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A Multi-Stage Expansion Planning Method for Rural Distribution Networks with Flexible Interconnection

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Received: 14 October 2025; Accepted: 23 December 2025; Published: 12 July 2026

ABSTRACT: With the increasing penetration of distributed generations and continuous growth of loads, traditional rural distribution networks face severe challenges in both hosting capacity and reliability. Addressing these issues requires planning approaches that strike a balance between economic efficiency in infrastructure development and resilience in operation. Considering the dynamic growth of distributed generations and rural loads over the planning horizon, this paper presents a multi-stage expansion planning approach that coordinates flexible interconnection devices (FIDs) with substation and line construction to improve both economic performance and system reliability. The proposed method account for the time-varying growth of DGs and loads, as well as the declining investment cost of power electronic devices across multiple planning stages. The model holistically considers both economic efficiency and operational reliability, formulating the problem as a mixed-integer second-order cone programming (MISOCP) model to ensure computational efficiency. Case studies conducted on a practical 138-node rural distribution network in Guangxi, China, demonstrate the effectiveness of the proposed method. Compared to traditional single-stage or single-resource planning strategies, results indicate that the proposed multi-stage coordinated strategy achieves a significant reduction in total annualized cost while simultaneously enhancing system reliability, effectively mitigating voltage violations, and achieving a 100% PV accommodation rate without curtailment. This work provides a practical and adaptive planning framework for rural distribution networks, offering valuable insights for achieving cost-effective and resilient network development under rural energy transition.

KEYWORDS: Rural distribution networks; coordinated planning; flexible interconnection device; multi-stage planning; reliability

1 Introduction

Driven by dual-carbon goals and rural revitalization strategy, rural areas in China are undergoing rapid energy transition and electrification [1]. The extensive integration of distributed renewable energy, particularly photovoltaics [2], is transforming rural distribution networks from traditional passive and unidirectional systems into active networks with bidirectional power flows and complex spatiotemporal characteristics [3]. While this transformation facilitates the development and consumption of renewable energy, it also poses significant challenges to planning and operation of rural distribution networks [4]. In remote regions, historical underinvestment and geographic constraints have led to persistently weak network structures and long power supply radii [5]. These issues result in severe voltage fluctuations, high power losses [6], and poor fault recovery capability [7]. As distributed generator (DG) penetration continues to rise, issues such as reverse power flow and voltage violations become increasingly prominent. These developments



underscore the inadequacy of traditional planning approaches in meeting future demands for safe, efficient, and flexible rural distribution networks [8].

To enhance power supply capacity, traditional distribution network planning has primarily focused on substation siting optimization [9] and feeder expansion [10]. Although such solutions can improve network capacity, they suffer from long investment cycles, diminishing marginal returns, and limited adaptability to the dynamic hosting capacity requirements driven by rapid changes in generation and load patterns [11]. In recent years, advancements in flexible power electronics and energy storage technologies have offered new perspectives for distribution planning. A multi-criteria decision framework for optimal deployment of mobile solid-state hydrogen energy storage systems is proposed in [12] to enhance the reliability and sustainability of renewable-powered distribution networks. In particular, flexible interconnection devices (FIDs) can provide power regulation [13], voltage support [14] and load transformation [15] without altering the physical topology [16]. Research has demonstrated the multiple benefits of FIDs in active distribution networks. FIDs have been widely explored to enhance distribution network performance under high photovoltaic penetration. For instance, they have been used to improve hosting capacity [17], reduce operational losses [18], and support multi-objective robust optimization of network resources [19]. FIDs have also been leveraged to enhance fault recovery and self-healing capabilities, enabling rapid load restoration during power outages [20].

At the planning level, several studies have incorporated FIDs into expansion decision-making. A siting and sizing model of FIDs is established in [21], effectively enhancing the economy, flexibility, and reliability of distribution networks. Research in [22] demonstrates that FIDs can simultaneously mitigate harmonic distortion, voltage deviation, and power losses under renewable and load uncertainties through optimal allocation and coordinated active/reactive power control. Moreover, recent efforts have explored the coordinated planning of FIDs with conventional grid assets to exploit their complementary strengths in flexibility and capacity [23]. A mixed-integer linear programming model for collaborative expansion planning of multi-energy distribution networks is developed in [24], which adeptly manages the complexities involved in the construction and reinforcement of multiple resources.

However, current research still exhibits notable limitations. First, most planning models overlook the functional complementarity and investment synergy potential between configurations of FIDs and traditional capacity expansion measures such as new substation construction and line reinforcement. Second, existing models are predominantly based on single-stage static assumptions, failing to adequately account for the dynamic growth of generations and loads over time in rural areas, as well as the time-varying characteristics of equipment investment costs and technical parameters across different planning stages.

To address the aforementioned issues, this paper proposes a multi-stage, multi-resource planning method tailored for rural distribution grids. It aims to achieve coordinated, dynamic, and cost-effective allocation of FIDs and traditional resources such as substations. The main contributions are summarized as follows: (1) A multi-stage expansion planning framework is developed to account for the growth of distributed generations and loads. This framework allows investment decisions to be adjusted dynamically across multiple planning stages, enabling the planning process to better accommodate the evolution of sources and load patterns, as well as the declining tendency of FID costs. (2) A multi-resource collaborative optimization model is established that considers the coordinated allocation of FIDs, substations, and line construction simultaneously. The proposed model incorporates a comprehensive objective function that integrates stage-dependent investment costs, operational losses, and reliability assessments considering FID-assisted fault restoration. Thereby, economic objectives and reliability objectives are unified within the multi-stage framework. (3) Case studies based on a real rural distribution network in Guangxi, China, are conducted to validate the effectiveness of the proposed method. The results demonstrate that the proposed

method can significantly reduce the total cost over the planning horizon and substantially improve reliability compared to single-stage planning approaches.

