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Synergistic Effects of Dual Source–Sink Pairs and $\text{Al}_2\text{O}_3\text{-Cu-TiO}_2\text{/Water}$ Ternary Hybrid Nanofluid on Heat Transfer Enhancement in a Square Cavity

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ABSTRACT: This study demonstrates that natural convection and heat transfer in a square cavity can be significantly enhanced using a water-based $\text{Al}_2\text{O}_3\text{-Cu-TiO}_2$ ternary hybrid nanofluid (THNF) with dual heat source–sink pairs. Using the finite volume method under a single-phase nanofluid model with the Boussinesq approximation, we investigate the effects of Rayleigh number ($10^3 \leq \text{Ra} \leq 10^6$), nanoparticle volume fraction ($\phi \leq 0.1$), nanofluid composition, and source–sink configuration on flow structures, temperature distribution, and thermal performance. Validation against benchmark and literature data confirms the accuracy of our simulations. Results show that the optimal alternating source–sink arrangement with THNF increases the average Nusselt number by 41–45% over Cu/water nanofluid and 18–21% over $\text{Al}_2\text{O}_3\text{-Cu/water}$ hybrid nanofluid at $\phi = 0.1$, especially at low Rayleigh numbers. These findings reveal a strong synergy between ternary hybrid nanofluids and source–sink positioning, highlighting the combined effects of nanoparticle interactions and flow patterns. The study provides practical guidance for designing efficient thermal management systems in electronics, energy, and industrial applications, emphasizing both performance enhancement and potential environmental benefits.

KEYWORDS: Ternary hybrid nanofluid; heat transfer enhancement; square cavity; source-sink; finite volume method

1 Introduction

Natural convection heat transfer has attracted considerable attention due to its relevance in various thermal engineering applications, including solar collectors, thermal energy storage systems and cooling of electronic components. In buoyancy-driven flows within enclosed cavities, the fluid-wall heat exchange governs energy transport efficiency, making the cavity design and working fluid selection critical for achieving high heat transfer rates [1,2]. Conventional nanofluids, suspensions of single nanoparticles in base fluids, have been shown to enhance thermal properties, but they often suffer from aggregation and stability limitations, particularly under practical operating conditions [3,4].

Hybrid nanofluids, obtained by dispersing two types of nanoparticles in a base fluid, have been proposed to overcome these limitations by exploiting synergistic thermal effects, improving both thermal conductivity and convective heat transfer [5]. Despite these advantages, the relatively slow heat exchange at fluid-wall interfaces remains a limiting factor in natural convection applications. To enhance convective heat transfer, several strategies have been developed, including complex cavity geometries [6,7]. In particular, the study of U-shaped enclosures filled with Newtonian hybrid nanofluids at moderate Rayleigh numbers has attracted significant interest recently [8–10]. Other widely explored methods include strip heaters [11], application of magnetic fields [12] and porous media inserts [13].

Recently, ternary hybrid nanofluids, composed of three different nanoparticle species, have emerged as a promising approach for further enhancing heat transfer. The combination of three nanoparticles can significantly improve thermal conductivity, colloidal stability and other thermophysical properties compared to binary hybrids or mono nanofluids [14]. Such ternary formulations are particularly relevant for modern thermal management systems, including heat exchangers, electronic cooling units and solar energy devices, where high-performance fluids are needed to maximize heat transfer efficiency [15].

Recent studies demonstrate the growing interest in ternary hybrid nanofluids (THNFs) because of their enhanced thermophysical and heat transfer characteristics. Experimental investigations by Cakmak et al. (2020) [16] and Sahoo (2021) [17] confirmed that combining multiple nanoparticles within a base fluid significantly improves thermal conductivity and overall thermal performance compared with conventional nanofluids. Numerical studies have further examined THNF behavior under various physical conditions. Elnaqeeb et al. (2021) [18] analyzed three-dimensional ternary-hybrid nanofluid flow under suction and dual-stretching effects, while Shao et al. (2022) [19] reported enhanced cooling performance in porous enclosures with movable hot baffles. Alshahrani et al. (2022) [20] highlighted the influence of slip and thermal jump conditions on THNF transport characteristics.

In buoyancy-driven systems, Thirumalaisamy and Reddy (2023) [21] as well as Roy and Pop (2024) [22] demonstrated that dispersing ternary hybrid nanofluids (THNF) into porous cavities significantly enhances natural convective heat transfer under magnetic effects and cavity inclination, respectively, while Rajesh and Öztop (2024) [23] showed that magnetic fields, phase change, and non-uniform heating strongly affect thermal circulation and entropy generation in THNF-filled enclosures. More recently, Hanif (2025) [24], Hussein et al. (2025) [25], Moon et al. (2025) [26], and Anwar et al. (2025) [27] extended THNF applications to complex configurations involving triangular cavities, Joule heating, entropy generation, and bio-convective magnetized flows. In addition, the recent review by Pandian and Boobalan (2025) [28] emphasized the rapid development of THNFs, particularly regarding synthesis methods, thermophysical properties, machine-learning-based prediction models, and emerging thermal engineering applications.

Despite significant progress in ternary hybrid nanofluids (THNFs) and their enhanced thermo-physical performance, a critical research limitation still exists regarding the coupled interaction between (i) advanced THNFs and (ii) internal thermal forcing generated by multiple heat source–sink arrangements [29]. This limitation is particularly relevant in modern thermal systems such as high-power electronic cooling, compact heat exchangers, and solar thermal receivers, where heat generation and extraction are inherently non-uniform. In electronic cooling applications, distributed hotspots induce strong localized thermal gradients that degrade performance and reliability [30]. In compact heat exchangers, spatially varying thermal loads significantly modify flow structure and heat transfer efficiency [31]. In solar thermal receivers, non-uniform irradiance causes uneven temperature fields, thermal stress, and efficiency losses [32,33]. These practical constraints highlight the necessity of studying convection under heat source–sink arrangements.

As such, the present study provides a comprehensive numerical investigation of steady-state natural convection in a square cavity filled with an $\text{Al}_2\text{O}_3\text{--Cu--TiO}_2$ /water ternary hybrid nanofluid subjected to dual heat source–sink pairs. The key novelty of this work lies not only in the implementation of a ternary hybrid nanofluid, but more importantly in the simultaneous coupling of multi-region thermal forcing induced by heat source–sink arrangements with enhanced effective thermophysical properties, a configuration that has not, to the best of our knowledge, been previously explored in the literature within a unified framework. The outcomes of this study provide important insights for the design and optimization of next-generation thermal management systems, particularly in applications requiring high heat removal rates and precise thermal control. These include microelectronic cooling systems with spatially distributed heat sources, compact heat exchangers used in energy conversion systems, and solar thermal receivers operating under non-uniform heat flux distributions, where improving thermal uniformity and enhancing convective transport are critical for performance and reliability [30].

