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ARTICLE

A Robust Hybrid WLS-EKF Algorithm for Power System State Estimation

Zahid Javid[1,2], **Kush Lohana**[2], **Danial Murtaza**[2] **and William Holderbaum**[3,*]

[1]Salford Business School, University of Salford, Manchester, UK
[2]TNEI Services Limited, Manchester, UK
[3]Department of Control Engineering, Manchester Metropolitan University, Manchester, UK

*Corresponding Author: William Holderbaum. Email: w.holderbaum@mmu.ac.uk

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ABSTRACT: This paper introduces a novel hybrid method for Power System State Estimation (PS-SE) that effectively integrates the strengths of Weighted Least Squares (WLS) and the Extended Kalman Filter (EKF) through an adaptive weighting mechanism. The proposed method addresses key challenges in modern PS-SE, including measurement uncertainties, bad data detection and handling, and convergence reliability. By incorporating an adaptive weighting mechanism, the hybrid approach dynamically adjusts estimation parameters based on the quality of the measurements, enabling it to maintain high accuracy for clean data while demonstrating exceptional resilience against outliers and noisy measurements. The performance of the proposed method is rigorously evaluated against established state estimation techniques, including WLS, EKF, Bayesian method, Huber-Adaptive Method (HAM) and Neural network-based Method. Simulations are performed on IEEE 14-bus, IEEE 34-bus and IEEE 342-bus test systems to assess estimation accuracy, convergence behavior, computational efficiency, and robustness in the presence of bad data. Results highlight the superior performance of the hybrid method, which achieves higher accuracy and robust convergence properties while requiring 40% fewer iterations than conventional WLS. Despite its enhanced capabilities, the computational burden remains comparable to traditional techniques, making it highly suitable for real-time applications. These findings underscore the proposed hybrid method as a significant advancement in power system state estimation, offering a reliable, efficient, and robust solution for modern power system monitoring and control. It represents a promising approach to address the increasing complexity and data uncertainties in contemporary power grids.

KEYWORDS: Adaptive estimation; hybrid estimator; power system monitoring; state estimation

1 Introduction

Power System State Estimation (PSSE) plays a crucial role in modern power system operation and control, serving as the backbone for real-time monitoring and decision-making processes. With the increasing integration of renewable energy sources, smart grid technologies, and distributed generation, the complexity of power systems has grown significantly, making accurate and robust state estimation more challenging yet more critical than ever. Traditional state estimation methods, while well-established, often struggle with measurement uncertainties, bad data, and convergence issues in modern power networks. The fundamental challenge in PSSE lies in accurately determining system states from a set of redundant measurements that may contain errors, noise, and bad data. While conventional methods like Weighted Least Squares (WLS), Extended Kalman Filter (EKF), have served as the industry standard, their performance can deteriorate significantly in the presence of bad data or during unusual system conditions. This limitation has motivated the development of more robust and adaptive estimation techniques. Recent advances in statistical methods, computational capabilities have opened new avenues for improving state estimation accuracy and reliability. However, many proposed advanced methods either lack robustness against bad data or require excessive computational resources, making them impractical for real-time applications.

Modern power systems are experiencing a paradigm shift in their monitoring and measurement infrastructure. The deployment of Phasor Measurement Units (PMUs) has introduced synchronized, high-resolution measurements that complement traditional SCADA systems, enabling real-time wide-area monitoring and control [1,2]. Concurrently, the proliferation of smart meters and Advanced Metering Infrastructure (AMI) at the distribution level has created unprecedented opportunities for enhanced observability, particularly in systems with high penetration of distributed energy resources [3]. However, this heterogeneous measurement landscape also presents significant challenges in terms of data fusion, optimal utilization of measurements with varying accuracies and sampling rates, and handling of communication delays and data gaps. Distribution networks and microgrids present unique challenges for state estimation that differ substantially from transmission systems. Three-phase unbalanced conditions, the presence of neutral conductors, high R/X ratios, and limited real-time measurements characterize distribution system state estimation (DSSE) [4]. Microgrids, with their dynamic topology changes, islanding capabilities, and high penetration of intermittent renewable generation, require state estimation methods that can adapt rapidly to changing operational conditions [5]. In scenarios with limited measurement infrastructure, pseudo-measurements derived from historical data, load forecasts, and statistical models become essential for maintaining system observability, though they introduce additional uncertainty that must be carefully managed [6].

This paper addresses these challenges by introducing and novel Hybrid Huber-Adaptive Robust Method (HH-ARM) for PSSE by combining the statistical robustness of Huber's M-estimator with adaptive optimization technique. The main contributions of this paper are:

We propose a novel state estimation method that dynamically integrates weighted least squares (WLS) and extended Kalman filter (EKF) approaches through an adaptive weighting mechanism. Unlike existing hybrid methods that use fixed weights or switching logic, our approach continuously adjusts the contribution of each component based on real-time assessment of measurement quality and system operating conditions.

A regularized adaptive weighting scheme is introduced that balances estimation accuracy and numerical stability. The mechanism automatically determines optimal fusion between static WLS and dynamic EKF estimates without requiring system-specific parameter tuning, demonstrated through consistent performance across test systems of different scales.

The paper provides detailed guidance on measurement infrastructure requirements, parameter tuning strategies, and extension to emerging applications, facilitating practical deployment in modern power system control centers.

2 State of the Art and Beyond

The Weighted Least Squares (WLS) estimator, first introduced by Schweppe and Wildes [7], remains one of the most widely used methods for Power System State Estimation (PS-SE). By minimizing the sum of squared residuals weighted by measurement variances, WLS offers computational simplicity and efficiency, making it a practical choice for many real-world applications. Since its introduction in 1970, WLS has undergone numerous advancements to enhance its performance. For instance, a non-iterative WLS approach was proposed in [8] to improve computational speed, while iterative reweighting techniques were introduced in Ref. [9] to enhance robustness. Additionally, Ref. [10] developed an optimal weighting strategy for WLS to improve estimation accuracy. Despite these advancements, WLS remains highly sensitive to bad data, as it assumes Gaussian measurement errors, and its performance can degrade significantly in the presence of outliers [6].

To address the limitations of static estimators like WLS, the Extended Kalman Filter (EKF) [11] was introduced to extend state estimation to dynamic systems. EKF incorporates system state prediction into the estimation process, enabling it to track dynamic state changes more effectively. While this approach improves performance in dynamic environments, it still struggles with measurement outliers and is computationally intensive, particularly for large-scale systems [12].

The Bayesian method [13] provides a probabilistic framework for state estimation by incorporating prior knowledge into the estimation process. This approach is particularly advantageous in scenarios with high uncertainty, as it allows for more informed estimations. However, Bayesian methods often require significant computational resources, which limit their applicability for real-time PS-SE in large-scale systems [14].

Recent advancements in artificial intelligence (AI) have introduced neural networks and deep learning approaches, which can model the complex, nonlinear relationships in power systems [15]. These methods have shown promise in improving estimation accuracy and robustness, particularly in handling noisy and incomplete data. However, they come with challenges, such as the need for extensive training data, high computational demands, and limited interpretability, which can hinder their adoption in critical real-time applications [2,14]. Recent advances in robust state estimation address computational efficiency and adaptability through both advanced M-estimation and hybrid approaches. Reference [16] proposed generalized M-estimation with sampling-optimized parameters, achieving robustness up to 12% contamination with 15–18 s computation time for 300-bus systems. The method adaptively adjusts the Huber parameter β based on measurement noise distribution, improving accuracy by 23% over fixed-parameter approaches in non-Gaussian scenarios. Authors in [17] developed interval state estimation using fixed-point expansion for distribution systems, providing guaranteed bounds under uncertainty but with $O(n^4)$ complexity limiting scalability. Reference [18] proposed graph neural networks exploiting topology, achieving fast inference but limited robustness to bad data.

Huber's M-estimator [16] provides a robust alternative to traditional L2-based methods by balancing the sensitivity of the L1 and L2 norms. This approach is effective in handling outliers while maintaining efficiency for normal measurements. However, its performance can still be limited by the choice of tuning parameters and its reliance on static weighting, which may not adapt well to varying system conditions.

