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MHD Effect on Relative Motion of Two Immiscible Liquid Spheres

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Abstract: We examine the motion of the two concentric immiscible liquid spheres with different viscosities in an electrically conducting fluid in the presence of transverse magnetic field. The inner sphere is assumed to move at a constant velocity. The Stoke's equation along with the Lorentz force is considered to model the resulting fluid flow, analytical solutions being obtained by the similarity solution method in terms of modified Bessel's functions. Streamlines related to the fluid circulation in the annulus between the two liquid spheres and inside the inner liquid sphere are presented for different combinations of the governing non-dimensional parameters.

Keywords: Similarity solutions, immiscible liquids, magnetic field.

1 Introduction

The motion of particles relative to a fluid is often of interest in the field of chemical, biomedical, environmental engineering and sciences. In these transport phenomena's, the flow characteristics can be developed to understand many practical systems and industrial process. Such as sedimentation, flotation of oil fields / reservoirs during oil recovery, electrophoresis, agglomeration, lubrication and motion of blood cell in artery or vein. Further in many realistic applications like in dispersion of fine particles in geophysical flows and planetary atmosphere, floc sedimentation, paints and lubricant industries, etc. the force experienced by a body or fluid, when

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they are in relative motion is of great practical importance. In these kinds of motion, the multiple particle systems are more important than the single particle, in determining the effect of neighboring particles or boundaries on the movement of the particles. In the literature several investigations are reported on the flow of Newtonian fluids while considering the relative motion of solid or liquid spheres/cylinder.

Happel (1958) studied the slow motion of fluids relative to beds of spherical particles. The relation for the rate of sedimentation or pressure drop as a function of void volume was obtained. Also, the results are in good agreement with Carman-kozeny equation. Bernner (1961) analyzed the viscous flow for steady motion of a solid sphere towards or away from a plane surface of infinite extent. The exact solution was obtained using bi-polar co-ordinate system and was applied to determine the end-effect correlation in falling-ball viscometer. Wacholder and Weihs (1972) presented the study of Stokes flow in and around a spherical fluid particle in the presence of another fluid particle or of a plane surface normal to the settling velocity. The exact solutions are obtained using bi-spherical co-ordinates. The motion of two spherical particles (solid particle or liquid droplets or gas bubbles) along their line of centers was studied by Rushton and Davies (1978). They carried out these results for quasi-steady creeping flow condition and presented the results for drag coefficients and streamlines. The results are applied to the gravity settling of droplets and coalescence of droplets.

The hydrodynamic deformation of a solid elastic sphere, immersed in a viscous fluid and in close motion towards another sphere or plane solid surface was presented by Davis et al (1986). The rotational criteria that a solid particle will stick or rebound subsequent to the impact was established with intend of application in the course of filtration or coagulations where viscous forces are important. Masliyah et al (1987) solved the creeping flow of an incompressible fluid past an isolated composite sphere with a permeable shell for constant permeability and arbitrary thickness. The continuity of velocity and stress across the interface between porous and clear fluid are used as the boundary conditions. Palaniappan et al. (1994) stated and proved the theorem for non-axi symmetric Stokes flow of a viscous fluid for a sphere. It was noticed that the expression for drag on the fluid sphere was a linear combination of rigid and shear free drags. They also obtained the existing sphere theorem as a special their results. The uniform motion of a drop of one fluid moving in another immiscible unbounded fluid was presented by Paranjape and Paranjape (1996). They analyzed the condition of normal stress balance across the interface of the two fluid spheres.

Palaniappan (2000) presented a general solution for the creeping flow equations which are bounded by a non-deforming planar interface. The expression for general reflection theorem was derived for a fluid-fluid interface containing Lorentz reflection formula. The theorem allows a better interpretation of the image system for various singularities in the presence of a planar interface. Relative motion of liquid spheres of different viscosities when the surface of the outer sphere was free has been considered by Bhatt and Shirley (2002). The two dimensional viscous flows in a granular material with void of arbitrary shape was carried out by Rajasekhar and Sano (2003) because of its importance in formation of waterway networks and estimation of underground water velocity. An experimental study of the wetting and evaporation of sessile drops under the influence of atmospheric pressure was presented by Sefiane (2005). Adaptive sharp interface numerical technique was used by Sussman and Ohta (2007) to solve two phase flows.

