 FDMP, vol.6, no.4, pp.419-429, 2010

Finite Element Analysis of Elastohydrodynamic Cylindrical Journal Bearing

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Abstract: This paper presents a short and focused analysis of the pressure development inside the fluid film related to a journal bearing (i.e. the pressure distribution in the the gap between the shaft, generally referred to as the "journal", and the bearing). The related flow is considered to be isotherm, laminar, steady and incompressible. The lubricant is assumed to be an isoviscous fluid. The Reynolds equation governing the lubricant pressure is derived from the coupled continuity and momentum balance equations written in the framework of the Stokes theory. The non linear system given by coupled equations for fluid pressure development (the aforementioned Reynolds equation) and solid deformation (linear elasticity model) is solved using a fully coupled Newton Raphson procedure. The Reynolds equation is numerically solved resorting to the Galerkin finite-element-method. The results show that, when the elastic deformation takes place, there are obvious changes in the film pressure distribution, in the highest film pressure and in the film-thickness distribution.

Keywords: Finite element method, Reynolds equation, Elastohydrodynamic effects, Pressure, Lubricant.

1 Introduction

A journal bearing, sometimes referred to as a "friction" bearing, is a simple bearing in which a shaft, or "journal", or crankshaft rotates in the bearing with a layer of fluid (the lubricant) separating the two parts through fluid-dynamic effects. The shaft and bearing are generally both simple polished cylinders with lubricant filling the gap.

It is a well-known fact that elastic deformation of the journal (and of the bearing material) induced by hydrodynamic-fluid-pressure effects can change the fluid film profile, modify the pressure distribution and, therefore, alter the performance characteristics of journal bearings [Higginson (1965)].

The use of surface coatings such as white metals and elastomers can lead to significant deformations of the surface of the bearing which can be about the size of thickness of lubricating film. These coatings, used with the aim of reducing wear, are generally characterized by low values of the modulus of elasticity [Jain and all (1982); Conway and all (1975)].

The effect of the deformation of the bearing shell on journal bearing performance characteristics was reported by Carl [Carl (1963)]. He experimentally demonstrated the effect of bearing deformation on pressure distribution in the clearance space. Benjamin and Castelli [Benjamin and Castelli (1971)] investigated the effect of deformation on the pressure distribution and load-carrying capacity of journal bearings by assuming full Sommerfeld boundary conditions [Sommerfeld (1904)].

In this work, we are interested in the study of the influence of the elastic strain of surface coatings on the pressure distribution of a long journal bearing lubricated by a piezoviscous and compressible Newtonian fluid, and namely the maximum pressure and the minimal thickness of oil film.

In particular, this study is based on a discretization of the nonlinear equation of Reynolds by the finite element method and a resolution of the resulting equations system by the Newton-Raphson method. The elastic thin layer model is used for the calculation of the field of elastic strain of the coating [Lahmar and Nicolas (2002)].

The laws of variation of the viscosity and the density of the fluid lubricating with the pressure are those of Barus [Barus (1893)] and [Dowson (1977)]. This work is concerned with the study of the effects of the different materials bearing, the viscosity, the bearing speed and the radial clearance on the pressure distribution.

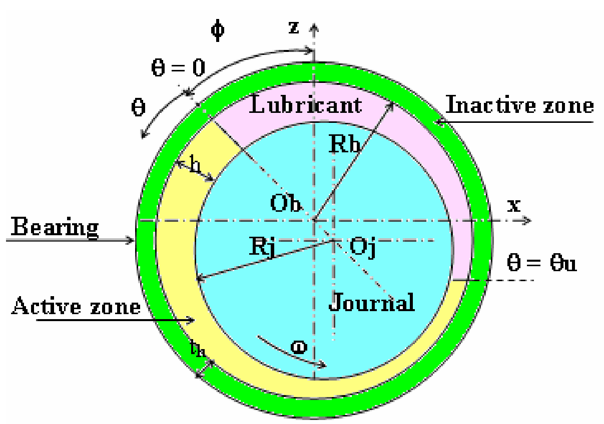

Figure 1: Journal bearing configuration and coordinate system

2 Elastohydrodynamic analysis

2.1 Reynolds equation

The Reynolds equation governing the lubricant pressure in the journal bearing [Reynolds (1886)], is an elliptic, partial and differential equation for the pressure in terms of the lubricant properties, density and viscosity, as well as the film thickness.

