



ARTICLE

A New Well-Testing Method for Pumping-Shutdown Data of Multi-Fractured Horizontal Wells: A Case Study from the Sichuan Shale Gas Basin

Xuefeng Yang^{1,2}, Chunyu Ren^{1,2}, Deliang Zhang^{1,2}, Huaicai Fan^{1,2}, Yue Chen^{1,2}, Yue Yang^{1,2}, Yan Zhang^{1,2}, Shuai Wu^{1,2}, Baoyun Zhang^{3,*} and Xin Zhao³

¹Shale Gas Research Institute, Southwest Oil & Gas Field Company, PetroChina, Chengdu, China

²Shale Gas Evaluation and Exploitation Key Laboratory of Sichuan Province, Chengdu, China

³Key Laboratory of Petroleum Resources and Engineering, China University of Petroleum (Beijing), Beijing, China

*Corresponding Author: Baoyun Zhang. Email: 2024315101@student.cup.edu.cn

Received: 22 October 2025; Accepted: 09 December 2025; Published: 12 July 2026

ABSTRACT: In southern Sichuan's deep shale gas development, multi-stage fractured horizontal wells are commonly used. Evaluating fracturing results is challenging due to complex fracture networks. This study classifies fracture systems into four types: single-wing, bi-wing, branched, and serial fractures. A discrete fracture model (DFM) combined with matrix-fracture flow is used to establish a single-stage well testing interpretation model. To address multi-solution issues in well testing, an equivalent fracture network model based on a trilinear flow model is proposed, adjusting crossflow coefficients and the fracture network volume ratio. The study finds significant differences in the pressure derivative dip and the second linear flow stage onset with changes in these parameters. Sensitivity analysis shows that inner zone permeability affects the early and middle stages of the well testing curve. Using shut-in pressure data, a numerical well testing model is applied, with results showing a fracture zone permeability of 0.95 mD, a fracture half-length of 87 m, and fracture network volume ratio of 5%. This research provides guidance for evaluating fracturing effects.

KEYWORDS: Deep shale gas; single-stage fracture; well test analysis; fracture network morphology; discrete fracture model

1 Introduction

With the improvement of the level of exploration and development in the middle and shallow layers, global oil and gas exploration and development are facing increasing challenges [1,2]. In recent years, the development of deep shale gas has become increasingly significant in the exploitation of unconventional oil and gas resources [3,4]. The southern Sichuan region is a hotspot for shale gas exploration and development in China, with diverse shale gas reservoir types and abundant data. The estimated proven shale gas reserves are nearly $1.76 \times 10^{12} \text{ m}^3$ [5,6]. In the deep layers with depths of 3500–4500 m, significant breakthroughs in shale gas have been achieved in regions such as Luzhou and Western Chongqing.

Unconventional oil and gas reservoirs usually rely on horizontal well fracturing development technology, volumetric fracturing is the key technology that enables deep shale gas to have commercial development value [7–9]. Therefore, the evaluation of post-fracturing fracture network effects is crucial for development decisions in deep shale gas reservoirs. For the complex fracture network assessment of volumetric fracturing, the main methods currently used are inter-well microseismic technology [10,11] and modern well testing analysis methods [12,13]. Inter-well microseismic technology can obtain fracture geometries



and locations [14,15]. However, this method lacks the ability to quantitatively evaluate parameters such as fracture permeability, fracture leakage capacity, and fracture conductivity. Modern well testing analysis methods can allow for obtaining fracture geometries and reservoir parameters by analyzing the unstable pressure response characteristics of fractured wells. These parameters include wellbore storage coefficient, skin factor, matrix permeability, fracture permeability, fracture leakage capacity, fracture conductivity, and effective fracture half-length. This allows for a quantitative assessment of the fracturing effect on both the well and the reservoir [16,17].

Currently, many scholars have conducted extensive research on the transient pressure response of fractured horizontal wells [18–21]. Reference [22] studied the unstable pressure characteristics of fractured horizontal wells with finite and infinite flow and multi-fracture systems. They also plotted the influence of fracture skin and wellbore storage effects on pressure. Reference [23] focused on the numerical simulation of the multi-stage hydraulic fracturing process in horizontal wells, and further studied the influence of perforation parameter optimization on fracture propagation and fracturing effectiveness. Reference [24] proposed a semi-analytical model based on the Green's function and source/sink method for transient pressure analysis of multi-stage fractured horizontal wells in closed box-type reservoirs. Subsequently, reference [25] based on the point-source solution model given by reference [26] for dual-porosity reservoirs with top and bottom closure and infinite boundaries, used the Green's function method, through mirror mapping and superposition principles, to obtain the Laplace-space solution for the bottom-hole pressure response of fractured horizontal wells with single or multiple fractures in fractured oil and gas reservoirs under top and bottom closed, infinite, and constant-pressure boundary conditions. These research results indicate that unstable well test analysis of fractured horizontal wells has made significant progress in theoretical models and numerical methods, providing strong technical support for the evaluation of hydraulic fracturing effectiveness and inversion analysis of reservoir parameters in oil and gas reservoirs.

The volume fracturing technology typically induces the continued expansion of natural fractures. As the pressure increases, a complex fracture network is formed, interconnecting hydraulic fractures. In recent years, scholars have established corresponding seepage mathematical models to characterize the fluid seepage characteristics in such complex fracture networks [27–31]. Reference [32] first explicitly proposed the concept of Stimulated Reservoir Volume (SRV) to characterize the relationship between the scale of volume. Reference [33] based on the trilinear flow model proposed by reference [34], established a horizontal well model for complex fracture networks under fracturing, dividing the fluid flow process into three distinct regions: flow in the formation, in natural fractures, and in artificial fractures. Reference [35] used a discrete fracture model (DFM) to characterize complex fracture geometry and achieved local refinement around the fracture, enabling the DFM to accurately capture transient flow behavior around the fracture. Although the DFM model ensures a certain level of computational accuracy, it greatly reduces computational efficiency. Reference [36] simulated complex fractures using the Embedded Discrete Fracture Model (EDFM). Unlike traditional discrete fracture models that require the grid to conform to the geometry of fractures, the EDFM allows fractures to be embedded within an existing grid without the need for grid alignment. This approach can accurately capture fluid flow between fractures and the surrounding matrix while significantly reducing computational cost. Reference [37] proposed a new analytically modified embedded discrete fracture model (AEDFM), which modifies the conductivity of EDFM to capture the transient pressure behavior of the well. Reference [38] considered the different connection relationships between artificial and natural fractures and established a multi-stage hydraulic fracturing horizontal well pressure transient analysis model. Research has shown that the pressure transient curve can be divided into six flow stages: bilinear flow, NFs fluid supply, linear flow, early pseudoradial flow, elliptical flow, and latter pseudoradial flow.

Deep shale gas reservoirs exhibit strong heterogeneity, and after fracturing, the fracture network morphology and distribution near the main fracture surface can be very complex [39,40]. The complex fracture network distribution severely restricts the parameter inversion and evaluation of the reservoir. In order to efficiently carry out reservoir parameter inversion, fracturing effect evaluation, and other tasks, it is urgent to establish an appropriate unstable pressure well-test interpretation model for deep shale gas reservoirs to quantitatively evaluate well and reservoir parameter information. Reference [41] interpreted the shut-in pressure drop data from the main fracturing stage of horizontal well volume fracturing using fracturing software. They used regression analysis namely G-function analysis, and fracture closure analysis namely FR-function analysis to obtain reservoir and fracturing parameters. Reference [42] introduced the flow coupling expressions of the main fracture-matrix, secondary fracture-matrix, and main fracture-secondary fracture into the traditional material balance equation of the main fracture. They established a “main fracture-secondary fracture-matrix” loss coupling flow model, and simultaneously coupled the stress sensitivity equation of the main and secondary fracture widths with the wellbore injection rate equation for the pressure-flow continuity solution. Reference [43] considered the stress sensitivity of the formation and the post-fracturing formation improvement effect, and established a linear heterogeneous composite zone fracturing well model suitable for shut-in pressure inversion. They used the established model and numerical well-test analysis to determine fracture and formation parameters. Reference [44] proposed a numerical well testing method based on the Discrete Fracture Model (DFM) to analyze the pressure transient response between vertical wells and natural fractures in response to the limitations of the dual-porosity and dual-permeability model (DPDK) in describing natural fracture reservoirs. However, this method has not yet taken into account the influence of fracture geometry on bottomhole pressure response.

Although various well testing models have been developed for analyzing complex fractures, it is difficult for these models to accurately characterize the complex and connected fracture network structure formed after fracturing, resulting in multiplicity in well testing inversion. To address this issue, this study first divides fracture geometry into four types based on their complexity from low to high: single wing fractures, double-wing fractures, branch fractures, and chain fractures, and constructs corresponding single-stage well testing interpretation models. Subsequently, to further address the ambiguity of fracture network inversion, an equivalent fracture network model is established based on the concept of trilinear flow. By adjusting key parameters such as the crossflow coefficient and the volume ratio of the fracture network, the equivalent characterization of the flow behavior of different fracture systems is achieved. Finally, by combining typical deep shale gas wells in southern Sichuan, the applicability and practical value of the modeling framework proposed in this study for well testing interpretation of complex fracture systems are verified.

2 Method

2.1 Fracture Morphology Classification

During the fracturing process, multiple parallel extending fractures, along with the continuous expansion of the fractures, cause secondary fractures near the main fracture to continue extending, thereby forming multiple branched fractures and creating a complex multi-branched fracture network. Due to factors such as geological and engineering conditions, the development of the fracture network after fracturing varies. Due to the presence of stress anisotropy, fracture opening is a common phenomenon during hydraulic fracturing. At this time, a hydraulic fracture is typically formed by multiple orthogonal fractures. When stress anisotropy is significant, fractures may sometimes open non-orthogonally, forming a tree-like fracture network. Multiple fractures may develop radially along the wellbore, forming a radial multi-fracture network type. In addition, some fracturing images display the complex geometry of a mutually orthogonal fracture network, which consists of fractures in two different and orthogonal directions. To better simulate the

development of fractures under different conditions, the above fracture morphologies are simplified: single fractures that do not form a fracture network are classified as single-wing fractures; two-wing fractures that do not form a network are classified as two-wing fractures; and depending on the degree of fracture network development, branched fractures and chain fractures are formed.