The motion of conducting fluids in an electromagnetic field finds applications in many physical, geophysical and industrial fields. In these practical applications there is a scope to control the motion of the fluid past solid bodies with Magnetohydrodynamics (MHD) effects. Thus Hartmann flow is a classical problem that has important application in MHD power generators and pumps, aerodynamics, heating, polymer technology, the petroleum industry, purification of crude oil, and design of various heat exchangers. Also, in the recent developments in rocketry and space craft have given rise to study of conducting fluids past solid bodies of various shapes. In the literature several works have been done on the flow of an electrically conducting fluid past a sphere / cylinder to understand the effect of applied magnetic field.

Stewartson (1956) analyzed the steady motion of a perfectly conducting sphere in an inviscid conducting fluid in the presence of a strong magnetic field. It was noticed that the streamlines outside the sphere are straight lines if the sphere moves in the direction of the field and execute sharp turns if it moves at right angles to the field. Childress (1963) has investigated the effect of magnetic field on the flow of a conducting fluid past a body of revolution when both the magnetic field and the streaming motion of the fluid at infinity are uniform and parallel to the axis of symmetry of the body. Nigam (1978) has studied the problem of two dimensional non-uniform flow of an infinitely conducting, inviscid, incompressible fluid past a non-conducting circular airfoil in the presence of magnetic field in the direction of the flow. The drag and lift experienced by the airfoil due to the external magnetic field have been discussed. Sanyal and Roy Chowdhury (1984) have studied two dimensional, non-uniform flow of an infinitely conducting, inviscid, incompressible fluid past a cylinder having the cross section bounded by two circular arcs in the presences of magnetic field along the flow direction. Anjali Devi and Raghavachar (1987) investigated the upward flow of a vertically stratified, electrically conducting fluid past a non conducting sphere in the presence of uniform magnetic field for diffusive medium. Quasi-steady approximation was made allowing for time dependent buoyancy force. Matching asymptotic expansion was employed to obtain the drag exerted on the sphere for small value of stratification parameter.

Kyrlidis et al. (1990) presented the study of conducting fluid past axi-symmetric bodies in the presence of magnetic field in the limit of small inertial and magnetic Reynolds numbers. The objective was to control the particle settling in metallic systems. The steady, viscous, electrically conducting fluid flow around a circular cylinder in the presence of magnetic field applied parallel to the main flow was investigated by Raghava Rao and Sekhar (2000). Sekhar et al. (2003) presented the effect of applied magnetic field parallel to the main flow for low and moderate Reynolds number of a steady, incompressible, viscous, conducting fluid flow past a sphere. A numerical simulation for liquid metal channel flow under inhomogeneous magnetic field was analyzed by Votyakov and Zienicke (2007). Keh and Hsieh (2010) presented the MHD effect on a translating and rotating colloidal sphere in an arbitrary electrolyte solution. A general flow field was considered for a steady state with uniform magnetic field. Stokes equation along with Lorentz force was used to describe the flow. The effect of MHD on the particle movement associated with translation and rotation of the particle and fluid flow are discussed. Pal and Talukdar (2011) analyzed the unsteady flow of a laminar two-dimensional oscillatory flow of an incompressible electrically conducting viscous fluid between two non-conducting parallel plane surfaces in the presence of suction / injection.

The use of magnetic field to control the flow processes in different domains under different types of boundaries place a vital role in modern metallurgical and metal working processes. This has led to considerable interest in the study of boundary layer flows subjected to an externally applied magnetic field. Keeping in view of vast area of practical importance, in this paper we present the analytical solution for relative motion of two concentric immiscible liquid spheres with different viscosity under the influence of applied magnetic field.