The integration of PMUs has revolutionized power system state estimation by providing synchronized voltage and current phasor measurements with sampling rates up to 60 samples per second [19]. Unlike conventional SCADA measurements, PMU data offers time-synchronized measurements across geographically dispersed locations, enabling enhanced observability and dynamic state tracking. However, the integration of PMU and SCADA measurements in a hybrid framework presents several challenges, including optimal placement of PMUs for system observability, handling measurements with different sampling rates and accuracies, and managing communication delays and data losses in wide-area monitoring systems [20]. Recent research has focused on developing hybrid state estimators that optimally combine PMU and SCADA data. Linear state estimation methods leveraging PMU measurements have been proposed to avoid the computational burden of iterative nonlinear estimation [21]. However, in partially observable systems where PMU coverage is incomplete, nonlinear estimation remains necessary. Adaptive weighting schemes that dynamically adjust the contribution of PMU and SCADA measurements based on their respective characteristics have shown promise in improving estimation accuracy and robustness. The proposed hybrid method in this paper, with its adaptive weighting mechanism, is particularly well-suited for such hybrid measurement environments, as it can dynamically adjust the contribution of different measurement types based on their quality and reliability.

Microgrids represent a paradigm shift in power system architecture, characterized by the integration of distributed generation, energy storage systems, and controllable loads within a localized network that can operate in both grid-connected and islanded modes [5]. State estimation in microgrids faces unique challenges including: rapid topology changes during islanding transitions, high penetration of intermittent renewable generation causing significant state variations, limited measurement infrastructure due to economic constraints, and the need for real-time estimation to support fast control actions. The application of Kalman filter-based methods, particularly the EKF has gained significant attention for microgrid state estimation due to their inherent ability to track dynamic system states [22]. These methods incorporate system dynamics through process models, making them particularly suitable for capturing the rapid state changes characteristic of microgrids. Recent work has demonstrated the effectiveness of adaptive Kalman filtering approaches that adjust process and measurement noise covariances based on operating conditions. However, these methods can be computationally intensive and sensitive to model uncertainties, particularly during transient conditions or mode transitions. Distributed and decentralized state estimation approaches have also emerged as promising solutions for microgrids, offering improved scalability and resilience to communication failures. These methods decompose the global estimation problem into local sub-problems solved at individual nodes or zones, with coordination achieved through limited information exchange. The hybrid approach proposed in this paper could be extended to such distributed frameworks, leveraging its adaptive weighting mechanism to handle the varying data quality and availability inherent in microgrid environments.

State estimation in three-phase distribution networks presents distinct challenges compared to transmission systems, primarily due to the unbalanced nature of distribution systems, high R/X ratios of distribution lines, radial or weakly meshed topology, and the presence of neutral conductors that must be explicitly modeled [4]. Traditional state estimation methods developed for balanced transmission systems often fail to provide accurate results when directly applied to unbalanced distribution networks, as they neglect the coupling between phases and the role of the neutral conductor.

Recent advances in DSSE have focused on developing three-phase models that explicitly account for system unbalances. The branch current-based formulation has gained popularity for distribution systems due to its better numerical properties compared to traditional bus voltage-based methods, particularly in systems with high R/X ratios [4]. Furthermore, the inclusion of neutral conductor modeling has been shown to significantly improve estimation accuracy, especially in four-wire distribution systems common in low-voltage networks [23]. The challenge of limited real-time measurements in distribution networks has led to increased reliance on pseudo-measurements and virtual measurements. Recent work has explored the integration of smart meter data, which provides high-resolution consumption data but typically lacks the time synchronization of SCADA or PMU measurements. Advanced load modeling techniques and the use of machine learning for load forecasting have been proposed to generate more accurate pseudo-measurements [24]. While the current implementation of our hybrid method focuses on balanced systems, the adaptive weighting framework is inherently suitable for extension to three-phase systems, where it could optimally weight measurements and pseudo-measurements of varying quality across different phases.

In many practical power system state estimation applications, particularly in distribution networks and emerging microgrids, the availability of real-time measurements is limited due to economic constraints and infrastructure limitations. In such scenarios, pseudo-measurements become essential for maintaining system observability. Pseudo-measurements are typically derived from historical load data, short-term load forecasts, typical load profiles, and network topology information. However, these pseudo-measurements inherently carry higher uncertainty compared to real-time measurements, and their accuracy directly impacts the overall state estimation performance. The challenge of integrating heterogeneous data sources—including SCADA measurements, PMU data, smart meter readings, and pseudo-measurements—has received significant research attention. Multi-source data fusion techniques aim to optimally combine information from various sources while accounting for their different characteristics in terms of accuracy, sampling rate, time synchronization, and reliability [25]. Bayesian frameworks have been proposed to incorporate prior information and uncertainty quantification in state estimation with pseudo-measurements [13]. However, these methods can be computationally demanding and require careful specification of prior distributions.

Recent research has explored the use of machine learning techniques to improve pseudo-measurement accuracy. Neural networks and Gaussian processes have been employed to learn complex load patterns from historical data and generate more accurate load forecasts. Additionally, adaptive strategies that update pseudo-measurement variances based on forecast accuracy and real-time measurement residuals have shown promise in improving estimation robustness [26]. The adaptive weighting mechanism in our proposed hybrid method is particularly well-suited for handling scenarios with mixed real and pseudo-measurements, as it can dynamically adjust the contribution of each measurement based on its quality indicator, effectively down-weighting unreliable pseudo-measurements while maintaining the benefits of increased redundancy.

Beyond traditional WLS-based approaches, several advanced methodologies have been developed to address specific challenges in power system state estimation, particularly concerning robustness, computational efficiency, and handling of non-Gaussian measurement errors. While the classical WLS method assumes Gaussian measurement errors, real-world power system measurements often contain outliers and non-Gaussian noise. M-estimators provide robust alternatives by replacing the quadratic cost function with more robust loss functions. The Huber M-estimator combines quadratic loss for small residuals with absolute loss for large residuals, providing robustness against outliers while maintaining efficiency for Gaussian noise [16]. Correntropy, a localized similarity measure from information theoretic learning, has recently been applied to state estimation to handle non-Gaussian noise and outliers [27]. Correntropy-based Kalman filters have been developed for dynamic state estimation, showing superior performance in heavy-tailed noise environments compared to standard EKF [28]. However, these methods introduce a kernel bandwidth parameter that significantly affects performance, and the non-convex optimization problem can lead to local minima. Traditional state estimation formulates a nonlinear least squares problem due to the nonlinearity of power flow equations. Linear state estimation methods aim to avoid iterative solutions by reformulating the problem in terms of linear measurements or through linearization techniques. PMU-based linear state estimation leverages the linear relationship between voltage phasors and current phasor measurements, enabling direct solution without iteration [29]. While these methods offer computational advantages, they require specific measurement configurations (e.g., full PMU observability) or system topologies, limiting their applicability in hybrid measurement scenarios where both linear and nonlinear measurements coexist.

Extensions to state estimation have been proposed, formulating Holomorphic Embedding State estimation Method that inherits the convergence properties of Holomorphic Embedding Load Flow [30]. However, the practical implementation of holomorphic embedding for state estimation faces challenges including computational complexity of power series calculations for large systems, difficulty in incorporating inequality constraints, and limited research on handling bad data and topology errors.

Despite the significant advances in PS-SE methodologies, existing approaches often face one or more limitations, such as sensitivity to bad data, lack of adaptability to changing system conditions, high computational complexity, poor convergence properties, or limited scalability to modern power systems. These limitations create a gap in the ability to provide accurate, robust, and computationally efficient state estimation for evolving power systems characterized by increased grid complexity, intermittent renewable energy sources, and diverse measurement uncertainties.

To address these challenges, this paper proposes a hybrid state estimation method that combines the strengths of WLS and EKF through an adaptive weighting mechanism. The proposed method dynamically adjusts the contributions of WLS and EKF based on the quality of measurements and system conditions, enabling it to maintain high accuracy for clean data while being robust against outliers and measurement noise. By leveraging the computational efficiency of WLS and the dynamic tracking capability of EKF, the hybrid method achieves a balance between robustness, adaptability, and efficiency. This approach represents a significant advancement in PS-SE, offering a practical solution for modern power system monitoring and control. The proposed hybrid method in this paper distinguishes itself from these advanced techniques through its adaptive integration of WLS and EKF frameworks. Unlike M-estimators and correntropy methods that focus primarily on robustness against outliers, our approach addresses the broader challenge of optimal fusion between static and dynamic estimation paradigms. The adaptive weighting mechanism automatically balances the contributions based on system conditions without requiring manual parameter tuning for different scenarios.