After a reservoir undergoes large-scale hydraulic fracturing, multiple secondary fractures extending from the primary fracture will form around the hydraulic main fracture. These secondary fractures continue to expand and create a multi-stage fracture system, resulting in a complex fracture network between the fractures. To accurately characterize the morphological features of each segment of the fractures after fracturing, this paper categorizes the fractures into four types based on the complexity of the fracture network: single-wing fracture, double-wing fracture, branched fracture, and chain-like fracture. As shown in Fig. 1, the equivalent fracture network model divides the reservoir into a complex fracture network zone and a matrix zone. The green part is the complex fracture network transformation zone, including complex fractures that communicate with each other, and the red part is the matrix zone.

- (1) Single-wing fracture: a single dominant fracture extending on one side of the wellbore. (Fig. 1a)
- (2) Double-wing fracture: two symmetric fractures extending in opposite directions from the wellbore. (Fig. 1b)
- (3) Branched fracture: a primary fracture connected to several secondary branches, forming a diverging pattern. (Fig. 1c)
- (4) Chain-like fracture: multiple fractures connected sequentially, forming a continuous channel-like structure. (Fig. 1d)

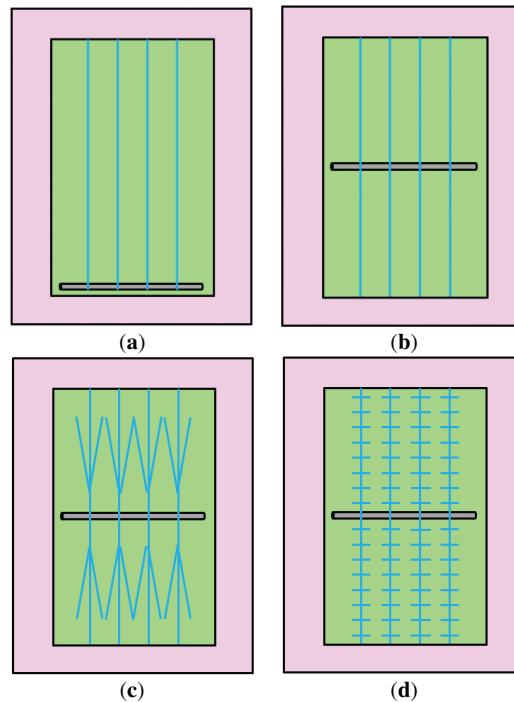


Figure 1: Four types of fracture morphology Schematic diagram: (a) Single-wing fracture (single segment) schematic diagram. (b) Double-wing fracture (single segment) schematic diagram. (c) Branch fracture (single segment) schematic diagram. (d) Chain fracture (single segment) schematic diagram

2.2 Discrete Fracture Network Model

2.2.1 Physical Model

Discrete Fracture Network (DFM) technology can explicitly characterize the attributes of fractures, including different orientations, sizes, spatial distributions, morphological features, and transmissivities. To study the impact of different fracture morphologies on typical well testing curves, this paper is based on discrete fracture descriptions and establishes corresponding single-segment numerical well testing interpretation models based on four different fracture systems. Fig. 2 illustrates the unstructured mesh characteristics of four fracture geometries represented by the discrete fracture models. The discretized mesh reflects how the fractures affect local grid refinement. The green zone contains a more refined mesh to capture the increased flow conductivity associated with the fracture structures.

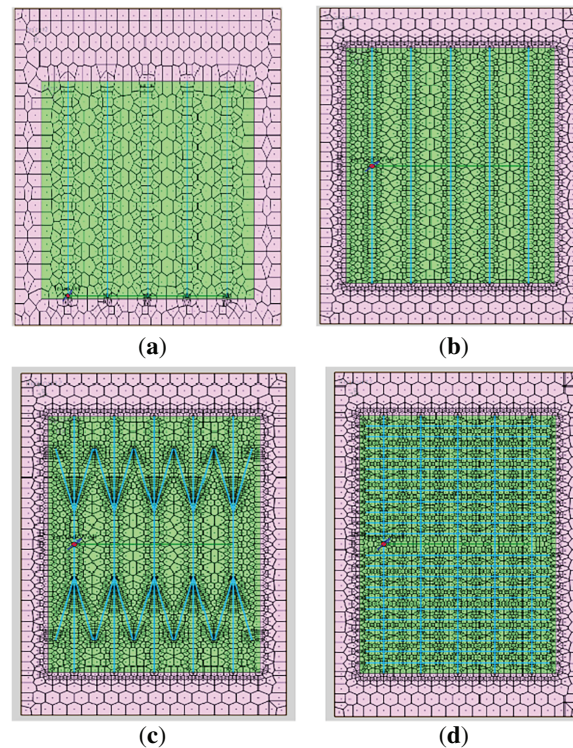


Figure 2: Four types of fracture—Discrete Fracture Model schematic diagram: (a) Single-wing fracture—Discrete Fracture Model schematic diagram. (b) Double-wing fracture—Discrete Fracture Model schematic diagram. (c) Branch fracture—Discrete Fracture Model schematic diagram. (d) Chain fracture—Discrete Fracture Model schematic diagram

The assumptions for the discrete fracture network (DFN) model well are as follows:

1. The reservoir is a closed homogeneous reservoir, bounded and horizontal with uniform thickness.
2. The fracture network is finite, with the reservoir penetrated by a finite number of conductive fractures in vertical planes, excluding factors such as fracture mechanical deformation.
3. There is fluid exchange between the reservoir matrix and fractures; pressure losses in the wellbore and the effects of gravity are not considered.
4. Only the stage of a single-stage fracturing in a horizontal well is considered, specifically the pumping stop stage.
5. The development degree of secondary fracture networks is low, and the contribution of external factors to the production in the stimulated area is considered.

2.2.2 Mathematical Model

To couple the flow between matrix-matrix, matrix-fracture, and fracture-fracture, the fluid exchange between matrix-fracture and fracture-fracture is added in the form of source and sink terms to their respective flow equations. This results in the analytical expressions for the matrix flow equation and the fracture flow equation. Among them, the matrix flow equation is shown in Eq. (1).

$$\frac{\partial(\phi^0 \rho)}{\partial t} + \nabla \cdot (\rho \vec{u}^0) - \rho \left[q^0 - \sum_{j=1}^{N_f} q^{0,j} \right] / V^0 = 0 \quad (1)$$

In the equation, the superscript denotes the fluid medium. When $k = 0$, it represents the matrix, and when $k \neq 0$, it represents the fracture.

Where N_f is the total number of fractures. ϕ^0 is the porosity of the matrix; ρ is the fluid density; q^0 is the source and sink term of the fluid in the matrix; V^0 is the volume of the matrix element; $\vec{u}^0$ is the seepage velocity of the fluid in the matrix; transfer function $q^{0,j}$ is the volume flow rate of fluid from the matrix to the j -th fracture.

The fracture flow equation is shown in Eq. (2).

$$\frac{\partial(\phi^k \rho)}{\partial t} + \nabla \cdot (\rho \vec{u}^k) - \rho \left[q^k - q^{k,0} - \sum_{j=1, j \neq k}^{N_f} q^{k,j} \right] / V^k = 0 \quad (2)$$

where ϕ^k is the porosity of the k -th fracture; q^k is the source and sink term of the fluid in the k -th fracture; V^k is the volume of the k -th fracture element; $\vec{u}^k$ is the seepage velocity of the fluid in the k -th fracture; the transfer function $q^{k,0}$ represents the volume flow rate of fluid from the k -th fracture to the matrix; the transfer function $q^{k,j}$ represents the volume flow rate of fluid from the k -th fracture to the j -th fracture.

The fluid in both the matrix and fractures follows Darcy's law, as shown in Eq. (3).

$$\vec{u}^k = -\frac{K^k}{\mu} (\nabla P^k - \rho g \nabla z) \quad (3)$$

where K^k is the absolute permeability of the fluid medium k ; μ is the viscosity of the fluid; ∇P^k is the pressure gradient of the fluid in the medium k ; ∇z is the altitude gradient; g is the gravitational acceleration. The model ignores the effect of gravity, therefore $\rho g = 0$.

The model takes into account the effects of wellbore storage and skin, and the inner boundary condition is assumed to be a constant production condition, as shown in Eq. (4).

$$\begin{cases} P_w = \left[P - s \cdot r \left(\frac{\partial P}{\partial r} \right) \right]_{r=r_w} \\ \left. \frac{K^k h}{\mu} 2\pi r_w \frac{\partial P_w}{\partial r} \right|_{r=r_w} = Bq + C \frac{\partial P_w}{\partial t} \end{cases} \quad (4)$$

where P_w is the bottom-hole pressure; P is the formation pressure; s is the skin factor; r is the formation radius; r_w is the wellbore radius; h is the formation thickness; B is the fluid volume factor; q is the source-sink term; C is the wellbore storage coefficient; t is time.

The model assumes that the outer boundary is a closed boundary, as shown in Eq. (5).

$$\left. \frac{\partial P^k}{\partial r} \right|_{\Gamma} = 0 \quad (5)$$

The model assumes that the initial reservoir pressure is uniform and equal to P_i , as shown in Eq. (6).

$$P^k = P_i \left(k \in [0, N_f] \right) \quad (6)$$

This paper discretizes the seepage equations using the finite volume method, and then obtains the discretized difference equations for the fracturing well model. The system of equations is solved using the Newton-Raphson iteration method to obtain the pressure response of the reservoir, as shown in Eqs. (7)–(9).

In order to make the discrete equation easy to understand and express, the discrete operators div and grad are used to represent the divergence operator and gradient operator.

$$\begin{cases} -\text{div} \left(\rho \frac{K^0}{\mu} [\text{grad} (P^{0(n+1)})] \right) + \rho \left[Q^0 - \sum_{k=1}^{N_f} Q^{0,k} \right] \\ = \frac{(\rho \phi^0 V^0)^{(n+1)} - (\rho \phi^0 V^0)^{(n)}}{\Delta t^{(n+1)}} \\ -\text{div} \left(\rho \frac{K^k}{\mu} [\text{grad} (P^{k(n+1)})] \right) + \rho \left[Q^k - Q^{k,0} - \sum_{j=1, j \neq k}^{N_f} Q^{k,j} \right] \\ = \frac{(\rho \phi^k V^k)^{(n+1)} - (\rho \phi^k V^k)^{(n)}}{\Delta t^{(n+1)}} \end{cases} \quad (7)$$

The fluid exchange between the matrix and the fractures is represented as:

$$\begin{cases} Q^{m,f} = \int_{V^m} q^{m,f} dV = T^{mf} \cdot \frac{1}{\mu} (P^m - P^f) \\ Q^{f,m} = \int_{V^f} q^{f,m} dV = T^{mf} \cdot \frac{1}{\mu} (P^f - P^m) \end{cases} \quad (8)$$

where T^{mf} is the conductivity between the matrix cells and the fracture cells, m and f respectively represent the matrix cells and fractures cells. Q^{mf} and Q^{fm} are the total flow exchange between fractures and matrix, respectively.

