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Optimization of Chemically Reactive Radiative MHD Casson Hybrid Nanofluid Flow over a Time-Dependent Stretching Surface Using Response Surface Methodology and ANOVA

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ABSTRACT: This study examines transient heat and mass transfer characteristics in a Casson-based hybrid nanofluid (Au–Cu/water) flowing over a time-dependent stretching elastic surface in the presence of porous media and viscous dissipation. The mathematical model further incorporates the effects of magnetic fields, thermal radiation, chemical reactions, and velocity slip conditions to capture realistic transport phenomena encountered in advanced thermal systems. Through suitable similarity transformations, the governing partial differential equations are reduced to a system of nonlinear ordinary differential equations, which are solved numerically using the MATLAB bvp4c solver. To identify optimal operating conditions, Response Surface Methodology (RSM) is employed to develop predictive correlations for skin-friction coefficient, heat transfer rate, and mass transfer rate. The statistical significance and adequacy of the developed models are assessed using Analysis of Variance (ANOVA). Validation against previously published results demonstrates excellent agreement, confirming the accuracy and robustness of the proposed formulation. The results reveal that variations in the magnetic field substantially modify the Lorentz force distribution, leading to pronounced changes in the velocity, temperature, and concentration profiles. Furthermore, magnetic field strength, slip effects, and hybrid nanofluid properties significantly influence surface shear stress, thermal transport, and concentration boundary-layer development. The RSM optimization framework successfully identifies parameter combinations that maximize heat and mass transfer performance while controlling frictional resistance.

KEYWORDS: RSM; Casson hybrid nanofluid; porous medium; Eckert number; magnetic field; chemical reaction; radiation; slip effects; bvp4c solver

1 Introduction

There are many different types of engineering devices, including electrodes for heat transfer and fuel cells, that require fluids in order to increase the amount of heat that is transmitted through them. In order to provide a more precise explanation of the movement of a particular fluid, models that are utilized in the study of fluid flow are constructed. According to the research that has been done, these structures are categorized as either Newtonian or non-Newtonian fluids studied by Lu et al. [1]. There are a few fundamental laws, such as Fourier's and Fick's, that play an important part in fluid movement issues. Rauf et al. [2] have focused a large amount of emphasis on the movement and transport processes

of non-Newtonian fluids. This is due to the fact that non-Newtonian fluids have an extensive variety of applications in the disciplines of industry, biology, technology, and chemistry. Among these are flow tracers, increased capacity, a component of friction in oil pipelines, and several cooling and heating systems examined by Khan et al. [3]. In the fields of mechanical engineering, business, and manufacturing, non-Newtonian materials have become more common than Newtonian ones in the last several decades. This is because, according to research by Abbas et al. [4], non-Newtonian fluids predict elastic characteristics more accurately than Newtonian fluids because they exhibit significant viscoelastic behavior. It is also well known that the majority of physiological fluids behave differently from Newtonian fluids.

The extraordinary thermal properties and potential medicinal uses of nanofluids have sparked a never-ending stream of research into these materials. Nanofluids may be used in place of more traditional fluids due to research showing that they are more effective heat transfer agents. Many researchers have shifted their attention to researching small fluids because of their superior thermal performance. Nanoparticles distinguish themselves from other fluid types and give them a competitive advantage due to their better thermal conductivity. Medical, transportation, electronics, food, and energy production are just a few of the many industries that rely on them because of this property. Choi & Eastman [5] further claim that the original liquids' conductivity properties improve with the introduction of these nanoparticles, which may be anywhere from one nanometre to one hundred nanometres in size. The results of the studies stated earlier support this. Improving particle properties, including size, medicinal content, and overall structure, can enhance their usefulness in numerous biomedical research sectors. Said et al. [6] research into the preparation of nanofluids has begun at both the industrial and academic levels, following the new door that the concept of nanofluids opened. Surface roughness for condensation flow inside microchannels was investigated by Rashidi et al. [7]. Nanofluids, as discussed by Jazaa et al. [8], are of paramount importance because of the vast range of industries that rely on them and the tremendous heat transmission capabilities they possess.

Hybrid nanofluids have recently supplanted conventional nanofluids due to their substantially higher capabilities for heat transfer. An analysis of nanofluid/hybrid nanofluid for improved heat transmission was conducted by Guo et al. [9]. Narankhishig et al. [10] reviewed many nanofluids (NFs) features related to convective heat transfer, with an emphasis on the function of magnetism in improving heat transfer. There are several possible benefits and uses for hybrid nanofluids, which have piqued the curiosity of researchers. Heat transfer efficiency and thermophysical properties are best achieved by hybrid nanofluids (HNFs) that combine several nanoparticles, as opposed to single-phase NFs [11,12]. Naganthran et al. [13] have taken a keen interest in hybrid nanofluids as a result of the wide variety of processes they use in a broad spectrum of scientific, industrial, and biological fields. An intriguing new option is hybrid nanofluids, which, because of their unusual makeup, have better thermal characteristics than competing materials. These materials have the potential to recover the capacity for heat transmission and thermal conductivity through the combined action of two sorts of nanoparticles, non-metallic and non-ferromagnetic, scattered in a standard fluid. Scientists and investigators from all around the world have taken to using the hybrid nanofluid model in their work. Research on the intricate rheological characteristics of non-Newtonian hybrid nanofluids may be conducted using a variety of distinct methods. The following kinds of fluids are known: Maxwell (Haneef et al. [14]), Casson (Gohar et al. [15]), Williamson (Rashad et al. [16]), Viscoelastic (Ummeda & Ontela [17]), Micropolar (Gumber et al. [18]), Jeffery (Asjad et al. [19]), Sutterby (Khan et al. [20]), and so on. Small gold particles were examined by Ellahi et al. [21] in their inquiry into the peristaltic blood-flowing behavior of catheters with pliable walls and two stress fluids.

The development of efficient, temperature-sensitive medication delivery devices depends on our ability to comprehend the behavior of nanofluids in porous media. Akbar et al. [22] studied the wave-like motion

created by the cyclic motion of the ducts' ability to contract and expand when their walls are flexible to move fluid. In engineering and physiology, this system plays a significant role in assisting digestion, slurry movement, and blood circulation. Important products like tube pumps and medical equipment use peristaltic motion to manage fluid transfer precisely. The intricacies of peristaltic flow have been the subject of mathematical investigations, which consider various fluid kinds and duct designs. Using an exponentially stretched sheet, by adding copper (Cu) and gold (Au) nanoparticles, Madiha Takreem et al. [23] investigate the flow behavior of a blood-based Casson HNF. Vishalakshi et al. [24] and Chahregh and Dinarvand [25] have shown that porous structures have the potential to greatly impact thermal performance and flow resistance. Rauf et al. [26] investigated the mass and heat transfer properties of a boundary layer of a two-dimensional mixed convective Maxwell nanofluid across a porous linearly stretched surface in the presence of an external magnetic field. The Dufour and Soret phenomena, together with the effects of thermal radiation, were also involved in the research. Heat systems that make use of hybrid nanofluids to work more efficiently were one of the 18 engineering domains studied by Hanif et al. [27]. Habib et al. [28] analyzed the impacts of thermal radiation on porous surfaces on the temperature transfer and flow properties of hybrid nanofluids composed of Graphene nanotubes with one or more walls. For the purpose of analyzing fluid flow issues, several methods, geometries, and assumptions have been put forward over the past many decades. Magnetohydrodynamic (MHD) fluid movement with a boundary layer conducting is one of the components. Khan [29] examined the impacts of a magnetic field on the movement of molybdenum disulfide nanofluids used in heat transfer enhancement treatments. The mixed convection motion in a square enclosure using a magnetic field was also investigated by Chamkha & Ismael [30]. Mabood et al. [31] detailed the many physical characteristics of a nanofluid that combines water and other substances, and studied the behavior of these qualities change when exposed to heat. A lot of investigation has been focused on the impacts of thermal radiation on the dynamics of the transmission of heat inside the hybrid nanofluid flows. In their study, Shatnawi et al. [32] investigated the behavior of magnetic and nonlinear radiation impacting the movement of Casson fluid in a hybrid nanofluid. The entropy production of nanofluid flows across heated sheets influenced by viscous dissipation and thermal radiation using the RSM technique was discussed in detail in [33,34].

Environmental management, biochemistry, food processing, atmospheric dynamics, and freezing damage reduction in crops are just a handful of the several fields that rely heavily on chemical interactions. When species exist in distinct phases of a system or at interfaces, it becomes very difficult to account for their molecular diffusion. Investigating the consequences of heat production, thermal radiation, and chemical processes, Agbaje and Leach [35] examined a viscoelastic Jeffrey NF's natural convectional descent through the porous surface. Fatunmbi and Salawu [36] discussed chemical reaction-induced micropolar nanofluid flow, mass flux, and multiple slip hybrid nanofluid flow. Ramzan et al. [37] investigated the thermophysical characteristics of the Casson fluid movement on an inclined surface. Several impacts, such as Dufour, Soret, and chemical reactions, were investigated in this study.

Viscous dissipation, sometimes called frictional heating, is the energy transfer from motion to heat in a fluid when it is subjected to forces that are both viscous and mechanically strong. The boundary sheet flow, which is formed when a surface is either shrunk or stretched, has many practical uses throughout the manufacturing and industrial phases, such as continuous glass casting, wire drawing, and metal or polymer extrusions [38]. Alzahrani et al. [39] investigated the viscosity effect of Casson fluid flow over a rotating tube. Alali and Megahed [40] formulated the notion of MHD Casson nanofluid concerning viscous dissipation in a liquid film over a stretching sheet, incorporating heat radiation and slip effects. Devi and Devi [41] developed the concept of augmenting heat transfer in hybrid nanofluid flow across an elongating sheet.

Gaps between This Study and Existing Literature

Current research indicates that there has been no examination of the two-dimensional time-dependent dynamics of a novel non-Newtonian system, which integrates Casson-type fluid flow of a hybrid nanofluid comprising gold and copper with water as the base fluid, through the stretched permeability of a porous medium. Consequently, the proposed research constitutes a vital strategy to bridge this gap and encourages academics to create an innovative methodology for examining the flow, temperature, and concentration distribution behavior over a stretched sheet. This research effectively examines the physical factors and their effects on the flow, thermal, and concentration properties of a hybrid nanoparticle-water fluid system over a stretched sheet within a permeable porous medium, utilizing numerical techniques, graphical representations, and tables. This comprehensive framework captures the interplay between nanoscale particle dynamics, slip effects, and stochastic electromagnetic forces, advancing the predictive modeling of nanofluid boundary layers with direct applications in thermal management, coating, materials processing, and biomedical engineering.

2 Formulation of the Physical System

This work aims to analyze the time-dependent dynamics of a novel hybrid non-Newtonian nanofluid system, which integrates a Casson base fluid, characterized by a Γ factor, with dual nanoparticle suspensions of copper (Cu) and gold (Au). An external field with a strength of $B = \frac{B_0}{\sqrt{1-ct}}$, radiative heat transfer with a coefficient of q_r and homogeneous chemical reactions with a factor of γ_1 . Govern the coupled physical mechanisms that drive the movement, which is represented by a temporally modulated extending surface with a momentum of $u_w = \frac{bx}{1-ct}$ where b and c are constants. What sets this work apart from traditional no-slip formulations is the incorporation of the interfacial slip velocity phenomena into the analysis, which occurs when the discontinuity in fluid-wall velocity profoundly influences boundary sheet formation. In addition, the solutal C_w and thermal T_w at the elastic sheet's surface are thought to change with both the position along the sheet (x) and the time (t), according to certain mathematical formulas that can be described below:

$$T_w - T_\infty = T_0 \left[\frac{u_w x}{v_f (1-ct)^{\frac{1}{2}}} \right]$$

$$C_w - C_\infty = C_0 \left[\frac{u_w x}{v_f (1-ct)^{\frac{1}{2}}} \right]$$

For the base fluid, the kinematic viscosity is denoted as v_f . T_0 is the slit's constant temperature, and C_0 is the equivalent constant concentration. Fig. 1 provides a comprehensive overview of the physical model that is the subject of this inquiry.

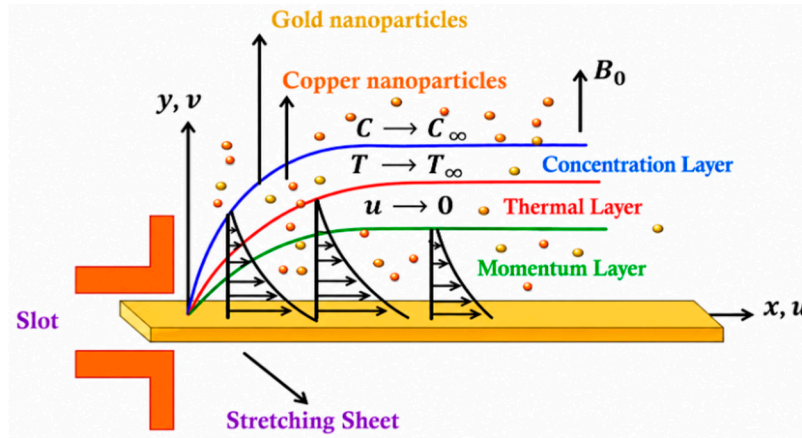


Figure 1: A schematic representation of the physical model.

