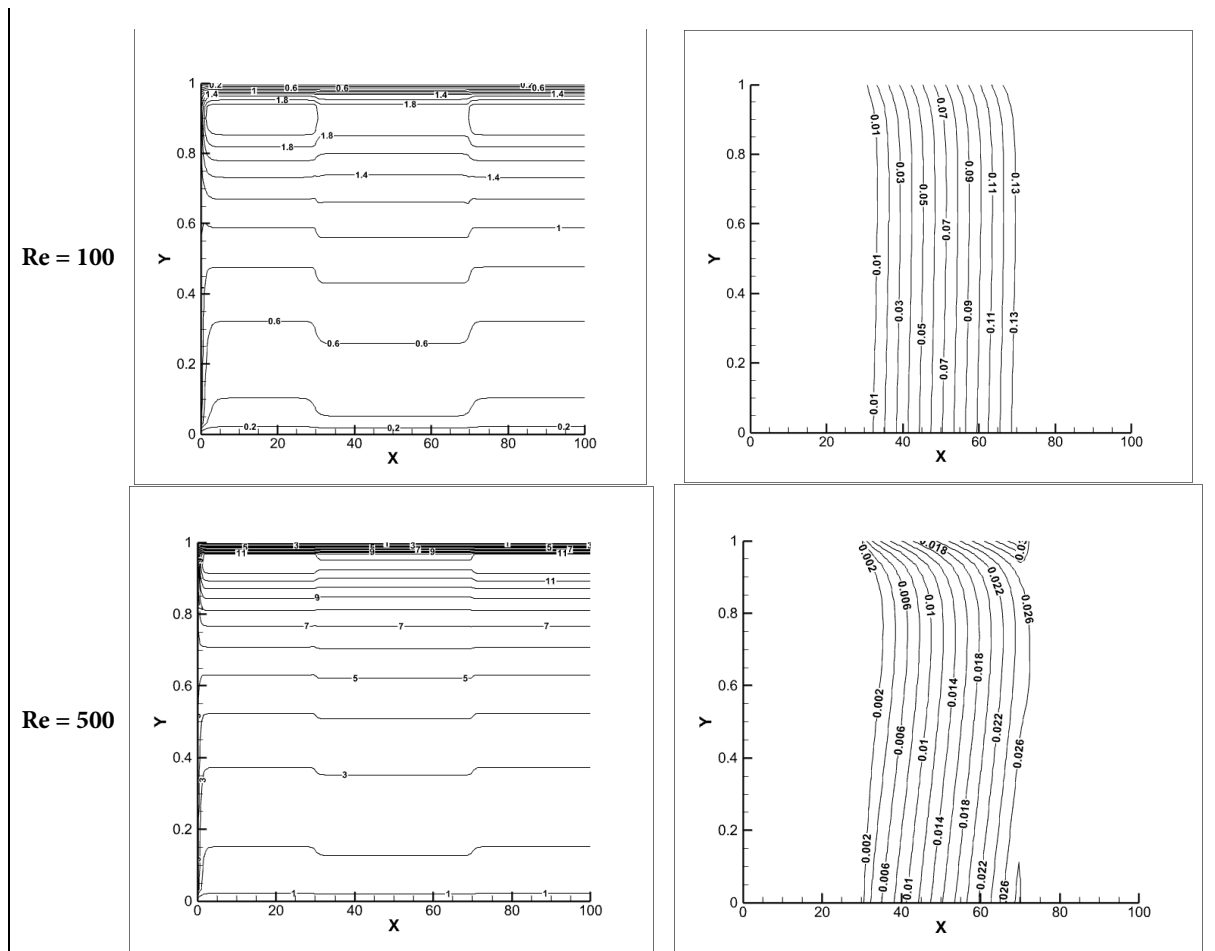


Figure 21: Velocity and temperature contour line inside microchannel with $Ha = 20$, $Da = 0.2$, and $\varphi = 0.02$.

Fig. 22 shows residual convergence graphs for the iterations performed for convergence of the governing equations. Convergence history of the continuity, momentum, and energy equations has been plotted as a separate figure. It is clear from the results that there was a steady decline in residuals with respect to convergence criteria, thus proving the numerically stable and accurate nature of the computational process. This further proves that the results presented do not have any iterative error in them.