2 Mathematical Formulation

The study of relative motion of the two concentric immiscible liquid spheres with different viscosity in an electrically conducting fluid is presented in the presence of transverse magnetic field. The inner liquid sphere of radius a with the viscosity μ_1, moving with the velocity V is embedded in another liquid sphere of radius b with the viscosity μ_2. Here the two liquid fluids are considered as immiscible fluids, thereby the flow region is divided into two regions namely region I from $0 < r \leq a$ and region II from $a < r \leq b$. Let the index i in the subscript of any flow property X_i, $i = 1, 2$ represents region I and region II respectively. Using Stokes approximation (see Happel (1958)) along with the additional body force in the momentum equation is given by:

$$\nabla \cdot \vec{q_i} = 0, \tag{1}$$

$$\nabla p_i = \mu_i \nabla^2 \vec{q_i} + \mu_h^2 \sigma_e \left(\vec{q_i} \times \vec{H} \right) \times \vec{H}, \tag{2}$$

where $\vec{q_i}\,(u_i\,,\, v_i\, w_i)$ is the velocity of the fluid, p_i is the hydrostatic pressure of the fluid, μ_i is the viscosity of fluids, μ_h is the magnetic permeability, σ_e is electrical conductivity and $\vec{H}$ is applied magnetic field.

The governing equations (1) and (2) are Non- dimensionalised using the transformations:

$$r^* = \frac{r}{a}, \quad \vec{q_i^*} = \frac{\vec{q_i}}{V}, \quad p_i^* = \frac{a p_i}{\mu V}, \quad \vec{H^*} = \frac{\vec{H}}{H_0}. \tag{3}$$

Where H_0 is the uniform magnetic field. Using the transformation from equation (3) in equation (1) and (2), for spherical polar co-ordinates in presence of transverse magnetic field and for axi-symmetry, takes the form:

$$\frac{\partial}{\partial r}\left(r^2 u_i\right) + \frac{r}{\sin\theta}\frac{\partial}{\partial \theta}\left(v_i \sin\theta\right) = 0, \tag{4}$$

$$\begin{aligned}\frac{\partial p_i}{\partial r} =& \frac{\mu_i}{\mu}\left\{\frac{\partial^2 u_i}{\partial r^2} + \frac{2}{r}\frac{\partial u_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_i}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u_i}{\partial \theta} - \frac{2u_i}{r^2} - \frac{2}{r^2}\frac{\partial v_i}{\partial \theta} - \frac{2 v_i \cot\theta}{r^2}\right\}\\ &- \frac{\mu_h^2 \sigma_e H_0^2 a^2}{\mu} u_i,\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{1}{r}\frac{\partial p_i}{\partial \theta} =& \frac{\mu_i}{\mu}\left\{\frac{\partial^2 v_i}{\partial r^2} + \frac{2}{r}\frac{\partial v_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_i}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial v_i}{\partial \theta} + \frac{2}{r^2}\frac{\partial u_i}{\partial \theta} - \frac{v_i \cos ec^2\theta}{r^2}\right\}\\ &- \frac{\mu_h^2 \sigma_e H_0^2 a^2}{\mu} v_i. \,(6)\end{aligned} \tag{6}$$

As the motion is axi-symmetrical, the present problem is readily treated by means of stream function, $\psi(r,\theta)$. In terms of stream function the velocity components in spherical polar co-ordinates $(r,\theta,0)$ take the form:

$$u_i = \frac{1}{r^2 \sin\theta}\frac{\partial \psi_i}{\partial \theta}; \quad v_i = \frac{-1}{r \sin\theta}\frac{\partial \psi_i}{\partial r}. \tag{7}$$

Using equation (7) in equations (5) and (6) and on cross differentiation, we get

$$\left(E^4 - M^2 E^2\right)\psi_i = 0, \tag{8}$$

where $E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)$ is the Laplacian operator and

$M = \sqrt{\frac{\mu_h^2\sigma_e H_0^2 a^2}{\mu}}$ is the Hartmann number.

The boundary conditions for the problem which are physically realistic and mathematically stable are:

The normal and tangential velocity remains finite as $r \to 0$, i.e. u_1 and v_1 remains constant for $r \to 0$,

at $r = a$,

$$\left.\begin{array}{l} u_1 = 0 = u_2 \\ v_1 = v_2 \\ \frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial\psi_1}{\partial r}\right) = \lambda\,\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial\psi_2}{\partial r}\right) \end{array}\right\}, \tag{9}$$

where $\lambda = \frac{\mu_2}{\mu_1}$is the viscosity ratio.