The remainder of this paper is organized as follows. [Section 2](#) details the mathematical model of the multi-stage coordinated planning problem. [Section 3](#) presents the case parameters and discusses the calculation results. [Section 4](#) summarizes the paper and outlines future research directions.

2 Multi-Stage Planning Model for Rural Distribution Networks

To accommodate the dynamic growth of generations and loads in rural distribution networks during the planning horizon, A multi-stage coordinated expansion planning model that integrates flexible interconnection devices with substation construction is established.

2.1 Planning Objectives Considering Both Economic Efficiency and Reliability

Considering the construction and operational costs of FIDs and substations, as well as the operational and outage losses of rural distribution networks, the comprehensive objective function can be formulated as follows:

$$\min f = f^{\text{FID}} + f^{\text{SL}} + f^{\text{E}} + f^{\text{R}} \quad (1)$$

2.1.1 Annual Construction and Operation Cost of FID

The FID annual construction and operating cost f^{FID} is regarded as the sum of the construction cost f^{C} and the operating cost f^{O} . The multi-stage construction cost of FIDs is shown in (2b)–(2d), and the multi-stage operational cost of FIDs is formulated as (2e) and (2f).

$$f^{\text{FID}} = f^{\text{C}} + f^{\text{O}} \quad (2a)$$

$$f^{\text{C}} = \sum_{r=1}^{N_r} f_r^{\text{C}} \quad (2b)$$

$$f_r^{\text{C}} = \frac{d(1+d)^y}{(1+d)^y - 1} \sum_{ij \in \Omega^{\text{FID}}} c_{r,ij}^{\text{FID}} S_{r,ij}^{\text{FID}}, r = 1 \quad (2c)$$

$$f_r^{\text{C}} = \frac{d(1+d)^y}{(1+d)^y - 1} \sum_{ij \in \Omega^{\text{FID}}} c_{r,ij}^{\text{FID}} (S_{r,ij}^{\text{FID}} - S_{r-1,ij}^{\text{FID}}), r \geq 2 \quad (2d)$$

$$f^{\text{O}} = \sum_{r=1}^{N_r} f_r^{\text{O}} / N_r \quad (2e)$$

$$f_r^{\text{O}} = \eta \sum_{ij \in \Omega^{\text{FID}}} c_{r,ij}^{\text{FID}} S_{r,ij}^{\text{FID}} \quad (2f)$$

2.1.2 Annual Construction and Operation Costs of Substations and Lines

The annual investment, construction, and operational maintenance costs for new substations and transmission lines f^{SL} is shown in (3a). It comprises the sum of the costs for new substations f_r^{Sub} , new constructed transmission lines f_r^{L1} , and reconstructed transmission lines f_r^{L2} .

$$f^{\text{SL}} = \sum_{r=1}^{N_r} (f_r^{\text{Sub}} + f_r^{\text{L1}} + f_r^{\text{L2}}) \quad (3a)$$

$$f_r^{\text{Sub}} = \begin{cases} (\xi + \gamma) \sum_{i \in \Omega^{\text{Sub}}} (c_r^{\text{Sub},0} \delta_{r,i}^{\text{Sub}} + c_r^{\text{Sub}} S_{r,i}^{\text{Sub}}), r = 1 \\ (\xi + \gamma) \sum_{i \in \Omega^{\text{Sub}}} (c_r^{\text{Sub},0} (\delta_{r,i}^{\text{Sub}} - \delta_{r-1,i}^{\text{Sub}}) + c_r^{\text{Sub}} (S_{r,i}^{\text{Sub}} - S_{r-1,i}^{\text{Sub}})), r \geq 2 \end{cases} \quad (3b)$$

$$f_r^{\text{L1}} = \begin{cases} (\xi + \gamma) c_r^{\text{L1}} \sum_{ij \in \Omega^{\text{L1}}} \delta_{r,ij}^{\text{L1}} L_{ij}^{\text{L1}}, r = 1 \\ (\xi + \gamma) c_r^{\text{L1}} \sum_{ij \in \Omega^{\text{L1}}} (\delta_{r,ij}^{\text{L1}} - \delta_{r-1,ij}^{\text{L1}}) L_{ij}^{\text{L1}}, r \geq 2 \end{cases} \quad (3c)$$

$$f_r^{\text{L2}} = \begin{cases} (\xi + \gamma) c_r^{\text{L2}} \sum_{ij \in \Omega^{\text{L2}}} \delta_{r,ij}^{\text{L2}}, r = 1 \\ (\xi + \gamma) c_r^{\text{L2}} \sum_{ij \in \Omega^{\text{L2}}} (\delta_{r,ij}^{\text{L2}} - \delta_{r-1,ij}^{\text{L2}}), r \geq 2 \end{cases} \quad (3d)$$

2.1.3 Annual Operational Loss Cost of Rural Distribution Network

The average annual operating loss cost f^E is shown in Eq. (4a). E_r^{LOSS} represents the expected daily operating losses of rural distribution network, encompassing both line losses and the operating losses of the FID installed on the distribution network.

$$f^E = 365 c^P \sum_{r=1}^{N_r} E_r^{\text{LOSS}} / N_r \quad (4a)$$

$$E_r^{\text{LOSS}} = \sum_{h=1}^{N_H} \sum_{t=1}^{N_T} \rho_h \left(\sum_{ij \in \Omega^b} r_{ij} I_{r,h,t,ij}^2 + \sum_{ij \in \Omega^{\text{FID}}} P_{r,h,t,ij}^{\text{FID,L}} \right) \Delta t \quad (4b)$$

2.1.4 Annual Power Outage Loss Cost of Rural Distribution Network

The annual power outage cost f^R is shown in Eq. (5a). The power outages E_r^{PSI} is calculated based on the power outage durations, as shown in Eqs. (5b) and (5c). The cost of power outage losses $E_r^{\text{LOSS,F}}$ is shown in Eq. (5d).