The fluid exchange between fractures is represented as:

$$\begin{cases} Q^{f_1,f_2} = \int_{V^{f_1}} q^{f_1,f_2} dV = T^{ff} \cdot \frac{1}{\mu} (P^{f_1} - P^{f_2}) \\ Q^{f_1,f_2} = \int_{V^{f_2}} q^{f_1,f_2} dV = T^{ff} \cdot \frac{1}{\mu} (P^{f_2} - P^{f_1}) \end{cases} \quad (9)$$

where T^{ff} is the conductivity between fracture cells. Q^{f_1,f_2} is the total flow exchange between fractures.

2.2.3 Typical Well Test Curve Plotting

First, discrete fracture models under four types of fracture network systems are established using the Discrete Fracture Network (DFN) module in the well test software Saphir. Then, based on the application context of the pressure drop during the shut-in stage of a single-stage fracturing operation, this paper only

considers the single-stage fracturing of a horizontal well. The effective reservoir flow area is designed as $100 \text{ m} \times 160 \text{ m} \times 20 \text{ m}$, and the well length is designed as 60 m , with 5 main fractures. At the same time, key parameters of the reservoir and matrix are input, as shown in [Tables 1](#) and [2](#).

Table 1: Basic reservoir parameters

Initial pressure (MPa)	Reservoir thickness (m)	Porosity (%)	Internal permeability (mD)	External permeability (mD)	Wellbore storage coefficient (m^3/MPa)	Skin factor
60.00	20.00	5.60	1.00	0.50	0.02	0.00

Table 2: Basic fracture parameters

Number of fracture clusters (Cluster)	Fracture width (m)	Fracture half length (m)	Fracture height (m)	Fracture conductivity ($\text{mD}\cdot\text{m}$)
5	0.01	60.00	20.00	60.00

Based on the model solution results, the single-stage double-logarithmic well test curves for single-wing fractures, two-wing fractures, branching fractures, and serial fractures are plotted, and the corresponding flow stages are divided.

As shown in [Figs. 3](#) and [4](#), the single-stage double-logarithmic well test curves for single-wing fractures and two-wing fractures follow similar patterns. The specific characteristics are as follows:

1. First stage: A slope of 1 and a 'hump' feature appear, representing the wellbore storage and skin effect.
2. Second stage: A slope of $1/4$ forms a straight line, indicating the fracture dual linear flow phase.
3. Third stage: A slope of $1/2$ forms a straight line, indicating the fracture linear flow phase.
4. Fourth stage: Transition from the fracture linear flow phase to the outer region flow phase.
5. Fifth stage: A rapid decline in pressure derivative, indicating the boundary flow phase.

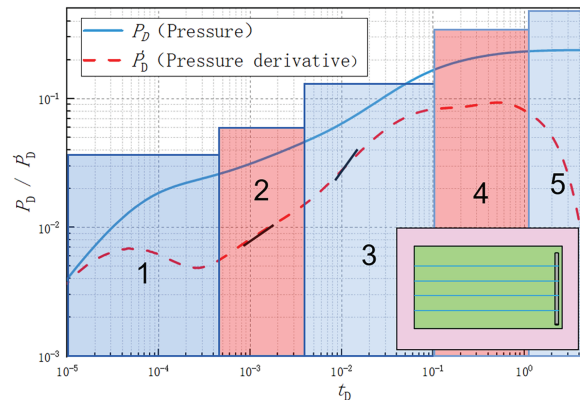


Figure 3: Single-wing fracture single segment double-logarithmic well test curve

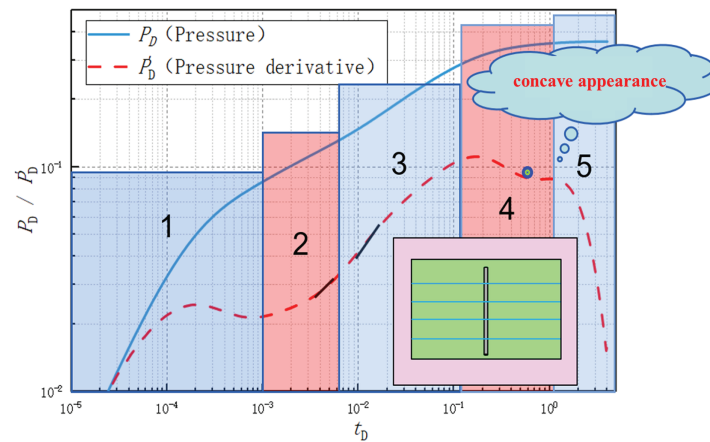


Figure 4: Double-wing fracture single segment double-logarithmic well test curve

As shown in Fig. 5, the single-stage double-logarithmic well test curve for branching fractures has the following specific characteristics:

1. First stage: A slope of 1 and a 'hump' feature appear, representing the wellbore storage and skin effect.
2. Second stage: A slope of 1/4 forms a straight line, indicating the fracture dual linear flow phase.
3. Third stage: A slope of 1/2 forms a straight line, indicating the fracture linear flow phase.
4. Fourth stage: A concave curve feature appears (although not very obvious), indicating the matrix to fracture network flow phase.
5. Fifth stage: A rapid decline in pressure derivative, indicating the boundary flow phase.

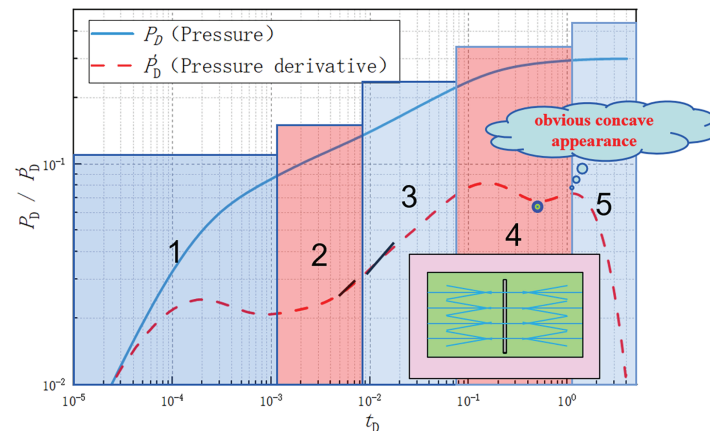


Figure 5: Branch fracture single segment double-logarithmic well test curve

As shown in Fig. 6, the single-stage double-logarithmic well test curve for serial fractures has the following specific characteristics:

1. First stage: A slope of 1 and a 'hump' feature appear, representing the wellbore storage and skin effect.
2. Second stage: A slope of 1/4 forms a straight line, indicating the fracture dual linear flow phase.
3. Third stage: A slope of 1/2 forms a straight line, indicating the fracture linear flow phase.
4. Fourth stage: A concave curve feature appears, indicating the matrix to fracture network flow phase.
5. Fifth stage: A slope of 1/2 forms a straight line, indicating the second linear flow phase in the outer region.

6. Sixth stage: A rapid decline in pressure derivative, indicating the boundary flow phase.

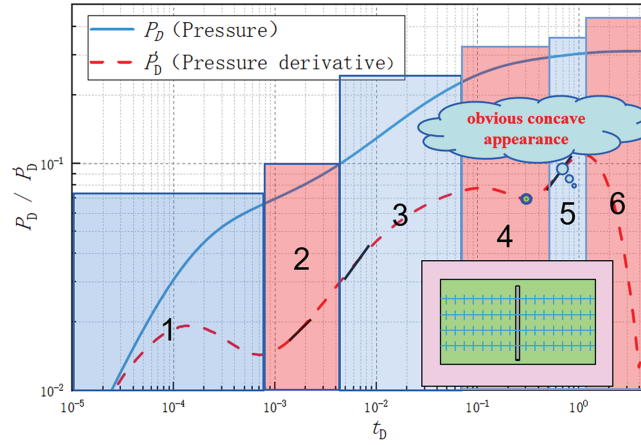


Figure 6: Chain fracture single segment double-logarithmic well test curve

Through the analysis of well test curves under different fracture network systems, the following insights and conclusions are drawn (Fig. 7):

1. Single-wing fractures have poor connectivity with the reservoir matrix, resulting in lower pressure and pressure derivative curves.
2. Single-wing fractures do not show matrix-fracture crossflow characteristics. As the fracture complexity increases, such as in double-wing fractures, branching fractures and serial fractures, the pressure derivative curve gradually shifts to the left and becomes more pronounced.

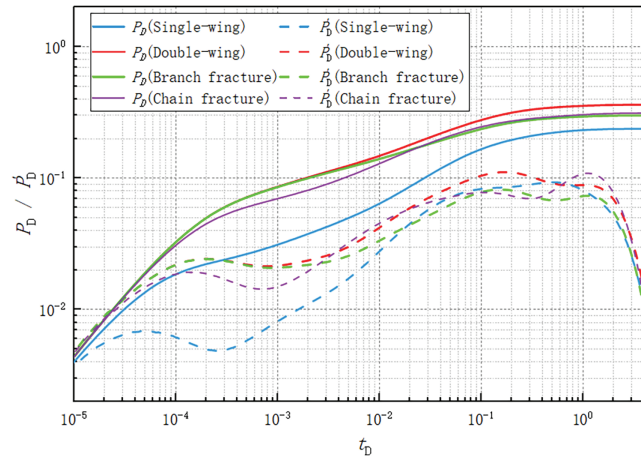


Figure 7: Double-logarithmic well test curve under different fracture network systems

2.3 Equivalent Seam Network Model

After the fracturing construction of the horizontal well, fractures are generated in the near-wellbore area, and these fractures communicate with each other to form a complex fracture network. Therefore, the model not only considers the primary fractures but also considers the complex fracture network and high permeability zones formed after fracturing. As shown in Fig. 8, the equivalent fracture network model divides the reservoir into a complex fracture network zone and a matrix zone. The green part is the complex fracture

network stimulated zone, including complex fractures that communicate with each other, and the red part is the matrix zone. This model is then compared and analyzed with four types of fracture systems: single-wing fractures, two-wing fractures, branched fractures, and serial fractures.