At $r = b$

$$\left.\begin{array}{l} u_2 = V\cos\theta \\ r\frac{\partial}{\partial r}\left(\frac{v_2}{r}\right) + \frac{1}{r}\frac{\partial u_2}{\partial\theta} = 0 \end{array}\right\}. \tag{10}$$

3 Method of Solution

The solution of the Eq. (8) is considered in the form of (see Pop et al. 2010):

$$\psi_i(r,\theta) = f_i(r)\sin^2\theta. \tag{11}$$

On substituting equation (11) in equation (8), the partial differential equation of fourth order in stream function $\psi_i(r,\theta)$ reduces to ordinary differential equation of order four in $f(r)$ as:

$$f_i^{1V} - \frac{4}{r^2}f_{i\prime\prime}^{\prime\prime} + \frac{8}{r^3}f_i^{\prime} - \frac{8}{r^4}f_i - M^2\left(f_i^{\prime\prime} - \frac{2}{r^2}f_i\right) = 0. \tag{12}$$

To find the solution for the equation (12), we take the transformation as

$$\sqrt{r}g_i(r) = f_i^{\prime\prime} - \frac{2}{r^2}f_i. \tag{13}$$

Substituting the equation (13) in the equation (12), we get

$$r^2 g_i^{\prime\prime} + r g_i^{\prime} - \left(\left(\frac{3}{2}\right)^2 + Mr^2\right)g_i = 0 \quad . \tag{14}$$

Equation (14) is the modified Bessel differential equation of order 3/2 and the solution in terms modified Bessel functions as:

$$g_i(r) = C_i I_{\frac{3}{2}}(rM) + D_i K_{\frac{3}{2}}(rM), \tag{15}$$

where $I_{\frac{3}{2}}(rM)$ and $K_{\frac{3}{2}}(rM)$ are the modified Bessel's function of first and second kind respectively of order 3/2, and C_i and D_i are arbitrary constants of integration. Substituting equation (15) in equation (13), fourth order ordinary differential equation reduces to second order with variable co-efficient as:

$$f_i'' - \frac{2}{r^2} f_i = C_i\sqrt{r} I_{\frac{3}{2}}(rM) + D_i\sqrt{r} K_{\frac{3}{2}}(rM). \tag{16}$$

Equation (16) is linear differential equation of order two with variable co-efficient form and is solved completely using method of variation of parameter, the solution is:

$$f_i = \frac{A_i}{r} + B_i r^2 + C_i\sqrt{rM} I_{\frac{3}{2}}(rM) + D_i\sqrt{rM} K_{\frac{3}{2}}(rM). \tag{17}$$

where A_i and B_i are also arbitrary constants. From Eq. (17) we can write the solution in both the regions as:

$$f_1 = \frac{A_1}{r} + B_1 r^2 + C_1\sqrt{rM} I_{\frac{3}{2}}(rM) + D_1\sqrt{rM} K_{\frac{3}{2}}(rM), \quad 0 \le r < a \tag{18}$$

$$f_2 = \frac{A_2}{r} + B_2 r^2 + C_2\sqrt{rM} I_{\frac{3}{2}}(rM) + D_2\sqrt{rM} K_{\frac{3}{2}}(rM), \quad a \le r \le b. \tag{19}$$

The validity of the solution, from equation (18) is true for the flow in the region I provided $A_1 = 0$ and $D_1 = 0$ when $r \to 0$. Therefore the solution in, region I, is given by

$$f_1(r) = B_1 r^2 + C_1\sqrt{rM} I_{\frac{3}{2}}(rM), \quad 0 \le r < a, \tag{20}$$

here B_1, C_1, A_2, B_2, C_2 and D_2 are arbitrary constants. Also the respective boundary conditions, from equations (9) and (10) in $f(r)$ as

at $r = a$

$$\left.\begin{array}{l} f_1(r) = 0 = f_2(r), \\ f_1'(r) = f_2'(r), \\ \left(r f_1''(r) - 2f_1'(r)\right) = \lambda \left(r f_2''(r) - 2f_2'(r)\right) \end{array}\right\} \tag{21}$$

at $r = b$

$$\left.\begin{array}{l} f_2(r) = \frac{r^2 V}{2}, \\ r^3 f_2''(r) - r^2 f_2'(r) + 2f_2(r) = 0 \end{array}\right\}. \tag{22}$$

The constants in the solution of equations (19) and (20) are obtained with the help of the boundary conditions from equations (21) and (22), and are given in the appendix.