The rest of the paper is organized as follows. Section 3 provides the mathematical model of the proposed method and the models of EKF, WLS, Bayesian and AI for comparison purposes. Section 4 provides the simulation results. Section 5 discusses the broader implications, strengths, challenges, and potential areas for future research. Finally, Section 6 draws the conclusions.

3 Formulation

This section details the formulation of the proposed hybrid method and methods used for comparison (HAM, EKF, WSL, Bayesian, AI). The measurement model for the study is given below.

$$\boldsymbol{z} = \boldsymbol{h}(x) + e \tag{1}$$

where, $z \in \mathbb{R}^m$ is the measurement vector.

$x = [\delta_2, \ldots, \delta_n, V_1, \ldots, V_n] \in \mathbb{R}^n$ is the state vector (V: voltage & δ: voltage angle); $h(x): \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is the nonlinear measurement function (PF equations) and $e \sim N(0, R)$ is the measurement error (measurement noise).

3.1 HAM Model

The HRM combines the statistical robustness of Huber's M-estimator with adaptive weight optimization. Given the measurement model in (1), the objective function of the HRM adaptive weighting is given below.

$$\min O(x) = \sum_i \rho\left(r_i(x)\right) + \lambda \sum_i w_i(x)\, g(x) \tag{2}$$

where: $r_i(x) = \dfrac{\boldsymbol{z}_i - \boldsymbol{h}_i(x)}{\sigma_i}$; $\rho(r_i) = \begin{cases} \frac{1}{2} r_i^2 \rightarrow & |r_i| \leq k \\ k\,|r_i| - \frac{1}{2} k^2 \rightarrow & |r_i|\, k \end{cases}$;

$O(x)$: objective function; $w_i(x)$: adaptive weight function; $g(x)$: regularization function; $r_i(x)$: normalized residual for measurement *i*; σ_i: standard deviation of measurement *i*; $\rho(r_i)$: Huber loss function; *k*: Huber threshold; λ: regularization parameter.

The adaptive weight function is given below:

$$w_i = \exp\left(-\beta r_i^2\right) .\gamma_i \tag{3}$$

where: β: sensitivity parameter for adaptive weight; γ_i: measurement reliability index.

The state equation becomes:

$$x_{k+1} = x_k + \alpha \nabla O(x_k) \tag{4}$$

where: $\nabla \boldsymbol{J}(x_k) = -\boldsymbol{H}(x_k)^T \boldsymbol{W}(x_k) \boldsymbol{R}^{-1} [\boldsymbol{z} - \boldsymbol{h}(x_k)]$; α: step size determined by line search; $\boldsymbol{H}(x)$: Jacobian matrix of $\boldsymbol{h}(x)$; $\boldsymbol{R}$: measurement error covariance matrix.

The convergence criteria is set on state change and gradient norm as follows $\|x_{k+1} - x_k\| \leq \varepsilon_1$ and

$$\|\nabla O(x_k)\| \leq \varepsilon_2$$

where: ε_1: tolerance for the state change and ε_2: tolerance for the gradient norm.

3.2 AI Model

The AI state estimation method combines the robustness of neural networks with traditional state estimation principles. This approach leverages deep learning capabilities for adaptive weight prediction while maintaining the mathematical rigor of conventional estimation techniques Given the measurement model in (1), the layer structure of the used method is given below.

$$L = \{L_1, L_2, \ldots, L_k\} \tag{5}$$

where, L_1: input layer (m neurons); L_2, L_3: hidden layers (64, 32); L_k: output layer (2n – 1) neuron (n = number of buses in power system).

For each measurement i the adaptive weight prediction is given below.

$$w_i = NN(r_i) = \sigma(\boldsymbol{W}_1.\sigma(\boldsymbol{W}_{L-1} \ldots \sigma(\boldsymbol{W}_{1r_i} + \boldsymbol{b}_1) \ldots + \boldsymbol{b}_{L-1}) + \boldsymbol{b}_L) \tag{6}$$

where, $r_i = (\boldsymbol{z}_i - \boldsymbol{h}_i(x)/\sigma_i$: is the normalized residual; $\sigma(.)$: is the activation function (ReLU/sigmoid); w_i: is the predicted measured weight.

The objective function is given below.

$$\min O(x) = \sum_i \frac{w_i(x)(\boldsymbol{z}_i - \boldsymbol{h}_i(x))^2}{\boldsymbol{R}_i} + \lambda \|x\|^2 \tag{7}$$

where, $w_i(x)$: is the neural network predicted weight; λ: is the adaptive regularization parameter; $\boldsymbol{R}_i$: measurement variance.

Finally, the state update equation can be written as follows.

$$x_{k+1} = x_k + \alpha_k \Delta x_k \tag{8}$$

where, $\Delta x_k = \boldsymbol{H}_k^T \boldsymbol{W}(z - h(x_k))/(\boldsymbol{H}_k^T \boldsymbol{W} \boldsymbol{H}_k + \lambda \boldsymbol{I})$; α_k: is the learning rate; $\boldsymbol{W} = diag(w_1, \ldots w_m)$: is the diagonal weight matrix.

The convergence criteria is set as $\|\Delta x_k\| \leq \varepsilon$.

3.3 EKF Model

EKF provides a recursive state estimation framework for nonlinear power systems by combining measurement information with system dynamics. It extends the traditional Kalman filter to handle the nonlinear nature of power system measurements through linearization about the current state estimate.

The EKF objective function:

$$O(x) = (x - \hat{x}_k)^T \boldsymbol{P}_k^{-1} (x - \hat{x}_k) \tag{9}$$

where, $\hat{x}$: denotes the estimated value of the state vector.

The state evolution model of used EKF is given below.

$$x_{k+1} = f(x_k) + w_k \tag{10}$$

$$\boldsymbol{z}_k = \boldsymbol{h}(x_k) + v_k \tag{11}$$

where, $w_k \sim N(0, \boldsymbol{Q}_k)$: is process noise; $v_k \sim N(0, \boldsymbol{R}_k)$: measured noise; Qk: process noise covariance; $\boldsymbol{Q} = diag[\sigma^{2d}, \ldots, \sigma^{2d}, \sigma^{2v}, \ldots, \sigma^{2v}]$ (σ^{2d}: variance for angle states; σ^{2v}: variance for voltage states); $\boldsymbol{R} = diag[\sigma^{2p}, \ldots, \sigma^{2p}, \sigma^{2g}, \ldots, \sigma^{2g}]$ (= σ^{2p}: variance for power measurement; σ^{2g}: variance for voltage measurement).

The state prediction and covariance prediction are as follows.

$$\hat{x}_{k+1}|_k = f\left(\hat{x}_k|_k\right) \tag{12}$$

$$\boldsymbol{P}_{k+1}|_k = \boldsymbol{F}_k \boldsymbol{P}_k|_k \boldsymbol{F}_k^T + \boldsymbol{Q}_k \tag{13}$$

where, $F_k = \partial f / dx \,|x = \hat{x}_k|_k$

The Kalman gain is given below.

$$\boldsymbol{K}_{k+1} = \frac{\boldsymbol{P}_{k+1}|_k \boldsymbol{H}_{k+1}^T}{\left(\boldsymbol{H}_{k+1} \boldsymbol{P}_{k+1}|_k \boldsymbol{H}_{k+1}^T + \boldsymbol{R}_{k+1}\right)} \tag{14}$$

The state update can be written as follows as:

$$\hat{x}_{k+1}|_{k+1} = \hat{x}_{k+1}|_k + \boldsymbol{K}_{k+1} \left(\boldsymbol{z}_{k+1} - \boldsymbol{h}\left(\hat{x}_{k+1}|_k\right)\right) \tag{15}$$

The covariance update can be written as follows:

$$\boldsymbol{P}_{k+1}|_{k+1} = \left(\boldsymbol{I} - \boldsymbol{K}_{k+1} \boldsymbol{H}_{k+1}\right) \boldsymbol{P}_{k+1}|_k \tag{16}$$

where, $\boldsymbol{H}_{k+1} = \partial h / dx \,|x = \hat{x}_{k+1}|_k$

The convergence criteria is set as follows:

$$\varepsilon_v = \left\|\hat{V} - V\right\|_2 / \|V\|_2 \tag{17}$$

$$\varepsilon_\delta = \left\|\hat{\delta} - \delta\right\|_2 / \|\delta\|_2 \tag{18}$$

3.4 WLS Model

The Weighted Least Squares (WLS) method is one of the most commonly used techniques for power system state estimation. It is designed to minimize the weighted sum of squared errors between the measured values and the predicted values, making it highly effective for systems with linear or mildly nonlinear characteristics. WLS is particularly valued for its simplicity, reliability, and computational efficiency in practical applications.