2 Mathematical Analysis

2.1 Problem Statement

As shown in Fig. 1, the flow configuration being considered here is that of a two-dimensional square enclosure with two pairs of heat source–sink covering the entire length of the enclosure bottom wall, while the other walls are thermally insulated. The heat sources are maintained at a constant temperature (T_H) that is higher than that of the heat sink T_C . The ternary hybrid nanofluid in the enclosure is modeled as a dilute solid–liquid mixture with a uniform volumetric composition (20:60:20) of both Al_2O_3 , Cu and TiO_2 nanoparticles (NPs), respectively. Simulations have been carried out for the pertinent parameters in the following ranges: the Rayleigh number ($10^3 \leq \text{Ra} \leq 10^6$), the Prandtl number fixed at $\text{Pr} = 6.2$, and the NPs overall volumetric fraction ($0\% \leq \varphi \leq 10\%$).

While experimental setups and strict hydrodynamic definitions often restrict the dilute regime to $\varphi \leq 5\%$ to avoid particle interaction and high viscosity penalties, the extended range up to 10% is adopted here as a parametric numerical investigation. This allows for a comprehensive characterization of the fluid flow trends and heat transfer behavior over a broader spectrum. Several recent CFD-based numerical studies have adopted this fraction in order to better map these thermodynamic behaviors [34,35], while some investigations have even reported concentrations reaching 15% [36] and 20% [37,38].

The ternary hybrid nanofluid is treated as a single-phase Newtonian fluid under thermal equilibrium conditions and no-slip assumption between the base fluid and nanoparticles. This monophasic approach is well justified for low nanoparticle concentrations, where the mixture behaves as an effective homogeneous fluid with negligible particle migration effects [39,40]. It is important to acknowledge that at the upper limit of the investigated range ($\varphi = 10\%$), the fluid enters a regime where particle-particle interactions and micro-convection could theoretically emerge. However, within the scope of this study, the single-phase model is utilized as a well-established parametric simplification to explore the theoretical heat transfer enhancement limits and asymptotic trends. The potential minor deviations from non-homogeneous particle distribution are assumed to be secondary to the overall thermal buoyancy effects. The flow is assumed laminar and incompressible, and buoyancy forces are modeled using the Boussinesq approximation, which is valid for moderate temperature differences commonly encountered in natural convection studies of nanofluids [41]. The effects of viscous dissipation and compression work are neglected, as their influence is insignificant in low-speed natural convection flows [42]. Advanced multiphase dynamics considering non-homogeneous distributions remain a prospective avenue for future studies.

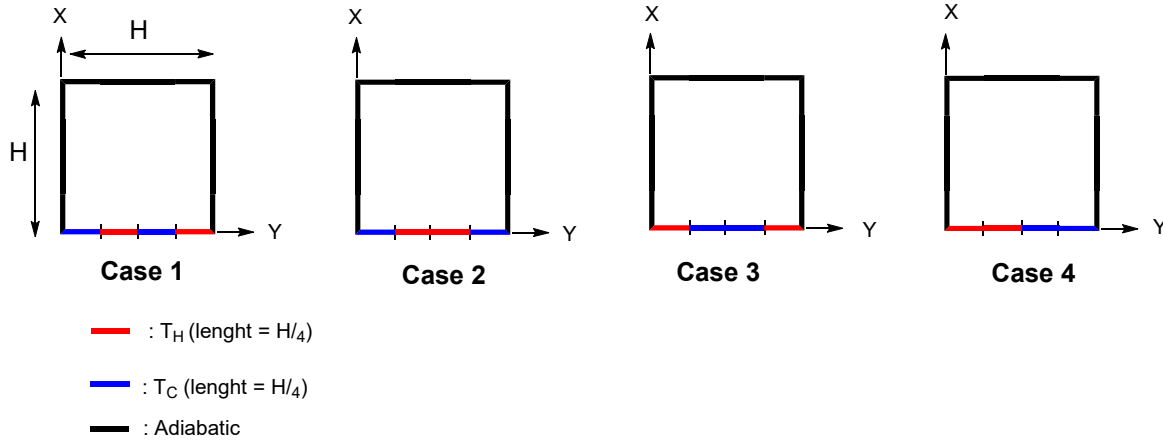


Figure 1: Physical domain.

2.2 Model Equation

Invoking the foregoing assumptions, the dimensionless fundamental equations for continuity, momentum, and energy can be expressed as follows [37,43,44]. Following the same modeling procedure validated by recent THNF numerical studies [35,45], the classical conservation forms are retained and adapted according to the specific system assumptions, while the physics of the ternary hybrid nanofluid is fully captured through its effective thermophysical properties embedded directly within the dimensionless numbers Ra_{thnf} and Pr_{thnf} .

$$U \frac{\partial U}{\partial X} + V \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + Pr_{thnf} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (2)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + Pr_{thnf} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - Pr_{thnf} Ra_{thnf} \theta \quad (3)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \quad (4)$$

The subscript *thnf* stands for the ternary hybrid nanofluid thermophysical properties.

The dimensionless variables used in Eqs. (1)–(4) are defined based on their dimensional variables as follows [35]:

X, Y : Dimensionless coordinates ($X = x/H, Y = y/H$)

U, V : Dimensionless x and y components of velocity ($U = u H / \alpha_{thnf}, V = v H / \alpha_{thnf}$)

P : Dimensionless pressure ($P = (pH^2) / (\rho_{thnf} \alpha_{thnf}^2)$)

θ : Dimensionless temperature ($\theta = (T - T_C) / (T_H - T_C)$)

where:

x, y : Dimensional coordinates

u, v : Dimensional x and y components of velocity

T, p : Dimensional temperature and pressure

H : Height of the enclosure

Ra_{thnf} , Pr_{thnf} : The THNF's Rayleigh and Prandtl numbers

These equations are solved subject to the following non-dimensional boundary conditions, commonly adopted in natural convection problems involving discrete heat source-sink configurations [46,47].

$$\begin{cases} \text{On the source : } U = V = 0, \theta = 1 \\ \text{On the sink : } U = V = 0, \theta = 0 \\ \text{On the adiabatic walls : } U = V = 0, \frac{\partial \theta}{\partial N} = 0 \end{cases} \quad (5)$$

where:

N : Normal direction to the walls

The Rayleigh and Prandtl numbers governing the buoyancy-driven flow [35] are defined as:

$$Ra_{thnf} = \frac{g\beta_{thnf}H^3\Delta T}{\alpha_{thnf}\vartheta_{thnf}}; \quad Pr_{thnf} = \frac{\vartheta_{thnf}}{\alpha_{thnf}} \quad (6)$$

where:

g : Gravitational acceleration

β_{thnf} , α_{thnf} , ϑ_{thnf} : THNF's thermal expansion coefficient, fluid thermal diffusivity and kinematic viscosity, respectively.

2.3 Ternary Hybrid Nanofluid Attributes

Based on recent studies on ternary hybrid nanofluids [27] and classical models developed for conventional solid-liquid mixtures [28], the effective thermophysical properties of the THNFs are summarized in Table 1.

Table 1: Ternary hybrid nanofluid attributes.