4 Results and Discussion

In the study, relative motion of two immiscible liquid spheres with the same origin of different viscosity is investigated for an electrically conducting fluid in the presence of applied magnetic field. The uniform magnetic field is applied in the direction perpendicular to the flow. The modified Stokes equation is considered to describe the flow in both the domains. The induced magnetic field is assumed to be negligible since the magnetic Reynolds number of the present study is very small. An analytical solution, in terms of stream function is obtained using similarity solution method. The obtained solution consists of modified Bessel's function of first and second kind of order 3/2. The constants are evaluated using Happel (1958) cell boundary conditions on the surface of the outer liquid sphere. The fluid flow is studied in terms of streamlines for various non-dimensional parameters present in the problem.

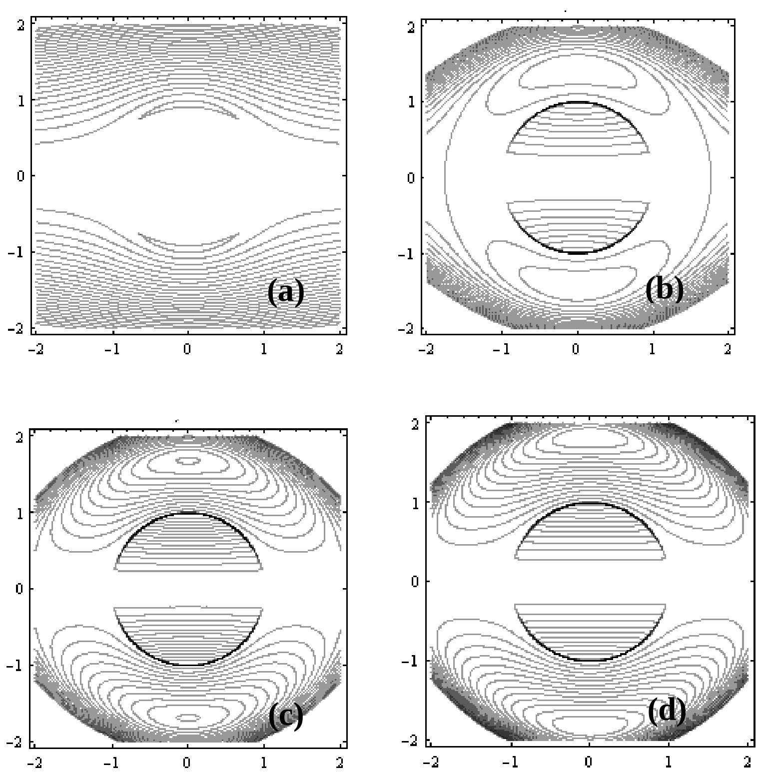

Figure 1: Streamlines for different values of M with $\lambda = 0.5$ (a) $M = 0$ (b) $M = 2$ (c) $M = 3$ (d) $M = 5$.

The effect of magnetic field is considered on the streamlines pattern for a fixed viscosity ratio λ. The streamline patterns for $\lambda = 0.5(\lambda < 1)$ are illustrated in Fig. 1. The flow is uniform for $M = 0$ when the inner fluid sphere is moving with the constant velocity V. As the magnetic field strength M increases to 2, the fluid is circulating in the inner fluid sphere and a uniform fluid flow is observed outside the sphere. Further, an increase in the Hartmann number to 3 and 5, the fluid circulation is visible in the outer sphere also. It is also, noted that the streamlines outside the sphere are concentrated on the surface of the inner sphere for$M = 3$. But streamlines are considerably free on the surface of the inner sphere for $M = 5$ and the amount fluid flow inside the sphere is also less.

The streamlines are studied for various Hartmann numbers when viscosities of the fluids remain same $(\lambda = 1)$, and the same is illustrated in Fig. 2. It is observed that the circulation of the fluid appear both in inner and outer sphere. Also the amount of fluid flow in the inner sphere is more when the Hartmann number is increased.