2.1 Dimensional Analysis Framework

The four coupled conservation principles that mathematically describe the system's dynamics are mass balance (continuity), momentum transport with non-Newtonian effects, energy transfer with radiative heat flux and thermal slip boundary conditions, and species transport with reaction kinetics. Within the specified physical limits, this formulation accurately depicts the interaction of hydrodynamics, heat, and chemical processes (Devi and Devi [41]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} = v_{hnf} \left(1 + \frac{1}{\Gamma} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{hnf} B_0^2(t)}{\rho_{hnf}} u - \frac{\mu_{hnf}}{\rho_{hnf} k_0} u, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \frac{\partial T}{\partial t} = \frac{1}{(\rho C_p)_{hnf}} \left[k_{hnf} \left(\frac{\partial^2 T}{\partial y^2} \right) - \frac{\partial q_r}{\partial y} \right] + \frac{\sigma_{hnf}}{(\rho C_p)_{hnf}} B_0^2 u^2 + \frac{\mu_{hnf}}{(\rho C_p)_{hnf}} \left(\frac{\partial u}{\partial y} \right)^2, \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + \frac{\partial C}{\partial t} - \gamma_1 (C_\infty - C) = \frac{\partial^2 C}{\partial y^2} \cdot D_B \quad (4)$$

The dimensional temperature (T), diffusion coefficient (D_B), electrical conductivity (σ), time (t), and Casson parameter (Γ) are all variables in this equation. Additionally, the Rosseland diffusion approximation is utilized to quantify the impacts of thermal radiation in the energy conservation analysis (3), which exposes a radiative transfer mechanism q_r , that is temperature dependent. In this method, the radiative flux is represented as

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} = -\frac{16\sigma^* T^3}{3k^*} \left(\frac{\partial T}{\partial y} \right), \quad (5)$$

here k^* signifies the factor of mean absorption, and σ^* denotes the Stefan-Boltzmann constant.

2.2 Physical System Boundary Definitions

Crucial interfacial phenomena driving fluid-wall interactions are captured by formally defining the domain's boundary constraints. Three crucial physical phenomena are included in these mathematically determined conditions: momentum slip, thermal slip, and solutal slip at the boundary surface. Their strict application guarantees that the solutions to the governing equations are physically compatible with the behavior of systems in the real world. A mathematical summary of these phenomena is:

$$u - u_w = L_1 \left(\frac{u_{hnf}}{v_f p_{hnf}} \right) \left(1 + \frac{1}{\Gamma} \right) \frac{\partial u}{\partial y}, \quad T - T_w = L_2 \left(\frac{\partial T}{\partial y} \right), \quad \text{at } y = 0, \quad (6)$$

$$C - C_w = L_3 \left(\frac{\partial C}{\partial y} \right), \quad v + v_0 = 0 \quad \text{at } y = 0. \quad (7)$$

$$C - C_\infty \rightarrow 0, \quad u \rightarrow 0, \quad T - T_\infty \quad \text{as } y \rightarrow \infty. \quad (8)$$

The suction velocity, denoted as v_0 is related to the velocity slip coefficients, L_1 , L_2 and L_3 , which are velocity, thermal, and solutal, respectively.

2.3 Canonical Dimensionless Quantities

We simplify the governing partial differential equations by applying a similarity transformation that captures the inherent scaling behavior of the boundary layer flow, thereby reducing these equations to a more manageable set of ordinary differential equations. Both transformations work together to reduce system complexity. There are three major benefits to this approach: better numerical stability due to the elimination of unit dependency, more physical understanding through characteristic parameter ratios, and easier pattern recognition in solution behavior. The full nondimensionalization process is described in the section that follows, which involves methodically obtaining each transformed equation from its dimensional equivalent. This method relies on the following structure (Devi and Devi [41]).

$$\eta = y \sqrt{\frac{b}{v_f(1 - ct)}}, \quad \Psi = x \sqrt{\frac{b v_f}{1 - ct}} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \quad (9)$$

2.4 Similarity-Transformed Equation Set

Both the characteristic scaling method and the system complexity reduction and the universalization of conclusions for broader physical interpretation are essential analytical aims of the transformation to dimensionless form. The key dynamics of the governing equations are revealed by introducing normalized variables, which in turn disclose the dominant dimensionless groups that drive the mechanisms of flow, thermal and mass transport. The transformed system, which includes the boundary restrictions and the ordinary differential equations, is a generalized form that may be applied to other scales and different types of operations. The improved mathematical foundation that underpins our following analysis is presented below:

$$\left(1 + \frac{1}{\Gamma} \right) f''' - \frac{\Delta_2}{\Delta_1} \left\{ \gamma \left(\frac{\eta}{2} f'' + f' \right) + (f')^2 - f f'' \right\} - \frac{\Delta_5}{\Delta_1} M f' - \frac{\Delta_2}{\Delta_1} K_p f' = 0, \quad (10)$$

$$\frac{1}{Pr} \left(\frac{R}{\Delta_4} + 1 \right) \theta'' + \left(\frac{\Delta_3}{\Delta_4} \right) (f\theta' - 2f'\theta) - \frac{1}{2} \left(\gamma \frac{\Delta_3}{\Delta_4} \right) (\eta\theta' + 3\theta) + \left(\frac{\Delta_5}{\Delta_4} \right) M Ec f'^2 + \left(\frac{\Delta_1}{\Delta_4} \right) Ec f''^2 = 0, \quad (11)$$

$$\frac{1}{Sc} \phi'' + (f\phi' - 2f'\phi) - \left(\frac{\gamma}{2} \right) (\eta\phi' + 3\phi) - C_r \phi = 0. \quad (12)$$

These formulas comprise the boundary edge conditions:

$$f = \beta, \quad f' = 1 + \lambda_1 \left(1 + \frac{1}{\Gamma} \right) \left(\frac{\Delta_1}{\Delta_2} \right) f'' \quad \text{at} \quad \eta = 0 \quad (13)$$

$$\theta(0) = 1 + \lambda_2 \theta', \quad H(0) = 1 + \lambda_3 H' \quad \text{at} \quad \eta = 0 \quad (14)$$

$$f' \rightarrow 0, \quad \theta \rightarrow 0, \quad H \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \quad (15)$$

The eight main control variables that govern the coupled transport phenomena are shown by the parametric analysis. These include the following: the magnetic field factor M , which modifies Lorentz forces; permeability of porous media K_p , the unsteadiness parameter γ , which dictates flow instability; the velocity, temperature and solutal slip constraints ($\lambda_1, \lambda_2, \lambda_3$) which regulate the way the fluid and wall interact, the chemical reaction factor C_r which controls species conversion; the radiation constraint R ; because of the interplay between these factors and their effects on boundary layer formation, the hydrodynamic behaviour, heat transport efficiency, and mass diffusion properties of the system are all determined by them; the viscous dissipation factor Ec ; which control the speed of the momentum of the fluid flow. In addition, these elements can be described as:

$$\gamma = \frac{c}{b}, \quad M = \frac{\sigma_f B_0^2}{\rho_f b}, \quad K_p = \frac{v_f}{b k_0}, \quad Ec = \frac{u_w^2}{(C_p)_f (T_w - T_\infty)}, \quad Sc = \frac{v_f}{D_B}, \quad C_r = \frac{\gamma_1}{b}, \quad Pr = \frac{v_f}{\alpha_f}, \quad R = \frac{16\sigma^* T_\infty^3}{3k^* k_f},$$

$$\lambda_1 = L_1 \left(\frac{b}{2v_f} \right), \quad \lambda_2 = L_2 \left(\frac{b}{2v_f} \right), \quad \lambda_3 = L_3 \left(\frac{b}{2v_f} \right),$$

$$\Delta_1 = (1 - \phi_1)^{-2.5} (1 - \phi_2)^{-2.5}, \quad \Delta_2 = \left[\left(\frac{\rho_{s1}}{\rho_f} \right) \phi_1 + \left(\frac{\rho_{s2}}{\rho_f} \right) \phi_2 + (1 - \phi_1) \right] (1 - \phi_2),$$

$$\Delta_3 = \left[(1 - \phi_1) + \phi_1 \left(\frac{(\rho c_p)_{s1}}{(\rho c_p)_f} \right) + \phi_2 \left(\frac{(\rho c_p)_{s1}}{(\rho c_p)_f} \right) \right] (1 - \phi_2), \quad \Delta_4 = \frac{k_{hnf}}{k_f}, \quad \Delta_5 = \frac{\sigma_{hnf}}{\sigma_f}.$$

2.5 Quantities of Interest

The momentum contour, thermal contour, and concentration contours for various values of the controlling parameters are the main quantities of interest in this investigation. In addition, accurate characterization of surface behavior and boundary layer structure is vital for engineering and industrial applications involving Casson-type hybrid nanofluid movement and thermal transport, such as advanced cooling technologies, materials processing, and magnetically guided drug delivery. To this end, it is important to quantify the heat, mass and momentum transfer at the linearly stretching surface. The

nondimensional parameters of practical relevance include the coefficient of skin friction Cf_x the Nusselt number Nu_x and the Sherwood number Sh_x which stand for surface shear stress, mass transfer rate, heat transfer rate, and mass transfer rate respectively. These numbers shed light on how slip effects, nanofluid characteristics, and magnetic field variations affect transport processes close to the wall.

2.5.1 Skin Friction Coefficient

The coefficient of skin friction Cf_x quantifies the wall shear stress and is defined as

$$Cf_x = \frac{\tau_w}{\rho u_w^2} = \frac{\mu}{\rho u_w^2} \left. \frac{\partial u}{\partial y} \right|_{y=0}$$

where τ_w is the wall shear stress, μ is the dynamic viscosity, and u_w is the velocity of the stretching surface. By using similarity variable, this becomes:

$$Cf_x \sqrt{Re_x} = -f''(0) \left(1 + \frac{1}{\Gamma} \right) \Delta_1$$

2.5.2 Local Nusselt Number

The local Nusselt number Nu_x characterizes the rate of heat transmission at the wall

$$Nu_x = \frac{x q_w}{\kappa (T_w - T_\infty)} = -\frac{x}{T_w - T_\infty} \left. \frac{\partial T}{\partial y} \right|_{y=0},$$

where q_w is the heat flux at the surface and κ is the thermal conductivity of the hybrid nanofluid. Using the similarity function for the temperature $\theta(\eta)$, we write

$$\frac{Nu_x}{\sqrt{Re_x}} = -\Delta_4 (1 + R) \theta'(0).$$

2.5.3 Local Sherwood Number

Similarly, the local Sherwood number Sh_x measures the mass transfer rate of hybrid nanofluid at the surface and is expressed as

$$Sh_x = \frac{x q_w}{D_B (C_w - C_\infty)} = -\frac{x}{C_w - C_\infty} \left. \frac{\partial C}{\partial y} \right|_{y=0},$$

where q_w is the nanoparticle mass flux and D_B is the mass diffusivity coefficient. Using the similarity function for the concentration $H(\eta)$, we write

$$\frac{Sh_x}{\sqrt{Re_x}} = -H'(0),$$

here $Re_x = \frac{u_w x}{\nu}$ is the local Reynolds number.

3 Computational Methodology

The solution of the current analysis is obtained by implementing the bvp4c code with respect to related boundary constraints. The Lobato scheme is used on the base of the finite difference method.

The MATLAB software built-in code is used here.

$$Y_1 = f, Y_2 = f', Y_3 = f'', Y_3' = f'''$$

$$Y_4 = \theta, Y_5 = \theta', Y_5' = \theta'', Y_6 = \phi, Y_7 = \phi', Y_7' = \phi''$$

The corresponding equations take the form of

$$Y_3' = \frac{\Gamma}{1+\Gamma} \left[\frac{\Delta_2}{\Delta_1} \left\{ \gamma \left(\frac{\eta}{2} Y_3 + Y_2 \right) + Y_2^2 - Y_1 Y_3 \right\} + \frac{\Delta_5}{\Delta_1} M Y_2 + \frac{\Delta_2}{\Delta_1} K_p Y_2 \right]$$

$$Y_5' = \frac{\text{Pr} * \Delta_4}{R + \Delta_4} \left[\frac{1}{2} \left(\gamma \frac{\Delta_3}{\Delta_4} \right) \{ \eta Y_5 + 3 Y_4 \} - \frac{\Delta_5}{\Delta_1} (Y_1 Y_5 - 2 Y_2 Y_4) - \frac{\Delta_5}{\Delta_4} M * Ec * Y_2^2 - \frac{\Delta_1}{\Delta_4} Ec Y_3^2 \right]$$

$$Y_7' = Sc \left[\frac{\gamma}{2} \{ \eta Y_7 + 3 Y_6 \} + Cr Y_6 - Y_1 Y_7 + 2 Y_2 Y_6 \right]$$

with the boundary conditions:

$$Y_1(a) = \beta, Y_2(a) = 1 + \lambda_1 \left(1 + \frac{1}{\Gamma} \right) \frac{\Delta_1}{\Delta_2} Y_3(a), Y_4(a) = 1 + \lambda_2 Y_5(a); Y_6(a) = 1 + \lambda_3 Y_7(a);$$

$$Y_2(b) = 0, Y_4(b) = 0, Y_6(b) = 0.$$

In the above equations, the subscripts a and b represent the equivalent constraints at $\eta = 0$, and $\eta \rightarrow \infty$, respectively. The flowchart of bvp4c, RSM and ANOVA is given as in Fig. 2. The thermophysical properties of the base fluid and the Au and Cu nanoparticles are given in Tables 1 and 2.