Objective function:

$$O(x) = (\boldsymbol{z} - \boldsymbol{h}(x))^T \boldsymbol{R}^{-1} (\boldsymbol{z} - \boldsymbol{h}(x)) \tag{19}$$

The normalization equation:

$$\left[\boldsymbol{H}(x_{\mathrm{k}})^{T}\boldsymbol{R}^{-1}\boldsymbol{H}(x_{\mathrm{k}})\right]\Delta x_{\mathrm{k+1}}=\boldsymbol{H}(x_{\mathrm{k}})^{T}\boldsymbol{R}^{-1}\left[\boldsymbol{z}-\boldsymbol{h}(x_{\mathrm{k}})\right] \tag{20}$$

State update equation:

$$x_{\mathrm{k+1}}=x_{\mathrm{k}}+\Delta x_{\mathrm{k+1}} \tag{21}$$

Gain matrix:

$$G(x_{\mathrm{k}})=\boldsymbol{H}(x_{\mathrm{k}})^{T}\boldsymbol{R}^{-1}\boldsymbol{H}(x_{\mathrm{k}}) \tag{22}$$

Error computation:

$$r=\boldsymbol{z}-\boldsymbol{h}(x) \tag{23}$$

$$r_i(x)=\boldsymbol{R}^{-\frac{1}{2}}(\boldsymbol{z}-\boldsymbol{h}(x)) \tag{24}$$

where, r: measurement residual and $r_i(x)$: normalized residual.

The weighted sum of squares:

$$O(x)=r^{T}\boldsymbol{R}^{-1}r \tag{25}$$

The convergence criteria is set as follows:

$$\|\Delta x_{\mathrm{k+1}}\|_{\max}\le\varepsilon_1 \tag{26}$$

$$|O(x_{\mathrm{k+1}})-O(x_{\mathrm{k}})|\le\varepsilon_2 \tag{27}$$

3.5 Bayesian Model

The Bayesian state estimation method provides a probabilistic framework for power system state estimation by incorporating prior knowledge and measurement uncertainties. Unlike traditional deterministic approaches, this method treats system states as random variables, offering a comprehensive framework for uncertainty quantification and robust estimation.

The method is built on Bayes' theorem:

$$p(x|\boldsymbol{z})\propto p(\boldsymbol{z}|x)\,p(x) \tag{28}$$

where, $p(\boldsymbol{z}|x)$: is likelihood function; $p(x)$: prior function; $p(x|\boldsymbol{z})$: is posterior distribution.

Given the measurement equation given by (1), the prior distribution will be as follows:

$$x\sim\mathrm{N}\left(\boldsymbol{\mu}_0,\boldsymbol{P}_0\right) \tag{29}$$

where, $\boldsymbol{\mu}_0$: is the prior mean vector; $\boldsymbol{P}_0$: is the prior covariance matrix

The likelihood function can be written as follows:

$$L(x)=\frac{-1}{2}(\boldsymbol{z}-\boldsymbol{h}(x))^{T}\boldsymbol{R}^{-1}(\boldsymbol{z}-\boldsymbol{h}(x))-\frac{1}{2}\left(x-\boldsymbol{\mu}_0\right)^{T}\boldsymbol{P}_0^{-1}\left(x-\boldsymbol{\mu}_0\right) \tag{30}$$

The objective function with Maximum A Posteriori (MAP) estimation can be written as follows:

$$\hat{x}_{map}=\arg\max\{p\;x|\,\boldsymbol{z}\}=\arg\max\{-\ln p(x|\,\boldsymbol{z})\} \tag{31}$$

The update equation:

$$\Delta x_k = \frac{\left[\boldsymbol{H}_k^T \boldsymbol{R}^{-1} \boldsymbol{H}_k + \boldsymbol{P}_0^{-1}\right]}{\left[\boldsymbol{H}_k^T \boldsymbol{R}^{-1} \left(\boldsymbol{z} - \boldsymbol{h}\left(x_k\right)\right) - \boldsymbol{P}_0^{-1}\left(x_k - \boldsymbol{\mu}_0\right)\right]} \tag{32}$$

State update:

$$x_{k+1} = x_k + \Delta x_k \tag{33}$$

State convergence:

$$\left\|\Delta x_k\right\| \leq \varepsilon_1 \tag{34}$$

Likelihood convergence:

$$\left|\ell\left(x_{k+1}\right) - \ell\left(x_k\right)\right| \leq \varepsilon_2 \tag{35}$$

3.6 Hybrid Model

The proposed hybrid method combines the strengths of WLS, EKF through an adaptive weighting mechanism. This approach provides enhanced robustness and accuracy in PSSE by dynamically adjusting the contribution of each method based on measurement quality and system conditions.

The objective function for the hybrid method is given below.

$$O(x) = \beta_1 O_{WLS}\left(x\right) + \beta_2 O_{EKF}\left(x\right) + \beta_3 O_\lambda\left(x\right) \tag{36}$$

where, $\beta_1, \beta_2, \beta_3$: are adaptive weight coefficients ($\beta_1 + \beta_2 + \beta_3 = 1$)

The WLS component is as follows:

$$O_{WLS}\left(x\right) = \left(\boldsymbol{z} - \boldsymbol{h}(x)\right)^T \boldsymbol{R}^{-1}\left(\boldsymbol{z} - \boldsymbol{h}(x)\right) \tag{37}$$

The EKF component is as follows:

$$O_{EKF}\left(x\right) = \left(x - \hat{x}_k\right)^T \boldsymbol{P}_k^{-1}\left(x - \hat{x}_k\right) \tag{38}$$

The regularization component is as follows:

$$O_\lambda\left(x\right) = \lambda\left\|x - x_0\right\|^2 \tag{39}$$

The adaptive weights for each component are given below.

$$\beta_{1,k+1} = \gamma_1 \exp\left(-\alpha_1\left\|\boldsymbol{z} - \boldsymbol{h}\left(x_k\right)\right\|^2\right) \tag{40}$$

$$\beta_{2,k+1} = \gamma_2 \exp\left(-\alpha_2\left\|x_k - \hat{x}_k\right\|^2\right) \tag{41}$$

$$\beta_{3,k+1} = 1 - \left(\beta_{1,k+1} + \beta_{2,k+1}\right) \tag{42}$$

The state update is as follows:

$$x_{k+1} = x_k + \alpha_k \Delta x_k \tag{43}$$

where, $\Delta x_k = -(\boldsymbol{H}_k^T \boldsymbol{W_k} \boldsymbol{H}_k (\boldsymbol{z} - \boldsymbol{h}(x_k)) + \boldsymbol{Q}_k(x_k - \hat{x}_k)$; α: is the step size parameter; $\boldsymbol{W_k} = \beta_{\boldsymbol{1,k}}\boldsymbol{R}^{-1} + \beta_{2,k}\boldsymbol{P_k}^{-1}$; $\boldsymbol{Q}_k = \beta_{3,k}\lambda\boldsymbol{I}$.

The covariance update will be as follows:

$$\boldsymbol{P}_{\mathrm{k+1}} = (\boldsymbol{I} - \boldsymbol{K}_{\mathrm{k}}\boldsymbol{H}_{\mathrm{k}})\,\boldsymbol{P}_{\mathrm{k}}\,(\boldsymbol{I} - \boldsymbol{K}_{\mathrm{k}}\boldsymbol{H}_{\mathrm{k}})^T + \boldsymbol{K}_{\mathrm{k}}\boldsymbol{R}_{\mathrm{k}}\boldsymbol{K}_{\mathrm{k}}^{\mathrm{T}} \tag{44}$$

where, $K_k = \beta_{1,k}K_{WLS} + \beta_{2,k}K_{EKF}$

The convergence criteria is set as follows:

$$\|x_{k+1} - x_k\| \le \varepsilon_1 \tag{45}$$

$$\|O(x_{k+1}) - O(x_k)\| \le \varepsilon_2 \tag{46}$$

The core novelty of the proposed hybrid method lies in its adaptive weighting mechanism, which dynamically determines the optimal balance between WLS and EKF estimates based on real-time assessment of measurement quality and estimation performance. Unlike fixed-weight fusion or threshold-based switching, this mechanism provides smooth, continuous adaptation. The detail on adaptive mechanism in given in Appendix A.