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right)=6\mu U\frac{\partial h}{\partial x} \tag{1}$$

To simplify the numerical analysis for journal bearing, the following dimensionless parameters and variables are used, Fig. 1:

$$\bar{h}=\frac{h}{C};\ \theta=\frac{x}{R};\ \bar{y}=\frac{y}{l};\ \bar{p}=\frac{p}{6\mu_0\omega\left(R/C\right)^2};\ \lambda=L/D;\ \bar{\mu}=\frac{\mu}{\mu_0};\ \bar{\rho}=1+\frac{0.6\bar{p}}{1+1.7\bar{p}} \tag{2}$$

The viscosity of most oils increases with pressure and the following relationship is assumed [Molimard and Le Richea (2003)]

$$\bar{\mu}=e^{\bar{\alpha}\bar{p}} \tag{3}$$

Where:
$\bar{\alpha}=\mu_0\omega\left(R/C\right)^2\alpha$ μ is the viscosity of the oil,
μ_0 is the zero pressure viscosity of the ambient temperature,
α is the piezo-viscosity coefficient.

After simplification, the modified Reynolds equation can be expressed:

$$\frac{\partial}{\partial\theta}\left(\frac{\bar{\rho}\bar{h}^3}{\bar{\mu}}\frac{\partial\bar{p}}{\partial\theta}\right)+\frac{1}{4\lambda^2}\frac{\partial}{\partial\bar{y}}\left(\frac{\bar{\rho}\bar{h}^3}{\bar{\mu}}\frac{\partial\bar{p}}{\partial\bar{y}}\right)=\frac{\partial\bar{\rho}\bar{h}}{\partial\theta} \tag{4}$$

2.2 The film thickness equation

2.2.1 The film thickness equation of a rigid journal bearing

The dimensionless local oil film thickness equation of a rigid journal bearing can be written as:

$$\bar{h}=1+\varepsilon\cos\theta \tag{5}$$

where $\varepsilon=e/C$ is the eccentricity ratio, C is the radial clearance, e denotes the eccentricity and θ is the angular position, measured from the diameter of the bearing that also passes through the centre of journal. θ is zero where h is maximum.

2.2.2 The film thickness equation of a compliant journal bearing

The dimensionless film thickness expression of a compliant journal bearing is written [Rhode (1975)] :

$$\bar{h} = 1 + \varepsilon \cos\theta + \bar{L}_0 \bar{p} \tag{6}$$

Where

$$\bar{L}_0 = \frac{(1+\nu)(1-2\nu)}{1-\nu} \bar{C}_d \bar{t}_h \tag{7}$$

and

$$\bar{C}_d = \frac{\mu_0 \omega \left(\frac{R}{C}\right)^3}{E} \tag{8}$$

$\bar{p}$ is the dimensionless hydrodynamic pressure generated in the lubricating film, $\bar{L}_0$ is the dimensionless operator of compliance, E and ν are respectively the Young modulus and the Poisson's ratio of the elastic layer, $\bar{t}_h = t_h/R$ is the relative thickness of the elastic layer and $\bar{C}_d$ is the coefficient of deformation.