Table 1: Thermophysical features of nanoparticles and base fluid [Shaiq [42]].

Properties	Cu	Au	Water
ρ (kg/m ³)	8933	19,300	997.1
C_p (J/kg·K)	385	129	4179
k (W/m·K)	386	318	0.613
σ	5.96×10^7	4.1×10^7	0.05
Pr	-	-	6.2

Table 2: Physical thermo properties of HNF (Cu-Au/Water) (Shaiq [42]).

Properties	HNF (Cu-Au/Water)
Density (ρ)	$\rho_{hnf} = ((1 - \phi_2) \{ (1 - \phi_1) \rho_f + \phi_1 \rho_{s1} \} + \phi_2 \rho_{s2})$.
Viscosity (μ)	$\mu_{hnf} = \frac{\mu_f}{(1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5}}$.
Heat Capacity (ρC_p)	$(\rho C_p)_{hnf} = (1 - \phi_2) \{ (1 - \phi_1) (\rho C_p)_f + \phi_1 (\rho C_p)_{s1} \} + \phi_2 (\rho C_p)_{s2}$.
Thermal conductivity (k)	$\frac{k_{hnf}}{k_{bf}} = \frac{k_{s2} + 2k_{bf} - 2\phi_2(k_{bf} - k_{s2})}{k_{s2} + 2k_{bf} - \phi_2(k_{bf} - k_{s2})}$, where $\frac{k_{nf}}{k_f} = \frac{k_{s1} + 2k_f - 2\phi_1(k_f - k_{s1})}{k_{s1} + 2k_f - \phi_1(k_f - k_{s1})}$.
Electrical Conductivity (σ)	$\frac{\sigma_{hnf}}{\sigma_{nf}} = \frac{\sigma_{s2} + 2\sigma_{nf} - 2\phi_2(\sigma_{nf} - \sigma_{s2})}{\sigma_{s2} + 2\sigma_{nf} - \phi_2(\sigma_{nf} - \sigma_{s2})}$, where $\frac{\sigma_{nf}}{\sigma_f} = \frac{\sigma_{s1} + 2\sigma_f - 2\phi_1(\sigma_f - \sigma_{s1})}{\sigma_{s1} + 2\sigma_f - \phi_1(\sigma_f - \sigma_{s1})}$.

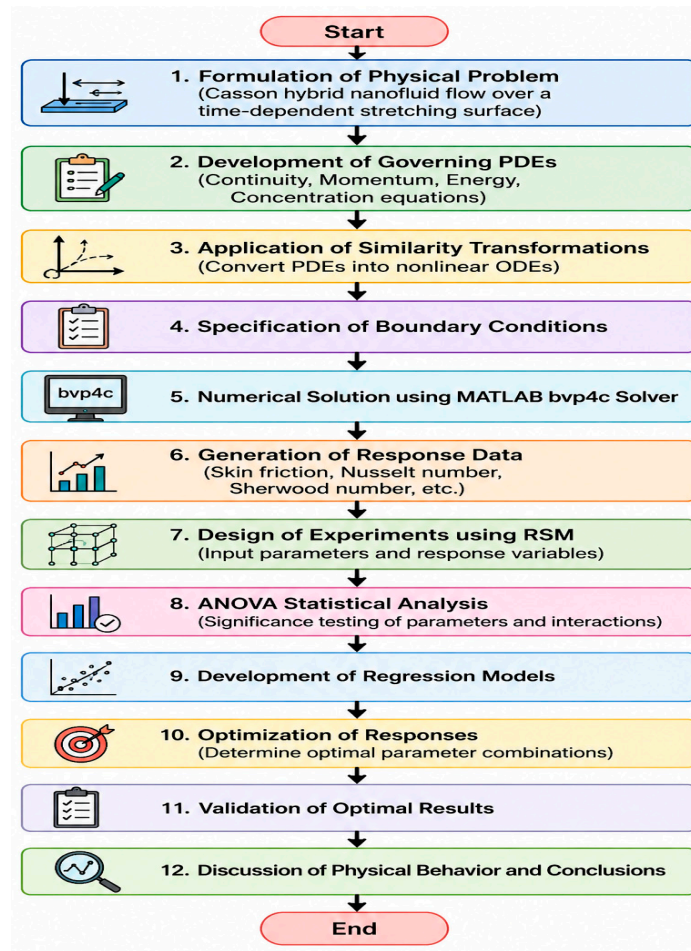


Figure 2: Flow chart of solution and optimization technique procedure.

Validation of the Results

Numerical comparison with current numerical results is done to confirm the accuracy of the generated simulator. Devi & Devi [41] reported in the literature for relevant limiting cases. We implement those problems in our simulator and calculate heat transfer. We present the generated data by our simulator and the existing data available in the literature in Table 3. Numerical outcomes for heat transfer rate in terms of $-\theta'(0)$ show an extraordinary deal between the present results and previously issued data, confirming the desired accuracy of the developed simulator and numerical techniques. Thus, the validation cases collectively confirm the robustness of the proposed model incorporating the Casson non-Newtonian fluid via the permeability of porous medium along with complex phenomena of thermal radiation, Eckert number, and slip border conditions in hybrid nanofluid flow over a linearly elongating surface.

Table 3: Values of $-\vartheta(0)$ for distinct values of Pr when $\Gamma \rightarrow \infty$, $\gamma = \beta = \lambda_1 = \lambda_2 = \lambda_3 = Rd = \phi_1 = \phi_2 = 0$, and at constant wall temperature.

Pr	Present Study	Devi & Devi [41]
20.0	3.352000	3.35390
2.00	0.910035	0.91135
6.13	1.749682	1.75968
7.00	1.895399	1.89540

4 Results and Discussions

The numerical results obtained by solving the transformed system of nonlinear ordinary differential equations regulating the nanofluid flow over a linearly stretching sheet are presented and analyzed in this section using graphs. The influence of the physical constraints, such as Casson parameter (Γ), magnetic field parameter (M), thermal radiation (Rd), Viscous dissipation (Ec), chemical reaction parameter (Cr), unsteadiness constraints (γ), suction constraints (β), thermal slip (λ_2), momentum slip (λ_1), solutal slip (λ_3), Schmidt number (Sc), incorporated into the solution system governing our model flow problem is systematically investigated in the existence of convective slip effects via porous medium modeled via the `bvp4c`. We use the `bvp4c` method to compute the momentum, thermal, and solutal profiles, along with key physical quantities such as the coefficient of skin friction, Nusselt number, and Sherwood number. The effects of each parameter are demonstrated through graphical representations, pointing out the insight into physical mechanisms and patterns that emerge from the complex interaction between slip conditions, nanofluid properties, and spatially varying magnetic field behavior. It is of great significance for the purpose of determining the behavior of hybrid nanofluids (HNFs) to take into consideration the thermophysical properties of both water and nanoparticles. Gold (Au) and copper (Cu) nanoparticles are combined with water in this research to serve as the base fluid in order to improve their qualities, such as density, specific heat capacity, and thermal conductivity. The incorporation of these nanoparticles results in a significant development in the performance of heat transfer.

The effect of M on momentum and thermal distribution is depicted in Figs. 3 and 4. When the magnetic field is stronger, the Lorentz force is increased, which diminishes fluid movement and lowers momentum while also snowballing the efficiency of thermal energy dissipation. As a result of the Ohmic heating impacts that enhance the MHD heating mechanism, Joule heating is responsible for a significant surge in the temperature profile. In general, this suppression of momentum upsurges the thermal boundary sheet, thickness, which results in an upsurge in the thermal of the fluid near the surface due to a declination of convective heat transfer. An upsurge in the value of M results in a decline in fluid flow and a rise in thermal, both of which negatively impact the heat transfer rate within the system. In border layer flows, the Lorentz force also influences a species' solutal contour, especially in Magnetohydrodynamic (MHD) flows, which are studied in Fig. 5 and involve the interaction of a fluid that conducts electricity with a magnetic field. It is evident from this graph that the Lorentz force tends to cause the concentration boundary sheet to thicken. The species diffuses deeper into the fluid before reaching the free-stream solutal because of the decreased velocity. Finally, as M values increase, the solutal contour declines.

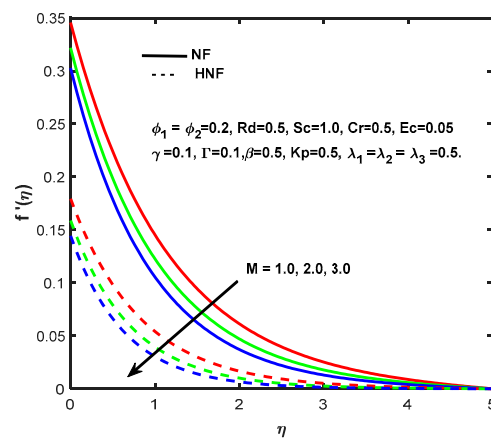


Figure 3: Fluctuations of M on $f'(\eta)$.

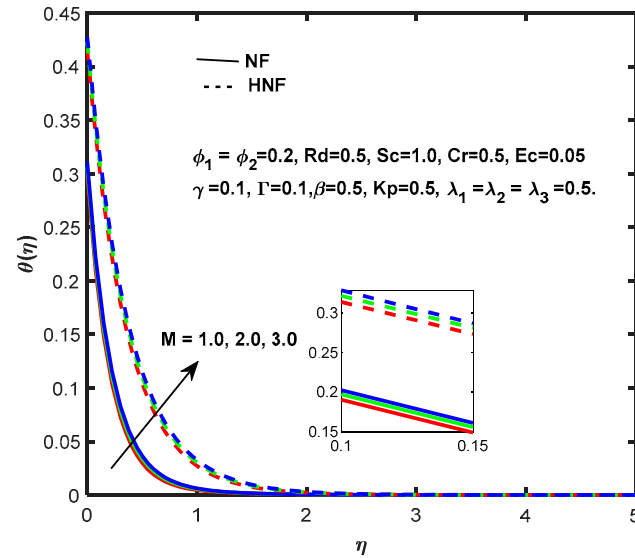


Figure 4: Fluctuations of M on $\theta(\eta)$.

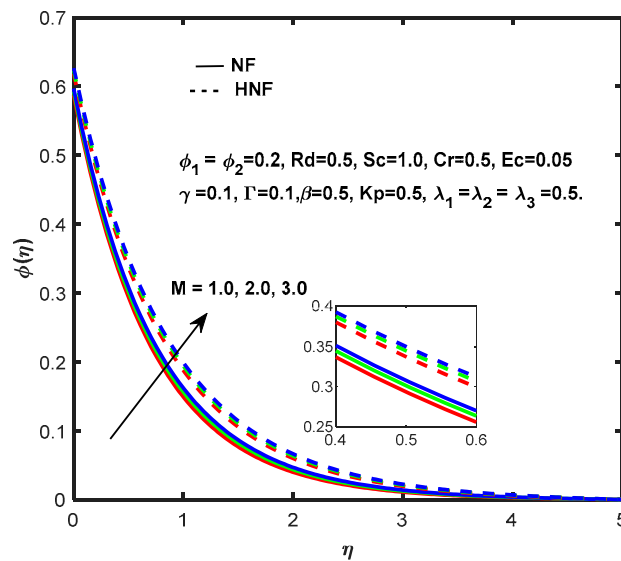


Figure 5: Fluctuations of M on $\phi(\eta)$.

The porosity parameter K_p gives a quantifiable measurement of the proportion of the whole volume of the medium that is comprised of pores. Fig. 6 is strategized in order to understand the influence of the porosity term on the momentum contour, and it is noticed that the fluid experiences resistance as a result of the solid matrix of the medium. The degree of this resistance is determined by the value of the porosity constraint. When porosity is lower, there is more resistance to flow; when porosity is higher, there is less resistance. This behavior of the porosity constraint's influence on the thermal contour in boundary sheet movements is demonstrated by Fig. 7. Because the effective thermal conductivity is higher in a material with lower porosity, the thermal range is steeper, and the temperature boundary sheet is smaller, as seen in this graph. When porosity is higher, the temperature boundary sheet is thicker, and the thermal difference is more gradual.

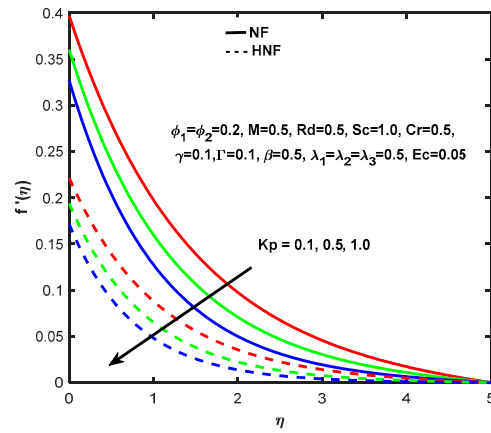


Figure 6: Fluctuations of Kp on $f'(\eta)$.

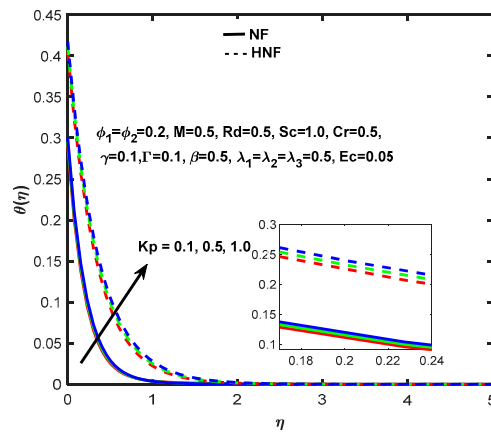


Figure 7: Fluctuations of Kp on $\theta(\eta)$.