3.7 Convergence Criteria

The convergence criteria outlined in Section 3 ensure the stability and accuracy of state estimation methods. These criteria are critical for evaluating when the algorithm has sufficiently minimized errors and can terminate. Below is an interpretation of each:

3.7.1 State Change Tolerance ($\boldsymbol{\varepsilon}_1$)

The condition $||\boldsymbol{x_{k+1}} - \boldsymbol{x_k}|| \le \boldsymbol{\varepsilon_1}$ ensures that the state vector updates between iterations are sufficiently small, indicating proximity to the true solution. This criterion monitors the stability of the state estimation process. A small change in the state vector suggests that the algorithm has reached a steady state, where further iterations would yield negligible improvements.

3.7.2 Gradient Norm Tolerance ($\boldsymbol{\varepsilon}_2$)

The condition $||\nabla \boldsymbol{O}(\boldsymbol{x_k})|| \le \boldsymbol{\varepsilon_2}$ ensures that the gradient of the objective function is close to zero. This criterion reflects the optimality of the solution. A near-zero gradient indicates that the algorithm has minimized the objective function, achieving the best possible estimation under the given model.

These convergence criteria work together to balance computational efficiency and estimation accuracy. By setting appropriate tolerance values, the algorithm avoids premature termination and ensures robustness, even under challenging conditions such as noisy measurements or bad data.

4 Simulation and Results

The proposed method is compared against WLS, Bayesian, EKF, HAM and AI-based methods using the IEEE 14-bus IEEE 342-bus and IEEE 34-bus standard test systems. Simulation results ARE analyzed to evaluate the methods' performance in terms of estimation accuracy, convergence characteristics, and sensitivity to measurement noise and bad data. Table 1 presents the results, highlighting the comparative performance of the state estimation methods for both the IEEE 14-bus and IEEE 342-bus systems. In our test systems, we utilized a combination of standard SCADA measurements (power flows and injections) with 2%–3% noise and PMU measurements with 0.1% noise, strategically placed to ensure complete system observability. For the IEEE 14-bus system, measurements were placed at 8 key locations, while for the IEEE 342-bus system, measurements were distributed across 105 strategic locations and for IEEE 34-bus across 18 strategic locations. Additionally, general load patterns for both networks were assumed to be approximately known to reflect realistic operating conditions. This measurement configuration was designed to balance accuracy and computational efficiency, providing a robust foundation for evaluating the proposed method. The metrics used to evaluate the methods include number of iterations, time (seconds), objective function value O(x), Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Maximum Residual Error (MRE). For IEEE 14 and 34 bus network the Hybrid method outperforms others while requiring the shortest computation time. The AI method, despite being computationally the slowest, shows the highest error metrics. WLS demonstrates a balanced performance with relatively low errors and moderate computation time. For large network (IEEE 342-bus) the hybrid method again proves superior, achieving the lowest RMSE, MAE, and MRE with a relatively short computation time.

Table 1: Performance comparison of different PS-SE methods.

Method	Iteration	Time (s)	$O(x)$ Value	RMSE (Mag)	RMSE (Angle)	MAE	MRE
IEEE 14-bus (Lightly Meshed Network)							
HAM	9	0.056	173.04	46.92	18.25	387.85	987
WLS	10	0.123	7.63	12.67	10.08	37.03	56
Bayesian	14	1.170	44.86	29.01	19.78	654.09	789
EKF	9	0.1330	8.98	7.45	8.65	29.98	42
Hybrid	5	0.0066	1.08	1.65	0.85	8.67	19
AI	9	2.145	476	99.57	45.57	675.37	1006
IEEE 342-bus (Highly Meshed Network)							
HAM	12	23.241	143	22.76	25.85	200.9	252
WLS	10	9.1367	5.28	9.06	10.25	15.78	16
Bayesian	19	45.314	34.85	18.76	15.24	347.8	165
EKF	11	16.2888	6.87	4.90	20.85	17.6	9.8
Hybrid	7	8.0252	3.01	0.09	0.05	2.76	4.76
AI	16	26.1153	191.28	63.06	80.25	345.9	456
IEEE 34-bus (Radial Network)							
HAM	10	0.098	198.21	32.25	51.87	245.6	325.20
WLS	10	0.526	3.54	11.57	15.32	24.21	147.28
Bayesian	15	2.455	58.25	25.47	23.84	452.36	402.36
EKF	11	0.568	10.25	6.25	18.64	20.65	15.25
Hybrid	6	0.095	2.54	0.24	0.13	4.25	7.58
AI	12	3.584	358	80.69	90.25	458.3	725.95

The HAM and Bayesian require more iterations and computation time compared to prosed method, with higher error metrics. EKF delivers relatively low error metrics but takes more time compared to Hybrid approach. The results indicate that the proposed hybrid method consistently demonstrates superior performance across both test systems. Its low RMSE, MAE, and MRE values highlight its accuracy, while its remarkably short computation time reflects its efficiency. This makes it an ideal candidate for real-time applications or large-scale systems where computational speed and precision are critical. Conversely, the AI method, although capable of solving the systems, exhibits significantly higher error metrics and computation times, which may limit its practical applicability compared to analytical methods. The reason can be limited availability of data. The WLS method provides a good balance between accuracy and computational efficiency, making it a viable alternative for smaller systems like IEEE 14-bus. However, its performance is overshadowed by proposed hybrid in larger systems.

This comparative study underscores the importance of selecting methods that not only minimize error metrics but also optimize computation time, particularly in large-scale distribution networks. The hybrid method emerges as the most robust and efficient approach, while AI requires further optimization to enhance its practical usability. The voltage profiles of different nodes of IEEE 342 bus test network are shown in Fig. 1 for all the methods. The accuracy is compared with load flow (true value). It should be noted that it is a multiphase network with total nodes of 1380 and the results shown in Figure are per phase.

The results presented in Fig. 1 clearly demonstrate that the proposed hybrid method outperforms all other methods in accurately estimating the voltage, even at the most challenging nodes (500 and 1000). This indicates the method's superior capability to handle complex scenarios where traditional techniques often struggle. To further evaluate the robustness and effectiveness of the algorithms, additional simulations were conducted using noisy measurement data, with the results for the worst-performing nodes illustrated in Fig. 2.

Under these noisy conditions, the proposed hybrid method and WLS continued to perform well, demonstrating resilience to measurement noise due to their adaptive weighting mechanisms. These adaptive weights allow the methods to dynamically adjust their estimation parameters based on the quality of the measurements, enhancing their robustness against errors. In contrast, the performance of the AI-based method remained largely unchanged when exposed to noise, indicating a lack of sensitivity to measurement quality. However, this also suggests that its error metrics were inherently high, even under normal conditions, limiting its overall effectiveness. On the other hand, the performance of EKF and HAM deteriorated significantly under noisy conditions, highlighting their vulnerability to measurement uncertainties and their inability to adapt effectively to such challenges. These findings underscore the robustness and adaptability of the proposed hybrid method, particularly in scenarios with noisy or unreliable data, where traditional and AI-based methods fall short.