$$f^R = \sum_{r=1}^{N_r} (c^R E_r^{\text{PSI}} + c^P E_r^{\text{LOSS,F}}) / N_r \quad (5a)$$

$$E_r^{\text{PSI}} = \sum_{k \in \Omega^f} \lambda_k \sum_{i \in \Omega_k^L} T_{r,k,i} P_{r,k,i}^{\text{LOAD}} \quad (5b)$$

$$T_{r,k,i} = (1 - \mu_{r,k,i}) t^{\text{rep}} + \mu_{r,k,i}^0 t^{\text{iso}} + (\mu_{r,k,i} - \mu_{r,k,i}^0) (t^{\text{iso}} + t^{\text{FID}}) \quad (5c)$$

$$E_r^{\text{LOSS,F}} = \sum_{k \in \Omega^f} \lambda_k (t^{\text{rep}} - t^{\text{iso}} - t^{\text{FID}}) \times \left(\sum_{ij \in \Omega^b} r_{ij} I_{r,k,ij}^2 + \sum_{ij \in \Omega^{\text{FID}}} P_{r,k,ij}^{\text{FID,L}} \right) \quad (5d)$$

2.2 Multi-Stage Configuration Constraints of FIDs and Substations

The multi-stage planning for rural distribution networks requires coordinated configuration of FIDs and substations, and construction of transmission lines.

2.2.1 Multi-Stage Configuration Constraints of Flexible Interconnection Devices

During the planning process, configuration constraints of FIDs must be considered. The minimum configuration capacity for FIDs should be established as shown in Eqs. (6a) and (6b). Eqs. (6c) and (6d)

represent the installation quantity limits for FIDs. Additionally, the capacity of FID at the current stage is always greater than or equal to that of the previous stage, as illustrated in Eqs. (6e) and (6f).

$$m_{ij} \leq \delta_{r,ij}^{\text{FID}} M^{\text{FID}} \quad (6a)$$

$$S_{r,ij}^{\text{FID}} = s^m m_{ij} \quad (6b)$$

$$\sum_{ij \in \Omega^{\text{FID}}} \delta_{r,ij}^{\text{FID}} \leq N^{\text{FID,g}} \quad (6c)$$

$$\sum_{ij \in \Omega^{\text{FID}}} \delta_{r,ij}^{\text{FID}} \leq N^{\text{FID,n}}, \forall i \in \Omega^n \quad (6d)$$

$$\delta_{r-1,ij}^{\text{FID}} \leq \delta_{r,ij}^{\text{FID}}, \forall r \geq 2 \quad (6e)$$

$$S_{r-1,ij}^{\text{FID}} \leq S_{r,ij}^{\text{FID}}, \forall r \geq 2 \quad (6f)$$

2.2.2 Multi-Stage Configuration Constraints of Substations and Lines

The capacity constraints of substations must be considered during the planning process, as shown in Eq. (7a). The configuration of new substations will result in constructions of new lines and new inter-connections, as shown in Eqs. (7b) and (7c). Eqs. (7d) and (7e) represent multi-stage substation capacity constraints, indicating that the capacity will not be reduced in subsequent planning stages. Eqs. (7f) and (7g) represent multi-stage line construction constraints, allowing only line additions during the planning process.

$$0 \leq S_{r,i}^{\text{Sub}} \leq M \times \delta_{r,i}^{\text{Sub}} \quad (7a)$$

$$\sum_{ij \in \Omega^{\text{L1}}} \delta_{r,ij}^{\text{L1}} = \delta_{r,i}^{\text{Sub}} \quad (7b)$$

$$\sum_{ij \in \Omega^{\text{L2}}} \delta_{r,ij}^{\text{L2}} = \delta_{r,i}^{\text{Sub}} \quad (7c)$$

$$\delta_{r-1,i}^{\text{Sub}} \leq \delta_{r,i}^{\text{Sub}}, \forall i \in \Omega^{\text{Sub}}, \forall r \geq 2 \quad (7d)$$

$$S_{r-1,i}^{\text{Sub}} \leq S_{r,i}^{\text{Sub}}, \forall i \in \Omega^{\text{Sub}}, \forall r \geq 2 \quad (7e)$$

$$\delta_{r-1,ij}^{\text{L1}} \leq \delta_{r,ij}^{\text{L1}}, \forall ij \in \Omega^{\text{L1}}, \forall r \geq 2 \quad (7f)$$

$$\delta_{r-1,ij}^{\text{L2}} \leq \delta_{r,ij}^{\text{L2}}, \forall ij \in \Omega^{\text{L2}}, \forall r \geq 2 \quad (7g)$$

2.3 Operational Constraints of Rural Distribution Networks

2.3.1 Operational Constraints under Normal Conditions

(1) Power Flow Constraints

The branch flow method is employed to model the fundamental power flow constraints of rural distribution networks. Eqs. (8a) and (8b) represent the active and reactive power balance constraints at nodes, respectively. Eqs. (8c)–(8e) denote the branch flow constraints for general nodes, nodes of lines to be constructed, and nodes of lines to be reconstructed, respectively. The active and reactive power constraints for substation nodes are shown in Eqs. (8g)–(8j).

$$\sum_{ji \in \Omega^b} (P_{r,h,t,ji} - r_{ji} I_{r,h,t,ji}^2) + P_{r,h,t,i} = \sum_{il \in \Omega^b} P_{r,h,t,il} \quad (8a)$$

$$\sum_{ji \in \Omega^b} (Q_{r,h,t,ji} - x_{ji} I_{r,h,t,ji}^2) + Q_{r,h,t,i} = \sum_{il \in \Omega^b} Q_{r,h,t,il} \quad (8b)$$

$$U_{r,h,t,i}^2 - U_{r,h,t,j}^2 + (r_{ij}^2 + x_{ij}^2) I_{r,h,t,ij}^2 - 2(r_{ij} P_{r,h,t,ij} + x_{ij} Q_{r,h,t,ij}) = 0, \forall ij \in \Omega^b \setminus (\Omega^{\text{L1}} \cup \Omega^{\text{L2}}) \quad (8c)$$