Fig. 8 demonstrates that raising the porous medium constraint Kp lowers species concentration in the border layer. Furthermore, the use of a hybrid nanofluid (HNF) amplifies this impact, resulting in a more dramatic upsurge in the concentration contour than a standard nanofluid (NF).

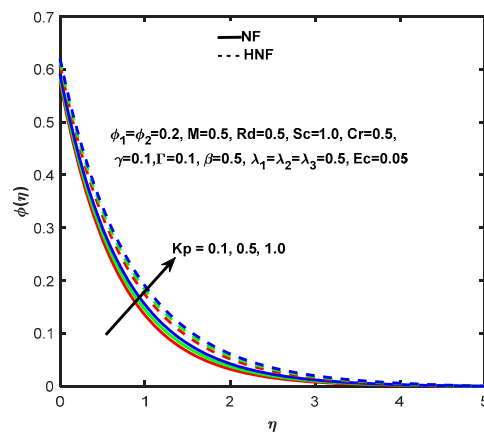


Figure 8: Fluctuations of Kp on $\phi(\eta)$.

Fig. 9 depicts the difference of $f'(\eta)$ for dissimilar values of the Casson constraint Γ . It is observed that increasing Γ from 0.5 to 2.0 decreases the momentum contour, leading to a thinner momentum boundary sheet. Additionally, the hybrid nanofluid (HNF) constantly has a lower momentum than the nanofluid (NF), suggesting that the hybrid nanofluid offers more flow resistance.

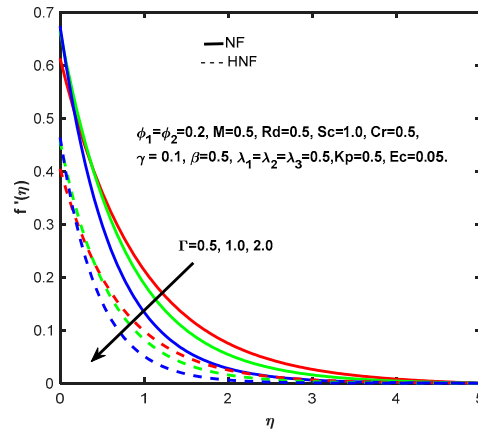


Figure 9: Fluctuations of Γ on $f'(\eta)$.

Fig. 10 exemplifies the impact of the Casson constraint Γ on the temperature contour $\theta(\eta)$ for both nanofluid (NF) and hybrid nanofluid (HNF). It is evident that increasing Γ from 0.5 to 2.0 diminishes the thermal distribution, which corresponds to a decline in the thermal boundary sheet thickness. Furthermore, the hybrid nanofluid exhibits higher temperature values than the nanofluid, suggesting improved heat retention capacity as a result of the nanoparticles' synergistic contribution. The near-wall temperature behavior, where the influence of Γ is more noticeable, is highlighted in the magnified inset.

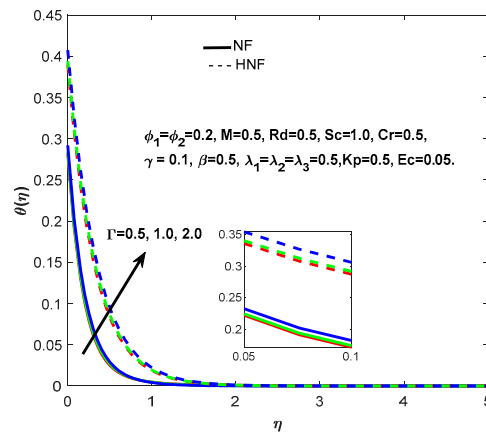


Figure 10: Fluctuations of Γ on $\theta(\eta)$.

Fig. 11 displays the effect of the Casson parameter Γ on the concentration contour $\phi(\eta)$ for nanofluid (NF) and hybrid nanofluid (HNF). It is observed that an upsurge in Γ from 0.5 to 2.0 results in a reduction in solutal, thereby decreasing the concentration boundary sheet thickness. Moreover, the hybrid nanofluid consistently maintains higher solutal levels compared to the nanofluid, suggesting improved mass transfer resistance due to the combined effect of nanoparticles. The inset highlights the near-wall variation, where the sensitivity of $\phi(\eta)$ to Γ is more evident.

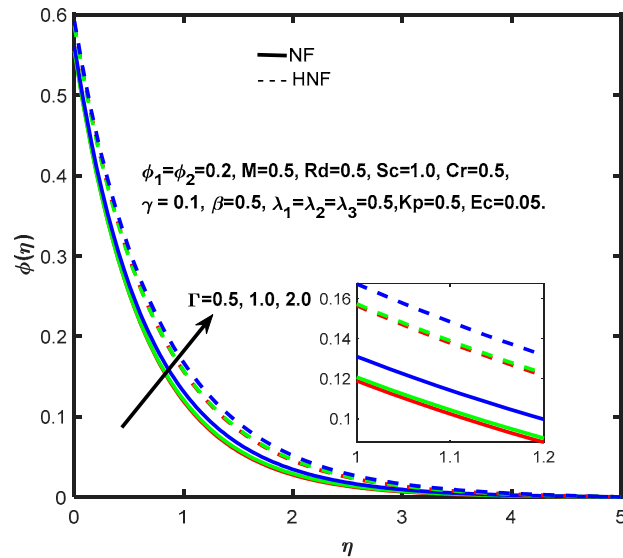


Figure 11: Fluctuations of Γ on $\phi(\eta)$.

The fluid's velocity is significantly decelerated by the unsteadiness constraint γ , as shown in Fig. 12. The hybrid nanofluid (HNF) exhibits a more noticeable flow retardation compared to the ordinary nanofluid (NF). The velocity drops at this point, sealing the deal with the constraint γ . Fig. 13 illustrates that the constraint γ serves to decline the thermal of the fluid. When it comes to the cooling impact, the use of a hybrid nanofluid is substantially more pronounced, and the heat dissipation of this method is always superior to that of the standard nanofluid. The value of γ can be seen in Fig. 14. This indicates that the hybrid nanofluid has distinct mass diffusivity properties, which result in a more substantial fall in concentration within the boundary layer when compared to the traditional nanofluid. The constraint γ serves to decrease the solutal of the fluid.

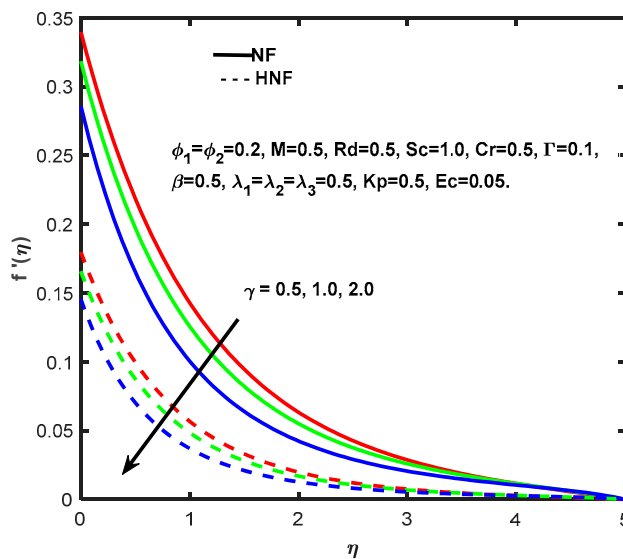


Figure 12: Fluctuations of γ on $f'(\eta)$.

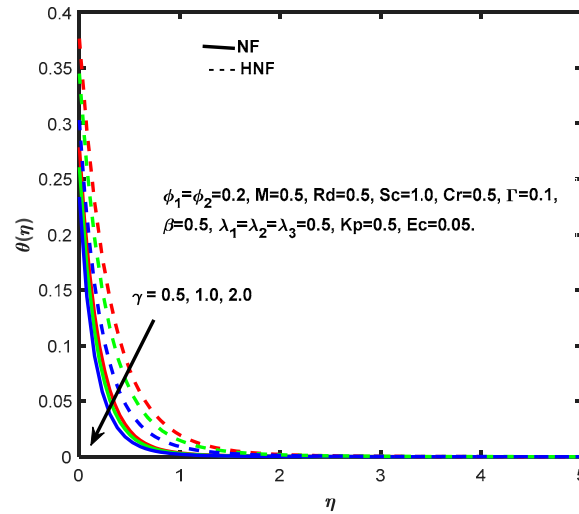


Figure 13: Fluctuations of γ on $\theta(\eta)$.

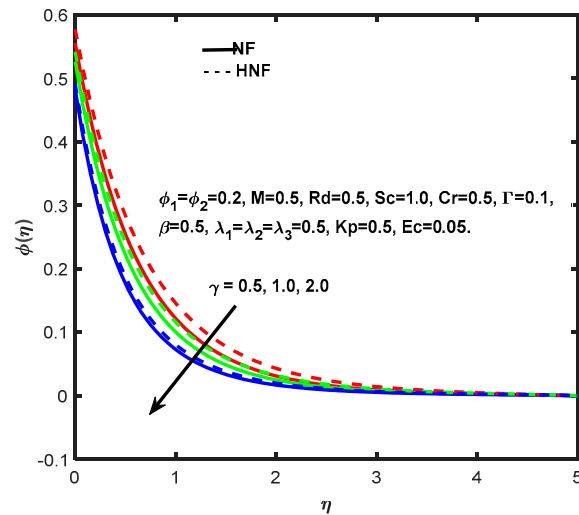


Figure 14: Fluctuations of γ on $\phi(\eta)$.

Fig. 15 displays a decreasing trend in the momentum contour as a function of the momentum slip constraint λ_1 . The slip condition causes this drop because there is a difference in fluid velocity between the surrounding flow and the stretching surface, which results in less momentum transfer close to the surface. Here, the momentum slips constraint λ_1 values upsurge then the momentum contour declines. The effects of the temperature leap constraint λ_2 on the thermal contour are established in Fig. 16. The fluid thermal in the border sheet region declines as λ_2 upsurges. A decrease in the thickness of the temperature border sheet and a corresponding upsurge in the heat transfer rate result from this increase in thermal slip, which improves fluid flow along the upward boundary. Fig. 17 shows how the solutal varies with the solutal slip constraint λ_3 . It is easily observable that the solutal contour gradually declines as the solutal slip constraint upsurges. The barrier resistance to particle flow tends to decline as the slip constraint upsurges. Particles may be rapidly displaced from the boundary as a result, leading to a thinner boundary sheet and a lower total solutal near the boundary.

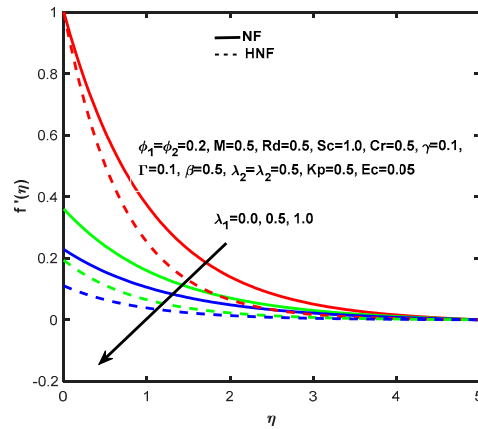


Figure 15: Fluctuations of λ_1 on $f'(\eta)$.

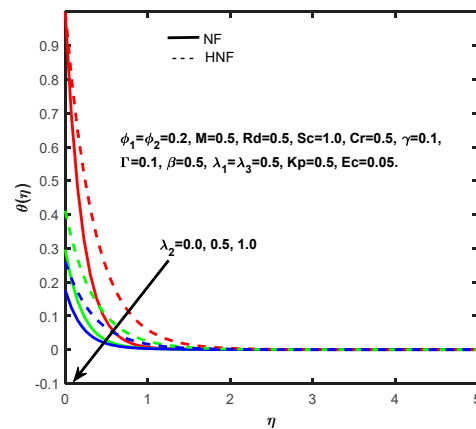


Figure 16: Fluctuations of λ_2 on $\theta(\eta)$.

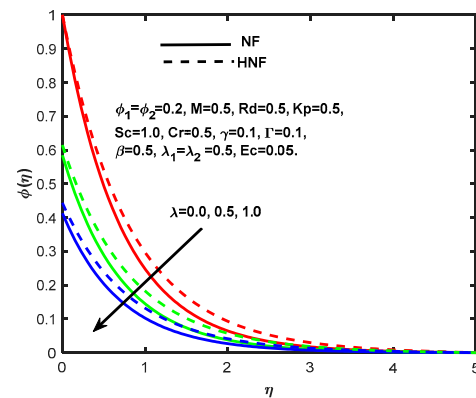


Figure 17: Fluctuations of λ_3 on $\phi(\eta)$.

The velocity change with suction constraint β is seen in Fig. 18. Higher β encourages momentum suppression, which thins the boundary layer and reduces velocity. When Joule heating is present, temperature effects worsen because too much heating negates suction cooling and reduces movement stability. The border sheet gets thinner as a result. Furthermore, the velocity border sheet sticks to the slip more firmly, halting the flow's velocity and lowering the fluid speed. As the β rises, the NF and HNF velocities fall.