Attributes	Symbol	Expression	
Density [Kg m ⁻³]	ρ_{thnf}	$\rho_{thnf} = (1 - \varphi_3)[(1 - \varphi_2)\{(1 - \varphi_1)\rho_f + \varphi_1 \rho_{s1}\} + \varphi_2 \rho_{s2}] + \varphi_3 \rho_{s3}$	(7)
Thermal volume capacity [KJ m ⁻³ K ⁻¹]	$(\rho C_p)_{thnf}$	$(\rho C_p)_{thnf} = (1 - \varphi_3)[(1 - \varphi_2)\{(1 - \varphi_1)(\rho C_p)_f + \varphi_1(\rho C_p)_{s1}\} + \varphi_2(\rho C_p)_{s2}] + \varphi_3(\rho C_p)_{s3}$	(8)
Volume heat expansion parameter [Kg m ⁻³ K ⁻¹]	$(\rho\beta)_{thnf}$	$(\rho\beta)_{thnf} = (1 - \varphi_3)[(1 - \varphi_2)\{(1 - \varphi_1)(\rho\beta)_f + \varphi_1(\rho\beta)_{s1}\} + \varphi_2(\rho\beta)_{s2}] + \varphi_3(\rho\beta)_{s3}$	(9)
Viscosity [N s m ⁻²]	μ_{thnf}	$\mu_{thnf} = \mu_{tf}(1 - \varphi_1)^{-2.5} (1 - \varphi_2)^{-2.5} (1 - \varphi_3)^{-2.5}$	(10)
Thermal conductivity [W m ⁻¹ K ⁻¹]	k_{thnf}	$k_{thnf} = k_{hnf} \left[\frac{(k_{s3} + 2k_{hnf})^{-2\varphi_3}(k_{hnf} - k_{s3})}{(k_{s3} + 2k_{hnf}) + \varphi_3(k_{hnf} - k_{s3})} \right]$	(11)
		where: $\begin{cases} k_{hnf} = k_{nf} \left[\frac{(k_{s2} + 2k_{nf})^{-2\varphi_2}(k_{nf} - k_{s2})}{(k_{s2} + 2k_{nf}) + \varphi_2(k_{nf} - k_{s2})} \right] \\ k_{nf} = k_f \left[\frac{(k_{s1} + 2k_f)^{-2\varphi_1}(k_f - k_{s1})}{(k_{s1} + 2k_f) + \varphi_1(k_f - k_{s1})} \right] \end{cases}$	(12)

where φ_1 , φ_2 , φ_3 are volume fraction of Al₂O₃, Cu and TiO₂ NPs, respectively. The subscripts *thnf*, *hnf*, *nf*, *f*, *s*₁, *s*₂, *s*₃ denote ternary hybrid nanofluid, hybrid nanofluid, simple nanofluid, host liquid, Al₂O₃, Cu and TiO₂ NPs correspondingly.

The physical attributes of the ternary hybrid fluid components are exhibited in Table 2 as follow [48,49]:

Table 2: Physical attributes $\text{Al}_2\text{O}_3\text{-Cu-TiO}_2/\text{water}$ ternary hybrid nanofluid.

Attributes	H_2O (f)	Al_2O_3 (s1)	Cu (s2)	TiO_2 (s3)
ρ [Kg/m^3]	997.1	3970	8933	4250
C_p [$\text{J}/\text{Kg K}$]	4179	765	385	686.2
k [$\text{W}/\text{m K}$]	0.631	40	401	8.9538
β [K^{-1}]	21×10^{-5}	0.85×10^{-5}	1.67×10^{-5}	0.9×10^{-5}

2.4 Nusselt Number Calculation

The average Nusselt number can be computed using surface integrals over the wall [37,43] in question:

$$Nu_{avg} = \int_0^1 Nu(N) dN \quad (13)$$

Nu : is the local Nusselt number. It is defined as:

$$Nu = - \left(\frac{k_{thnf}}{k_f} \right) \frac{\partial \theta}{\partial N} \quad (14)$$

3 Numerical Approach

The sets of non-dimensional form of the mass, momentum and energy conservation equations (Eqs. (1)–(4)) with corresponding boundary conditions (Eq. (5)) are discretized using the finite volume technique. A Second Order Upwind approach is employed to approximate the convection–diffusion terms while the Simple procedure is adopted to couple the velocity–pressure fields as described by Patankar (1980) [50]. The solution employs an iterative process that can be continued till fulfilling the convergence criterion which is 10^{-8} . A structured non-uniform grid distribution is used to discretize the computational domain. As illustrated in Fig. 2a, the mesh structure is refined near the boundaries to accurately capture the high gradients within the boundary layers, while a coarser distribution is maintained at the core region of the cavity. To achieve an optimum balance between numerical precision and computational economy, a grid independence study was initially performed by monitoring the temperature profile evolution, as shown in Fig. 2b.

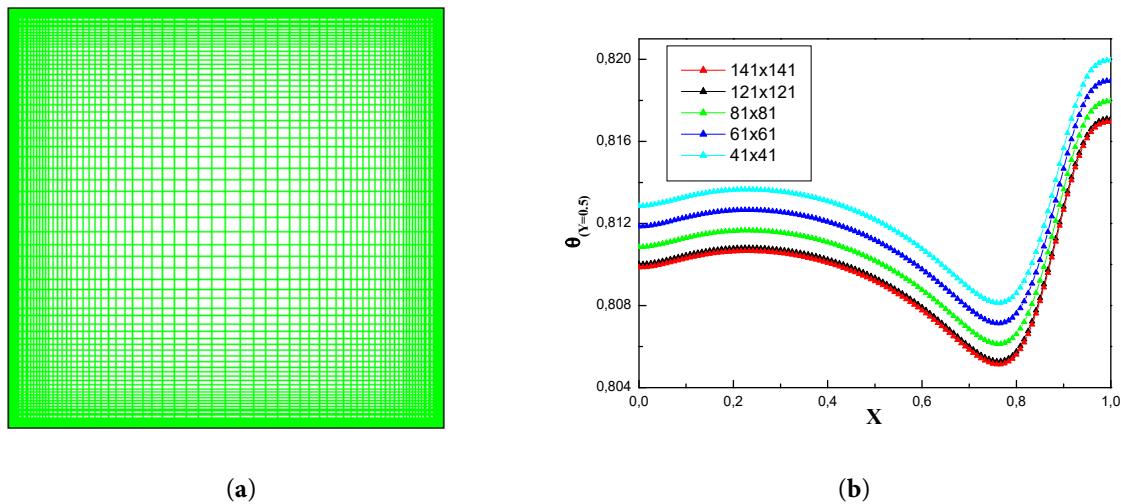


Figure 2: Grid numerical analysis: (a) Mesh structure, (b) Grid independence study (Case 1, $Ra = 10^6$, $\phi = 0.0$).