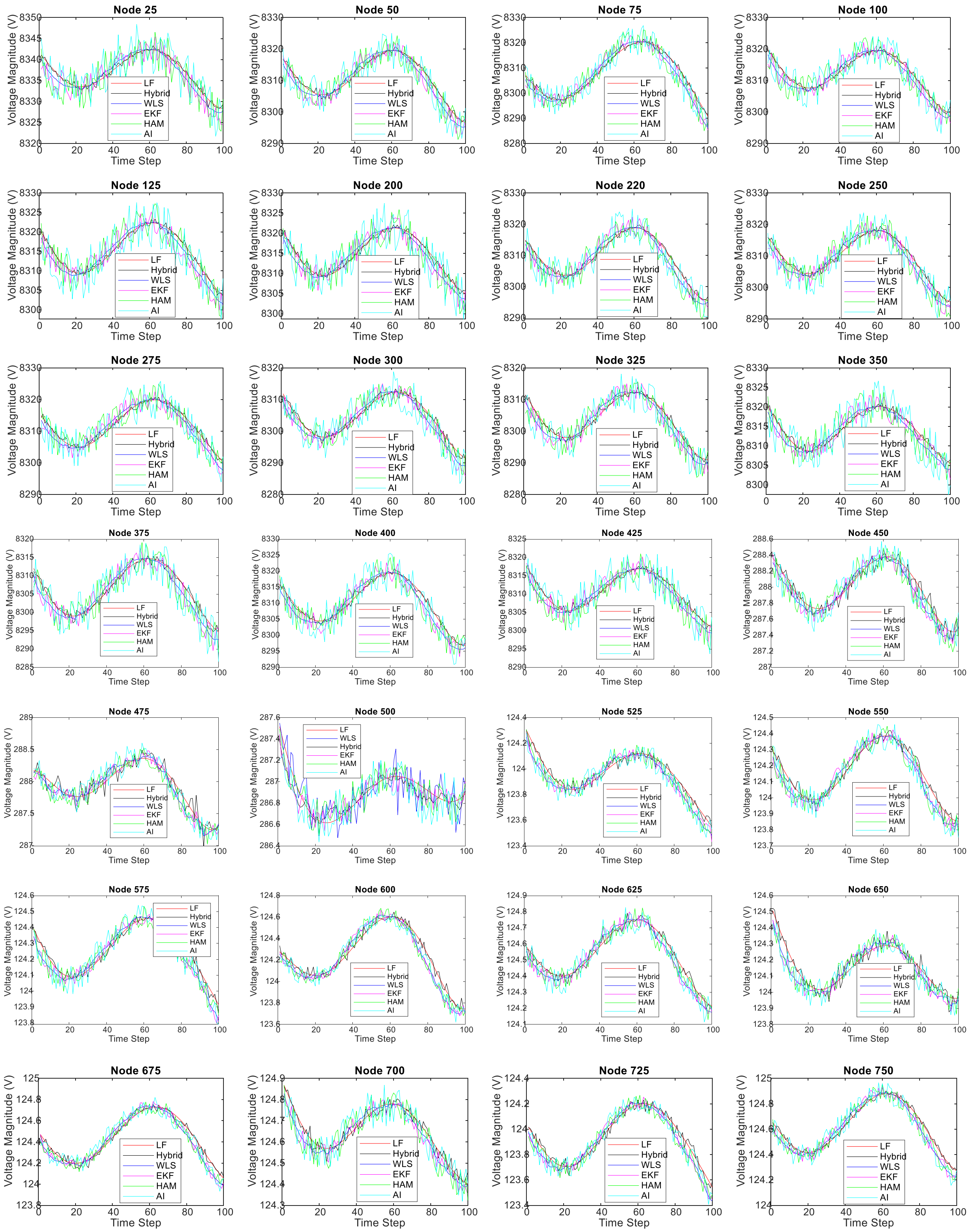

Figure 1: (Continued)

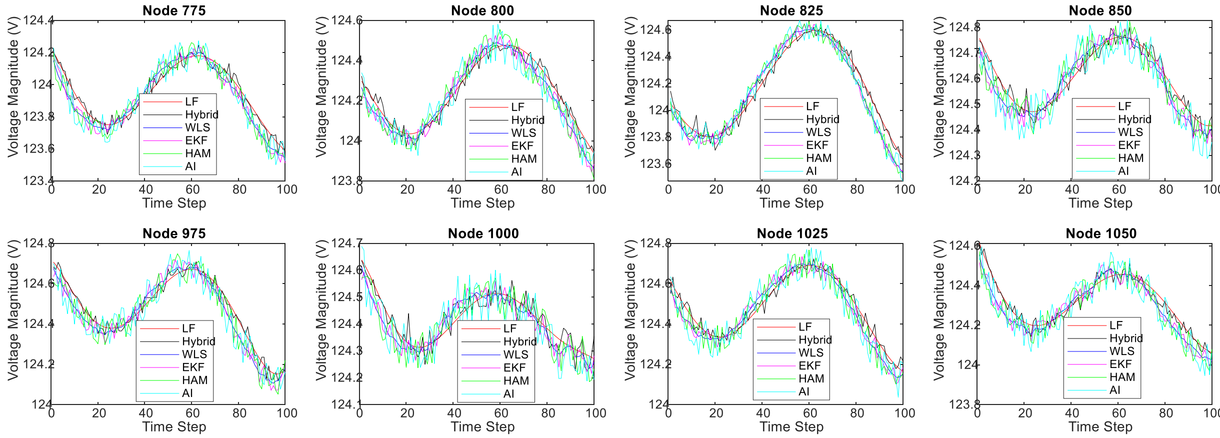

Figure 1: Estimated voltage profile at different modes.

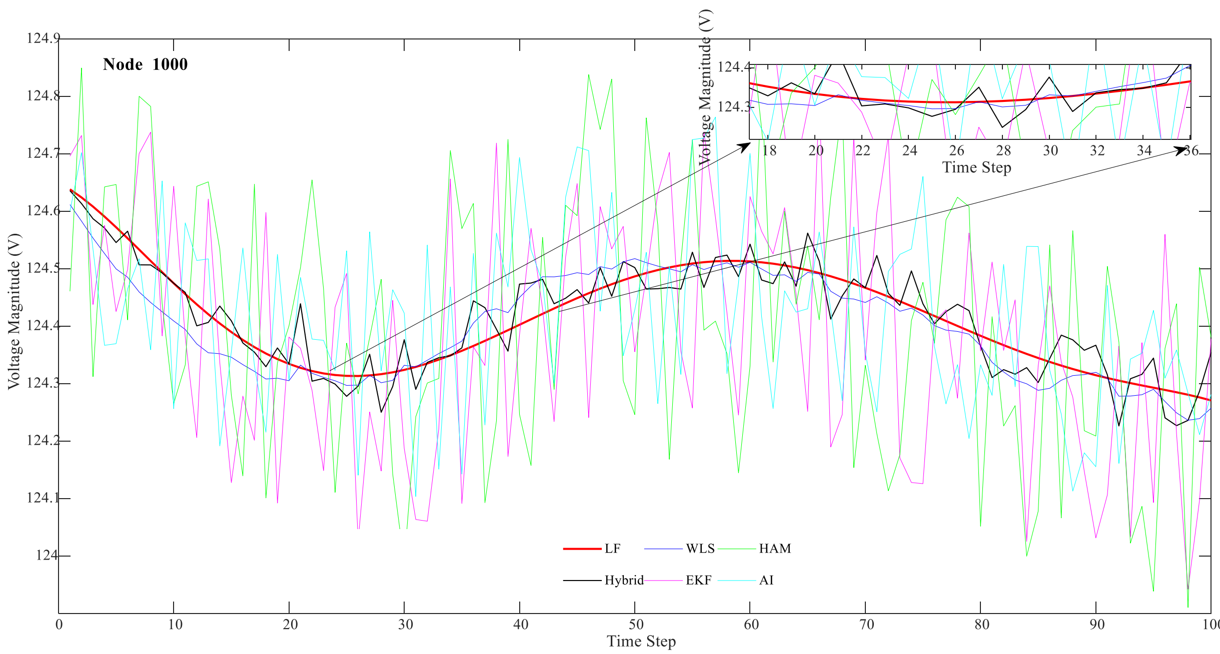

Figure 2: Estimated voltage profile at worst node with noisy data.