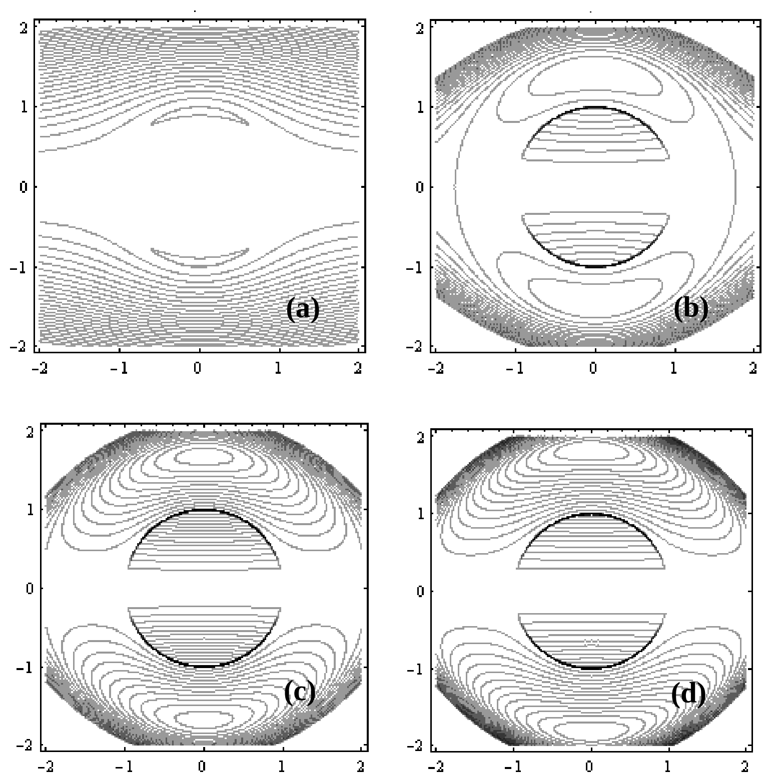

Figure 2: Streamlines for different values of M with $\lambda = 1$ (a) $M = 0$ (b) $M = 2$ (c) $M = 3$ (d) $M = 5$.

Further, Fig. 3 reveals the streamlines for different Hartmann numbers when the viscosity of the fluid for outer sphere is more than that of inner sphere. In this case it has been noticed that the circulation of the fluid in and around the inner sphere and the streamlines are uniform in absence of magnetic field. The increase in magnetic field strength leads to the fluid circulation in the inner and outer sphere. The meandering of streamlines around the inner sphere amplifies by amplifying the Hartmann number. Thus, it can be concluded that the effect of magnetic field is to reduce the flow circulation for a fixed value of viscosity ratio.

Streamlines are also discussed for various viscosity ratios with fixed Hartmann number and are illustrated in Fig. 4. The fluid flow past an inner sphere is observed for lower values of viscosity ratio, but the circulation of the fluid in the inner spherical region is noticed and illustrated in Fig. 4.