$$|U_{r,h,t,i}^2 - U_{r,h,t,j}^2 + (r_{ij}^2 + x_{ij}^2) I_{r,h,t,ij}^2 - 2(r_{ij} P_{r,h,t,ij} + x_{ij} Q_{r,h,t,ij})| \leq M \times (1 - \delta_{r,ij}^{L1}), \forall ij \in \Omega^{L1} \quad (8d)$$

$$|U_{r,h,t,i}^2 - U_{r,h,t,j}^2 + (r_{ij}^2 + x_{ij}^2) I_{r,h,t,ij}^2 - 2(r_{ij} P_{r,h,t,ij} + x_{ij} Q_{r,h,t,ij})| \leq M \times \delta_{r,ij}^{L2}, \forall ij \in \Omega^{L2} \quad (8e)$$

$$P_{r,h,t,ij}^2 + Q_{r,h,t,ij}^2 = I_{r,h,t,ij}^2 U_{r,h,t,i}^2 \quad (8f)$$

$$|P_{r,h,t,i}^{DG} + P_{r,h,t,i}^{FID} - P_{r,h,t,i} - P_{r,h,t,i}^{LOAD}| \leq M \times \delta_{r,i}^{Sub}, \forall i \in \Omega^{Sub} \quad (8g)$$

$$P_{r,h,t,i} = P_{r,h,t,i}^{DG} + P_{r,h,t,i}^{FID} - P_{r,h,t,i}^{LOAD}, \forall i \in \Omega^n \setminus \Omega^{Sub} \quad (8h)$$

$$|Q_{r,h,t,i}^{DG} + Q_{r,h,t,i}^{FID} - Q_{r,h,t,i} - Q_{r,h,t,i}^{LOAD}| \leq M \times \delta_{r,i}^{Sub}, \forall i \in \Omega^{Sub} \quad (8i)$$

$$Q_{r,h,t,i} = Q_{r,h,t,i}^{DG} + Q_{r,h,t,i}^{FID} - Q_{r,h,t,i}^{LOAD}, \forall i \in \Omega^n \setminus \Omega^{Sub} \quad (8j)$$

(2) Secure Operation Constraints

To ensure the safe operation of rural distribution networks, node voltage constraints and branch current constraints are specified as shown in Eqs. (9a) and (9b), respectively. Additionally, considering substation reverse power flow rate limitations, the reverse power flow capacity of 10 kV rural distribution networks must not exceed 80% of the capacity of the transformer of the superior 110 kV substations, as indicated in Eq. (9c).

$$(U^{\min})^2 \leq U_{r,h,t,i}^2 \leq (U^{\max})^2 \quad (9a)$$

$$0 \leq I_{r,h,t,ij}^2 \leq (I^{\max})^2 \quad (9b)$$

$$-0.8S_{tran} \leq P_{r,h,t,ij}, \forall tran \in \Omega^{Sup}, ij \in \Omega^{tr} \quad (9c)$$

(3) Operational Constraints of FIDs

FID operation must satisfy the active power balance constraint, as shown in Eq. (10a), where equipment losses are given by Eq. (10b). Furthermore, the apparent power during FID operation must not exceed the equipment capacity, as indicated in Eq. (10c).

$$P_{r,h,t,i}^{FID} + P_{r,h,t,j}^{FID} + P_{r,h,t,i}^{FID,L} + P_{r,h,t,j}^{FID,L} = 0 \quad (10a)$$

$$A_i^{FID} \sqrt{(P_{r,h,t,i}^{FID})^2 + (Q_{r,h,t,i}^{FID})^2} = P_{r,h,t,i}^{FID,L} \quad (10b)$$

$$\sqrt{(P_{r,h,t,i}^{FID})^2 + (Q_{r,h,t,i}^{FID})^2} \leq S_{r,ij}^{FID} \quad (10c)$$

2.3.2 Operation Constraints under Fault Condition

(1) Radial Constraint of Rural Distribution Networks

Except for branches interconnected by FIDs, power restoration of rural distribution networks after faults should adhere to the principle of radial operation as follows.

$$\alpha_{r,k,ij} = \beta_{r,k,ij} + \beta_{r,k,ji}, \forall ij \in \Omega^b \cup \Omega^{FID} \quad (11a)$$

$$\alpha_{r,k,ij} = \delta_{r,ij}^{L1}, \forall ij \in \Omega^{L1} \setminus \Omega_k \quad (11b)$$

$$\alpha_{r,k,ij} = 1 - \delta_{r,ij}^{L2}, \forall ij \in \Omega^{L2} \quad (11c)$$

$$\sum_{ij \in \Omega^b \cup \Omega^{FID}} \beta_{r,k,ij} = 1 - \theta_{r,k,j}, \forall j \in \Omega^n \setminus \Omega^0 \quad (11d)$$

$$\sum_{ij \in \Omega^b \cup \Omega^{FID}} \beta_{r,k,ij} = 0, \forall j \in \Omega^0 \quad (11e)$$

(2) Power Restoration Constraints based on FIDs

Considering that the FID can switch to the $U_{ac}\theta$ control mode during faults to provide voltage support for power outage areas, as shown in Eqs. (12a)–(12c). Furthermore, it is assumed that nodes in the power outage areas can only be restored when supported by a stable voltage source, as depicted in Eqs. (12d)–(12h).