Additionally, the grid independence was quantitatively assessed using the average Nusselt number Nu_{avg} for various mesh sizes (Table 3). The results show that numerical variations become negligible as the grid is refined, with the 121×121 mesh providing an optimal balance between accuracy and computational cost. Thus, this resolution was chosen for all simulations. Furthermore, to ensure the reliability of the numerical code, the present outcomes were validated against the benchmark numerical work of Aminossadati & Ghasemi (2011) [11]. As illustrated in Fig. 3, the compared profiles show an identical behavior with the same orders of magnitude for the velocity fields. This excellent agreement confirms the accuracy of the implemented numerical model. Crucially, validating the code for a standard nanofluid confirms its reliability for the THNF, since the tri-hybrid properties are integrated solely as effective thermophysical inputs into the same conservation equations.

Table 3: Grid independence test based on the average Nusselt number variation of Al_2O_3 -Cu-TiO₂/water nanofluid (Case1, $Ra = 10^3$, $\phi = 0.05$).

Grid Seize	41×41	61×61	81×81	121×121	141×141
Nu_{avg}	5.3015	5.3089	5.3203	5.3260	5.3261
Relative error %	–	0.1396%	0.2147%	0.1071%	0.0019%

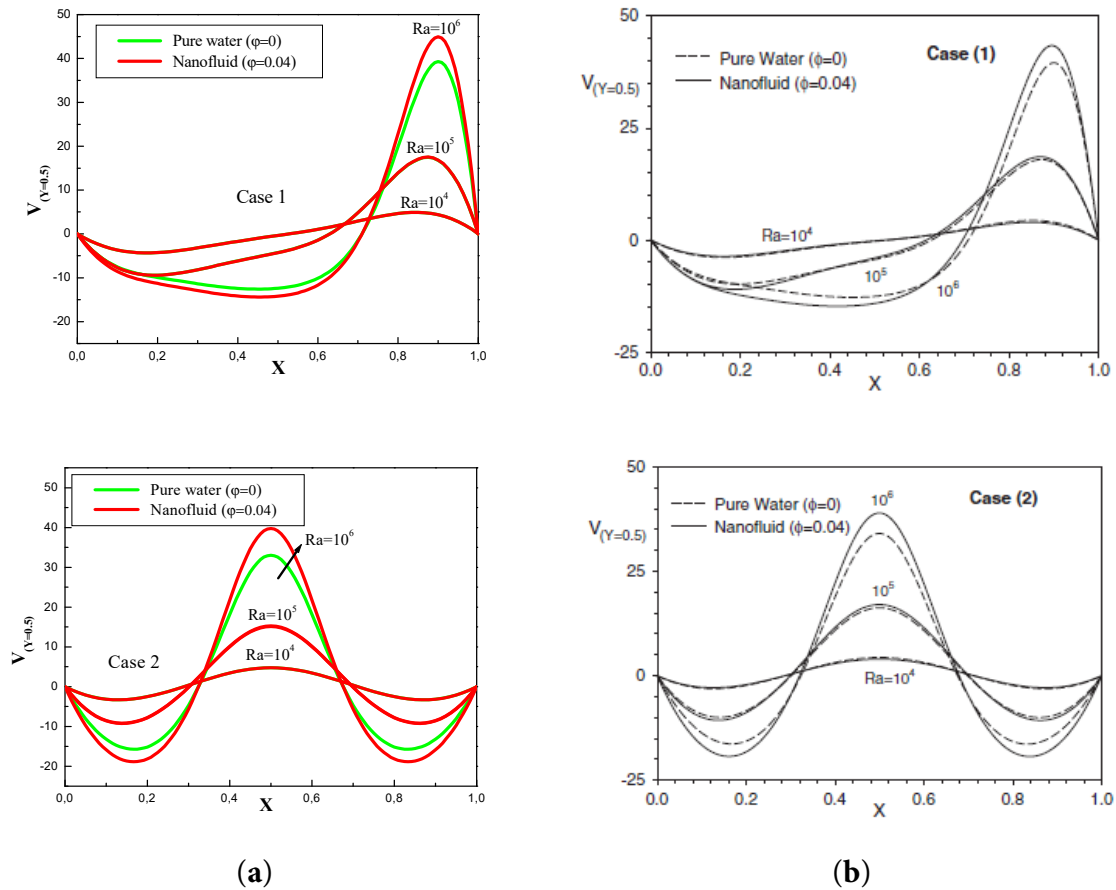


Figure 3: Variation of vertical velocity component at the cavity mid-section at different Ra for water and CuO/water nanofluid with $\phi = 0.04$ (a) present study, (b) Results of Aminossadati and Ghasemi (2011) [11].

4 Results and Discussion

In view to explore the THNF flow and heat transfer patterns, isotherms and streamlines as well as temperature and velocity fields are presented. Moreover, the evolution of the average Nusselt number is analyzed subject to different parameters including the Ra number, the source-sink position and the THNF volume concentration.

4.1 Isotherms and Streamlines Variation

Figs. 4 and 5 illustrate, respectively, the effects of heat source-sink positioning on the isotherms and streamlines of the ternary hybrid nanofluid (THNF) in comparison with pure water, for different Rayleigh numbers. It is clearly observed that both heat transfer characteristics and flow structures exhibit a pronounced sensitivity to key governing parameters, namely the Rayleigh number, the heat source-sink configuration, and the THNF volume fraction. Similar sensitivities have been reported in previous studies on nanofluid-filled enclosures, confirming the strong coupling between buoyancy forces and thermophysical property enhancement [37,43].

As the Rayleigh number increases, the isotherms become increasingly compressed near the active walls, indicating a significant rise in the temperature gradient and, consequently, an enhancement of convective heat transfer. This trend is consistent with the classical transition from conduction-dominated to convection-dominated regimes reported in natural convection studies for nanofluids and hybrid nanofluids [44,51]. The enhanced distortion of isotherms at higher Ra values reflects the intensification of buoyancy-driven circulation within the enclosure.

It is further observed that a quasi-stagnant fluid region develops in the upper part of the cavity due to thermal stratification induced by density gradients. This region is characterized by nearly uniform temperature and expands with increasing Rayleigh number. Similar stagnant layers associated with thermal stratification have been documented in enclosure flows and are attributed to the competition between vertical buoyancy forces and horizontal confinement effects [52]. Despite the presence of this stagnant zone, the overall convection strength increases with Ra, leading to vigorous mixing in the core region and an intensification of the internal thermal transport.

The streamline patterns depicted in Fig. 5 reveal a strong dependence on the heat source-sink arrangement. For asymmetric configurations (Fig. 5a,d), a single dominant eddy occupies most of the enclosure, rotating counterclockwise in case 1 and clockwise in case 4 as a direct consequence of the asymmetric thermal boundary conditions. This behavior agrees well with earlier numerical investigations highlighting the role of thermal asymmetry in controlling flow circulation direction and intensity [11].

In contrast, for symmetric heat source-sink configurations (Fig. 5b,c), the main vortex splits into two counter-rotating and symmetric eddies located in the left and right halves of the cavity. Such flow bifurcation is a well-known feature of symmetrically heated enclosures and has been reported for both conventional fluids and nanofluids [11]. The symmetry of the streamlines with respect to the cavity mid-plane further confirms the dominant role of boundary condition symmetry in shaping the hydrodynamic structure.