3 Approximate Solution for Long Bearings

In this condition, the approximation of Sommerfeld [Sommerfeld (1904)] supposes that the flow in the y direction is neglected ($\partial \bar{p}/\partial \bar{y} = 0$). When the length to diameter ratio of bearing λ is more than $\lambda > 4$, the bearing is said to be infinitely long ($\lambda = L/D \to \infty$). The dimensionless pressure distribution of the fluid film in the journal bearing can be expressed by the Sommerfeld function:

$$\bar{p} = \frac{\varepsilon \sin\theta \,(2 + \varepsilon \cos\theta)}{(2+\varepsilon^2)(1+\varepsilon\cos\theta)^2} \tag{9}$$

4 Finite element analysis

For the equation (4) and the boundary conditions, we can write the following weighted residual expression [Zienkiewicz and Taylor (1988)]:

$$W(\bar{p}) = \int_{\vartheta} \delta \bar{p} \left[\frac{\partial}{\partial\theta}\left(\frac{\bar{\rho}\bar{h}^3}{\bar{\mu}} \frac{\partial \bar{p}}{\partial\theta} \right) + \frac{1}{4\lambda^2} \frac{\partial}{\partial\bar{y}} \left(\frac{\bar{\rho}\bar{h}^3}{\bar{\mu}} \frac{\partial\bar{p}}{\partial\bar{y}} \right) - \frac{\partial\bar{\rho}\bar{h}}{\partial\theta} \right] d\vartheta = 0 \tag{10}$$

Integrating (10) by parts, we have:

$$
\begin{aligned}
W(\bar{p}) = & \int_{\vartheta} \left\{ \frac{\bar{\rho}\bar{h}^3}{\bar{\mu}} \left(\frac{\partial(\delta\bar{p})}{\partial\theta}\frac{\partial\bar{p}}{\partial\theta} + \frac{1}{4\lambda^2}\frac{\partial(\delta\bar{p})}{\partial\bar{y}}\frac{\partial\bar{p}}{\partial\bar{y}} \right) - \frac{\partial\bar{\rho}\bar{h}}{\partial\theta}(\delta\bar{p}) \right\} (d\vartheta) \\
& - \int_S \left(\frac{\bar{\rho}\bar{h}^3}{\bar{\mu}}\frac{\partial\bar{p}}{\partial\theta} \right)(\delta\bar{p})\,dS - \frac{1}{4\lambda^2}\int_S \left(\frac{\bar{\rho}\bar{h}^3}{\bar{\mu}}\frac{\partial\bar{p}}{\partial\bar{y}} \right)(\delta\bar{p})\,dS \\
& = 0
\end{aligned} \tag{11}
$$

Finally, equation (11) can be written as:

$$
\begin{aligned}
& \int_{\vartheta} \left[\frac{\bar{\rho}\bar{h}^3}{\bar{\mu}} \left(\frac{\partial(\delta\bar{p})}{\partial\theta}\frac{\partial\bar{p}}{\partial\theta} + \frac{1}{4\lambda^2}\frac{\partial(\delta\bar{p})}{\partial\bar{y}}\frac{\partial\bar{p}}{\partial\bar{y}} \right) - \frac{\partial\bar{\rho}\bar{h}}{\partial\theta}(\delta\bar{p}) \right] d\vartheta \\
& \qquad - \frac{\bar{\rho}\bar{h}^3}{\bar{\mu}} \int_{S_f} (\delta\bar{p})\, f_s dS = 0
\end{aligned} \tag{12}
$$

Lubricant pressure and its variation can be interpolated as follows:

$$
\bar{p} = <N> \{\bar{p}\}\,; \delta\bar{p} = <N> \{\delta\bar{p}\} \tag{13}
$$

where N are the shape functions, $\{\bar{p}\}$ and $\{\delta\bar{p}\}$ are nodal pressures and pressures variations, respectively. The discretization shape of the elementary integral is obtained by the substitution of relations (13). One finds:

$$
W^e = <\bar{p}_n> \begin{pmatrix} \int \frac{\bar{\rho}\bar{h}^3}{\bar{\mu}} \left[\left\{ \frac{\partial N}{\partial\theta} \right\} < \frac{\partial N}{\partial\theta} > + \frac{1}{4\lambda^2} \left\{ \frac{\partial N}{\partial\bar{y}} \right\} < \frac{\partial N}{\partial\bar{y}} > \right] \{\bar{p}_n\}\, d\vartheta \\ - \int_{\vartheta^e} \frac{\bar{\rho}\bar{h}^3}{\bar{\mu}} \frac{\partial\bar{h}}{\partial\theta} \{N\}\, d\theta - \frac{\bar{\rho}\bar{h}^3}{\bar{\mu}} \int_{S_f^e} f_s \{N\}\, dS \end{pmatrix} \tag{14}
$$