$$(U^{Vf,\min})^2 - M^{Vf}(1 - \beta_{r,k,ij}) \leq U_{r,k,j}^2 \leq (U^{Vf,\max})^2 + M^{Vf}(1 - \beta_{r,k,ij}), \forall ij \in \Omega^{FID} \quad (12a)$$

$$\alpha_{r,k,ij} \leq \delta_{r,ij}^{FID}, \forall ij \in \Omega^{FID} \quad (12b)$$

$$\beta_{r,k,ij} = 0, \forall ij \in \Omega^{FID}, i \in \Omega_k^{NS} \quad (12c)$$

$$s_{r,k,i} \leq \sum_{ji \in \Omega^b \cup \Omega^{FID}} (\beta_{r,k,ji} s_{r,k,j}), \forall i \in \Omega^n \setminus (\Omega^0 \cup \Omega^{Sub}) \quad (12d)$$

$$s_{r,k,i} \leq 1 - \theta_{r,k,i} \quad (12e)$$

$$\mu_{r,k,i} \leq M^{Sup} s_{r,k,i}, \forall i \in \Omega_k^{NS} \quad (12f)$$

$$s_{r,k,i} = 1, \forall i \in \Omega_0 \quad (12g)$$

$$s_{r,k,i} = \delta_{r,i}^{Sub}, \forall i \in \Omega^{Sub} \quad (12h)$$

Additionally, in fault scenarios, the subscripts of variables subject to power flow constraints should be adjusted accordingly. For instance, $P_{r,k,i}$ represents the active power at node i during the k th fault scenario in the r th stage, and $P_{r,k,i}^{LOAD}$ represents the load restoration amount at node i during the k th fault scenario in the r th stage. Due to page limitations, further elaboration is omitted here.

3 Case Study

The case study is based on a real 10 kV rural distribution network in Guangxi, China. The model is implemented in MATLAB R2022b using YALMIP and solved with the commercial solver CPLEX. The optimality tolerance is set to 1% to balance computational efficiency and solution accuracy. All simulations are run on a computer with an Intel Core Ultra 7265K processor (3.90 GHz), 32 GB RAM, and Windows 10. To capture the temporal characteristics of distributed generation and load, two representative days are selected, one from winter and the other from summer. Each typical day is divided into six time periods, with corresponding photovoltaic output and load power curves used as input data.

3.1 The 138-Node Rural Distribution Network

A practical rural distribution network in Guangxi, China, is used to validate the proposed model. The system comprises four 10 kV feeders. Feeders A and B exhibit high photovoltaic penetration rates. The distribution lines of Feeder C and D are long and heavily-loaded. The topology of the system is shown in Fig. 1. The case study comprises a total of 138 nodes and 134 branches, with photovoltaic (PV) connections at 24 nodes. The load and DG conditions for each area in the case study are summarized in Table 1.

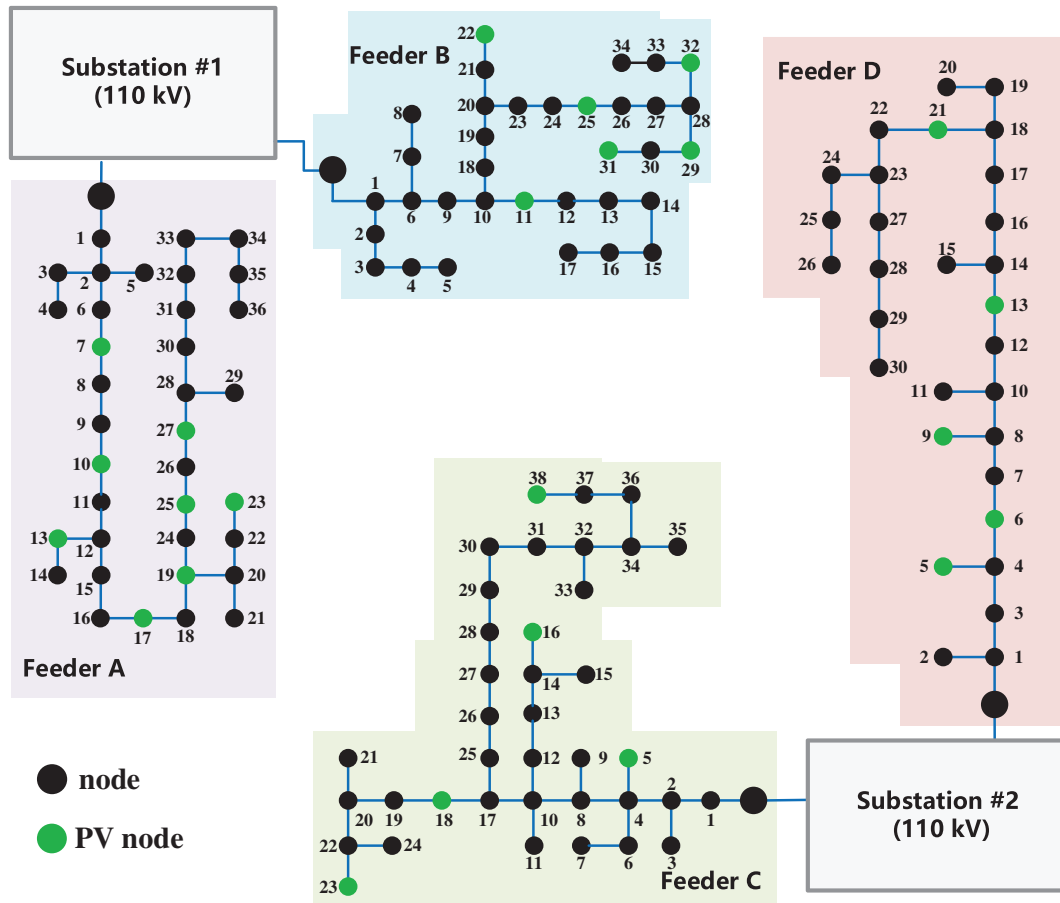


Figure 1: Topology of the 138-node rural distribution network

Table 1: Network parameters of the case

Area	Node amount	Positive load (MW)	Negative load (MW)	DG (MW)
Feeder A	36	4.4841	2.5862	4.4
Feeder B	34	4.3675	2.6791	3.2
Feeder C	38	7.1365	5.6964	0.8
Feeder D	30	5.2392	4.4388	0.8
Substation 1	70	8.8516	5.2653	7.6
Substation 2	68	12.3757	10.1352	1.6
Total	138	21.2273	15.4005	9.2

The maximum installation capacity for FID is 5 MV·A, with a unit configuration capacity of 0.1 MV·A. The loss coefficient is 0.02, and the initial unit investment cost for FID is 800 CNY/(kV·A). The discount rate of FID is 0.08, with a service life of 20 years and an operation cost factor of 0.01. To ensure safe and stable distribution network operation, the current amplitude upper limit for 10 kV lines is set at 1000 A. The safe operating voltage range of rural distribution network is set to 0.95 to 1.05 p.u. Load and DG output curves are

shown in Fig. 2. Two typical daily operating curves are considered as representative scenarios for planning, each comprising 6 time slots.