Moreover, increasing the Rayleigh number significantly intensifies the streamline density for both pure water and THNF, indicating stronger circulation and higher momentum transport. However, the enhancement is more pronounced for THNF due to its superior effective thermal conductivity and modified viscosity, which collectively strengthen buoyancy effects. This observation is in close agreement with recent studies on hybrid and ternary hybrid nanofluids, where improved thermophysical properties were shown to substantially enhance convective flow strength and heat transfer rates [53]. From a physical standpoint, the progressive compression of isotherms with increasing Rayleigh number reflects the thinning

of the thermal boundary layer and the growing dominance of buoyancy forces over viscous resistance. The resulting modification of vortex intensity and structure is therefore governed by the balance between density-gradient-induced driving forces and hydrodynamic confinement, which ultimately determines the global heat transfer rate and the redistribution of thermal energy inside the cavity.

A more detailed quantitative analysis of the influence of THNF volume fraction on the thermal and hydrodynamic fields is presented and discussed in the subsequent section.

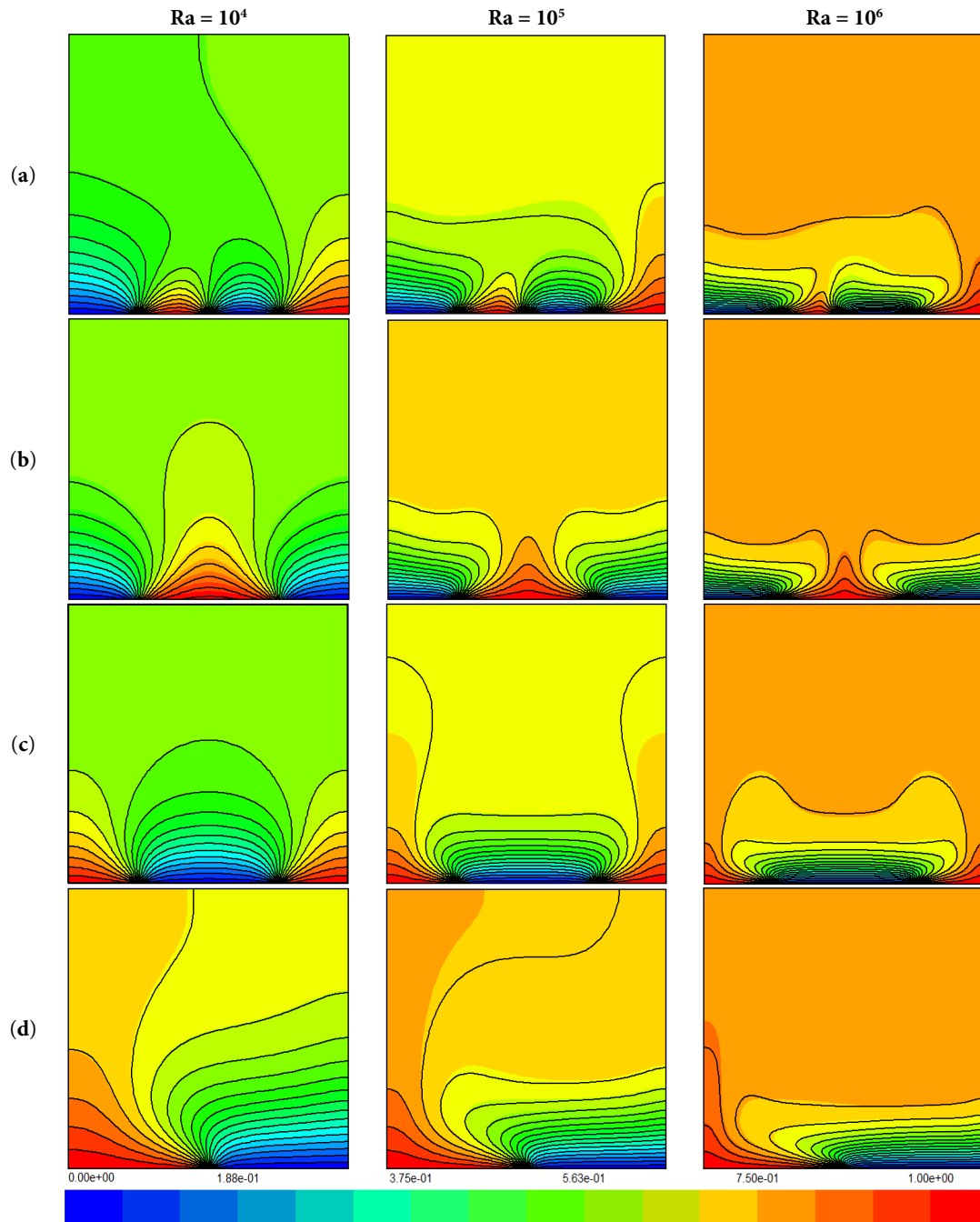


Figure 4: Isotherms for different Ra numbers for water (Black lines) and $\text{Al}_2\text{O}_3\text{-Cu-TiO}_2$ THNF with $\varphi = 0.1$ (Floods), (a) case 1, (b) case 2, (c) case 3 and (d) case 4.