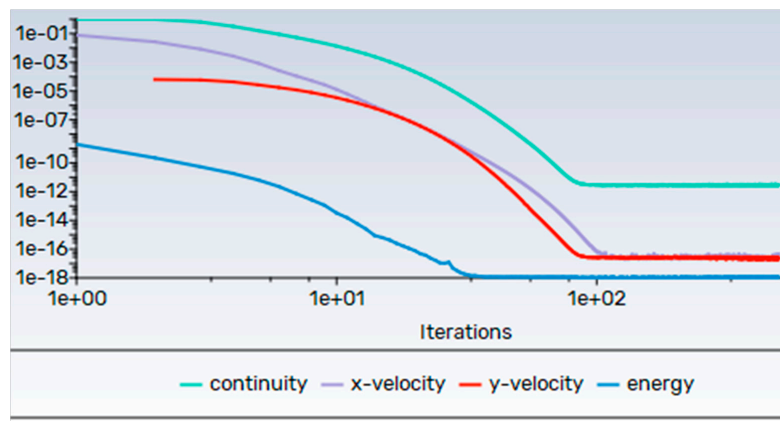


Figure 22: Residual convergence plots illustrating the iterative convergence behavior of the governing equations.

6 Conclusion

Non-uniform porosity strongly affects both the flow and heat transfer characteristics inside the microchannel. When porosity varies along the channel, the fluid experiences additional resistance, which reduces the velocity because of increased porous drag and magnetic effects. The slower flow allows the fluid to remain longer in the heated region, promoting greater heat absorption and leading to higher temperature and enthalpy levels. At the same time, the porosity gradient contributes to a larger pressure drop as the fluid moves downstream. Overall, using non-uniform porosity offers a practical way to regulate fluid motion while enhancing thermal performance in porous microchannels.

The numerical results presented provide a comprehensive understanding of how magnetic field strength, Reynolds number, porosity distribution, Darcy number, and hybrid nanoparticle volume fractions collectively shape the hydrodynamic and thermal behavior of hybrid Cu–Al₂O₃/water nanofluid in a non-uniform porous microchannel. The velocity analyses consistently show that introducing porosity and applying a transverse magnetic field suppresses fluid motion, reducing the centerline velocity from approximately 1.04 at Re = 500 to around 0.76 at Re = 10. Non-uniform porosity generates the strongest damping effect, especially within the heated region, where the velocity plateaus significantly decrease compared to the clear-channel case. Increasing the Reynolds number weakens these resistive effects, and the flow becomes increasingly convection-dominated, with sharp acceleration zones at Re = 500 and smoother, diffusion-dominated profiles at lower Reynolds numbers.

The temperature distributions demonstrate that the thermal response is highly sensitive to heating location, magnetic damping, and nanoparticle concentration. Temperatures remain constant in the adiabatic inlet (L_1) and rise sharply in the heated section (L_2). Higher Hartmann numbers ($Ha = 20\text{--}40$) produce slightly elevated temperatures due to reduced convective sweeping and longer residence time, whereas increasing Reynolds number suppresses temperature rise, reducing the maximum dimensionless temperature from around 0.14 at Re = 10 to approximately 0.03 at Re = 500. Vertical temperature profiles reveal strong thermal asymmetry: the upper heated wall ($Y = 1$) consistently shows the highest temperatures.

Hybrid nanofluid effects become particularly evident with change in volume fraction. Increasing Cu and Al₂O₃ nanoparticle concentrations enhance the fluid's effective thermal conductivity and improves heat absorption. The combination $\phi_1 = 0.04$ (Cu) and $\phi_2 = 0.04$ (Al₂O₃) produces the highest wall temperatures and the highest Nusselt numbers, with Nu_s increasing from ~36 for no nanoparticles to above 42 for the highest hybrid loading. Velocity increases modestly with higher hybrid volume fractions, but the thermal improvement is significantly more pronounced, confirming that Cu provides strong conductivity enhancement while Al₂O₃ improves suspension stability, producing a synergistic effect that outperforms single nanoparticle nanofluids.

The flow and thermal characteristics are governed by the interplay of three mechanisms, which compete with each other. While the inertial effect, captured in the Reynolds number, is aimed at increasing the speed and convective heat transfer in the fluid, the Lorentz force, created by the external magnetic field, acts against the flow direction and reduces momentum transport. At the same time, the drag force created by the porous matrix is an additional force acting on the flow, and its strength is proportional to the matrix permeability. Thus, the resulting velocity and temperature profiles are dictated by the competition of these three effects, while the dominance of inertia leads to an increase in convection and Nusselt number.

The analysis performed shows that the effect of the magnetic field cannot be analyzed separately from the effects of the Reynolds and Darcy numbers presented in Table 5. For low values of the Reynolds number, the Lorentz force plays a very significant role in changing the velocity profile and the thermal boundary layer; however, an increase in permeability counteracts the drag due to the porous medium. On the other

hand, for high Reynolds numbers, inertial forces play the key role and minimize the effect of magnetic damping.

Table 5: Combined sensitivity analysis of (Re, Ha, Da) interaction.