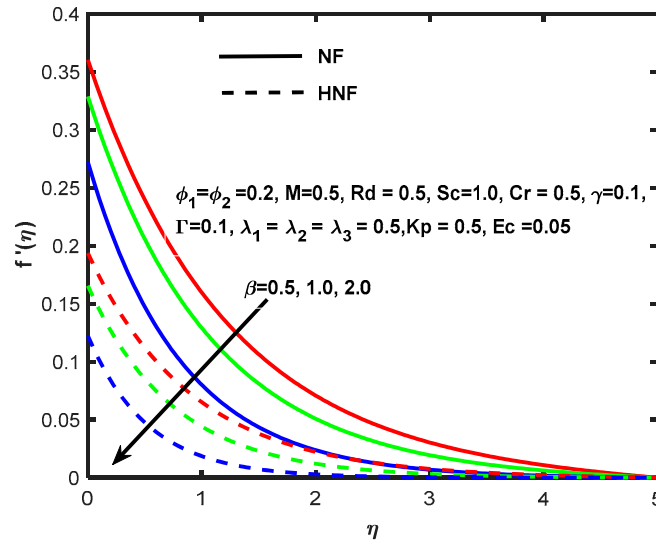


Figure 18: Fluctuations of β on $f'(\eta)$.

Fig. 19 exemplifies the effect of the temperature radiation parameter (Rd) on the thermal contour $\theta(\eta)$ that is dimensionless for both a conventional Nanofluid (NF) and a Hybrid Nanofluid (HNF). For both the NF and HNF the temperature contour $\theta(\eta)$ upsurges as the temperature radiation constraint Rd is enlarged from 0.0 to 1.0. A higher Rd value corresponds to a higher temperature across the boundary layer.

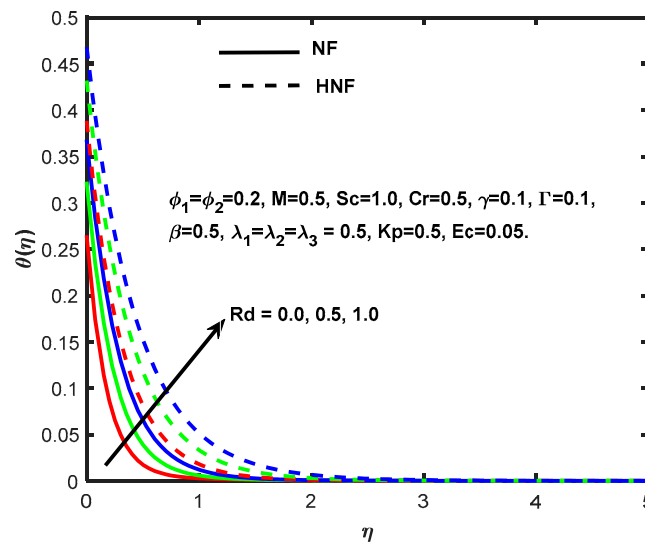


Figure 19: Fluctuations of Rd on $\theta(\eta)$.

Fig. 20 illustrates the impact of the viscous dissipation constraint Ec on the thermal contours. Physically, viscous dissipation in the boundary layer represents Kinetic energy conversion for moving fluid into internal thermal energy due to the viscous fluid friction within the fluid layers. Moreover, the viscous-dissipation source term appears in the energy equation in our problem of interest, and thereby the increased value of viscous dissipation parameter Ec raises the fluid thermal and increases the temperature border sheet.

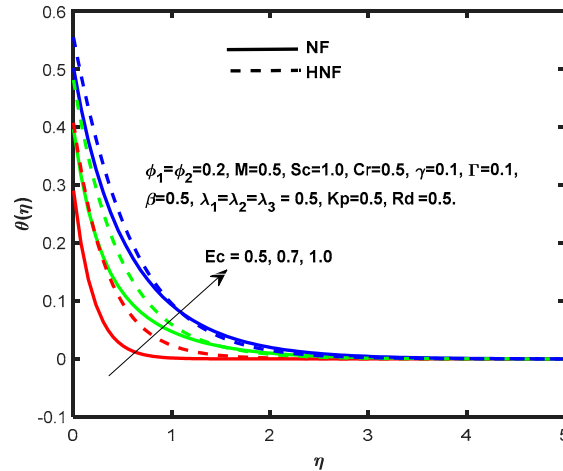


Figure 20: Fluctuations of Ec on $\theta(\eta)$.

A reduction in the thermal contour is exemplified in Fig. 21, which displays thermal contours for a range of Prandtl number (Pr) values (3–10) as Pr increases. As Prandtl numbers upsurges, thermal contours that are physically smaller are formed. When it comes to practical applications, selecting a fluid on the basis of its Prandtl number value is of the utmost importance in order to get the best possible thermal performance. To give an example, in systems that necessitate effective surface cooling, oils with high the impact of the changing magnetic field and the Prandtl number on the thermal and movement behavior of Cu-Au/water HNFs in porous media is inspected in this study. The variable magnetic field is essential in MHD because it has the ability to affect the flow of fluid by modifying the momentum contour, minimizing turbulence, and changing the way that heat is transferred. It is particularly helpful in applications such as reactors and electronic cooling, where controlling the magnetic field is essential to the management of temperature and movement properties. The Prandtl number is an important variable because it symbolizes the balance between thermal contour and momentum contour in a fluid. The study is able to signify the ways in which modifications to the properties of fluids have an impact on heat transport by employing a variable Prandtl number. Heat transfer is improved when the temperature boundary sheet is narrower, as indicated by a low Prandtl value. Conversely, a high Prandtl number results in a bigger thermal boundary layer, which lowers heat transmission efficiency.

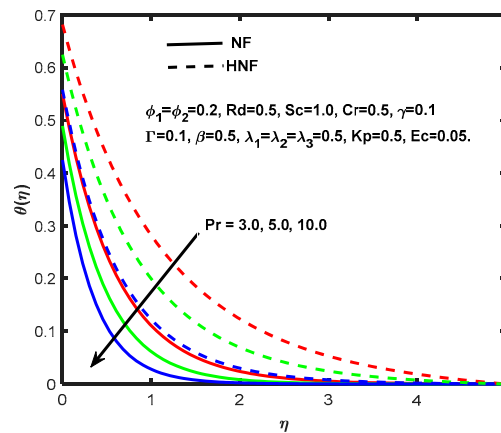


Figure 21: Fluctuations of Pr on $\theta(\eta)$.

Fig. 22 displays the concentration curve of $\phi(\eta)$ as a function of the Schmidt number (Sc). Additionally, it has been noted that the solutal contour diminishes as the value of the (Sc) upsurges. In physics, the symbol Sc represents the ratio of mass diffusivity to momentum diffusivity. An upsurge in the Schmidt number corresponds to a decline in the mass diffusivity of the fluid relative to its momentum diffusivity, which means that the scalar diffusivity decreases, resulting in a reduction in diffusion and a decline in the solutal changes that occur in the fluid medium. The influence of the chemical reaction restriction Cr on the solutal contour is illustrated in Fig. 23. It is observed that the solutal declines when there is an upsurge in Cr . In terms of physical properties, when the solutal of Cr upsurges, the rate of disintegration of reactant species intensifies as a result of the destructive chemical reaction. The solutal gradient is increased as a result of this, and the solutal border sheet close to the surface is thicker as a consequence.

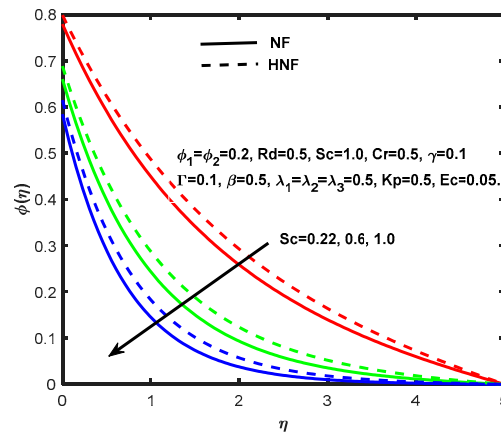


Figure 22: Fluctuations of Sc on $\phi(\eta)$.

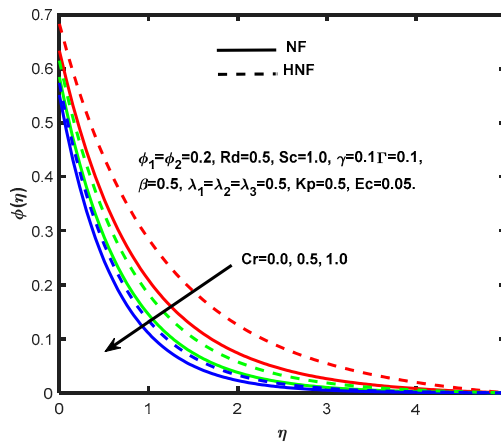


Figure 23: Fluctuations of Cr on $\phi(\eta)$.

The skin-friction factor is revealed in Fig. 24 as a function of M and λ_1 . It is observed that the skin-friction factor declines as M and λ_1 upsurges. The fluctuations of M and λ_1 with respect to the Nusselt number is portrayed in Fig. 25. When M and λ_1 are raised, the Nusselt number is revealed to decline. The fluctuations of M and λ_1 is demonstrated in Fig. 26, along with the Sherwood number. When both M and λ_1 are increased, the Sherwood number is shown to decline. The fluctuation in R and λ_2 is seen in Fig. 27 as a change in the Nusselt number. It is discovered that increasing R , Nusselt number also enhances, while

the reverse trend can be seen with λ_2 . The Sherwood number is revealed in Fig. 28 as a function of Cr and solutal slip factor variation λ_3 . It is discovered that the Sherwood number drops as the solutal slip factor λ_3 .

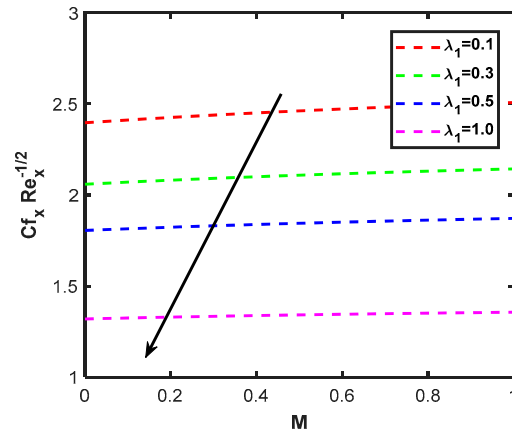


Figure 24: Impact on skin friction with M and λ_1 .

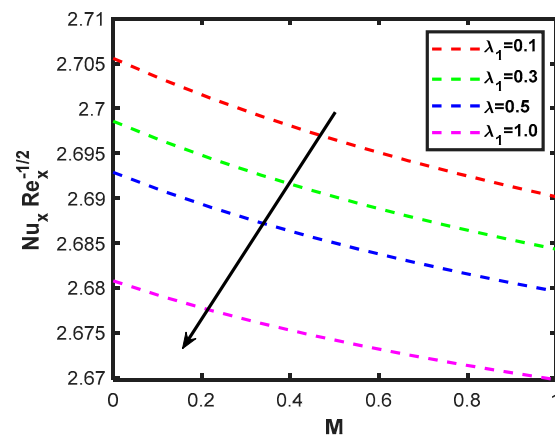


Figure 25: Impact on Nusselt number with M and λ_1 .

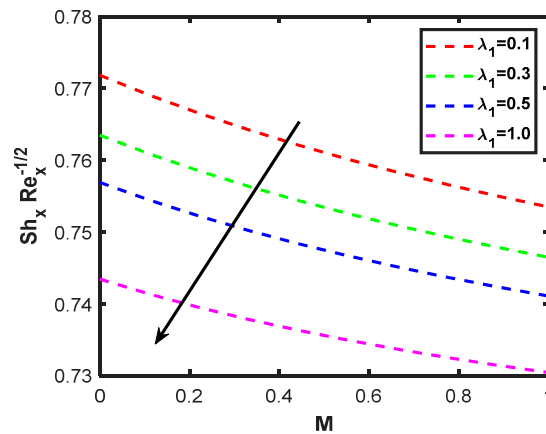


Figure 26: Impact on Sherwood number with M and λ_1 .

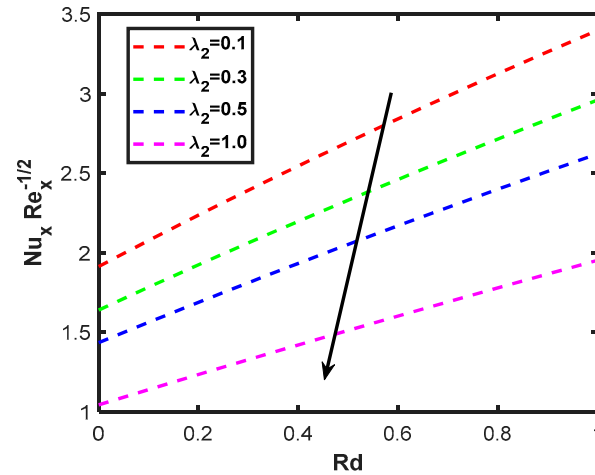


Figure 27: Impact on Nusselt number with Rd and λ_2 .