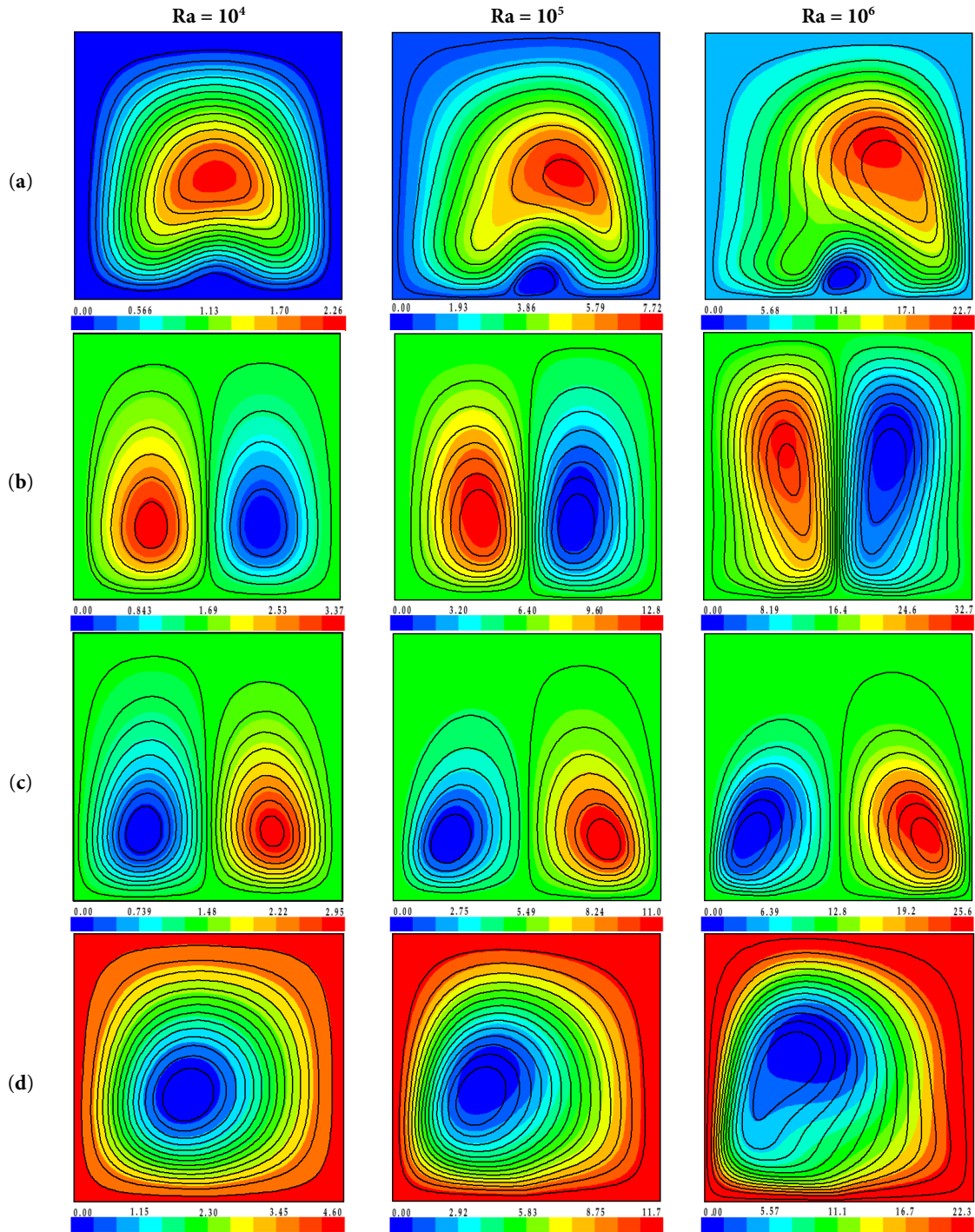


Figure 5: Streamlines for different Ra numbers for water (Black lines) and Al_2O_3 -Cu- TiO_2 THNF with $\phi = 0.1$ (Floods), (a) case 1, (b) case 2, (c) case 3 and (d) case 4.

4.2 Temperature and Velocity Profiles

To gain deeper insight into the thermal and hydrodynamic behavior of the ternary hybrid nanofluid (THNF) inside the enclosure, the variations of the dimensionless temperature and the vertical velocity

component along the horizontal mid-section are examined for both pure water and THNF, under different Rayleigh numbers and volume fractions.

From Fig. 6, it is observed that increasing the $\text{Al}_2\text{O}_3\text{-Cu-TiO}_2/\text{water}$ ternary hybrid nanofluid (THNF) volume fraction leads to a pronounced increase in the dimensionless temperature, with a markedly stronger effect in Case 2 where dual heat source–sink pairs are centrally located along the bottom wall. For identical nanoparticle loadings, the temperature growth rate in Case 2 is nearly twice that of Case 1, highlighting the critical coupling between buoyancy-driven convection structures, nanoparticle-induced thermal transport, and source–sink positioning. This monotonic local temperature enhancement with increasing nanoparticle concentration reflects the altered thermal capacity and enhanced effective thermal conductivity of the mixture, which locally redistributes the internal thermal fields [37,41,53], rather than acting as a direct indicator of global wall heat exchange efficiency. Recent investigations demonstrate that ternary hybrid nanofluids significantly alter these localized temperature distributions through synergistic interactions among multiple nanoparticles, particularly under non-uniform or localized heating configurations [54]. Finally, as expected by the corresponding isotherms (Fig. 4b), the dimensionless temperature profile exhibits symmetry with respect to the vertical median plane.

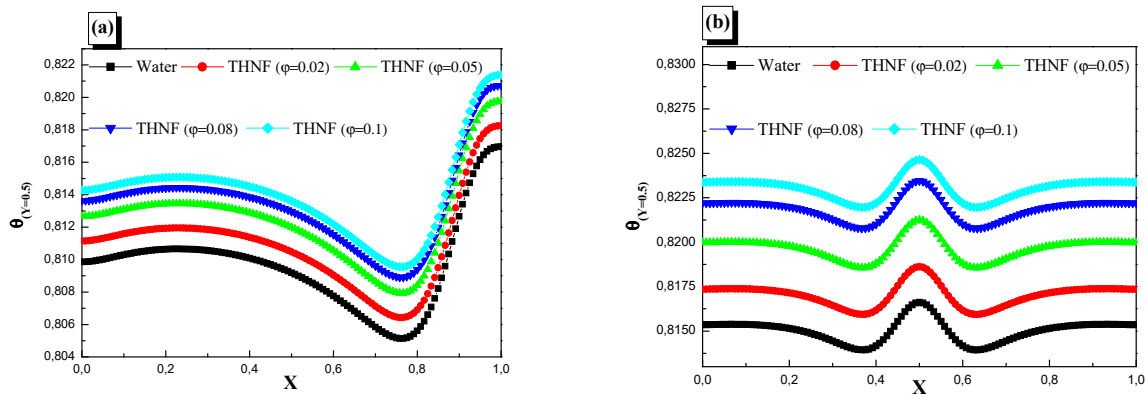


Figure 6: Variation of dimensionless temperature profile of $\text{Al}_2\text{O}_3\text{-Cu-TiO}_2/\text{water}$ THNF at the cavity mid-section for $\text{Ra} = 10^6$, (a) case 1 and (b) case 2.

Fig. 7 shows the effect of the Rayleigh number on the velocity field distribution in configuration C3, where two pairs of heat sinks are positioned at the middle bottom wall of the cavity. For the host liquid, the maximum velocity magnitudes increase from 0.5 to 20.28 as Ra rises from 10^3 to 10^6 , revealing a pronounced strengthening of the vertical flow component. This trend is consistent with benchmark numerical solutions for natural convection in differentially heated cavities, which report an order-of-magnitude increase in velocity when Ra increases from 10^4 to 10^6 [46], as well as with subsequent numerical and experimental investigations showing similar buoyancy-driven flow intensification and counter-rotating vortex formation [37]. The lower fluid density near the heated source pairs on the sidewalls induces upward motion of the hot fluid and its circulation toward colder regions, leading to two counter-rotating eddies in agreement with the isocurrent contours in Fig. 5c, thereby confirming the physical consistency of the present results.

For a better description of the ternary hybrid nanofluid (THNF) effect on the velocity field distribution, Fig. 8 is analyzed for total nanoparticle volume fractions ranging from 0% to 10%. At $\text{Ra} = 10^4$, the introduction of nanoparticles induces complex competing effects between the enhancement of buoyancy forces due to improved thermal conductivity and the increase in viscous resistance. While the local velocity profiles along

the mid-plane display minor variations, the core fluid circulation remains closely coupled with the modified thermophysical properties [55,56]. At higher Rayleigh numbers, the intense thermal energy transfer within the flow promotes altered momentum transport, modifying the local core velocity magnitudes.