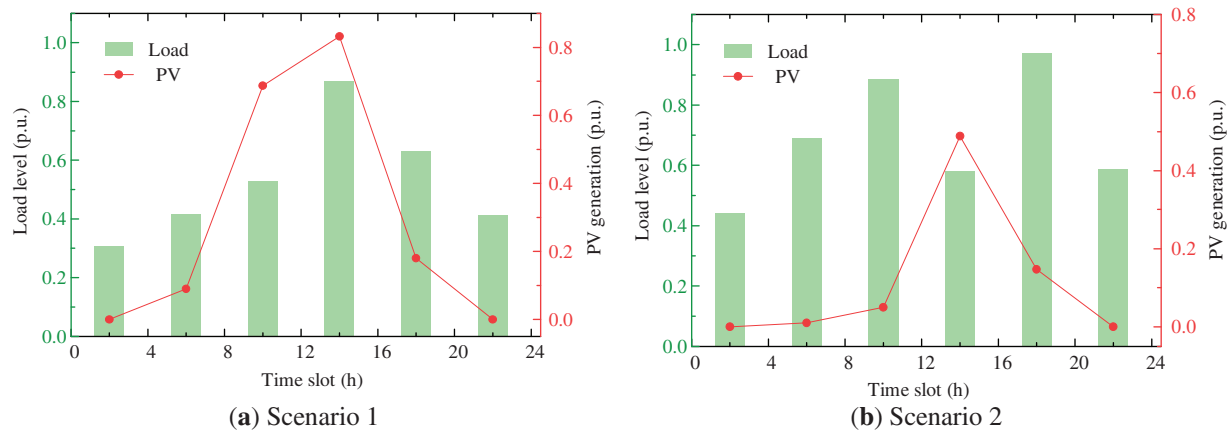


Figure 2: Typical scenarios for photovoltaic generation and load level

3.2 Scheme Settings

The proposed method is applied to plan the rural distribution network over a 15-year horizon. The line fault between node 27 and node 28 of feeder A is used as a fault scenario for reliability assessment. Table 2 lists the investment costs of different equipment or construction items in each planning stage. Table 3 shows the projected growth of load and distributed generation across the stages.

Table 2: Equipment and construction costs in each planning stage

Item	Stage 1	Stage 2	Stage 3
FID unit cost (CNY/kVA)	800	600	500
Substation fixed cost (10^4 CNY)	200	210	220
Substation unit cost (10^4 CNY/MVA)	10	11	12
New feeder unit cost (10^4 CNY/km)	20	22	24
Feeder upgrade fixed cost (10^4 CNY)	10	11	12

Table 3: Load and distributed generation growth across planning stages

Quantity	Stage 1	Stage 2	Stage 3
Load (p.u.)	1.00	1.10	1.19
Distributed Generation (p.u.)	1.00	1.21	1.41

Based on the above parameters, the following four schemes are defined.

- Scheme 1: The original operational state of the system without planning.
- Scheme 2: Considering the economic efficiency, substations and FIDs are configured with a planning horizon of 15 years in order to evaluating the economic benefit of infrastructure investment.
- Scheme 3: Considering both the economic efficiency and reliability, substations and FIDs are configured with a planning horizon of 15 years, evaluating the economic and reliability improvement.

- Scheme 4: Considering both the economic efficiency and reliability, substations and FIDs are configured with a planning horizon of 15 years in three stages for the accommodation of the source and load variation.

3.3 Results Analysis

The detailed configurations of FIDs and substations in Schemes 2 and 3 are presented in Table 4.

Table 4: The detailed configurations of FIDs and substations in Schemes 2 and 3

Schemes	Location of the FID	Capacity(MVA)	Location of the substation
S2	A29-C21	1.4	D18
	B28-C35	1.4	
S3	A29-C21	1.9	D18
	B28-C35	1.2	

The optimal configurations of FIDs and substations in each stage of Scheme 4 are shown in Fig. 3 and the detailed configurations are presented in Table 5.

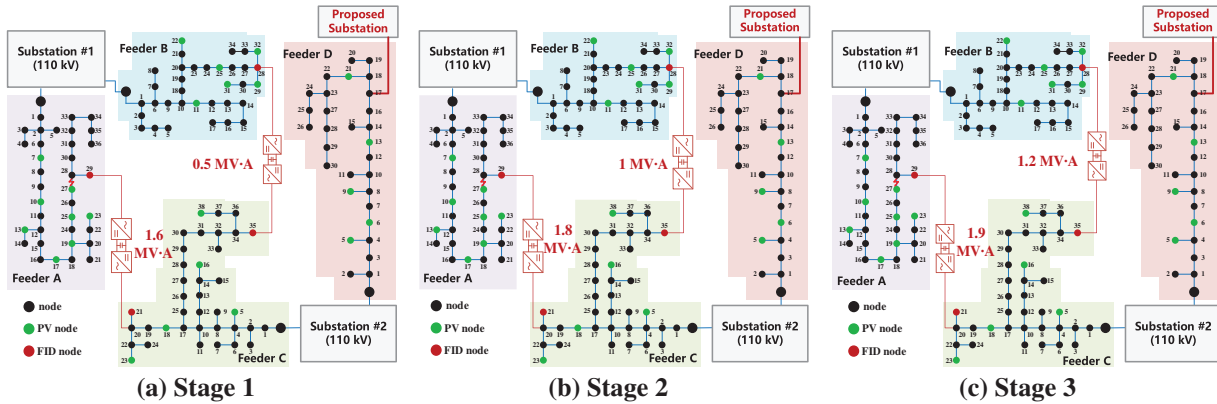


Figure 3: The optimal configurations of FIDs and substations

Table 5: The detailed configurations of FIDs and substations in Scheme 4

Schemes	Location of the FID	Capacity(MVA)			Location of the Substation
		Stage 1	Stage 2	Stage 3	
S4	A29-C21	1.6	1.8	1.9	D18
	B28-C35	0.5	1	1.2	