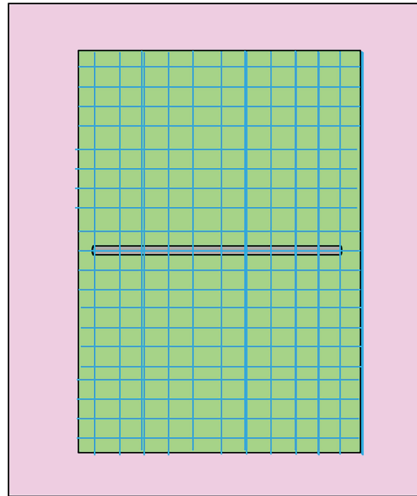


Figure 8: Schematic diagram of the equivalent fracture network model

2.3.1 Physical Model

The assumptions for the single-stage well model under the trilinear flow model are as follows: a closed, homogeneous reservoir with bounded, horizontally layered, and equal thickness; the fluid is a slightly compressible, single-phase fluid flowing in an isothermal, Darcy-permeable manner; the reservoir is vertically penetrated by finite-conductivity fractures; the pressure drop in the wellbore and gravity effects are neglected; the well is producing at a constant rate, considering skin effect and wellbore storage; the secondary fracture network is well-developed, and external contributions to the production from the stimulated zone are taken into account.

The trilinear flow model divides the reservoir into three physical regions, as shown in Fig. 9:

1. High permeability region: Represents the complex fracture network near the fracture after the hydraulic fracturing.
2. Reservoir internal region: Represents the area between the artificially created fractures, i.e., the region where the fractures have extended.
3. Reservoir external region: Represents the low permeability area unaffected by the fractures.

The trilinear flow model is highly symmetrical, making it suitable for scenarios where reservoir stimulation is effective and fractures are densely distributed. Therefore, the trilinear flow model is chosen to characterize the fluid flow characteristics under a complex fracture network pattern, and based on well testing theory, an unstable single-stage well test model for complex fracture networks is established.

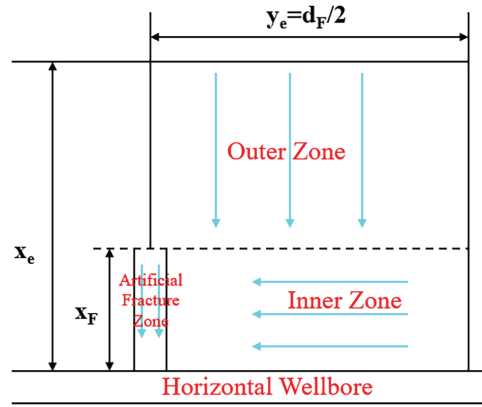


Figure 9: Trilinear flow physical model (single segment)

2.3.2 Mathematical Model

To describe the properties of the fractured stimulation zone, the following parameters are introduced:

Fracture network volume ratio: $\omega = \frac{(\phi C_t)_f}{(\phi C_t)_f + (\phi C_t)_m}$ it is equivalent to the density of artificial fractures.

Matrix cross-flow capacity coefficient: $\lambda = \alpha r_w^2 \frac{k_m}{k_f}$ it characterizes the speed of pressure propagation.

Shape factor: $\alpha = \frac{4n(n+2)}{l^2}$ it represents the distribution of the fracture network within the reservoir matrix.

Gas pseudo-pressure: $m = \int_0^p \frac{2p}{\mu_q(p)Z(p)} dp$ it characterizes the pressure of the gas.

Where ϕ is the porosity; C_t is the compressibility coefficient; k_m and k_f are the matrix and fracture permeabilities.

For the shale gas reservoir multi-stage fracturing horizontal well model, establish its mathematical model. First, based on symmetry, establish the corresponding mathematical models for the main fractures and the formation flow patterns.

The mathematical model for the flow in the affected zone of distant secondary fractures:

The governing equation:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial m}{\partial r} \right) = \frac{\phi \mu C_t}{3.6k} \frac{\partial m}{\partial t}, \Omega \in \Omega_3 \quad (10)$$

where μ is viscosity; m is pseudo-pressure of gas.

Boundary condition at the outer boundary:

$$\frac{\partial m_{r_3}}{\partial r} = 0 \quad (11)$$

The initial condition:

$$m|_{(t=0)} = m_i \quad (12)$$

Mathematical model of seepage in the near-well fracture network reconstruction zone:

The governing equation of the fluid:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial m}{\partial r} \right) + q_m = \omega \frac{\phi \mu C_t}{3.6k} \frac{\partial m}{\partial t}, \Omega \in \Omega_{1,2} \quad (13)$$

where ω is the fracture network volume ratio, and the matrix flow term in the near-well fracture network zone is considered with a value of 1 when the fracture network is not considered.

The boundary conditions for the inner and outer boundaries:

$$\frac{r h k}{1.842 \times 10^{-3} \mu B} \frac{\partial m}{\partial r} \Big|_{r \rightarrow 0} = q_{sc}, \Omega \in \Omega_{1,2} \quad (14)$$

where h is the reservoir thickness; B is the volume factor; q_{sc} is the production under standard conditions.

Linear fluid flow from the secondary fracture zone to the fracture network zone:

The governing equation for the fluid:

$$3.6 \frac{k_F}{\mu} \frac{\partial m_F^2}{\partial y^2} + \frac{B q_F}{24 W_F h_F} = \frac{\mu \phi C_t}{k_F} \frac{\partial m_F}{\partial t} \quad (15)$$

The boundary conditions:

$$3.6 \frac{k_F h_F W_F}{\mu} \frac{\partial m_F}{\partial y} \Big|_{y \rightarrow y_o} = \frac{B q_w}{24} \quad (16)$$

The initial conditions:

$$m \Big|_{t=0} = m_i \quad (17)$$

The solution to the multi-stage fracturing horizontal well test analysis model for shale gas reservoirs is obtained using a trilinear flow model. To simplify the equation solving, the model is first simplified using dimensionless variables.

Dimensionless pressure:

$$m_D = \frac{k_2 h}{1.842 \times 10^{-3} \mu q_{sc} B} (m_i - m), m_{2D} = \frac{k_2 h}{1.842 \times 10^{-3} \mu q_{sc} B} (m_i - m_2) \quad (18)$$

$$m_{mD} = \frac{k_m h}{1.842 \times 10^{-3} \mu q_{sc} B} (m_i - m_m), m_{f1D} = \frac{k_{f1} h}{1.842 \times 10^{-3} \mu q_{sc} B} (m_i - m_{f1}) \quad (19)$$

where m_D , m_{mD} , m_{2D} and m_{fD} are the dimensionless pseudo pressure, dimensionless pseudo pressure in the matrix region, dimensionless pseudo pressure in the second region and dimensionless pseudo pressure in the fracture region.

Secondary fracture network volume ratio and matrix crossflow coefficient:

$$\omega = \frac{(\phi C_t)_f}{(\phi C_t)_f + (\phi C_t)_m}, \lambda = \alpha x_F^2 \frac{k_m}{k_f} \quad (20)$$

Dimensionless time:

$$t_D = \frac{3.6 k_2 t}{\mu \left[(\phi C_t)_f + (\phi C_t)_m \right] x_F^2} \quad (21)$$

Dimensionless flow rate:

$$q_D = \frac{q}{q_{sc}} \quad (22)$$

where q_D is the dimensionless flow rate, q is the actual flow rate, q_{sc} is the flow rate in the standard conditions.

Dimensionless distance:

$$x_D = \frac{x}{x_F}, x_{wD} = \frac{x_w}{x_F}, r_{mD} = \frac{r_m}{R_m}, y_D = \frac{y}{x_F}, w_{FD} = \frac{w_F}{x_F} \quad (23)$$

Dimensionless Fracture Conductivity:

$$C_{FD} = \frac{k_F w_F}{k_2 x_F} \quad (24)$$

Diffusivity Ratio:

$$\eta_{12} = \frac{(k/\mu\phi C_t)_1}{(k/\mu\phi C_t)_2}, \eta_{F2} = \frac{(k/\mu\phi C_t)_F}{(k/\mu\phi C_t)_2} \quad (25)$$

After simplifying the model, it can be obtained:

Fracture Effective Zone Equation:

$$\begin{cases} \frac{\partial^2 m_{2D}}{\partial y_D^2} = \frac{\partial m_{2D}}{\partial t_D} \\ m_{2D}|_{t_D=0} = 0 \\ \frac{\partial m_{2D}}{\partial y_D} \Big|_{y_D=y_{eD}} = 0, m_{2D} \Big|_{y_D=1} = m_{f1D} \Big|_{y_D=1} \end{cases} \quad (26)$$

Fracture Treatment Zone Matrix Equation:

$$\lambda (m_{fD} - m_{mD}) = (1 - \omega) \frac{\partial m_{mD}}{\partial t_D} \quad (27)$$

Secondary Fracture Network Equation in the Fracture Treatment Zone:

$$\begin{cases} \frac{\partial^2 m_{f1D}}{\partial x_D^2} \frac{1}{\eta_{12}} \left[\lambda (m_{fD} - m_{mD}) + \omega \frac{\partial m_{f1D}}{\partial t_D} \right] \\ m_{f1D} \Big|_{t_D=0} = 0 \\ \frac{\partial m_{f1D}}{\partial x_D} \Big|_{x_D=x_{eD}} = 0, m_{f1D} \Big|_{x_D=w_{FD}/2} = m_{FD} \Big|_{x_D=w_{FD}/2} \end{cases} \quad (28)$$

Main Fracture Equation:

$$\begin{cases} \frac{\partial^2 m_{FD}}{\partial y_D^2} + \frac{2}{C_{FD}} \frac{\partial m_{f1D}}{\partial x_D} \Big|_{x_D=w_{FD}/2} = \frac{\omega}{\eta_{F2}} \frac{\partial m_{FD}}{\partial t_D} \\ m_{FD} \Big|_{t_D=0} = 0 \\ \frac{\partial m_{FD}}{\partial y_D} \Big|_{y_D=1} = 0, \frac{\partial m_{FD}}{\partial y_D} \Big|_{y_D=0} = -\frac{\pi}{\eta_{12} C_{FD}} \end{cases} \quad (29)$$