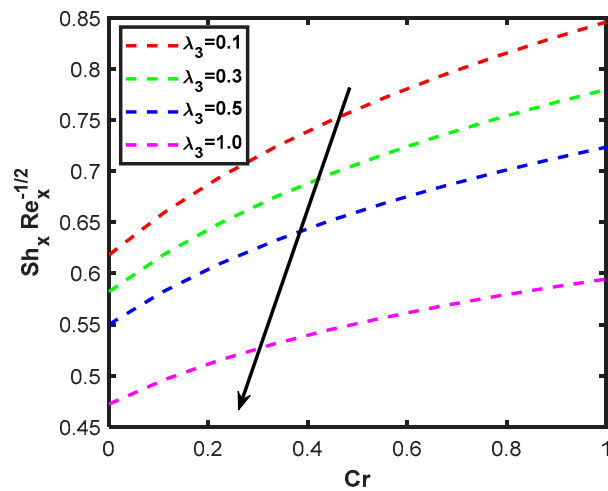


Figure 28: Impact on Sherwood number with Cr and λ_3 .

5 Optimization Analysis Using Response Surface Method

In the MHD flow of Casson hybrid nanofluid, the RSM was used to improve the efficiency of heat and mass transfer as well as skin friction factors. The results showed that, especially at low levels of thermal radiation, the sensitivity of heat and mass movement decreased as important parameters increased. Additionally, a greater Forchheimer parameter increased the sensitivity of the friction factor rate, whereas the Casson-fluid parameter showed the opposite effect.

5.1 Response Surface Regression: Cf_x Versus M , Kp and λ_1

Overview of the model

S	R ²	R ² (Adj.)	R ² (Pred.)
0.0066002	99.99%	99.99%	98.98%

Variance examination

Source	DF	Adj. SS	Adj. MS	F-Value	p-Value
Model	9	22.2993	2.4777	56,876.36	2.4×10^{-22}
Linear	3	21.5777	7.1926	165,107.35	9.1×10^{-24}
<i>M</i>	1	0.0420	0.0420	963.33	2.8×10^{-11}
<i>Kp</i>	1	0.0306	0.0306	702.93	1.3×10^{-10}
λ_1	1	21.5051	21.5051	493,655.79	8.4×10^{-25}
Square	3	0.7085	0.2362	5421.01	2.4×10^{-16}
<i>M</i> * <i>M</i>	1	0.0000	0.0000	0.91	0.363
<i>Kp</i> * <i>Kp</i>	1	0.0000	0.0000	0.51	0.491
λ_1 * λ_1	1	0.3979	0.3979	9133.86	3.9×10^{-16}
2-Way Interaction	3	0.0132	0.0044	100.71	9.0×10^{-8}
<i>M</i> * <i>Kp</i>	1	0.0004	0.0004	8.51	0.015
<i>M</i> * λ_1	1	0.0074	0.0074	169.74	1.3×10^{-7}
<i>Kp</i> * λ_1	1	0.0054	0.0054	123.87	5.9×10^{-7}
Error	10	0.0004	0.0000		
Lack-of-Fit	5	0.0004	0.0001	*	*
Pure Error	5	0.0000	0.0000		
Total	19	22.2997			

*: not applicable (NA).

Regression equation

$$Cf = \alpha_0 + \alpha_1 x + \alpha_2 y + \alpha_3 z + \alpha_{11} x^2 + \alpha_{22} y^2 + \alpha_{33} z^2 + \alpha_{12} xy + \alpha_{13} xz + \alpha_{23} yz$$

$$Cf_x = 12.1608 + 0.3984 M + 0.683 Kp - 14.319 \lambda_1 - 0.0152 M * M - 0.0456 Kp * Kp + 6.0861 \lambda_1 * \lambda_1 - 0.0545 M * Kp - 0.2432 M * \lambda_1 - 0.4155 Kp * \lambda_1.$$

$$Cf_x = 12.1608 + 0.3984 M + 0.683 Kp - 14.319 \lambda_1 + 6.0861 \lambda_1 * \lambda_1 - 0.0545 M * Kp - 0.2432 M * \lambda_1 - 0.4155 Kp * \lambda_1.$$

A response surface regression analysis was conducted in order to evaluate the effect of various parameters on the skin friction coefficient in Fig. 29. The analysis revealed a highly accurate model, achieving an R^2 value of 99.99%, a modified R^2 of 99.99% and an anticipated R^2 of 98.98%, illustrating a strong correlation between the predicted and actual results. The ANOVA findings validated the model's significance, with an overall F-value of 56,876.36 and a p -value below 0.0001, confirming its reliability. The linear effects of *M* ($F = 963.33$, $p = 2.8 \times 10^{-11}$), *Kp* ($F = 702.93$, $p = 1.3 \times 10^{-10}$) and λ_1 ($F = 493,655.79$, $p = 8.4 \times 10^{-25}$) were found to be highly significant, highlighting their strong influence on Cf_x . Among the quadratic terms, λ_1^2 ($F = 9133.86$, $p = 3.9 \times 10^{-16}$) was statistically significant, whereas M^2 ($F = 0.91$, $p = 0.363$) and Kp^2 ($F = 0.51$, $p = 0.491$) were insignificant. The interaction term *M* * *Kp* ($F = 8.51$, $p = 0.015$), *M* * λ_1 ($F = 169.74$, $p = 1.3 \times 10^{-7}$), and *Kp* * λ_1 ($F = 123.87$, $p = 5.9 \times 10^{-7}$) were also significant, demonstrating the combined effect of these parameters on Cf_x . Providing a precise mathematical relationship for predicting Cf_x . The contour and surface plot (Figs. 30 and 31) visually represent the variation of Cf_x with *M* and *Kp*, showing that Cf_x increases with both *M* and *Kp* while decreasing with λ_1 . The nonlinear contours confirm the interactive effects observed in ANOVA. These findings indicate that optimizing Cf_x requires balancing

M , Kp and λ_1 with RSM serving as an effective tool for predicting and improving skin friction efficiency while minimizing frictional losses (Figs. 32 and 33).

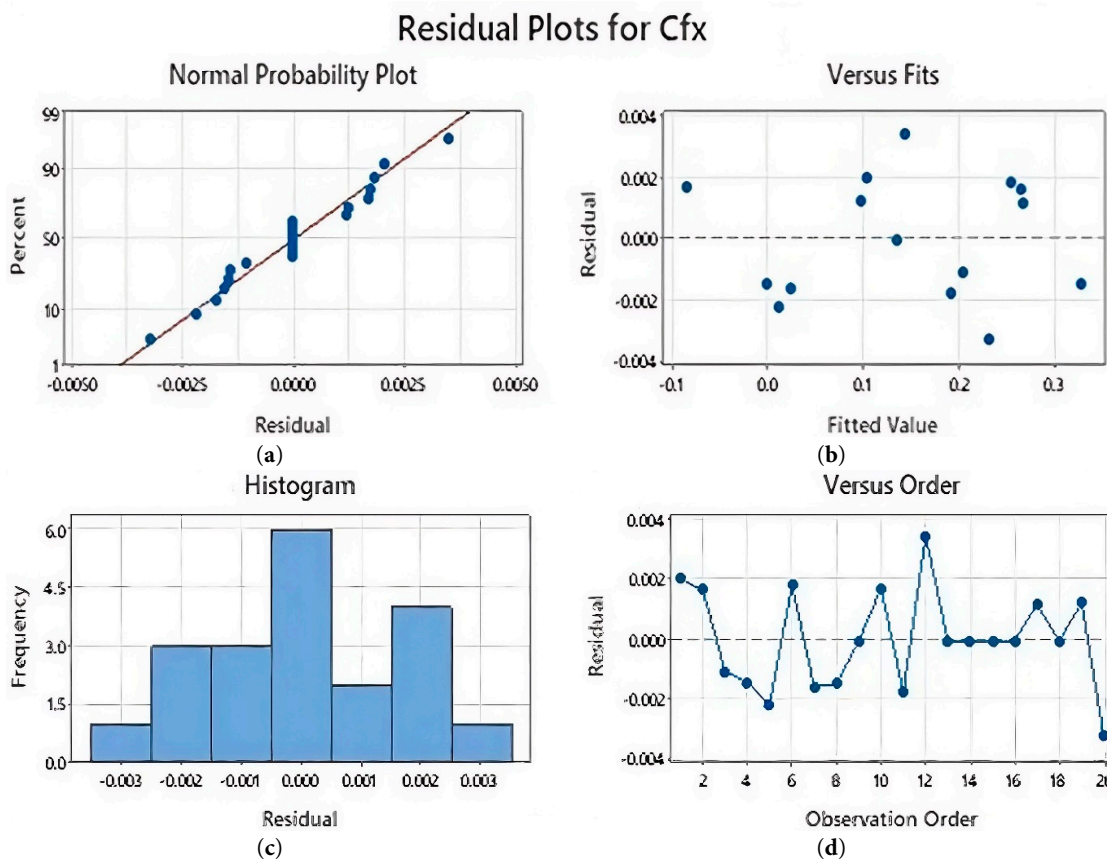


Figure 29: Residual analysis plots for the skin-friction coefficient (C_{fx}): (a) Normal probability plot, (b) Residuals versus fitted values, (c) Histogram of residuals, and (d) Residuals versus observation order.

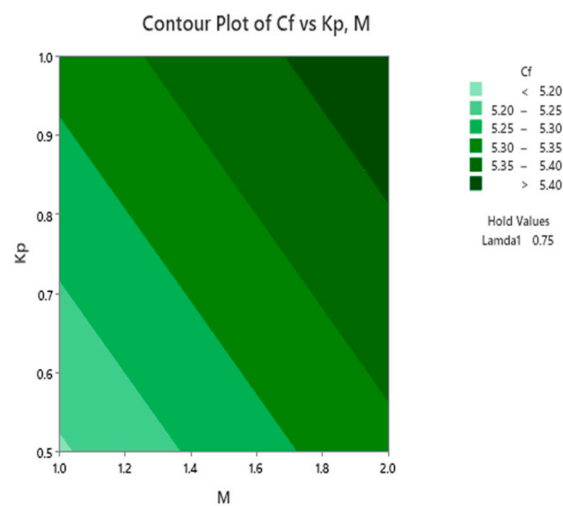


Figure 30: (Color online) contour plot of C_{fx} versus M , Kp .

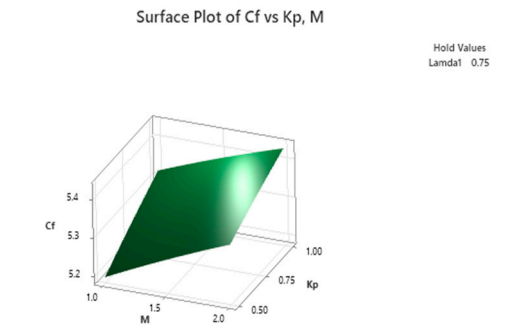


Figure 31: (Color online) surface plot of Cf_x versus M , Kp .

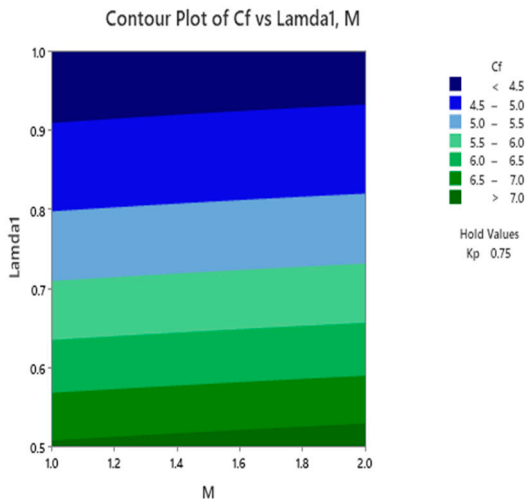


Figure 32: (Color online) contour plot of Cf_x versus M , λ_1 .

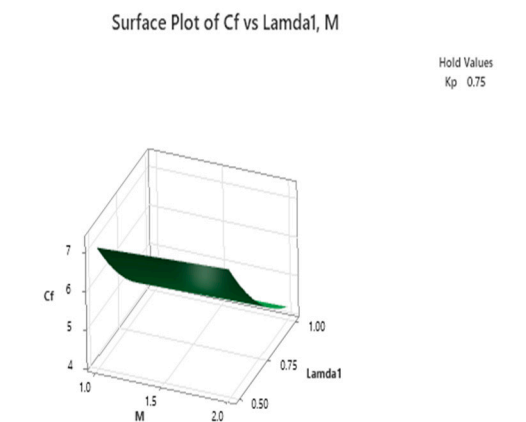


Figure 33: (Color online) surface plot of Cf_x versus M , λ_1 .

5.2 Response Surface Regression: Nu_x Versus Rd , Ec , λ_2

Model Summary

S	R ²	R ² (Adj.)	R ² (Pred.)
0.369036	99.75%	99.53%	97.67%

Analysis of variance

Source	DF	Adj. SS	Adj. MS	F-Value	p-Value
Model	9	546.741	60.749	446.07	8.0×10^{-12}
Linear	3	456.192	152.064	1116.58	6.3×10^{-13}
Rd	1	58.111	58.111	426.70	1.6×10^{-9}
Ec	1	8.738	8.738	64.16	1.2×10^{-5}
λ_2	1	389.343	389.343	2858.88	1.3×10^{-13}
Square	3	70.790	23.597	173.27	6.4×10^{-9}
$Rd * Rd$	1	0.009	0.009	0.07	0.797
$Ec * Ec$	1	0.002	0.002	0.01	0.922
$\lambda_2 * \lambda_2$	1	39.973	39.973	293.52	9.7×10^{-9}
2-Way Interaction	3	19.759	6.586	48.36	2.9×10^{-6}
$Rd * Ec$	1	0.458	0.458	3.36	0.097
$Rd * \lambda_2$	1	16.324	16.324	119.87	6.9×10^{-7}
$Ec * \lambda_2$	1	2.977	2.977	21.86	0.000874
Error	10	1.362	0.136		
Lack-of-Fit	5	1.362	0.272	*	*
Pure Error	5	0.000	0.000		
Total	19	548.103			

*: not applicable (NA).