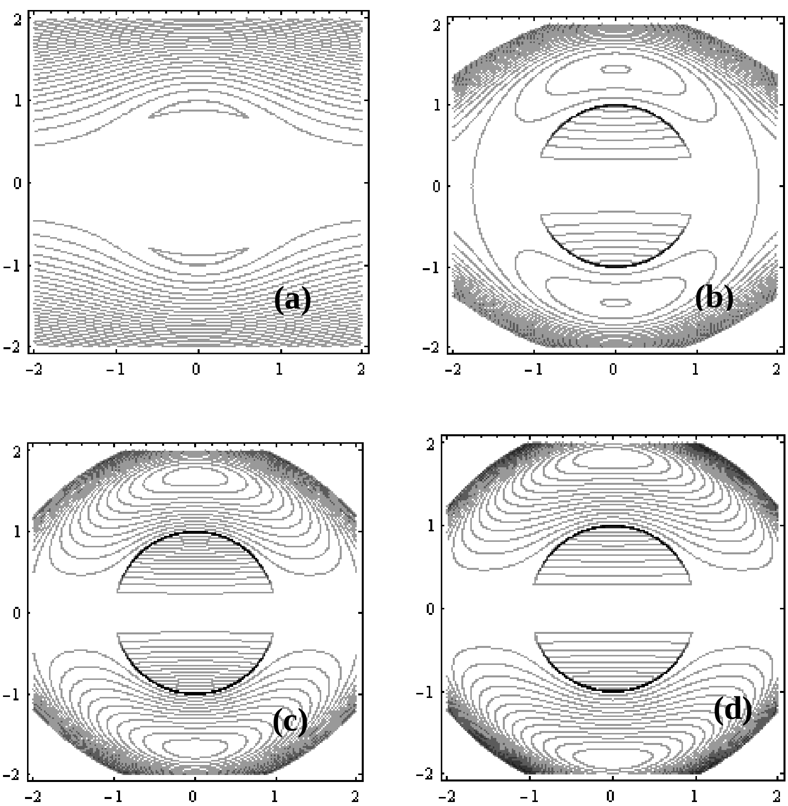

Figure 3: Streamlines for different values of M with $\lambda = 1.5$ (a) $M = 0$ (b) $M = 2$ (c) $M = 3$ (d) $M = 5$.

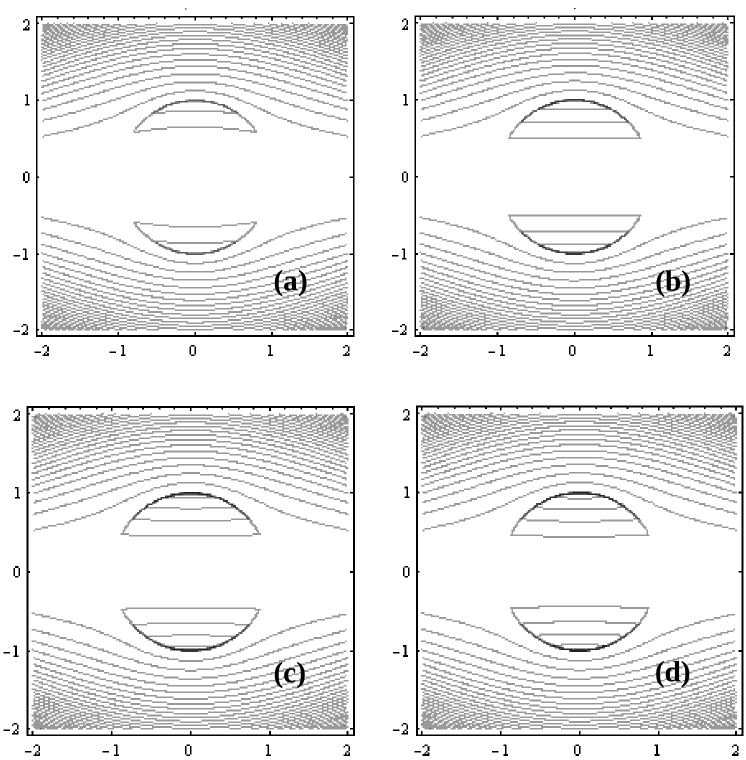

Figure 4: Streamlines for different values of λ with $M = 1$ (a) $\lambda = 0.001$ (b) $\lambda = 0.1$ (c) $\lambda = 0.5$ (d) $\lambda = 1.0$.

5 Conclusions

The phenomenon, concerning the flow of immiscible fluids has a definite role in chemical engineering and medicine. In the view of this, here we have considered the steady flow related to two immiscible fluid spheres of different viscosities in the presence of a transverse magnetic field for the case in which the inner sphere moves with a constant velocity. The modified Stokes equations have been considered to describe the flow field. The flow has been assumed two dimensional and axi-symmetric. Hence, the analytical solution has been obtained in terms of stream function by a similarity solution method (the obtained solution is a function of modified Bessel's function of order 3/2).

Streamlines have been plotted to understand the fluid flow behavior when the strength of the magnetic field is increased.