Using the Laplace transform method to solve the linearized well testing mathematical model, the coupled model yields the bottom-hole pressure solution as follows:

$$\bar{p}_{wD} = \frac{\pi}{\eta_{12} C_{FD} s \sqrt{f_F(s)}} \frac{1}{\tanh \left[\sqrt{f_F(s)} \right]} \quad (30)$$

$$\begin{cases} f_F(s) = \frac{2}{C_{FD}} \sqrt{f_1(s)} \tanh \left[\sqrt{f_1(s)} (x_{eD} - w_{FD}/2) \right] + \frac{s}{\eta_{F2}} \\ f_1(s) = \frac{1}{\eta_{12} x_{eD}} \left\{ s + \sqrt{s} \tanh \left[\sqrt{s} (y_{eD} - 1) \right] \right\} + \frac{s}{\eta_{12}} \frac{\omega s (1 - \omega) + \lambda}{s (1 - \omega) + \lambda} \end{cases} \quad (31)$$

Furthermore, based on Eq. (30), by using the superposition principle, the bottom-hole pressure solution considering the wellbore storage effect and skin effect can be obtained.

$$\bar{m}_{wD}(s, S, C_D) = \frac{S + s \bar{m}_{wD}}{s + C_D s^2 (s \bar{m}_{wD} + S)} \quad (32)$$

Using Stehfest numerical inversion, the Laplace space bottom-hole pressure can be transformed into the real space bottom-hole pressure as follows:

$$m_{wD}(t, S, C_D) = L^{-1} [\bar{m}_{wD}(s, S, C_D)] \quad (33)$$

where m_{wD} is the dimensionless bottom-hole pseudo pressure; C_{FD} is the dimensionless fracture conductivity; s is the Laplace space variable; S is the skin factor; C_D is the dimensionless wellbore storage coefficient.

3 Results

Based on the established equivalent fracture network single stage well testing model, further verify the descriptive ability of the equivalent fracture network model for the well testing characteristics of fractured wells. As shown in Tables 3 and 4, key parameters of the reservoir and matrix were set, and the single segment double logarithmic well test curve under equivalent fracture network conditions was plotted by solving the bottomhole pressure response in the model. Based on the characteristics of the curve shape, each typical flow stage was identified and divided. The results indicate that the model can effectively reflect the pressure changes during different flow stages.

Table 3: Basic reservoir parameters

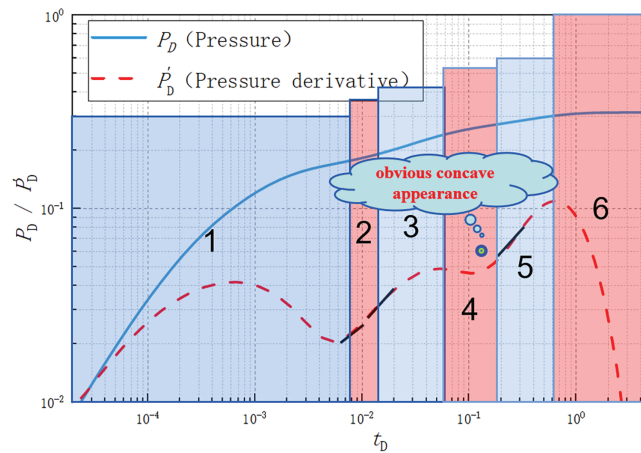
Initial pressure (MPa)	Reservoir thickness (m)	Porosity (%)	Internal permeability (mD)	External permeability (mD)	Wellbore storage coefficient (m ³ /MPa)	Segment length (m)
60.00	18.00	4.80	1.00	0.10	0.02	50.00

Table 4: Basic fracture parameters

Fracture width (m)	Fracture half length (m)	Fracture height (m)	Fracture conductivity (mD·m)	Volume ratio of fracture network (%)	Cross flow coefficient
0.01	60.00	20.00	60.00	10.00	4.35×10^{-4}

As shown in Fig. 10, the specific characteristics of the single-segment double-logarithmic well test curve for the equivalent fracture network model are as follows:

1. First stage: A slope of 1 and a 'hump' characteristic appear, representing wellbore storage and skin effect;
2. Second stage: A slope of 1/4 line appears, indicating the fractured bilinear flow stage, influenced by parameters such as the inner zone permeability and conductivity;
3. Third stage: A slope of 1/2 line appears, indicating the fractured linear flow stage, influenced by parameters such as the inner zone permeability and conductivity;
4. Fourth stage: A concave curve characteristic appears (concave is evident and complete), representing the matrix to fracture network flow stage, influenced by fracture conductivity;
5. Fifth stage: A slope of 1/2 line appears, representing the outer zone secondary linear flow stage, which is complete and more influenced by parameters such as inner zone permeability and fracture conductivity;
6. Sixth stage: A rapid pressure derivative decline appears, indicating the boundary flow stage.

**Figure 10:** Equivalent fracture network model single segment double-logarithmic well test curve

As shown in Fig. 11, by comparing the theoretical curves of well testing interpretation under complex fracture networks with the theoretical curves of well testing interpretation under four different types of fracture network systems mentioned earlier, the theoretical curves under complex fracture networks also exhibit obvious concave characteristics.

Under different combinations of fracture network volume ratio and crossflow coefficient parameters, the degree of concavity and the appearance time of the pressure drop well test curve for the equivalent fracture network model vary. As shown in Figs. 12 and 13, both the fracture network volume ratio and the cross-flow coefficient primarily affect the early and middle stages of the curve. As the fracture network volume ratio increases, the fracture system dominates the early fluid flow in the reservoir, which results in

the radial flow stage within the fractures lasting longer. Consequently, the “concave” feature of the curve becomes less pronounced. On the other hand, as the cross-flow coefficient increases, cross-flow between the matrix and fractures occurs earlier, causing the “concave” feature of the curve to shift to the left. At the same time, the outer zone secondary linear flow stage also appears. Therefore, the equivalent fracture network single-segment well test model can simultaneously represent single-wing fractures, double-wing fractures, branched fractures, and serial fractures in a single-segment well test model.

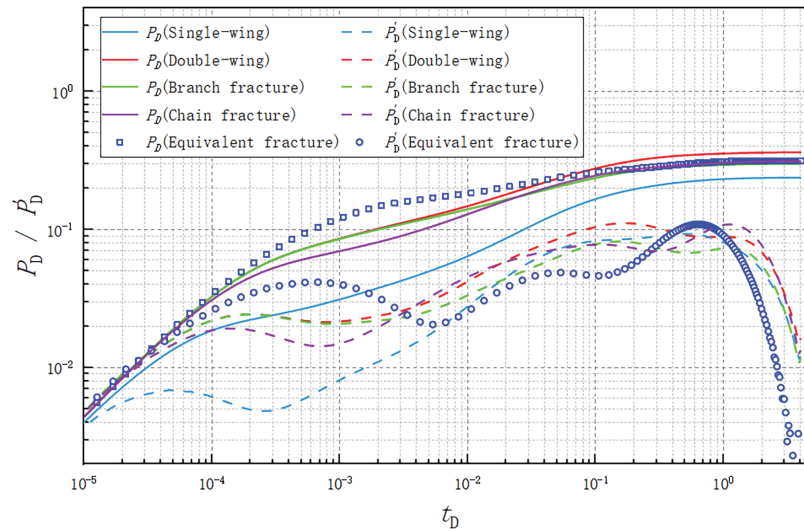


Figure 11: Comparison of theoretical well test curves under different fracture network systems

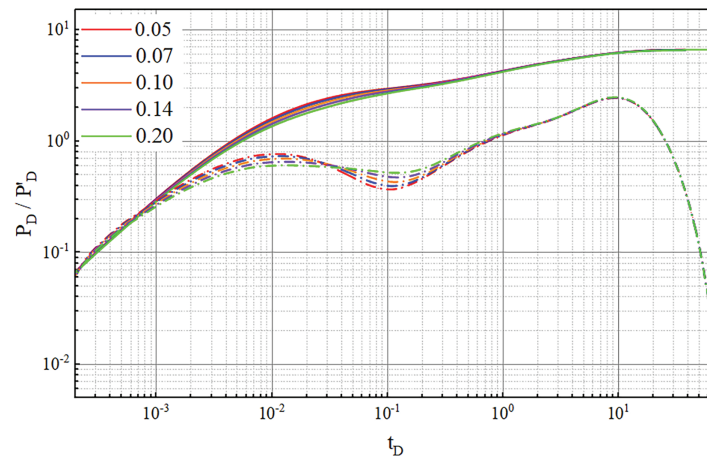


Figure 12: Schematic diagram of the curve's concave feature under different fracture network volume ratios

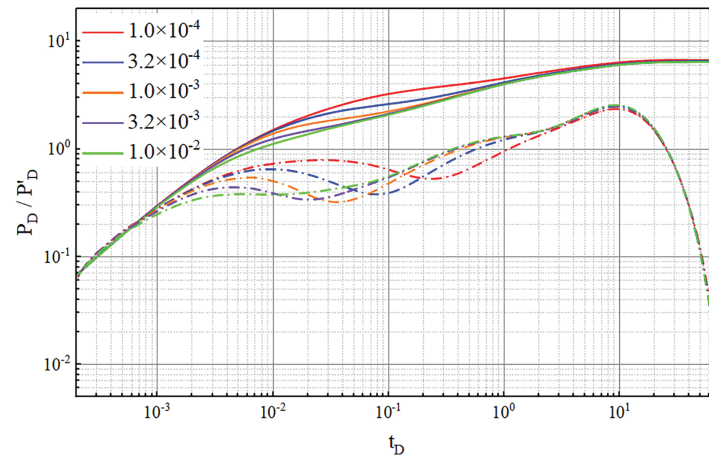


Figure 13: Schematic diagram of the curve's concave feature under different cross-flow coefficients

4 Discussion

This paper conducts a sensitivity analysis based on an established equivalent fracture network single-stage fracturing well test model. The sensitivity analysis is carried out for parameters such as inner zone permeability (0.5~2 mD), outer zone permeability (0.1~1 mD), outer zone diffusivity coefficient (7.39~29.56 m²/h), fracture conductivity (30~120 mD·m), wellbore storage coefficient (0.005~0.1 m³/MPa), skin factor (1×10^{-5} ~1), fracture network volume ratio (5%~20%), and crossflow coefficient (1×10^{-4} ~0.01).