Regression equation

$$Nu_x = \alpha_0 + \alpha_1 x + \alpha_2 y + \alpha_3 z + \alpha_{11} x^2 + \alpha_{22} y^2 + \alpha_{33} z^2 + \alpha_{12} xy + \alpha_{13} xz + \alpha_{23} yz.$$

$$Nu_x = 18.64 + 11.14 Rd - 4.92 Ec - 34.76 \lambda_2 - 0.28 Rd * Rd - 0.42 Ec * Ec + 18.83 \lambda_2 * \lambda_2 - 2.13 Rd * Ec - 7.054 Rd * \lambda_2 + 5.42 Ec * \lambda_2$$

$$Nu_x = 18.64 + 11.14 Rd - 4.92 Ec - 34.76 \lambda_2 + 18.83 \lambda_2 * \lambda_2 - 2.13 Rd * Ec - 7.054 Rd * \lambda_2 + 5.42 Ec * \lambda_2.$$

In order to assess the influence of the thermal radiation parameter, Eckert number, and thermal slip factor on the response variable, the response surface regression analysis was implemented in Fig. 34. The model's statistical significance was confirmed by an R^2 value of 99.75%, an adjusted R^2 of 99.53%, and a predicted R^2 of 97.67%, which indicates an exceptional model fit. The ANOVA results demonstrated that most of the linear, quadratic, and interaction terms were statistically significant ($p < 0.0001$). However, the quadratic terms Rd^2 and Ec^2 were not statistically significant, as their p -values were greater than 0.0001. The most significant effect was exhibited among the linear terms ($F = 426.70$), followed by Ec ($F = 64.16$) and ($F = 2858.88$). The strongest nonlinear effect was indicated by the quadratic term ($F = 293.52$), which exhibited the highest influence. $Rd*$ ($F = 119.87$) was the most prevalent interaction effect, followed by $Ec*$ ($F = 21.86$) and $Rd * Ec$ ($F = 3.36$). The interaction effects were substantial. The model's adequacy is confirmed by a nonsignificant lack of fit, as the final regression equation effectively predicts system behavior. The Nusselt number's variation with Ec and Rd is illustrated in the contour and surface plot Figs. 35–38. The Nusselt number (Nu_x) increases as the Eckert number (Ec) and the radiation parameter (Rd) increase, indicating that the convective heat transfer rate in the flow is enhanced by increased viscous dissipation and thermal radiation. An increase in λ_2 leads to greater thermal slip at the wall, resulting in

a reduced thermal gradient close to the surface. Since the Nusselt number $Nu \propto -\left(\frac{\partial T}{\partial y}\right)_{y=0}$, a reduced temperature gradient results in a lower Nu_x .

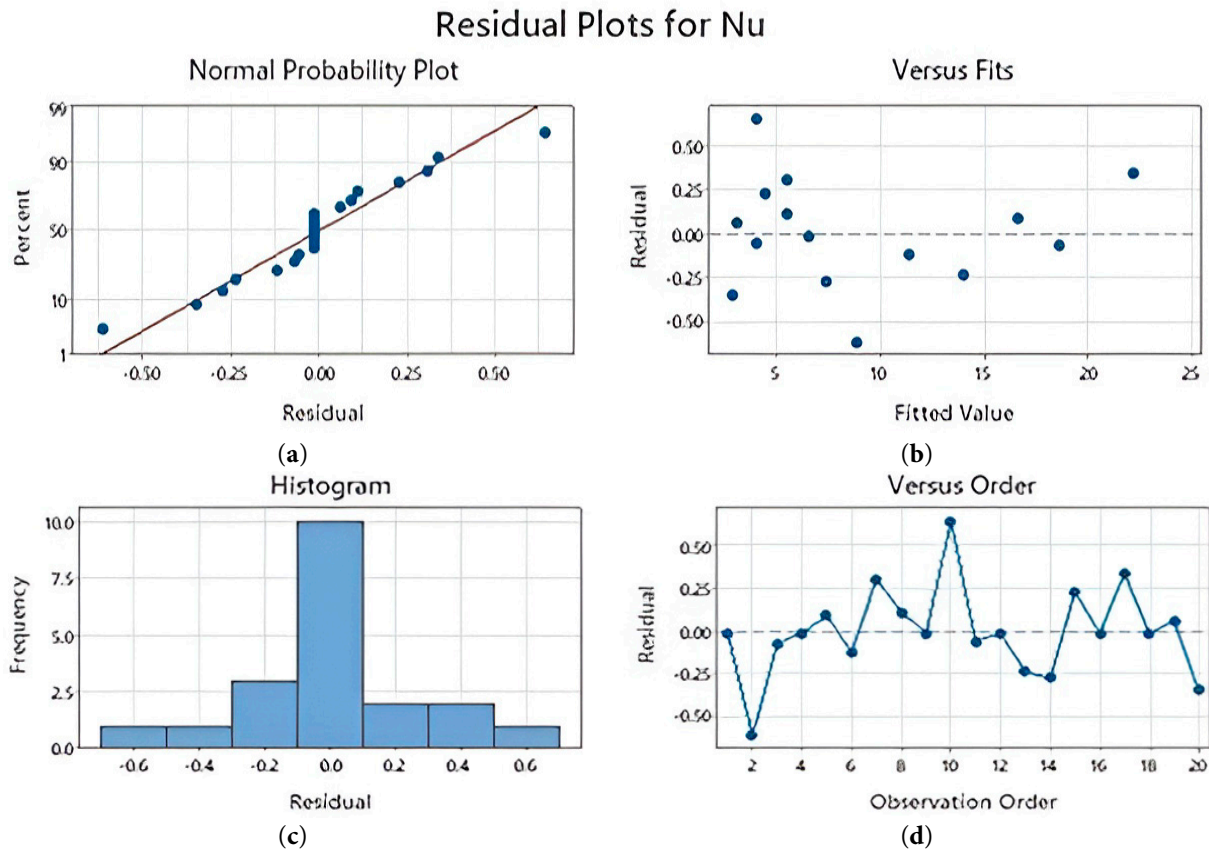


Figure 34: Residual analysis plots for the Nusselt number (Nu): (a) Normal probability plot, (b) Residuals versus fitted values, (c) Histogram of residuals, and (d) Residuals versus observation order.

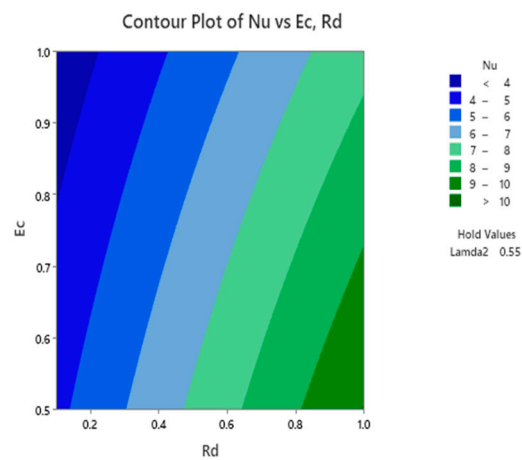


Figure 35: (Color online) contour plot of Nu_x versus Ec , Rd .

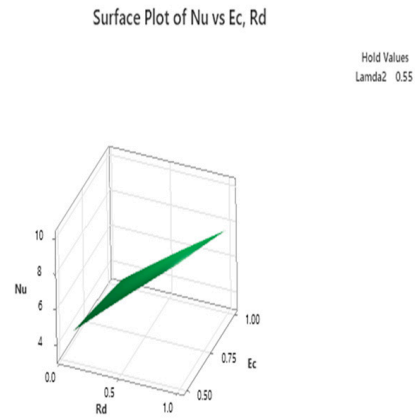


Figure 36: (Color online) surface plot of Nu_x versus Ec , Rd .

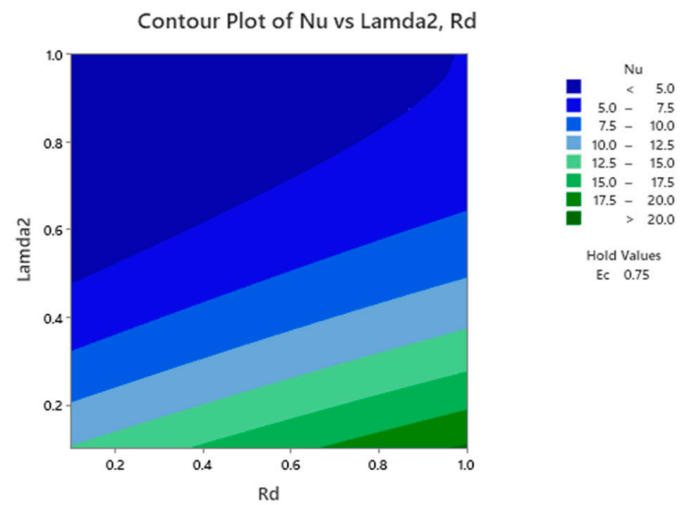


Figure 37: (Color online) contour plot of Nu_x versus λ_2 , Rd .

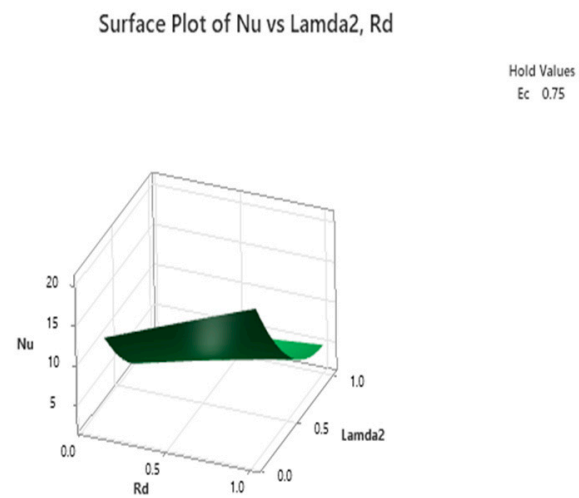


Figure 38: (Color online) surface plot of Nu_x versus λ_2 , Rd .

5.3 Response Surface Regression: Sh_x Versus Cr , Sc and λ_3

Summary of the model

S	R ²	R ² (Adj.)	R ² (Pred.)
0.0373484	99.31%	98.69%	97.31%

Analysis of variance

Source	DF	Adj. SS	Adj. MS	F-Value	p-Value
Model	9	2.00917	0.223242	160.04	1.3×10^{-9}
Linear	3	1.71121	0.570404	408.92	9.3×10^{-11}
Sc	1	0.83901	0.839008	601.48	2.9×10^{-10}
Cr	1	0.03647	0.036471	26.15	0.000455
λ_3	1	0.83573	0.835732	599.13	3.0×10^{-10}
Square	3	0.03492	0.011641	8.35	0.004466
Sc * Sc	1	0.01033	0.010326	7.40	0.022
Cr * Cr	1	0.00005	0.000051	0.04	0.853
$\lambda_3 * \lambda_3$	1	0.03090	0.030899	22.15	0.021549
2-Way Interaction	3	0.26304	0.087680	62.86	0.845489
Sc * Cr	1	0.00072	0.000720	0.52	0.487358
Sc * λ_3	1	0.25145	0.251448	180.26	1.0×10^{-7}
Cr * λ_3	1	0.01087	0.010872	7.79	0.019085
Error	10	0.01395	0.001395		
Lack-of-Fit	5	0.01395	0.002790	*	*
Pure Error	5	0.00000	0.000000		
Total	19	2.02312			

*: not applicable (NA).

Regression equation

$$Sh_x = \alpha_0 + \alpha_1 x + \alpha_2 y + \alpha_3 z + \alpha_{11} x^2 + \alpha_{22} y^2 + \alpha_{33} z^2 + \alpha_{12} xy + \alpha_{13} xz + \alpha_{23} yz$$

$$Sh_x = 0.2634 + 1.696 Sc + 0.226 Cr - 0.527 \lambda_3 - 0.383 Sc * Sc - 0.021 Cr * Cr + 0.523 \lambda_3 * \lambda_3 + 0.0527 Sc * Cr - 0.9849 Sc * \lambda_3 - 0.1820 Cr * \lambda_3.$$

Fig. 39 illustrates the residual analysis for the Sherwood number, and the response surface regression analysis indicates a highly accurate model fit, achieving an R^2 value of 99.31%, an altered R^2 of 98.69% and an expected R^2 of 97.31%. The ANOVA results confirm the statistical significance of most of the terms ($p < 0.0001$), except for the quadratic term $Cr * Cr$ ($p = 0.853$) and $Sc * Cr$ ($p = 0.487358$). Among the linear terms, the Schmidt number Sc exhibits the highest influence on the response variable, with an F-value of 601.48, followed by the concentration slip factor λ_3 with an F-value of 599.13. Quadratic terms also significantly impact the response, particularly $\lambda_3 * \lambda_3$ ($F = 22.15$), while the interaction term $Sc * \lambda_3$ ($F = 180.26$) indicates strong interdependence between these variables. The residual charts confirm the normality and randomness of residuals, validating the model's reliability. The contour and surface plot highlights the variation in Sherwood number Sh_x concerning Sc , Cr , and λ_3 in Figs. 40–43.