Re	Ha	Da	Dominant Mechanism	Expected Heat Transfer Behavior
10	0	0.2	Conduction + porous resistance	Low (Nu)
10	40	0.2	Strong magnetic damping	Moderate (Nu)
10	40	0.8	Reduced porous resistance	Moderate–High (Nu)
100	0	0.2	Mixed convection–diffusion	Moderate (Nu)
100	20	0.5	Balanced interaction	High (Nu)
100	40	0.8	Reduced magnetic influence	Higher (Nu)
500	0	0.2	Convection dominated	High (Nu)
500	20	0.5	Inertia dominates	Very high (Nu)
500	40	0.8	Weak relative MHD effect	Maximum (Nu)

Finally, the contour plots illustrate a clear transition from diffusion-dominated behavior at low Reynolds numbers to convection-dominated behavior at high Reynolds numbers. At $Re = 10$, velocity contours show recirculation, and isotherms are nearly vertical, while at $Re = 500$, velocity lines become parallel and isotherms dramatically tilt downstream—indicating intensified convective transport.

Overall, the combined results demonstrate that:

1. Non-uniform porosity markedly suppresses the flow velocity while promoting a more uniform temperature distribution within the microchannel.
2. Magnetic fields in the range of $Ha = 20–40$ tend to slow down the fluid motion by suppressing the momentum.
3. Higher Reynolds numbers ($Re = 500$) strengthen convection, increasing velocity and lowering temperature.
4. Darcy numbers strongly affect near-wall damping and velocity recovery.
5. Hybrid nanoparticles ($Cu + Al_2O_3$) provide the best thermal performance, increasing Nu_s by 15–20% compared with single-particle nanofluids.

Recommendations and Future Work

The study suggests hybrid nanofluids with optimized and non-uniform porosity distributions for thermal management and microfluidic systems for improved flow uniformity and heat transfer efficiency, based on it. As a result, tuning the Darcy number and porosity ratio in the presence of magnetic fields may help to achieve the aforementioned improvements in temperature management as well as having hybrid nanoparticle mixtures of $Al_2O_3–Cu$ offer a higher thermal conductivity and stability in hybrid nanofluids than that for single nanofluids. In order to this end, efforts can be made to generalize the current numerical model to three-dimensional and time-dependent environments, along with experimental verification at different magnetic field strengths and nanoparticle concentrations. Further studies may also investigate other hybrid nanoparticle combinations, the effects of non-Newtonian fluid behavior, and variable wall heating conditions. Combination of hybrid nanofluids could also lead to more sustainable and effective heat treatment in renewable energy and intelligent cooling systems.

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Availability of Data and Materials: Data available on request from the authors.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

Abbreviation	Description
A	Cross-sectional area of the surface (m^2)
B_0	Magnitude of the externally applied magnetic field
C_p	Heat capacity at constant pressure, expressed in ($\text{J kg}^{-1} \text{K}^{-1}$)
D	Characteristic diameter or representative length scale, in meters
h	Microchannel height (m)
H	Microchannel height ($h/h = 1$) (dimensionless)
Ha	Hartmann number ($B_0 h \sqrt{\frac{\sigma_{hf}}{\rho_{hf} \nu_f}}$)
k	Thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)
L	Microchannel length (l/h) (dimensionless)
Nu_s	Local Nusselt number
R	Viscous resistance of the porous medium
P	Pressure ($\frac{P}{\rho^2 u C_p}$) (dimensionless)
Pr	Prandtl number (ν_f / α_f)
Ec	Eckert number ($\frac{u_0^2}{C_{p(T)}}$)
q_0	Uniform heat flux (Kw/m^2)
Re	Reynolds number ($u_c H / \nu_f$)
T	Temperature (K)
T_c	Inlet flow temperature (K)
u, v	Velocity components in x and y directions (m s^{-1})
u_0	Inlet flow velocity (m s^{-1})
U, V	Velocity components (u/u_c), (v/u_c) (dimensionless)
x, y	Cartesian coordinate (m)
X, Y	Coordinates (x/h , y/h) (dimensionless)
Greek Letter	Description
α	Thermal diffusivity ($k/\rho C_p$) ($\text{m}^2 \text{s}^{-1}$)
φ_1	Cu solid volume fraction
φ_2	Al_2O_3 solid volume fraction
θ	Temperature ($(T - T_c)/\Delta T$) (dimensionless)
ν	Kinematic viscosity, (μ/ρ) ($\text{m}^2 \text{s}^{-1}$)
ρ	Density (kg m^{-3})
μ	Viscosity
ϕ_0	Mean porosity
ϕ_Y	Change in porosity in the y-direction
Subscripts	Description
f	Pure fluid
nf	Nanofluid

hnf	Hybrid nanofluid
min	Minimum
out	Outlet
s_1	Cu nanoparticle
s_2	Al_2O_3 nanoparticle

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