The sensitivity analysis of wellbore storage coefficient values ranging from 0.005 to 0.1 m³/MPa and skin factor values from 1×10^{-5} to 1 was conducted. From Figs. 14 and 15, it can be observed that both the wellbore storage coefficient and skin factor primarily affect the early stage of the curve. As the wellbore storage coefficient increases, the energy released by the fluid within the wellbore becomes more significant, which leads to an extended duration of the 45-degree slope section in the early stage of the curve. As the skin factor increases, it indicates more severe damage to the reservoir properties near the wellbore, requiring a larger pressure differential for the fluid to flow toward the wellbore in the early stage. This results in an upward shift of the curve and a higher “hump” in the curve.

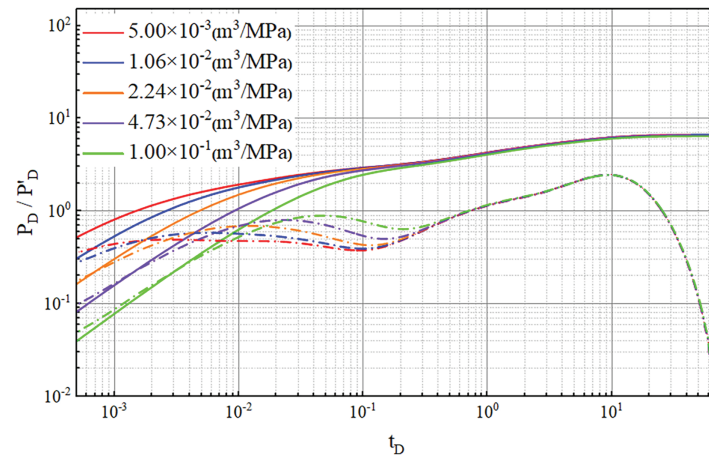


Figure 14: wellbore storage coefficient sensitivity analysis

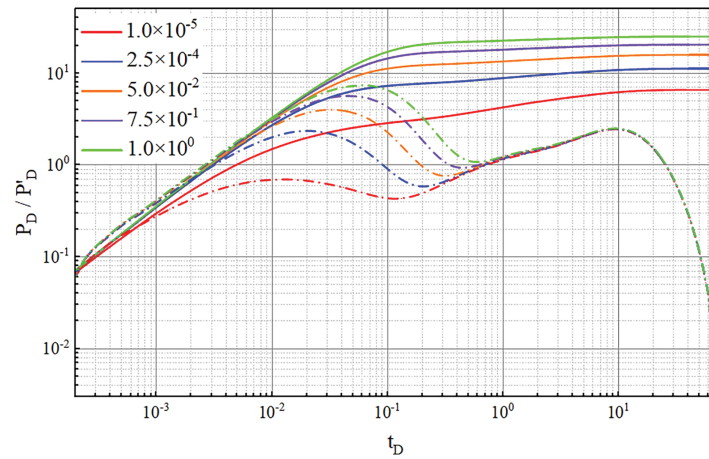


Figure 15: Skin factor sensitivity analysis

The sensitivity analysis was conducted by setting the inner zone permeability values from 0.5 to 2 mD and the outer zone permeability values from 0.1 to 1 mD. The analysis focused on the impact of inner zone permeability and outer zone diffusivity coefficient. The inner zone permeability primarily affects the early and middle stages of the curve. As the inner zone permeability increases, the matrix-to-fracture flow stage occurs earlier. At the same time, with the increase in inner zone permeability, the curve reaches the boundary flow stage more quickly, as shown in Fig. 16. The outer zone permeability mainly influences the middle and later stages of the curve. As both inner and outer zone permeabilities decrease, the well test curve is more likely to show the second linear flow characteristic in the outer zone during the middle and later stages. Additionally, the inner zone exhibits characteristics of pseudo-radial flow, as shown in Fig. 17.

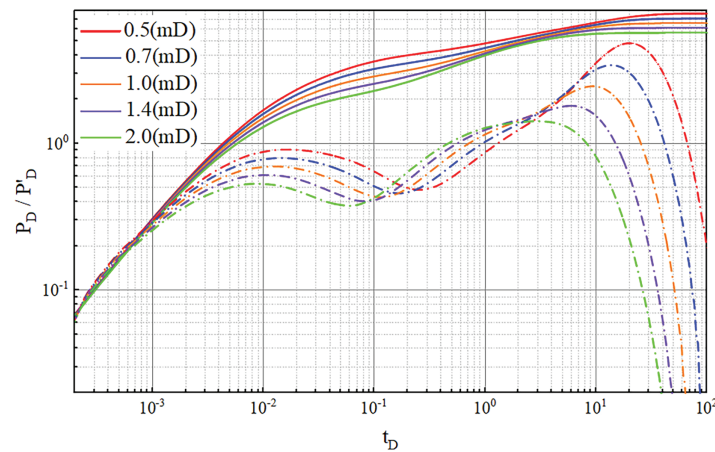


Figure 16: Inner region permeability sensitivity analysis

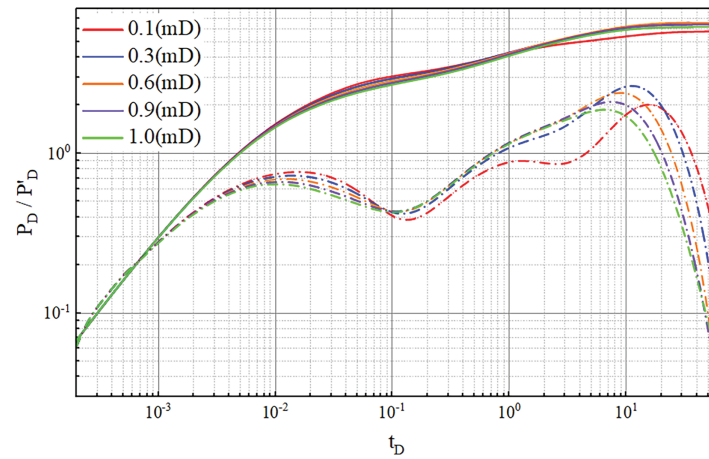


Figure 17: Outer region permeability sensitivity analysis

A sensitivity analysis was conducted by setting the outer zone pressure transmission coefficient values from 7.39 to 29.56 m²/h and the fracture conductivity values from 30 to 120 mD was examined. The outer zone pressure transmission coefficient primarily affects the later stage of the curve. As the outer zone diffusivity coefficient increases, the pressure wave propagates more quickly through the reservoir, which leads to the boundary flow stage appearing sooner, as shown in Fig. 18. Fracture conductivity primarily influences the early and middle stages of the curve. As fracture conductivity increases, the fluid flow between the matrix and fractures increases, leading to a more pronounced dip in the curve. At the same time, the entire curve shifts downward, as shown in Fig. 19.

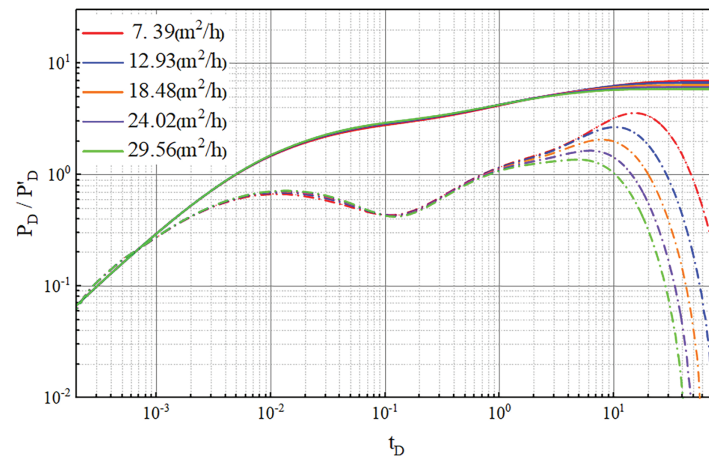


Figure 18: Outer region pressure conductivity coefficient sensitivity analysis

A sensitivity analysis was conducted by setting the fracture network volume ratio values from 0.05 to 0.2 and the cross-flow coefficient values from 0.0001 to 0.01. As shown in Figs. 12 and 13, both the fracture network volume ratio and the cross-flow coefficient primarily affect the early and middle stages of the curve. As the fracture network volume ratio increases, the fracture system dominates the early fluid flow in the reservoir, which results in the radial flow stage within the fractures lasting longer. Consequently, the dip of the curve becomes less pronounced. On the other hand, as the cross-flow coefficient increases, cross-flow between the matrix and fractures occurs earlier, causing the dip of the curve to shift to the left.

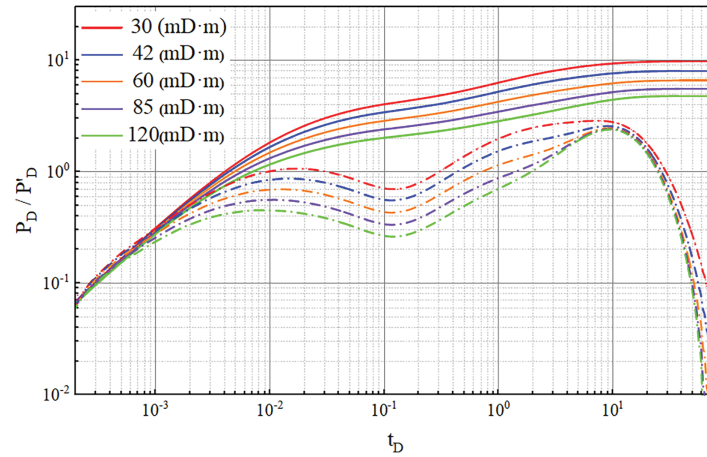


Figure 19: Fracture flow capacity sensitivity analysis

5 Practical Application

After large-scale hydraulic fracturing of deep shale gas reservoirs, complex and unevenly distributed fracture networks are often formed near the wellbore. Therefore, coupling the matrix-fracture flow characteristics to effectively evaluate the post-fracturing performance is crucial. However, wells with fractures that exhibit strong heterogeneity between different fractured stages still face the following major issues: (1) There is no systematic evaluation method for the fracturing performance of heterogeneous fracture gas reservoirs. (2) The complex fracture morphology severely restricts the inversion and evaluation of its parameters. In response to the above issues, it is urgent to research and establish a single-stage well test interpretation model suitable for heterogeneous fracture wells, and clarify the impact of key parameters such as reservoir and fracture properties on the well test interpretation results. This will improve the accuracy and reliability of fracturing performance evaluation for such wells.