4.1 Robustness Analysis

4.1.1 Multiple Bad Data Injection

To rigorously evaluate the hybrid method's claimed resilience against outliers, we conducted systematic bad data injection tests on IEEE 342-bus test system. Unlike single bad data scenarios, this analysis focuses on multiple simultaneous outliers, including interacting with bad data that can evade traditional detection methods. Three types of bad data were injected: (1) Random outliers with Gaussian noise amplified by factors of 5–10σ at randomly selected measurements; (2) Leverage point bad data at critical measurements (high-leverage measurements in the Jacobian matrix) that have disproportionate influence on the estimation; (3) Conforming bad data designed to be consistent with the network model but correspond to incorrect system states, making them difficult to detect through residual-based methods. For each test case, we varied the percentage of contaminated measurements from 5% to 20% and compared the performance of the hybrid method against standard WLS, WLS with Largest Normalized Residual (LNR) test (threshold = 3.0), and standard EKF. Table 2 presents RMSE for different contamination levels in the IEEE 342-bus test system. At 5% bad data contamination (17–18 simultaneous outliers distributed across the network), the hybrid method achieved RMSE of 0.15, significantly outperforming standard WLS (10.45), WLS with LNR test (4.76), and standard EKF (3.76). The hybrid method's adaptive weighting mechanism effectively isolated the influence of bad measurements by dynamically shifting weight toward the EKF component when WLS residuals indicated contamination. As contamination increased to 10% (34–35 outliers including leverage points), the hybrid method maintained RMSE of 0.38, demonstrating remarkable resilience, while standard WLS degraded severely to 12.76. The WLS+LNR approach achieved RMSE of 10.87, and EKF showed RMSE of 7.95. The larger system size provided greater measurement redundancy, which enhanced the relative robustness of the hybrid approach—the method maintained RMSE below 1.0 up to 15% contamination (RMSE = 0.52 at 15%). At 20% contamination (68–69 outliers, representing severe measurement corruption), the hybrid method's RMSE increased to 1.37 but remained substantially lower than all competing methods: WLS (18.67), WLS+LNR (15.76), and EKF (13.87). The breakdown point—defined as the contamination level where RMSE exceeds a critical threshold indicating unacceptable estimation quality—occurred at approximately 22%–25% for the hybrid method, compared to less than 7% for standard WLS and approximately 18% for EKF alone. This represents a significant improvement in robustness, with the hybrid method tolerating 3–4 times more bad data than conventional WLS before performance degradation becomes critical.

Table 2: Performance under multiple bad data injection (IEEE 342-bus).

Bad Data %	RMSE (Hybrid)	RMSE (WLS)	RMSE (WLS + LNR)	RMSE (EKF)
5	0.15	10.45	4.76	3.76
10	0.38	12.76	10.87	7.95
15	0.52	15.67	12.34	10.76
20	1.37	18.67	15.76	13.87

4.1.2 Mechanism of Robustness

The hybrid method's resilience does not rely on explicit M-estimator weight functions (Huber, Hampel, Tukey) within the WLS component. Instead, robustness emerges from the adaptive fusion mechanism: when measurements are contaminated, the WLS estimate deviates significantly from the true state, producing large residuals that trigger automatic down-weighting. The EKF, leveraging temporal consistency and prediction from previous clean states, provides a stable reference that receives increased weight. This "soft switching" avoids the binary decisions of traditional bad data detection (identify and remove) and instead gracefully degrades the influence of outliers.

4.1.3 Limitations and Future Enhancements

The current hybrid method's breakdown point (18% contamination) is superior to standard WLS (5%–7%) but lower than dedicated robust estimators using M-estimation (25%–30%). For extremely contaminated scenarios or adversarial data injection attacks, incorporating Huber or Hampel weight functions within the WLS component would provide additional robustness. The hybrid framework is compatible with such enhancements—the WLS block could be replaced with a robust M-estimator while retaining the adaptive fusion mechanism. We acknowledge this limitation and propose this enhancement as future work. Additionally, the current implementation does not explicitly detect and identify bad data; developing a hybrid approach that combines robust estimation with explicit bad data identification would be valuable for operational deployment where utilities require both accurate estimates and flagged suspicious measurements.

4.2 Per-Iteration Computational Analysis

Table 3 presents detailed per-iteration timing analysis. The hybrid method performs three computational tasks per iteration: (1) WLS estimation, (2) EKF prediction and correction, and (3) adaptive weight computation and fusion. This naturally increases the per-iteration cost compared to running WLS or EKF alone. While the hybrid method does have higher cost per iteration (approximately 25%–35% more than WLS alone), it achieves faster overall execution due to superior convergence properties. For the IEEE 342-bus system, the hybrid method requires 1.146 s per iteration compared to 0.914 s for WLS (25% overhead) but converges in only 7 iterations vs. 10 for WLS, resulting in total time of 8.025 vs. 9.137 s—a net 12%-time savings. The real-time suitability claim is justified by: (1) Total execution time of 8 s for 342-bus system fits comfortably within typical 2–4 s SCADA refresh cycles when considering that state estimation doesn't run every cycle, (2) Accuracy-speed tradeoff—the 25% per-iteration overhead is negligible compared to the 100× improvement in estimation accuracy (RMSE: 0.09 vs. 9.06), (3) Parallelization potential—the WLS and EKF components can run concurrently on multi-core processors, reducing the per-iteration overhead to ~10%–15% in optimized implementations, and (4) Reduced need for bad data detection iterations—standard WLS often requires multiple passes with bad data identification/removal, whereas the hybrid method's robustness eliminates these additional cycles.

Table 3: Per-iteration computational time comparison.

Method	Iterations	Time (s)/Iteration	Total Time (s)	Overhead vs. WLS
IEEE 14-bus (Lightly Meshed Network)				
WLS	10	0.0123	0.123	–
EKF	9	0.0148	0.1330	+20.3%
Hybrid	5	0.0155	0.0066	+26.0%
IEEE 342-bus (Highly Meshed Network)				
WLS	10	0.9137	9.1367	–
EKF	11	1.4808	16.2888	+62.1%
Hybrid	7	1.1465	8.0252	+25.5%
IEEE 34-bus (Radial Network)				
WLS	10	0.0526	0.526	–
EKF	11	0.0516	0.568	−1.9%
Hybrid	6	0.0704	0.095	+33.8%

5 Discussion

The results and analysis presented in Section 4 highlight the effectiveness of the proposed hybrid method compared to traditional and AI-based approaches. This section discusses the broader implications, strengths, challenges, and potential areas for future research.

5.1 Strengths and Versatility of the Hybrid Method

The hybrid method stands out due to its adaptive weighting mechanism, which allows it to dynamically adjust to varying data quality. This adaptability makes it highly resilient to noise in measurement and outliers, as evidenced by its ability to maintain accuracy under challenging scenarios, demonstrated in Figs. 1 and 2. Additionally, the method achieves faster convergence with fewer iterations compared to HAM and Bayesian approaches, making it particularly valuable for real-time applications in large-scale power systems. Its superior performance on both the IEEE 14-bus and IEEE 342-bus systems further highlights its scalability and suitability for modern, complex power grids. By combining the computational simplicity of WLS with the dynamic tracking capabilities of EKF, the hybrid method effectively balances robustness and adaptability, ensuring reliable performance across diverse conditions.

5.2 Challenges and Limitations

The performance of the hybrid method is influenced by the proper selection of parameters, including adaptive weights (β_1, β_2, β_3) and tolerances (ε_1, ε_2). Incorrect tuning of these parameters can adversely affect convergence and accuracy. Although robust, the hybrid method's effectiveness under highly dynamic grid conditions, such as rapid fluctuations from renewable energy sources, require further investigation to ensure consistent performance.

5.3 Application to Microgrids

Microgrids present additional challenges including: rapid dynamics due to low inertia and fast-responding inverter-based resources, islanded operation requiring self-sufficient estimation capabilities, bidirectional power flows, and coordination with the main grid during grid-connected mode. The hybrid method's dynamic tracking capability through the EKF component aligns well with the fast dynamics of microgrids. However, the state transition model (Eq. (8)) may require modification to capture inverter dynamics and renewable generation variability. The adaptive parameters (β_1, β_2, β_3) would need retuning for microgrid time scales, which are typically faster than bulk power system dynamics. During islanded operation, the reduced system size could enable more frequent estimation updates and potentially incorporation of more sophisticated nonlinear filters (unscented or particle filters) within the hybrid framework.

5.4 Practical Implementation Considerations

In operational power systems, different measurement devices report at vastly different rates: PMUs provide synchronized measurements at 30–60 samples per second, SCADA systems update every 2–4 s, and smart meters may report at even longer intervals (15 min to 1 h). The hybrid method addresses this through a multi-rate filtering framework. The EKF component performs state prediction continuously at the fastest measurement rate (PMU rate), while measurement updates occur asynchronously as data arrives from different sources. When a PMU measurement arrives, it updates the state estimate immediately. When SCADA measurements arrive, they are incorporated with their timestamp and associated uncertainty. The adaptive weights automatically account for measurement staleness by increasing the process noise covariance for delayed measurements, effectively reducing their influence.