The global system equation is obtained by assembling the elementary integrals (14). It can be expressed in matrix form as follows:

$$
[K]\{\bar{p}\} = \{Q\} \tag{15}
$$

where $[K]$ is modified to take into account the pressure boundary conditions. Flow boundary conditions have already been included in $\{Q\}$, the resultant or net vector of nodal flows. Often, to save computer storage space, the procedure used to solve equation (15) takes advantage of the fact that $[K]$ is banded and symmetric. The following boundary conditions are used to obtain the dimensionless pressure dimensionless distribution of the fluid film in the bearing.

Table 1: Calculating conditions.

Geometrical characteristics			**Operating conditions**		
Rj,	Journal radius (m)	0.025	μ_0,	Viscosity of the lubricant(Pa.s)	0.03
Rb,	Bearing radius (m)	0.02505	ρ0,	Density of the lubricant(kg/m3)	870
th,	Thickness of bush (m)	0.01	N,	Journal speed (rpm)	1000
L,	Bearing length (m)	0.07	ν,	Poisson's ratio	0.30
C,	Radial clearance (μm)	50	EBr,	Elastic modulus(GPa)	117

5 Results and Discussions

Table 1 describes the calculating conditions related to the journal bearing considered in the present study.

The numerical resolution of the Reynolds equation gives the pressure at any point between the journal and the bearing. The Reynolds boundary conditions are used to analyze the pressure distribution in the film between the journal and the bearing [Dammak (2008)].

The lubricated zone $\theta = 0 \rightarrow 7\pi/6$ has been divided into 34 pressure elements with 35 nodes. One pressure element contains 14 sub-elements; therefore, the total number of sub-elements is 476. The film pressure distribution of the lubricant fluid, for $\varepsilon = 0.8$, $\lambda = 1.4$ and $N = 1000rpm$ on the long journal bearing, is plotted in Figure 2.

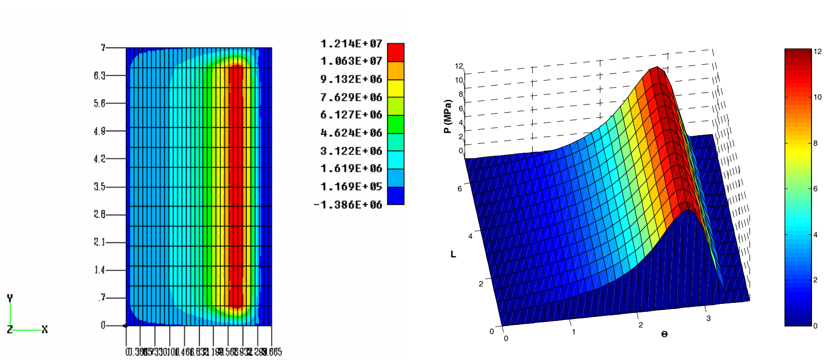

Figure 2: Circumferential pressures distribution of rigid bearing, with $\varepsilon = 0.8$ and $N = 1000rpm$.

It is obvious that the pressure builds up gradually from $\theta = 0$ to the maximum value, and then fades away more quickly to satisfy the Reynolds conditions.

Figure 3 shows a comparison between the pressure function of Sommerfeld and the numerical result of the Reynolds equation in the case of the infinite long bearing under the boundary conditions of Gümbel [Gümbel (1925)]. It can be seen that, there is a little difference between the two positive pressure zones.