Based on the single-stage shut-in data of the N1 well in a deep shale gas reservoir in the southern Sichuan region, a numerical well test interpretation model for a well with a complex fracture networks has been established using this method. Input parameters for the N1 well, such as reservoir and fracture properties, are shown in Table 5.

Table 5: Key parameter input

Reservoir thickness (m)	Porosity (%)	Number of fracture clusters (Cluster)	Segment length (m)	Reservoir top depth (m)	Wellbore radius (m)
18.00	4.80	4	45.40	4200	0.092

Using the discrete fracture network well test model for single-stage shale gas fracturing horizontal wells, the shut-in data for the 29th stage of the N1 well were used to plot and fit the double logarithmic curves. The well test interpretation results for the N1 well's single-stage were calculated. Taking the 29th stage of the N1 well as an example, as shown in Fig. 20.

As shown in Table 6, according to the single-stage well test interpretation results for the 29th stage of the N1 well, the permeability in the fracture network area is 0.95 mD, the effective fracture half-length is 87 m, the fracture network volume ratio is 0.05, and the crossflow coefficient is 1.00×10^{-6} .

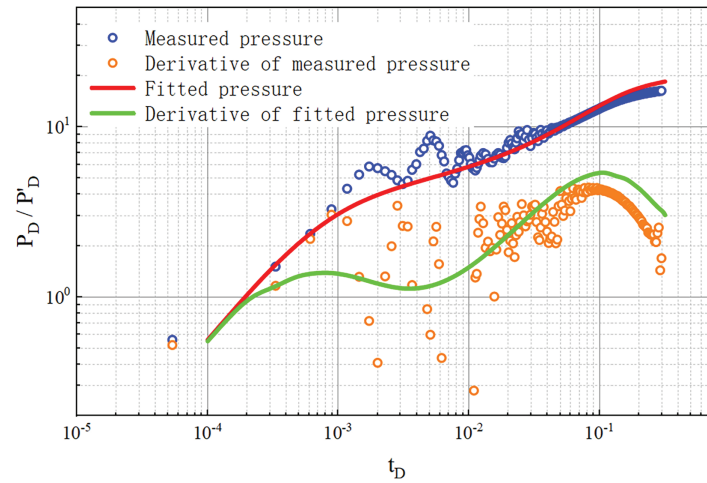


Figure 20: N1 Well 29-Stage Single Segment Log-Log Curve

Table 6: Key parameter fitting results

Initial pressure (MPa)	Permeability of fracture network area (m)	Fracture half length (m)	Fracture width (m)	Volume ratio of fracture network (%)	Cross flow coefficient
60	0.95	87.00	0.01	5.00	1.00×10^{-6}

6 Conclusion

(1) Based on the characteristics of deep shale gas fracture networks, the fracture network system was classified into single-wing fractures, two-wing fractures, branched fractures, and chain fractures. Four different single-stage well test models for different fracture systems are established, and theoretical log-log logarithmic curves are plotted. The results show: ① Single-wing fractures have poor connectivity with the reservoir matrix, and the pressure curve and pressure derivative curve are relatively low; ② No matrix-fracture crossflow characteristics are observed in single-wing fractures and two-wing fractures; ③ Branched fractures and chain fractures show a “dip” feature on the well test curves. As the fracture complexity increases, the pressure derivative curve gradually shifts to the left and becomes more pronounced.

(2) Considering the multiplicity of well test inversion, an equivalent fracture network model was established to represent the four types of fracture systems. Under different combinations of fracture network volume ratio and crossflow coefficient parameters, the depth of the dip fall off and appearance time of the equivalent fracture network model pressure drop well test curve vary, presenting an outer zone secondary linear flow stage. Therefore, the equivalent fracture network single-stage well test model can represent the single-stage well test models of single-wing fractures, two-wing fractures, branched fractures, and chain fractures simultaneously.

(3) A sensitivity analysis of key parameters was conducted based on the equivalent fracture network single-stage fracturing well test model. The results show: ① Wellbore storage coefficient and skin factor mainly affect the appearance time and height of the curve's peak; ② As the fracture conductivity increases, the curve moves downward in the early to middle stages; ③ As the inner zone permeability increases, the curve reaches the boundary flow stage more quickly; ④ As the fracture network volume ratio increases, the dip of the curve becomes less pronounced; ⑤ As the crossflow coefficient increases, the dip of the curve

shifts to the left; ⑥ As the outer zone permeability decreases, the secondary linear flow feature of the outer zone becomes more apparent in the late stage of the well test curve; ⑦ As the outer zone pressure coefficient increases, the boundary flow stage appears more quickly.

(4) Based on the reservoir and fracture information of the N1 well in a specific block of the Sichuan-South deep shale gas reservoir, combined with the application background of pressure drop during the shut-in phase after single-stage fracturing, a typical well single-stage well test interpretation model was established using the equivalent fracture network model. By plotting and fitting the log-log logarithmic curves of the N1 well's 29 single-stage fracturing shut-in data, the results show that the fracture network permeability of the N1 well area is 0.95 mD, the effective fracture flow half-length is 87 m, the fracture network volume ratio is 0.05, and the crossflow coefficient is 1.00×10^{-6} .

7 Limitation

The equivalent seam network model established in this paper is a simplified representation method. Although it is more efficient than DFM in parameter inversion, it sacrifices the specific geometric details of the fracture network. At present, this model mainly analyzes the pressure drop data of single segment pump shutdown. When it is extended to explain the production data of the entire well containing multiple complex fracture interferences, further correction may be needed. Therefore, future research should aim to extend this single-stage equivalent model into a multi-stage coupled model to more accurately simulate the overall dynamic response of multi-stage fractured horizontal wells.

Acknowledgement: All authors express gratitude for the support and cooperation provided by their respective institutions.

Funding Statement: The authors received no specific funding.

Author Contributions: Xuefeng Yang, Deliang Zhang: conceptualization, methodology, analysis, software, writing—reviewing and editing; Chunyu Ren, Yan Zhang: data curation, writing—original draft preparation; Yue Yang, Shuai Wu: analysis, software; Huaicai Fan, Xin Zhao: data curation, validation; Yue Chen, Baoyun Zhang: supervision, validation, writing—reviewing and editing. All authors reviewed and approved the final version of the manuscript.

Availability of Data and Materials: Data and materials are available from the corresponding author upon reasonable request.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Nomenclatures

ϕ^0	Matrix porosity
ρ	Fluid density, g/cm ³
q^0	Source and sink terms of fluid in the matrix, cm ³ /s
V^0	Volume of the matrix element, cm ³
$\vec{u}^0$	Fluid flow velocity in the matrix, cm/s
$q^{0,j}$	Volumetric flow rate of fluid from the matrix to the j-th fracture, cm ³ /s
ϕ^k	Porosity of the k-th fracture
q^k	Source and sink terms of fluid in the k-th fracture, cm ³ /s
V^k	Volume of the k-th fracture element, cm ³

$\vec{u}^k$	Fluid flow velocity in the k -th fracture, cm/s
$q^{k,0}$	Volumetric flow rate of fluid from the k -th fracture to the matrix, cm ³ /s
$q^{k,j}$	Volumetric flow rate of fluid from the k -th fracture to the j -th fracture, cm ³ /s
K^k	Absolute permeability of fluid medium k , D
μ	Fluid viscosity, mpa·s
∇P^k	Pressure gradient of fluid in medium k , 0.1 MPa/cm
∇z	Altitude gradient, cm/cm
g	Gravitational acceleration, cm/s ²
P_w	Bottom hole pressure, 0.1 MPa
P	Reservoir pressure, 0.1 MPa
S	Skin factor
r	Reservoir radius, cm
r_w	Well radius, cm
h	Reservoir thickness, cm
B	Fluid volume factor, cm ³ /cm ³
C	Well storage coefficient, cm ³ /(0.1 MPa)
t	Time, s
C_t	Compressibility coefficient, MPa ⁻¹
α	Shape factor, m ⁻²
s	Laplace spatial variable
m	Pseudo-pressure, MPa
C_D	Dimensionless wellbore reservoir coefficient
C_{FD}	Dimensionless fracture conductivity
m_{wD}	Dimensionless bottom-hole pseudo-pressure
x_D	Horizontal dimensionless distance
y_D	Vertical dimensionless distance
x_{wD}	The dimensionless distance of the wellbore in the horizontal direction
r_{wD}	Dimensionless wellbore radius
w_{FD}	Dimensionless fracture width

References

1. Wang H, Huang H, Bi W, Ji G, Zhou B, Zhuo L. Deep and ultra-deep oil and gas well drilling technologies: progress and prospect. *Nat Gas Ind B*. 2022;9(2):141–57. doi:10.1016/j.ngib.2021.08.019.
2. Guo X, Hu D, Huang R, Wei Z, Duan J, Wei X, et al. Deep and ultra-deep natural gas exploration in the Sichuan Basin: progress and prospect. *Nat Gas Ind B*. 2020;7(5):419–32. doi:10.1016/j.ngib.2020.05.001.
3. Ayad A, Tagma T. Lithostratigraphic and seismic investigation of the weathered zone in the exploration of deep petroleum structures (Boujdour area, Morocco). *Egypt J Petrol*. 2024;33(3):12. doi:10.62593/2090-468.1044.
4. Ma X, Wang H, Zhou S, Shi Z, Zhang L. Deep shale gas in China: geological characteristics and development strategies. *Energy Rep*. 2021;7:1903–14. doi:10.1016/j.egyr.2021.03.043.
5. Long S, Feng D, Li F, Du W. Prospect analysis of the deep marine shale gas exploration and development in the Sichuan Basin, China. *J Nat Gas Geosci*. 2018;3(4):181–9. doi:10.1016/j.jnggs.2018.11.001.
6. Zhao J, Ren L, Lin C, Lin R, Hu D, Wu J, et al. A review of deep and ultra-deep shale gas fracturing in China: status and directions. *Renew Sustain Energy Rev*. 2025;209:115111. doi:10.1016/j.rser.2024.115111.
7. Jiang T, Bian X, Wang H, Li S, Jia C, Liu H, et al. Volume fracturing of deep shale gas horizontal wells. *Nat Gas Ind B*. 2017;4(2):127–33. doi:10.1016/j.ngib.2017.07.018.
8. Ma Y, Cai X, Yun L, Li Z, Li H, Deng S, et al. Practice and theoretical and technical progress in exploration and development of Shunbei ultra-deep carbonate oil and gas field, Tarim Basin, NW China. *Petrol Explor Dev*. 2022;49(1):1–20. doi:10.1016/S1876-3804(22)60001-6.