As shown in Table 5, the locations that substations and FIDs are installed remain unchanged across the three stages, while the capacity of the installed FIDs is progressively expanded over the planning stages. This suggests that incrementally increasing FID capacity in a staged manner while meeting stage-specific supply capacity and reliability constraints can achieve better economic performance without compromising planning objectives, as the unit investment cost of FIDs decreases.

Table 6 lists the planning performance metrics under the four schemes.

Table 6: Planning performance metrics under the four schemes

Annualized Cost ($\times 10^4$ CNY)	S1	S2	S3	S4
Total annualized cost	297.88	223.33	214.13	209.4
FID investment	0	56.82	62.91	57.07
Substation investment	0	40.12	40.12	40.12
Power outage costs	88.04	16.78	0.13	0.13
System average interruption duration index (SAIDI) (h)	3.91	1.92	0.51	0.51
Operational loss costs	209.84	109.61	110.97	112.08
Extreme voltage (p.u.)	0.88~1.05	0.94~1.04	0.95~1.04	0.95~1.04

As shown in Table 6, compared with Scheme 1, Scheme 2 significantly improves system performance by planning FIDs and substations. The node voltages are maintained within secure limits, and both operational losses and total cost are reduced, indicating that the coordinated deployment of FIDs and substations effectively enhances the network's operational efficiency and economic performance. The PV accommodation rate also increases from 84.38% to 99.7% in the final year of the planning period.

However, Scheme 2 optimizes only for economic cost and ignores reliability under fault conditions, resulting in relatively high outage losses. In contrast, Scheme 3 jointly considers economic and reliability objectives, reducing the average annual outage loss to 1.3 thousand CNY and lowering the corresponding SAIDI index. Meanwhile, the PV accommodation rate reaches 100%.

Scheme 4 introduces a multi-stage planning framework that divides the 15-year horizon into three stages (each stage lasts for five years) and dynamically adjusts investment decisions by evolving source-load profiles and equipment costs. Results show that Scheme 4 also reduces outage losses by 99.2% compared with Scheme 2. The investment cost is reduced by 9.3% compared with Scheme 3. Scheme 4 achieves a total annualized cost of 2.094 million CNY. This represents a significant reduction of 29.7% compared to Scheme 1, and also outperforms the other planning scenarios with cost reductions of 6.2% against Scheme 2 and 2.2% against Scheme 3. Furthermore, voltage violations are fully eliminated, indicating improved voltage stability and operational security. This indicates that Scheme 4 outperforms all other schemes across key metrics.

The multi-stage planning results under different FID cost reduction rate are shown in Table 7.

Table 7: Multi-stage planning results under different FID cost reduction rate

Case	FID Unit Cost (CNY/kVA)	FID Capacity (MVA)		
		Stage 1	Stage 2	Stage 3
C1	[800, 300, 200]	2.0	2.8	3.2
C2	[800, 600, 500]	2.1	2.8	3.1
C3	[800, 800, 800]	2.9	3.1	3.1

As shown in Table 7, when the cost of FID devices in case 1 decreases rapidly, the installed FID capacity in the first stage is relatively low. In contrast, the capacity constructed in the final stage is higher than other cases. This indicates that the investment in FIDs is postponed due to the rapid decline in equipment cost. In

case 3, where the FID unit cost remains constant, the FID installed capacity of the first stage is significantly higher than other cases. The results demonstrate that variations in FID cost trajectories can significantly influence the timing and scale of FID capacity expansion in the planning process. As the FID cost declines more rapidly, the proposed model tends to postpone FID installation to later stages with lower unit costs, thereby reducing the capacity planning in earlier, high-cost stages.

Table 8 presents the multi-stage planning results under different load growth rates. As shown in the table, a higher load growth rate leads to a larger total planned FID capacity, and the incremental FID capacity between consecutive stages also increases correspondingly. This indicates that as the load expands more rapidly, the model allocates additional FID installations to maintain network reliability and voltage stability across the planning horizon.

Table 8: Multi-stage planning results under different load growth rate

Case	Load level (p.u.)	FID Capacity (MVA)		
		Stage 1	Stage 2	Stage 3
C4	[1.00, 1.10, 1.21]	2.1	2.6	3.4
C5	[1.00, 1.16, 1.34]	2.1	3.0	4.1
C6	[1.00, 1.22, 1.48]	2.1	3.3	5.0

4 Conclusions

This paper presents a multi-stage expansion planning method for rural distribution networks with flexible interconnection. The proposed approach co-optimises economic and reliability costs accounting for the dynamic growth of load and distributed generation. By deploying FIDs and substations at critical locations through multiple planning stages, the method significantly improves system performance. Calculation results demonstrate effective reduction in both operational losses and outage costs. It effectively mitigates voltage violations and achieves a 100% PV accommodation rate without curtailment. The proposed framework provides a quantitative cost-benefit tool that enables planners to identify the most economical investment pathways. For grid operators, the proposed framework serves as a quantitative decision-support tool to determine the optimal timing and scale of FID, substation, and feeder investments under evolving demand and renewable penetration. For policymakers, the model enables cost-benefit assessment and investment prioritisation, supporting the formulation of regulatory and incentive mechanisms that promote flexible and resilient rural distribution networks. Beyond the specific case of Guangxi, the proposed approach can be readily adapted to different regional cases by adjusting local parameters such as investment costs, renewable generation characteristics, and reliability targets.