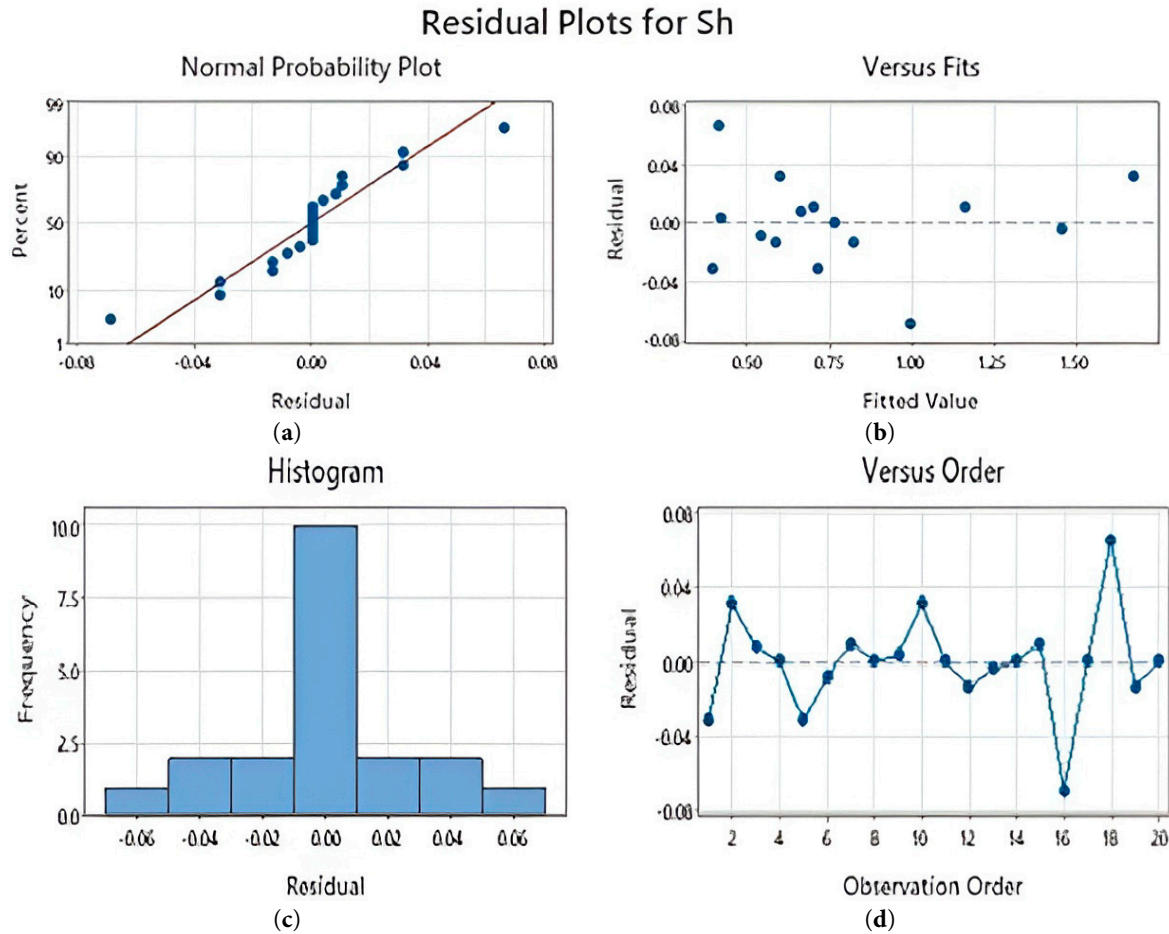


Figure 39: Residual analysis plots for the Sherwood number (Sh_x): (a) Normal probability plot, (b) Residuals versus fitted values, (c) Histogram of residuals, and (d) Residuals versus observation order.

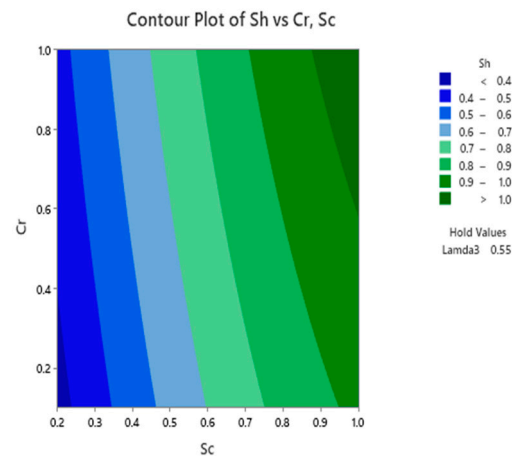


Figure 40: (Color online) contour plot of Sh_x versus Cr and Sc .

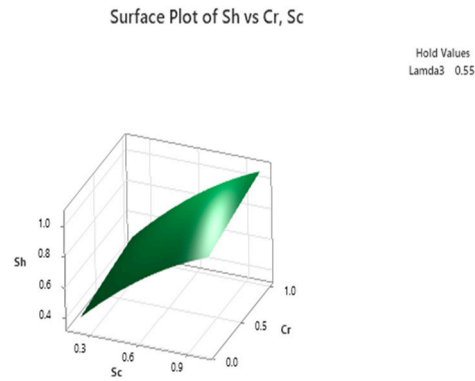


Figure 41: (Color online) surface plot of Sh_x versus Cr and Sc .

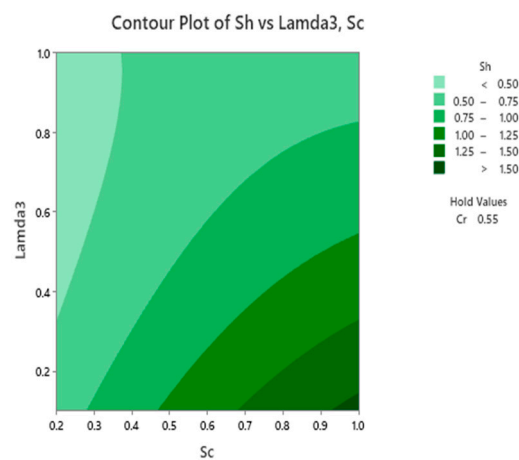


Figure 42: (Color online) contour plot of Sh_x versus λ_3 and Sc .

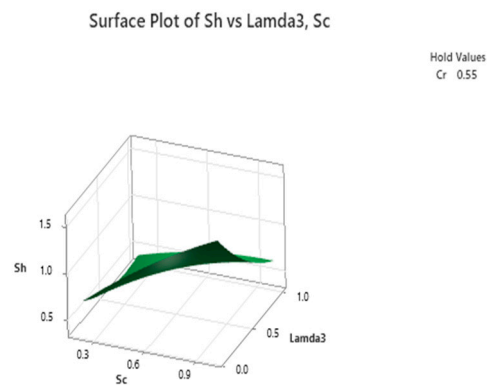


Figure 43: (Color online) surface plot of Sh_x versus λ_3 and Sc .

An elevated Sc indicates a fluid characterized by reduced mass diffusivity, resulting in a thinner concentration boundary sheet and a heightened solutal gradient at the wall. The chemical reaction improves diffusive transport by accelerating the rate at which species are eliminated (in the case of destructive reactions), i.e., both Sc and Cr have a positive influence on the Sherwood number (Sh_x). In contrast, a reverse trend is observable regarding the concentration slip factor.

6 Conclusions

This investigation examined the thermal and species transport phenomena in a Casson-type hybrid nanofluid that contained copper (Cu) and gold (Au) nanoparticles. The nanoparticles were propelled by an elongating surface that was time-dependent. The work objectively evaluates the synergistic effects of the Casson factor and the unsteadiness constraint on transport phenomena through a comprehensive graphical and tabular analysis. The results demonstrate the combined impact of the two factors on the momentum contour, nonlinear temperature distributions, and concentration gradients that are susceptible to both. The study is significant in that it illustrates the interaction between radiative flux and response rate, as well as velocity slip, temperature slip, and concentration slip, to establish multiple transport regimes. Certain findings provide novel perspectives on optimizing systems that exhibit operationally controllable characteristics.

1. Both the nanofluid (NF) and hybrid nanofluid (HNF) portray a decrease in momentum as the suction and unsteadiness constraint upsurges.
2. The Casson rheological parameter and velocity slip coefficient increase the thickness of thermal and solutal boundary sheets due to increased diffusive transport mechanisms that allow for higher heat and mass dispersion.
3. The porosity constraint shows an inverse relationship with momentum and a direct relationship with thermal and mass distribution.
4. In the absence of magnetic influences, movement designs show regularly spaced, organized streamlines that are typical of unobstructed motion. The introduction of a magnetic field causes Lorentz-force-induced disturbances, which result in dispersed and uneven streamlines that indicate movement retardation.
5. Chemical reactions and solutal slip work together to thin the species boundary sheet by encouraging reactive species consumption while reducing wall-normal diffusion.
6. The regression model exhibited remarkable precision, with R^2 values surpassing 99%, signifying a robust relationship between predicted and actual values.
7. The ANOVA results confirmed the model's statistical significance, highlighting the major influence of Schmidt number on mass transfer and the thermal slip factor on heat transfer.
8. Contour and surface plots demonstrated complex interactions among parameters, showing that the Nusselt number diminishes as the temperature slip constraint rises, whereas the Sherwood number escalates with the Schmidt number and chemical reaction.
9. The RSM accurately forecasts system performance, offering a framework for optimization that improves heat transfer efficiency and reduces frictional losses.

The results emphasize the promising biomedical uses of Casson hybrid nanofluids, especially in cancer treatments that rely on hyperthermia, where accurate thermal control is essential. The results of this study contribute to the development of effective hybrid nanofluid-based cooling and heating systems for medical and industrial uses.

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Shaik Mohammed Ibrahim; Software, Writing, review, visualization, editing; Bhavanam Naga Lakshmi; Investigation, resources and supervision: Shaik Mohammed Ibrahim and Giulio Lorenzini. All authors reviewed and approved the final version of the manuscript.

Availability of Data and Materials: All data are available within the manuscript.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

Nomenclature

a	constant stretching rate
b	Free stream rate
$(c)_f$	Specific heat at non varying pressure ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
u, v	Momentum-components in x, y directions ($\text{m}\cdot\text{s}^{-1}$)
U_w	Velocity at the wall ($\text{m}\cdot\text{s}^{-1}$)
U_∞	Ambient fluid velocity ($\text{m}\cdot\text{s}^{-1}$)
B_0	Intensity of magnetic field
Cr	Chemical reaction constraint
Pr	Prandtl number
β	Suction momentum
η	Similarity variable
μ_B	Viscosity of the plastic dynamic ($\text{kg}\cdot\text{M}^{-1}\cdot\text{s}^{-1}$)
π	Component deformation Product
g	Acceleration due to gravity ($\text{m}\cdot\text{s}^{-2}$)
C	Solutal of the fluid ($\text{kg}\cdot\text{m}^{-3}$)
C_w	Concentration level of fluid at surface ($\text{kg}\cdot\text{m}^{-3}$)
C_∞	solutal level of fluid far away from the surface ($\text{kg}\cdot\text{m}^{-3}$)
k^*	Absorption coefficient
κ	Fluid thermal conductivity ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$)
ψ	Stream function, ($\text{m}^2\cdot\text{s}^{-1}$)
χ_n	Characteristic function
f'	Dimensionless velocity
ρ_p	Nanoparticles mass density, ($\text{kg}\cdot\text{m}^{-3}$)
$(\rho c)_p$	Nanoparticles heat capacity
σ^*	Stefan-Boltzmann constant
ρ_f	fluid density
$(\rho c)_f$	fluid heat capacity
$(\sigma)_f$	Electrical conductivity, ($\text{S}\cdot\text{m}^{-1}$)
q_r	radiative heat flux
M	Magnetic field constraint
Kp	Porosity constraint
γ	Unsteadiness constraint
λ_1	Momentum slip constraint
λ_2	Temperature slip constraint
λ_3	Concentration slip constraint
Sc	Schmidt number
R	Thermal Radiation constraint
α	Fluid thermal diffusivity
Ec	Eckert number
Nu_x	Local Nusselt number
π_c	Product based non-Newtonian Critical value model
Sh_x	Sherwood number

Cf_x	Skin friction factor
T	Fluid thermal
T_w	Convective fluid thermal
T_∞	Ambient fluid temperature
q_w	Surface heat flux (Wm^{-2})
q_m	Surface mass flux
τ_w	Surface shear stress
f	Dimensionless stream function
Γ	Casson constraint
$\tau = \frac{(\rho c)_p}{(\rho c)_f}$	Nanofluid to base fluid heat Capability ratio
ν	kinematic viscosity ($\text{m}^2\cdot\text{s}^{-1}$)
μ	Dynamic viscosity

Subscripts

θ	Dimensionless Temperature
H	Dimensionless solutal
Hnf	Hybrid nano fluid
∞	Free stream
p	nanoparticle
f	fluid
w	wall

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