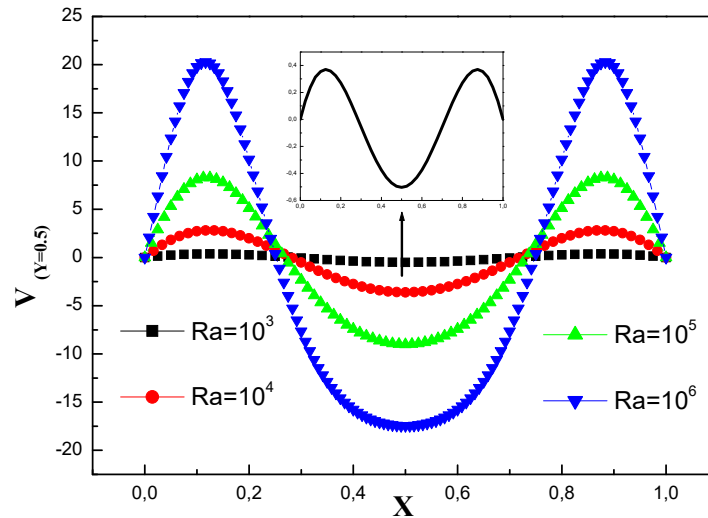


Figure 7: Variation of Y-velocity component of pure water at the cavity mid-section for different Ra numbers (Case 3).

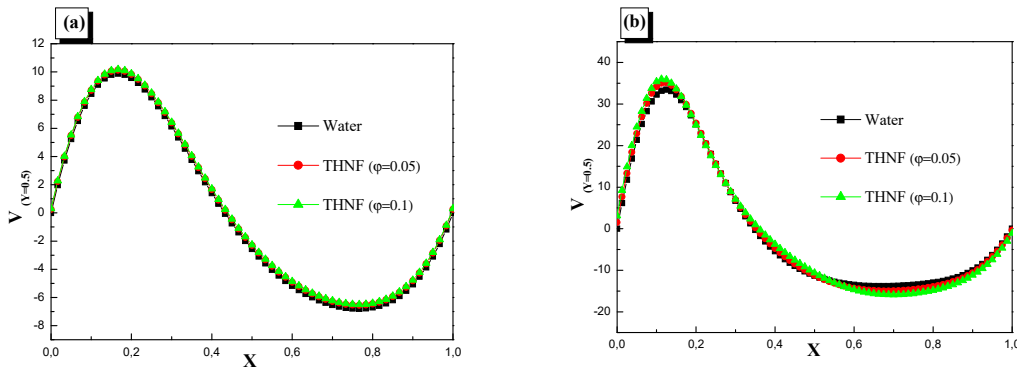


Figure 8: Variation of Y-velocity component against THNF volume fraction (Case 4), (a) $Ra = 10^4$, (b) $Ra = 10^5$.

In summary, the internal profiles shown in Figs. 6–8 indicate that the local flow and thermal fields are highly sensitive to the source–sink position parameter and nanoparticle loading. The modification of the mid-plane temperature and velocity with increasing volume fraction is a clear qualitative manifestation of the internal core energy redistribution and flow restructuring. To accurately evaluate whether these localized internal alterations translate into enhanced global thermal efficiency, a quantitative analysis of the wall-reported heat transfer metrics and Nusselt numbers is strictly required, as detailed in the subsequent section.

4.3 Average Nusselt Number

To quantitatively assess the heat transfer performance of THNF within the different cavity configurations, the average Nusselt number is evaluated as an indicator of the global heat exchange between the heat source–sink pairs. As shown in Fig. 9, the variation of average Nusselt number (at the bottom wall) is

presented as a function of Ra number. It can be observed from this figure that the average Nusselt number intensifies with increasing Ra number as a result of convection-dominated heat transfer regime [11,37]. As it can also be seen, the heat transfer performance within the four configurations is increasingly dependent on the THNF volume concentration. Under the same conditions, the first cavity with $T_C-T_H-T_C-T_H$ heat source-sink arrangement exhibits higher Nusselt numbers compared to the remaining cavities, offering better fluid mixing and thus enhanced convective heat transfer. This ranking is consistent with recent numerical studies on multi-source enclosures showing that alternating hot and cold segments along the wall intensify vortex interactions and disrupt thermal boundary layers, leading to higher Nu values [11,54].

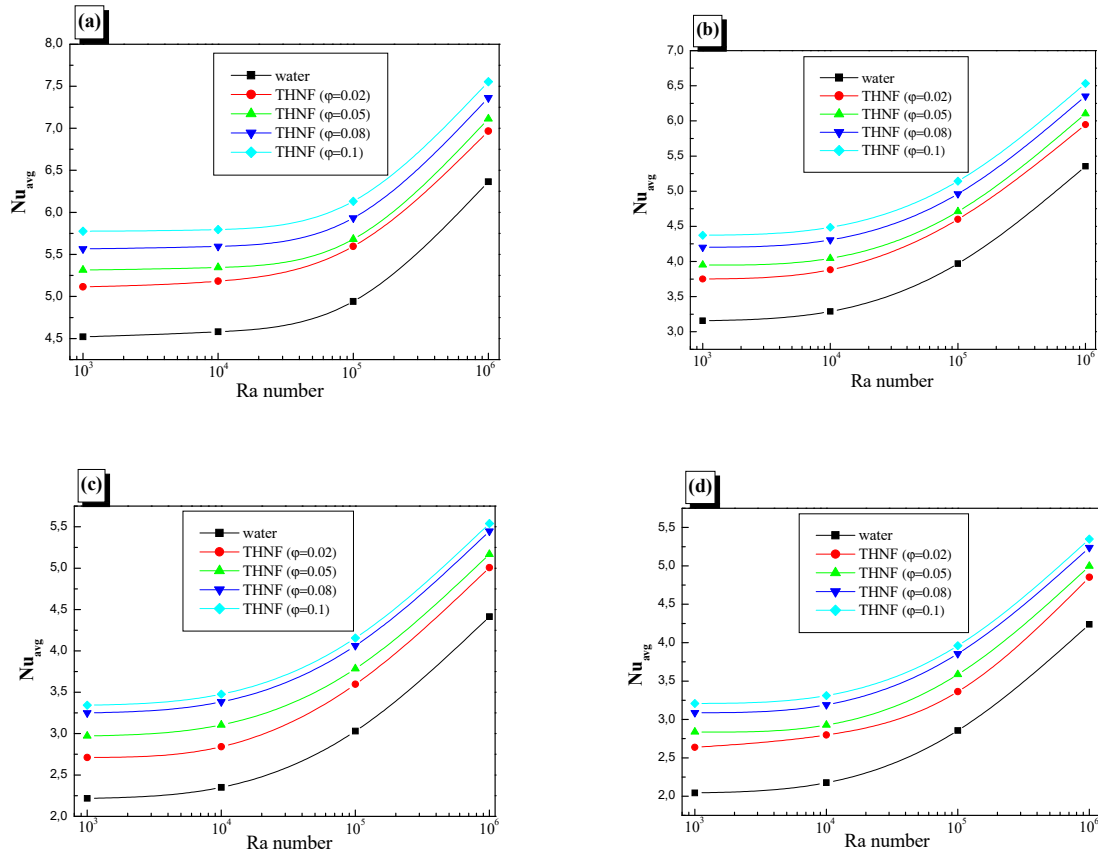


Figure 9: Variation of average Nusselt number at the cavity bottom wall (a) case 1, (b) case 2, (c) case 3 and (d) case 4.