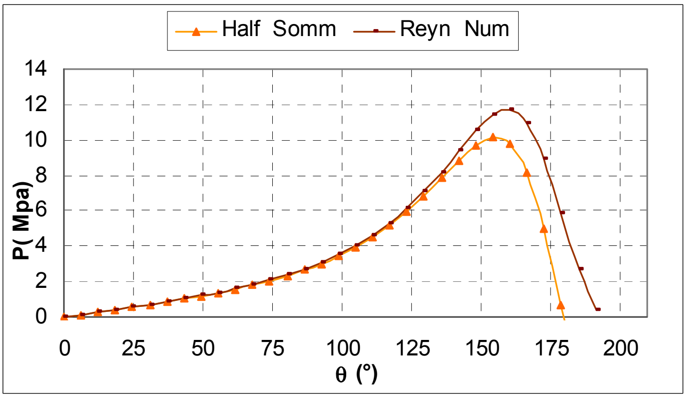

Figure 3: Circumferential pressures distribution θ in mid plane at $\varepsilon = 0.8$ and $\lambda = 1.4$.

5.1 *The effect of the materials on the pressure*

The comparison of the film pressure amongst the rigid ($E = \infty$), the bronze ($E_{Br} = 117GPa$), the Babbitt ($E_{Ba} = 29GPa$), and the Perspex ($E_P = 2.38GPa$) bearing is displayed in Figure 4, for $\varepsilon = 0.8$g and $N = 1000rpm$ at the middle of the journal bearing: $y = 3.5mm$. Apparently, the pressure p increases when the value of the elastic modulus E increases.

Figure 5 shows the influence of the pressure on the elastic strain of coatings of surface for different materials. The minimum film thickness increases when the elastic modulus E decreases.

The numerical values can be seen in the table 2.

5.2 *The effect of the viscosity on the pressure*

The effect of the viscosity on the pressure distribution is presented in Figure 6. It can be noted that when the viscosity μ, increases, the film pressure decreases.

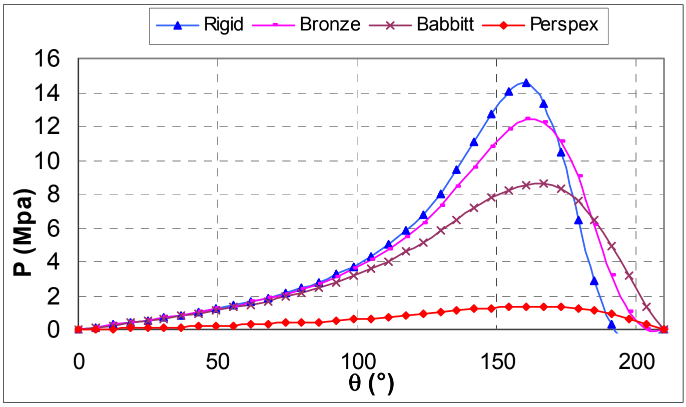

Figure 4: Circumferential pressures distribution in mid plane for different materials.

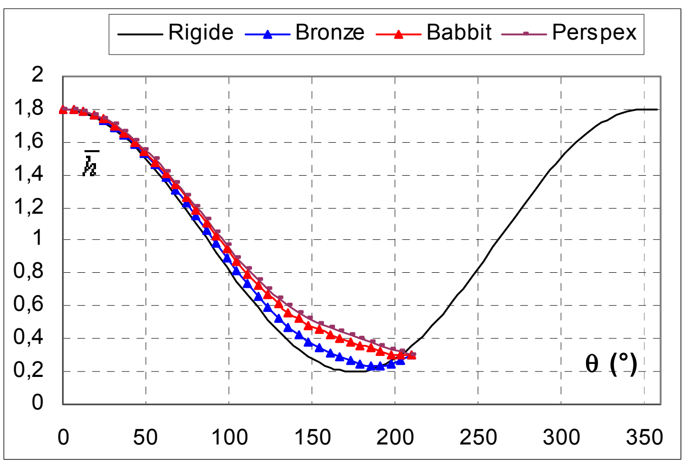

Figure 5: Dimensionless film thickness versus θ for different materials.