The proposed multi-stage coordinated planning framework enables adaptive investment pathways that align capacity expansion with evolving sources and loads and declining FID costs. However, the current model is based on deterministic load and PV generation curves. Future research will extend the proposed framework by integrating stochastic or scenario-based optimisation to capture uncertainties in renewable generation and demand evolution. In addition, multi-energy coordination such as energy storage system will be considered to further enhance system flexibility and economic performance. These extensions will strengthen the practical applicability of the proposed approach, and further enhance the hosting capacity and operational performance of rural distribution networks.

Nomenclature

$\Omega^{L1}, \Omega^{L2}, \Omega_k$	Set of lines to be constructed, lines to be reconstructed and faulty lines in fault scenario k .
$\Omega^b, \Omega^{FID}, \Omega^{tr}$	Set of branches, FID branches, branches connected to 110 kV substations.
$\Omega^n, \Omega^0, \Omega_k^L$	Set of nodes, source nodes and power-off nodes in fault scenario k .
Ω^f, Ω_k^{NS}	Set of fault scenarios and non-source nodes in fault scenario k .
$\Omega^{Sub}, \Omega^{Sup}$	Set of newly constructed substation nodes and superior 110 kV substation nodes.
$P_{r,h,t,i}^{FID}, Q_{r,h,t,i}^{FID}$	Active and reactive power injected by FID at node i at time t in scenario h of stage r .
$P_{r,h,t,ij}, Q_{r,h,t,ij}$	Active and reactive power flow from node i to node j at time t in scenario h of stage r .
$P_{r,h,t,i}, Q_{r,h,t,i}$	Active and reactive power injected at node i at time t in scenario h of stage r .
$P_{r,h,t,i}^{DG}, Q_{r,h,t,i}^{DG}$	Active and reactive power injected by DG at node i at time t in scenario h of stage r .
$P_{r,h,t,i}^{LOAD}, Q_{r,h,t,i}^{LOAD}$	Active and reactive load at node i at time t in scenario h of stage r .
$P_{r,h,t,ij}^{FID,L}$	Power loss of FID installed at branch ij at time t in scenario h of stage r .
$U_{r,h,t,i}^2, I_{r,h,t,ij}^2$	Square of the voltage amplitude of node i and the current amplitude of branch ij at time t in scenario h of stage r .
$\delta_{r,i}^{Sub}, \delta_{r,ij}^{FID}$	Binary variables indicating whether a new substation is to be constructed at node i and whether an FID is to be constructed between node i and node j in stage r .
$\delta_{r,ij}^{L1}, \delta_{r,ij}^{L2}$	Binary variables indicating whether a new line ij is to be constructed and whether line ij is to be reconstructed in stage r .
$\alpha_{r,k,ij}, \beta_{r,k,ij}, \theta_{r,k,j}$	Switch status of branch ij , relationship between node i and node j , and node state of node j in fault scenario k of stage r .
$s_{r,k,i}$	Binary variables indicating whether node i has power supply in fault scenario k of stage r .
$S_{r,i}^{Sub}, S_{r,ij}^{FID}$	Capacity of new substation i and FID installed at branch ij in stage r .
$T_{r,k,i}$	Power outage duration of node i in failure scenario k of stage r .
$\mu_{r,k,i}, \mu_{k,i}^0$	Restoration coefficient of node i with FID and without FID in fault scenario k of stage r .
λ_k	Annual average occurrence frequency of failure scenario k .
s^m, m_{ij}	Unit configuration capacity of FID and number of FID modules installed on branch ij .
ρ_h	Probability of scenario h .
S_{tran}	Capacity of the transformer of superior 110 kV substation.
$c_r^{Sub,0}, c_r^{Sub}, c_{r,ij}^{FID}$	Fixed cost and unit capacity cost for new substation, and unit capacity cost of FID located at branch ij in stage r .
c_r^{L1}, c_r^{L2}	Unit cost for line construction and fixed cost for line reconstruction in stage r .
c^P, c^R	Unit electricity price and unit cost of outage losses.
d, y, η	Discount rate, lifetime, and coefficient of the annual operational costs of FID.
U^{max}, U^{min}	Upper and lower limit of node voltage amplitude.
$U^{Vf,max}, U^{Vf,min}$	Upper and lower limit of voltage amplitude on the fault side of FID.
I^{max}	Upper limit of branch current.
$N^{FID,g}, N^{FID,n}$	Maximum number of FIDs installed in rural distribution network and at single node.
N_r, N_H, N_T	Number of planning stages, operational scenarios and time slots within a single scenario.
L_{ij}^{L1}	Length of newly constructed line ij .
A_i^{FID}	Loss coefficient of FID at node i .
M^{FID}	Installation limit of FID modules.
M, M^{Sup}, M^{Vf}	Sufficiently large constants in planning constraints and power recovery constraints.
r_{ij}, x_{ij}	Resistance and reactance of branch ij .
ξ, γ	Capital recovery factor and equipment maintenance factor.
$t^{rep}, t^{iso}, t^{FID}$	Required restoration time by repairing the faulty branch, by isolating the faulty branch using isolating switch and by FID.

Acknowledgement: Not applicable.

Funding Statement: This study was funded by the Smart Grid-National Science and Technology Major Project of China (2024ZD0800600).

Author Contributions: Conceptualization, Yueyang Ji and Haoran Ji; methodology, Yuxuan Chen; validation, Yaohui Peng and Xinran Na; Yaohui Peng and Xinran Na; writing—original draft preparation, Yueyang Ji, Xinran Na, Yuxuan Chen and Yaohui Peng; writing—review and editing, Wei Li and Shengbin Chen; supervision, Haoran Ji. All authors reviewed the results and approved the final version of the manuscript.

Availability of Data and Materials: Not applicable.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest to report regarding the present study.

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