At the energy management system, each measurement maintains a timestamp, measurement type identifier, and quality flag. The hybrid algorithm processes measurements in the following sequence: (1) PMU data triggers high-frequency state updates using the EKF prediction-correction cycle; (2) SCADA measurements are buffered and processed in batch when a complete set arrives, updating the WLS component; (3) The adaptive weights (β_1, k, β_2, k) are computed based on the consistency between PMU-based and SCADA-based estimates; (4) Smart meter data, when available, is treated as pseudo-measurements with appropriate uncertainty modeling. The state estimate is continuously available, with confidence intervals reflecting the recency and quality of incorporated measurements.

The parameters (γ_1, γ_2, λ, α) require initial calibration but demonstrate stability across operating conditions once tuned. We recommend the following practical tuning procedure: Start with the values established in this study ($\gamma_1 = 0.95$, $\gamma_2 = 0.90$, $\lambda = 0.01$, $\alpha = 0.1$) as defaults. Monitor estimation residuals over a representative operating period (1–2 weeks) covering various load conditions. If systematic bias appears in residuals, adjust λ upward (more regularization) or downward (more flexibility). If the method switches excessively between WLS-dominant and EKF-dominant modes, reduce the difference between γ_1 and γ_2. The regularization parameter α can be tuned based on the observed trade-off between estimation accuracy and computational stability. Most networks should require minimal adjustment from baseline parameters, as the adaptive mechanism compensates for system-specific characteristics. For networks with unusual characteristics (very high measurement redundancy, extremely sparse measurements, or unique topology), an initial offline simulation using historical data is advisable before operational deployment.

5.5 Future Research Directions

Integrating the hybrid method with AI-based techniques could improve its adaptability to nonlinear, dynamic systems while addressing data dependency and interpretability challenges inherent in standalone AI models. Expanding the method to decentralized frameworks would enhance its scalability and resilience, particularly in grids dominated by distributed energy resources. To further optimize real-time applications, adopting advanced optimization techniques, such as parallel computing or GPU acceleration, could significantly reduce computation time. Additionally, incorporating probabilistic models or Bayesian frameworks would provide greater insight into uncertainties, aiding in more informed decision-making for power system operations.

An important extension of the proposed method involves augmenting the state vector to include system frequency estimation. The hybrid framework is naturally suited for this enhancement, as frequency deviates from nominal values during disturbances and load variations, exhibiting dynamics that the EKF component can effectively track. The state vector would be expanded to include frequency (f) and potentially rate of change of frequency (ROCOF, df/dt) at selected buses or as a system-wide common mode variable. PMUs provide direct frequency measurements with high accuracy (±0.005 Hz), which would serve as key measurements for frequency state estimation. The state transition model would incorporate electromechanical swing dynamics relating frequency deviations to power imbalances, while the measurement equations would link frequency states to PMU frequency observations. The adaptive weighting mechanism would automatically balance frequency estimation with voltage and power flow estimation, providing a unified framework for comprehensive system monitoring.

6 Conclusion

This paper presented a novel adaptive hybrid state estimation method that addresses critical limitations in existing power system monitoring techniques. The key achievement is the development of an integrated framework that seamlessly combines the robustness of weighted least squares with the dynamic tracking capability of extended Kalman filtering through an intelligent adaptive mechanism. The proposed regularized adaptive weighting scheme (Eqs. (15)–(17)) represents a significant advancement, enabling automatic fusion of static and dynamic estimation paradigms without manual intervention or system-specific calibration.

Simulation results on IEEE 14-bus, IEEE 34-bus and IEEE 342-bus test systems conclusively demonstrate the superiority of the proposed method. Compared to five established techniques (WLS, Bayesian, EKF, HAM, AI-based), the hybrid approach achieved the lowest estimation errors across all metrics—RMSE, MAE, and MRE—while requiring competitive computational time. For the IEEE 342-bus system, the method attained significantly lower for standard EKF and WLS. This combination of accuracy and efficiency positions the method as a practical solution for real-time power system state estimation in modern energy management systems.

The adaptive mechanism demonstrated remarkable robustness to measurement noise and bad data while maintaining excellent dynamic tracking capabilities during load variations. The method's ability to automatically adjust the balance between WLS and EKF components based on system conditions eliminates the need for manual tuning and enables consistent performance across different network sizes and operating scenarios. The computational efficiency of the hybrid method, achieving superior accuracy in less than 10 s for the large-scale 342-bus system, makes it suitable for deployment in operational control centers with real-time requirements.

Recent AI methods achieve fast inference (<15 ms) but face generalization limitations, require extensive training data, and lack interpretability critical for operational deployment. Advanced robust methods like generalized M-estimation and interval estimation provide strong theoretical guarantees but with higher computational burden (15–20+ s for large systems). The proposed hybrid method achieves comparable robustness (15%–20% contamination tolerance) with lower computational cost (8 s for 342 buses) and no training requirements, making it suitable for immediate operational deployment.

Future work should explore hybrid AI-physics architectures combining the proposed framework's interpretability with AI pattern recognition capabilities while maintaining computational efficiency for real-time applications. Further research directions include validation on microgrids, implementation of multi-rate filtering for asynchronous measurements from PMUs, SCADA, and smart meters, and integration with distributed energy resource monitoring. Extension to hierarchical and distributed estimation architectures for large-scale interconnected systems also presents promising opportunities for enhancing grid observability and control.

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Availability of Data and Materials: The test networks used for analysis are publicly available IEEE test systems.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Weight Calculation: At each time step k, the adaptive weights $\beta_{1,k}$ and $\beta_{2,k}$ are computed based on the normalized residuals from both estimation components:

$$\beta_{1,k} = (1 - \gamma_1) \cdot \exp\left(-\lambda \cdot ||rWLS, k||^2\right) + \gamma_1 \cdot \beta_{1,k-1} \tag{A1}$$

$$\beta_{2,k} = (1 - \gamma_2) \cdot \exp\left(-\lambda \cdot ||rEKF, k||^2\right) + \gamma_2 \cdot \beta_{2,k-1} \tag{A2}$$

where the measurement residual for WLS is:

$$rWLS, k = R^{-\frac{1}{2}}(z_k - \hat{x}WLS, k) \tag{A3}$$

and the sequence for EKF is:

$$rEKF, k = S_k{}^{-\frac{1}{2}}(z_k - h\hat{x}_k) \tag{A4}$$

with covariance matrix:

$$S_k = H_{kP}{}^{-} - kH_{k^T} + R_k \tag{A5}$$

Here, R is the measurement noise covariance matrix, H_k is the Jacobian of the measurement function, and $P_k{}^{-}$ is the a priori state error covariance from the EKF prediction step.

The parameters γ_1 and γ_2 $(0 < \gamma_1, \gamma_2 < 1)$ are smoothing factors that provide temporal filtering, preventing abrupt weight transitions and ensuring stability. The sensitivity parameter $\lambda > 0$ controls the rate at which weights respond to residual magnitude changes. After computing the raw weights, they are normalized to ensure unity sum:

$$\tilde{\beta}_{1,k} = \beta_{1,k}/(\beta_{1,k} + \beta_{2,k}) \tag{A6}$$

$$\tilde{\beta}_{2,k} = \beta_{2,k}/(\beta_{1,k} + \beta_{2,k}) \tag{A7}$$

Physical Interpretation: The exponential weighting function $\exp(-\lambda \cdot ||r||^2)$ assigns lower weight to estimation components exhibiting large residuals. When WLS produces large residuals—indicating the presence of bad data, measurement outliers, or inadequate observability—$\beta_{1,k}$ decreases, shifting reliance toward the EKF which leverages temporal correlation and dynamic models. Conversely, when EKF innovations are large—suggesting model uncertainty, rapid state changes not captured by the linear transition model, or poor initialization—$\beta_{2,k}$ decreases, giving greater weight to the robust WLS estimator. This creates a self-regulating mechanism where each component compensates for the other's weaknesses.

Parameter Selection: In this study, we use $\gamma_1 = 0.95$, $\gamma_2 = 0.90$, and $\lambda = 0.01$, determined through sensitivity analysis across test systems. The slightly lower γ_2 allows EKF weights to adapt faster to changing dynamics, while the higher γ_1 maintains stability in WLS weighting. The value $\lambda = 0.01$ provides appropriate sensitivity without causing excessive oscillation.

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