9. Lei Q, Xu Y, Cai B, Guan B, Wang X, Bi G, et al. Progress and prospects of horizontal well fracturing technology for shale oil and gas reservoirs. *Petrol Explor Dev.* 2022;49(1):191–9. doi:10.1016/S1876-3804(22)60015-6.
10. Gong L, Liu K, Ju W. Editorial: advances in the study of natural fractures in deep and unconventional reservoirs. *Front Earth Sci.* 2023;10:1096643. doi:10.3389/feart.2022.1096643.
11. Ali N, Jamil S, Zaheer M, Hussain W, Hussain H, Iqbal SM, et al. Exploration and development of shale gas in China: a review. *Iran J Earth Sci.* 2022;14(2):87–103. doi:10.30495/ijes.2022.1940263.1652.
12. Zhou B, Chen Z, Zhao X, Wang B, Wang H, Sepehrnoori K. A three-dimensional numerical well-test model for pressure transient analysis in fractured horizontal wells with secondary fractures. *Phys Fluids.* 2024;36(4):046107. doi:10.1063/5.0203853.
13. Han J, Chen Z, Liu Y, Tang Z, Zhao X, Xin J. New approach for parameter evaluation of tight gas reservoirs stimulated with fracture networks. *Energy Fuels.* 2024;38(4):3105–17. doi:10.1021/acs.energyfuels.3c04650.
14. Chen D, Li N, Wang EY. Reliability assessment of the hydraulic fracturing process in coal mine based on the analysis of micro-seismic source parameters. *Nat Resour Res.* 2021;30(3):2317–32. doi:10.1007/s11053-021-09840-6.
15. Zhao J, Liu P, Li J, Chen Z, Li Y, Li F. Fracture propagation induced by hydraulic fracturing using microseismic monitoring technology: field test in CBM wells in Zhengzhuan region, Southern Qinshui Basin, China. *Front Earth Sci.* 2023;11:1130280. doi:10.3389/feart.2023.1130280.
16. Chen Z, Zhou B, Yu W, Sepehrnoori K. A numerical well test model for fractured wells in fractured shale gas reservoirs considering multiple flow mechanisms. *SPE J.* 2025;30(8):4995–5010. doi:10.2118/228296-pa.
17. Du D, Liu P, Ren L, Li Y, Tang Y, Hao F. A volume fracturing percolation model for tight reservoir vertical wells. *Processes.* 2023;11(9):2575. doi:10.3390/pr11092575.
18. Jahanbani A, Aguilera R. Well testing of tight gas reservoirs. In: *Proceedings of the Canadian International Petroleum Conference; 2008 Jun 17–19; Calgary, AB, Canada.* p. 161–74. doi:10.2118/2008-161.
19. Wang Q, Su X, Feng Y, Wang H, Song J. Experimental study of gas-water two-phase flow patterns in fracture: implication for enhancing coalbed methane production. *J Petrol Sci Eng.* 2021;207:109168. doi:10.1016/j.petrol.2021.109168.
20. Chen CC, Raghavan R. A multiply-fractured horizontal well in a rectangular drainage region. *SPE J.* 1997;2(4):455–65. doi:10.2118/37072-pa.
21. Horne RN, Temeng KO. Relative productivities and pressure transient modeling of horizontal wells with multiple fractures. In: *Proceedings of the Middle East Oil Show; 1995 Mar 11–14; Manama, Bahrain.* doi:10.2118/29891-ms.
22. Wu Y, Cheng L, Huang S, Jia P, Zhang J, Lan X, et al. A practical method for production data analysis from multistage fractured horizontal wells in shale gas reservoirs. *Fuel.* 2016;186:821–9. doi:10.1016/j.fuel.2016.09.029.
23. Zhao J, Chen X, Li Y, Fu B, Xu W. Numerical simulation of multi-stage fracturing and optimization of perforation in a horizontal well. *Petrol Explor Dev.* 2017;44(1):119–26. doi:10.1016/S1876-3804(17)30015-0.
24. Yao S, Zeng F, Liu H, Zhao G. A semi-analytical model for multi-stage fractured horizontal wells. In: *Proceedings of the SPE Canadian Unconventional Resources Conference; 2012 Oct 30–Nov 1; Calgary, AB, Canada.* doi:10.2118/162784-ms.
25. Yao J, Liu PY, Wu ML. Well test analysis of fractured horizontal well in fractured reservoir. *J China Univ Petrol Ed Nat Sci.* 2013;37(5):107–19. (In Chinese).
26. Ozkan E, Brown M, Raghavan R, Kazemi H. Comparison of fractured horizontal-well performance in conventional and unconventional reservoirs. In: *Proceedings of the SPE Western Regional Meeting; 2009 Mar 24–26; San Jose, CA, USA.* doi:10.2118/121290-ms.
27. Ouyang W, Sun H, Han H. A new well test interpretation model for complex fracture networks in horizontal wells with multi-stage volume fracturing in tight gas reservoirs. *Nat Gas Ind B.* 2020;7(5):514–22. doi:10.1016/j.ngib.2020.09.009.
28. Cipolla CL. Modeling production and evaluating fracture performance in unconventional gas reservoirs. *J Petrol Technol.* 2009;61(9):84–90. doi:10.2118/118536-jpt.

29. Cipolla CL, Fitzpatrick T, Williams MJ, Ganguly UK. Seismic-to-simulation for unconventional reservoir development. In: Proceedings of the SPE Reservoir Characterisation and Simulation Conference and Exhibition; 2011 Oct 9–11; Abu Dhabi, United Arab Emirates. doi:10.2118/146876-ms.
30. Jia P, Cheng L, Huang S, Liu H. Transient behavior of complex fracture networks. *J Petrol Sci Eng.* 2015;132:1–17. doi:10.1016/j.petrol.2015.04.041.
31. Mayerhofer MJJ, Lolon EPP, Warpinski NRR, Cipolla CLL, Walser D, Rightmire CMM. What is stimulated reservoir volume? *SPE Prod Oper.* 2010;25(1):89–98. doi:10.2118/119890-pa.
32. Mayerhofer MJ, Lolon EP, Youngblood JE, Heinze JR. Integration of microseismic fracture mapping results with numerical fracture network production modeling in the Barnett Shale. In: Proceedings of the SPE Annual Technical Conference and Exhibition; 2006 Sep 24–27; San Antonio, TX, USA. doi:10.2118/102103-ms.
33. Brown M, Ozkan E, Raghavan R, Kazemi H. Practical solutions for pressure-transient responses of fractured horizontal wells in unconventional shale reservoirs. *SPE Reserv Eval Eng.* 2011;14(6):663–76. doi:10.2118/125043-pa.
34. Lee ST, Brockenbrough JR. A new approximate analytic solution for finite-conductivity vertical fractures. *SPE Form Eval.* 1986;1(1):75–88. doi:10.2118/12013-pa.
35. Jiang J, Younis RM. Numerical study of complex fracture geometries for unconventional gas reservoirs using a discrete fracture-matrix model. *J Nat Gas Sci Eng.* 2015;26:1174–86. doi:10.1016/j.jngse.2015.08.013.
36. Xu Y, Cavalcante Filho JS, Yu W, Sepehrnoori K. Discrete-fracture modeling of complex hydraulic-fracture geometries in reservoir simulators. *SPE Reserv Eval Eng.* 2017;20(2):403–22. doi:10.2118/183647-pa.
37. Zhou B, Chen Z, Song Z, Tang Z, Wang B, Olorode O. A new analytically modified embedded discrete fracture model for pressure transient analysis in fluid flow. *J Hydrol.* 2024;636:131330. doi:10.1016/j.jhydrol.2024.131330.
38. Han J, Chen Z, Zhou B. Transient flow analysis of multistage fractured horizontal wells with varied natural fractures connection relationships. *SPE J.* 2025;30(4):1984–2001. doi:10.2118/224417-pa.
39. Yu W, Xu Y, Weijermars R, Wu K, Sepehrnoori K. A numerical model for simulating pressure response of well interference and well performance in tight oil reservoirs with complex-fracture geometries using the fast embedded-discrete-fracture-model method. *SPE Reserv Eval Eng.* 2018;21(2):489–502. doi:10.2118/184825-pa.
40. Cui Y, Jiang R, Wang Q, Liu X. Production performance analysis of multi-fractured horizontal well in shale gas reservoir considering space variable and stress-sensitive fractures. *J Petrol Sci Eng.* 2021;207:109171. doi:10.1016/j.petrol.2021.109171.
41. Xu P, Xiang Y, Liu R, Lu M, Chen S. Evaluation of temporary blocking fracturing effect in horizontal wells by pumping down pressure drop method. *Xinjiang Oil Gas.* 2022;18(4):79–83. (In Chinese). doi:10.12388/j.issn.1673-2677.2022.04.014.
42. Wang F, Xu J, Zhou T, Zhang S. Pump-stopping pressure drop model considering transient leak-off of fracture network. *Petrol Explor Dev.* 2023;50(2):473–83. doi:10.1016/S1876-3804(23)60402-1.
43. Wang T, Yu HY, Zhao PF, Li JS, Liu RM, Kou SY, et al. Productivity assessment of tight gas wells after fracturing based on unstable pressure well test analysis. *Spec Oil Gas Reserv.* 2023;30(4):122–30. (In Chinese).
44. Chen Z, Zhou B, Zhang S, Yu W. Pressure transient behaviors of discretely fractured reservoirs using a numerical discrete fracture model. *SPE Prod Oper.* 2022;37(4):748–61. doi:10.2118/210591-pa.