Table 2: Comparison of the minimum film thickness and the angular position for different materials.

Materials	Rigid	Bronze	Babbitt	Perspex
h_m	0.2004	0.2327	0.2974	0.3071
θ (°)	172°	191°	203°	210°

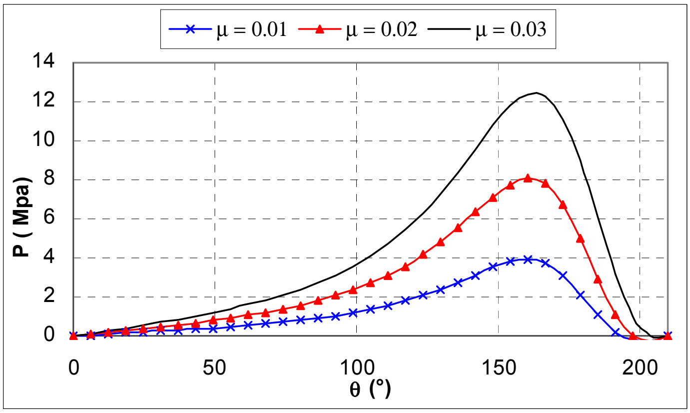

Figure 6: Circumferential pressure distribution for various viscosity.

5.3 The effect of the rotational speed on the pressure

The variation of the rotational speed has a very significant effect on the pressure distribution, which causes the modification of the characteristics of an elastohydrodynamic journal bearing as shown in Fig 7.

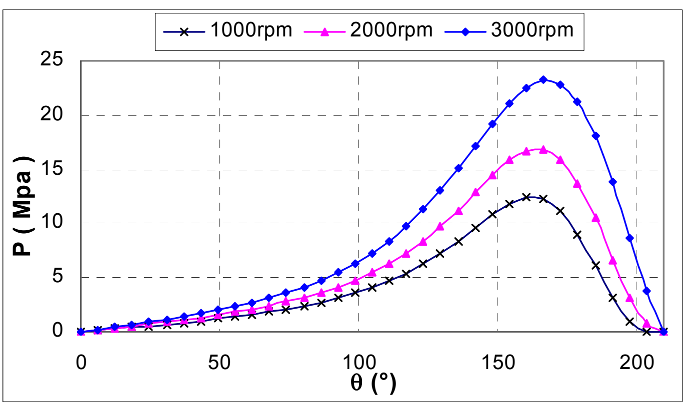

Figure 7: Circumferential pressure distribution for various rotational speeds.

5.4 The effect of the radial clearances on the pressure

Figure 8 shows the effect of the radial clearance (C) on the pressure distribution of the journal bearing. It is obvious that the pressure increases when the radial clearance decreases.

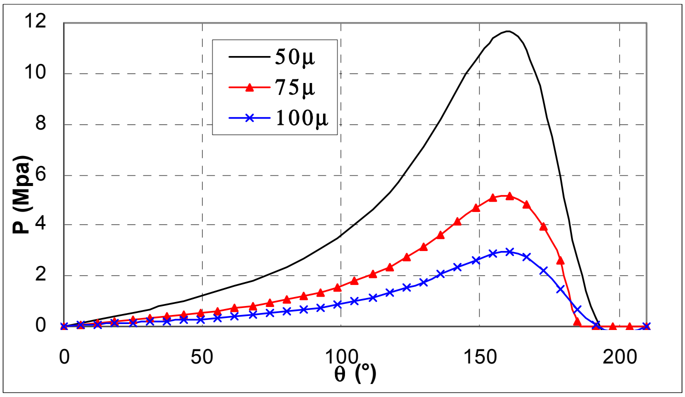

Figure 8: Circumferential pressure distribution for various radial clearances.

6 Conclusion

In this paper, we have investigated the influence of the elastic strain of surface coatings on the performances of a long journal bearing lubricated by a piezoviscous and compressible Newtonian fluid. The parametric study has shown that the surface coatings elasticity and the fluid rheology have significant effects on the journal bearing behavior.

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