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Appendix

$$D_2 = \frac{-n_3 n_4}{n_1 n_5 - n_4 n_2}, \quad C_2 = \frac{1}{n_1}\left(n_3 - n_2 D_2\right),$$

$$B_2 = \frac{1}{m_1 a^3 - m_2}\left\{(m_3 - m_1 l_2)C_2 + (m_4 - m_1 l_3)D_2\right\},$$

$$A_2 = \frac{-1}{m_1}\left\{m_2 B_2 + m_3 C_2 + m_4 D_2\right\},\ \mathrm{B}_1 = \frac{l_1 D_1}{a^3}$$

$$D_1 = \frac{1}{l_4 - 2l_1}\left[-A_2 + 2a^3 B_2 - l_5 C_2 + l_6 D_2\right],$$

$$n_1 = m_1^2\left(a^3 l_{10} - b^3 l_2\right) - m_1 m_2\left(l_{10} - l_2\right) - m_1 m_3\left(a^3 - b^3\right)$$

$$n_2 = m_1^2\left(a^3 l_{11} - b^3 l_3\right) - m_1 m_2\left(l_{11} - l_3\right) - m_1 m_4\left(a^3 - b^3\right)$$

$$n_3 = \left(m_1 a^3 - m_2\right) m_1 l_{12},$$

$$n_4 = m_1^2\left(l_{14} a^3 - 2b^2 l_2\right) - m_1 m_2\left(l_{14} - l_2\right) - m_1 m_3\left(a^3 - 2b^2\right)$$

$$n_5 = m_1^2\left(l_{15} a^3 - 2b^2 l_3\right) - m_1 m_2\left(l_{15} - l_3\right) - m_1 m_4\left(a^3 - 2b^2\right)$$

$$m_1 = l_7 + 10 l_1 - 4 l_4, \quad m_2 = -2a^3\left(l_7 + l_4\right), \quad m_3 = l_5 l_9 + 2 l_1 l_5 + l_8 l_4 - 2 l_1 l_8,$$

$$m_4 = -2 l_1 l_6 + l_4 l_9 - l_6 l_7 - 2 l_1 l_9, \quad l_1 = a\cosh M_1 a - \frac{\sinh M_1 a}{M_1},$$

$$l_2 = e^{-M_2 a}\left(a + \frac{1}{M_2}\right),$$

$$l_3 = a\cosh M_2 a - \frac{\sinh M_2 a}{M_2}, \quad l_4 = \left(M_1 a - 1\right) a \cosh M_1 a + \frac{\sinh M_1 a}{M_1},$$

$$l_5 = e^{-M_2 a}\left(M_2 a^2 + a + \frac{1}{M_2}\right),$$

$$l_6 = \left(M_2 a - 1\right) a \cosh \mathrm{M}_2 a + \frac{\sinh M_2 a}{M_2},$$

$$l_7 = a\left(5 - 2M_1 a\right)\cosh M_1 a + \left(\left(M_1 a_1\right)^3 - \left(M_1 a\right)^2 - 4\right)\frac{\sinh M_1 a}{M_1}$$

$$l_8 = \lambda\left[\left(M_2 a\right)^3 + \left(3M_2 a\right)^2 + 4M_2 a + 4\right]\frac{e^{-M_2 a}}{M_2},$$

$$l_9 = \lambda \text{ a } (\text{5-2M}_2 a)\cosh M_2 a + \lambda\left[\left(M_2 a\right)^3 - \left(M_2 a\right)^2 - 4\right]\frac{\sinh M_2 a}{M_2}$$

$$l_{10} = \left(b + \frac{1}{M_2}\right) e^{-M_2 b}, \quad l_{11} = \left(b\cosh M_2 b\right) - \frac{\sinh M_2 b}{M_2},$$

$$l_{12} = \frac{b^3 v}{2}, l_{13} = \left(3 + \frac{2}{b}\right),$$

$$l_{14} = \left[\left(bM_2\right)^3 + 2\left(bM_2\right)^2 + 3\left(bM_2\right) + 2M_2 + 2 + \frac{2}{b}\right]\frac{e^{-M_2 b}}{M_2}$$

$$l_{15} = \left(4b - M_2 b^2 + 2\right)\cosh M_2 b + \left[\left(M_2 b\right)^3 - \left(M_2 b\right)^2 - 3 - \frac{2}{b}\right]\frac{\sinh M_2 b}{M_2}.$$