The comparative analysis in Fig. 10 (Case 1, $\varphi = 0.1$) demonstrates that the $Al_2O_3-Cu-TiO_2$ /water ternary nanofluid outperforms single-component (Cu/water, Al_2O_3 /water, TiO_2 /water) and binary (Al_2O_3-Cu /water) nanofluids in terms of heat transfer enhancement. Over the range $Ra = 10^3-10^6$, the ternary nanofluid increases the Nusselt number by $\approx 41-45\%$ compared to Cu/water and by $\approx 18-21\%$ compared to Al_2O_3-Cu /water. These improvements reflect the synergistic interaction of nanoparticles with different thermal conductivities and dispersion characteristics, where Cu provides high thermal conductivity while Al_2O_3 and TiO_2 enhance stability and suspension uniformity. The maximum enhancement occurs at lower Rayleigh numbers, indicating that conductivity-driven mechanisms dominate when convection is weak, consistent with established nanofluid transport theories [39,57,58]. These gains are in line with previous experimental and numerical investigations on hybrid nanofluids, where Al_2O_3-Cu /water systems have reported Nusselt

number enhancements ranging from approximately 13% to 37%, depending on nanoparticle concentration, flow regime, and geometric configuration [59,60].

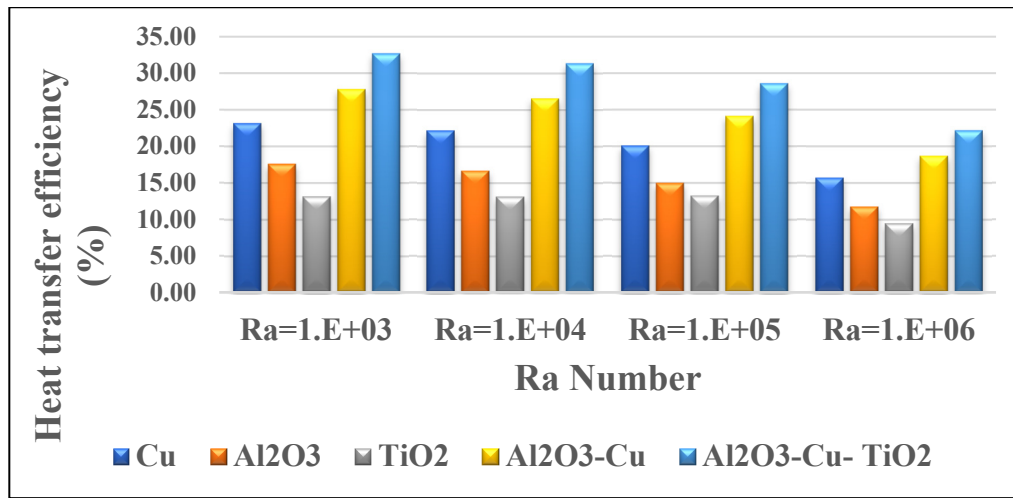


Figure 10: Comparative heat transfer performance of different nanofluids against various Ra number.

Recent investigations on ternary and hybrid nanofluids in natural convection cavities report comparable enhancement trends [26,29], which confirm that multi-component nanoparticle systems intensify conductive and convective transport mechanisms through cooperative thermophysical coupling.

Analysis of the present results confirms that the Al₂O₃-Cu-TiO₂/water ternary hybrid nanofluid significantly enhances heat transfer, particularly at lower Rayleigh numbers, and is highly relevant for practical applications such as electronic enclosures, compact heat exchangers, concentrated photovoltaic-thermal (PVT) modules, and thermoelectric generators [13,27]. The enhancement is primarily attributed to increased effective thermal conductivity and synergistic nanoparticle interactions [44]. At higher Ra, where convection dominates, the relative improvement decreases [61]. Furthermore, the present single-phase model neglects particle migration and slip effects [44]. Despite these limitations, the results provide physically consistent guidance for designing high-performance cavity-based thermal systems, and further experimental validation would reinforce their industrial applicability, confirming the robustness of ternary hybrid nanofluid strategies.

5 Conclusion

This study demonstrates the significant potential of a water-based Al₂O₃-Cu-TiO₂ ternary hybrid nanofluid (THNF) combined with an optimized source-sink arrangement to enhance natural convection in square cavities. Using a finite-volume numerical approach, the effects of Rayleigh number (10^3 – 10^6), nanoparticle volume fraction ($\phi \leq 0.1$), nanofluid type, and source-sink configuration were systematically examined.

The main findings of this investigation are highlighted as follows:

- **Geometric Configuration Effect:** The alternating source-sink arrangement (Case 1) yields the highest thermal performance among the studied geometries, proving that the spatial distribution of thermal sources crucially governs the fluid flow and heat transfer characteristics.
- **Nanofluid Type Performance:** The implementation of THNF provides a remarkable thermal enhancement compared to simpler fluids. Specifically, it increases the average Nusselt number by 41–45%

compared to the standard Cu/water nanofluid, and by 18–21% compared to the Al_2O_3 –Cu/water hybrid nanofluid.

- **Influence of Rayleigh Number:** The heat transfer enhancement triggered by the ternary hybrid matrix is particularly pronounced at lower Rayleigh numbers, where conduction dominates the heat transfer mechanism.
- **Synergistic Effects:** The outcomes highlight a critical synergy between the unique thermophysical properties of the THNF and the source–sink geometry, providing actionable guidance for designing efficient thermal management systems.

Looking ahead, further investigations are warranted to fully understand the underlying physics of these complex fluids. Specifically, optimizing nanoparticle selection, volume fractions, and spatial configurations will be crucial to maximizing the heat transfer benefits of THNFs in practical engineering applications.

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Nomenclature

C_p	Specific heat at constant pressure ($\text{kJ kg}^{-1} \text{K}^{-1}$)
g	Gravitational acceleration (m s^{-2})
H	Height of the enclosure (m)
k	Thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)
NPs	Nanoparticles
Nu	Local Nusselt number
Nu_{avr}	Average Nusselt number
PDEs	Partial differential equations
Pr	Prandtl number
Ra	Rayleigh number
T	Dimensional temperature (K)
u, v	Dimensional x and y components of velocity (m s^{-1})
U, V	Dimensionless x and y components of velocity
x, y	Dimensional coordinates (m)
X, Y	Dimensionless coordinates

Greek symbols

α	Fluid thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)
β	Thermal expansion coefficient (K^{-1})
φ	Nanoparticle volume fraction
ϑ	Kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
θ	Dimensionless temperature
Ψ	Dimensionless stream function
ρ	Density (kg m^{-3})
μ	Dynamic viscosity (N s m^{-2})

Subscripts

C	Cold
f	Fluid
H	Hot
hnf	Hybrid nanofluid
N	Normal direction to the walls
nf	Nanofluid
s	Solid
thnf	Ternary hybrid nanofluid

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