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Stackelberg Game-Based Low-Carbon Optimal Scheduling of IES with Dynamic Carbon Emission Factors Using Caterpillar Fungus Optimizer Algorithm

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ABSTRACT: To address multi-agent interest coordination in integrated energy parks, time-varying carbon responsibility of grid power purchases, and insufficient collaborative optimization of low-carbon coupling units, this paper proposes a Stackelberg game-based low-carbon optimal dispatch model for integrated energy systems (IESs) by incorporating dynamic carbon emission factors (DCEFs). A hybrid solution framework combining the caterpillar fungus optimizer (CFO) and CPLEX is employed to solve the resulting bilevel problem. The model establishes a bi-level Stackelberg game among an integrated energy system operator (IESO), an energy supplier (ES), and a load aggregator (LA). DCEFs and branch carbon emission flows trace carbon responsibility across power and heating networks, while carbon trading, carbon capture and storage (CCS), power-to-ammonia (P2A), methanation, and urea synthesis are integrated to improve carbon accounting and resource utilization. In the case study on an IEEE 69-bus power system and a 6-node heating network, compared with the best among the compared algorithms, the proposed CFO-CPLEX method improves the IESO profit by 18.0% and reduces carbon emissions by 13.2%, while its runtime is 15.2% longer than the fastest compared algorithm. Moreover, compared with scenario 3 (which includes the Stackelberg game and carbon trading but no DCEF), the proposed model increases the total park profit by 25.05%. Compared with scenario 1 (basic economic dispatch), the proposed model reduces carbon emissions by 11.76%.

KEYWORDS: Stackelberg game; integrated energy system; dynamic carbon emission factor; carbon emission flow; caterpillar fungus optimizer

1 Introduction

Against the backdrop of the global low-carbon energy transition, the clean, efficient, and intelligent transformation of energy systems has become essential for sustainable development [1]. As major energy consumers and carbon emitters, industrial parks are critical to achieving carbon neutrality. Integrated energy systems (IESs), which coordinate electricity, heat, gas, hydrogen, and other energy carriers, provide an effective pathway for improving energy efficiency and reducing emissions through multi-energy complementarity and cascade utilization [2,3].

Carbon capture and diversified utilization technologies have attracted increasing attention for deep emission reduction in parks. Refs. [4,5] integrate carbon capture and storage (CCS) with power-to-gas (P2G) to improve carbon resource recycling in multi-energy flow IES. Ref. [6] further introduces power-to-ammonia (P2A) for electricity-hydrogen-ammonia coupling, while Ref. [7] develops a mixed-time-scale

dispatch model for electricity-hydrogen-ammonia multi-agent systems. A Stackelberg game-based coordination framework for low-carbon parks with hydrogen blending was proposed in [8], where the integrated energy system operator (IESO) adjusts prices using carbon flow signals. However, existing studies mainly focus on single carbon conversion pathways and lack integrated modeling of the full “carbon capture-P2A-methanation-urea synthesis” chain.

Carbon emission trading (CET) has been widely used in IES optimal dispatch [9,10]. Ref. [11] shows that stepped CET can achieve greater emission reductions than traditional carbon pricing. However, most CET-based studies rely on total carbon emission accounting, making it difficult to trace carbon responsibility and guide load shifting toward low-carbon nodes. To address this issue, dynamic carbon emission factors (DCEF) have been introduced. Ref. [12] proposes a dynamic step-wise carbon trading model, and Ref. [13] develops a graph neural network-based real-time carbon emission monitoring method. Refs. [14,15] extend carbon emission flow theory to multi-energy systems, revealing the spatiotemporal distribution of emissions in electricity, heat, and gas networks. Ref. [16] combines DCEF with stepped carbon prices for electric-thermal-hydrogen IES dispatch, effectively reducing carbon emissions. Although DCEF can provide real-time nodal carbon intensity signals for load shifting and energy storage optimization, most existing studies use it mainly for post-hoc analysis rather than as part of a closed-loop dispatch feedback.

In park-level IES pricing and dispatch, Refs. [17–19] apply the Stackelberg game to multi-microgrid and electric-thermal IES, improving system economy. Refs. [20–22] combine cooperative games with carbon emission factors to develop source-load collaborative dispatch methods based on nodal carbon potential and price responses. Related studies on predictive control and transportation electrification also indicate that flexible loads can provide additional operational flexibility for low-carbon energy systems [23].

To handle non-convexity from lower-level mixed-integer programming and upper-level bilinear terms in the Stackelberg game, Ref. [24] studies multi-timescale capacity configuration, while Refs. [25,26] use Karush-Kuhn-Tucker (KKT) conditions and strong duality to reformulate bilevel optimization as a single-level Mathematical Program with Equilibrium Constraints. Refs. [27,28] further introduce linear relaxation to obtain a single-level mixed-integer linear programming (MILP) model solved by CPLEX. Robust optimization methods [29–31] address uncertainties but do not provide exact equilibrium transformation, and data-driven stochastic robust management of liquefied natural gas cold energy [32] extends multi-energy coupling to cold energy. However, solving large-scale single-level MILP models remains computationally demanding. For the upper-level pricing problem, the high dimensionality of time-of-use electricity and heat prices and their nonlinear interactions with lower-level responses make global strategy search challenging. This motivates the use of bio-inspired metaheuristic algorithms. Ref. [33] developed a distributed optimization framework for shared energy storage and multi-energy microgrids using an asymmetric Nash bargaining mechanism, reducing operating cost by 14.2% and carbon emissions by 51.5%. Ref. [34] proposed a greedy selection-enhanced RIME algorithm for integrated energy systems with multi-energy carbon trading, lowering daily cost by 10.17% and emissions by 39.20%. The above studies demonstrate that metaheuristic algorithms can effectively handle complex, non-convex, and high-dimensional optimization problems in IES. Among them, the caterpillar fungus optimizer (CFO) has shown particular potential for solving the proposed Stackelberg game model due to its parasite-host co-evolution mechanism, which balances global exploration and local exploitation [35]. To clarify the differences between existing studies and the present work, Table 1 summarizes representative literature in terms of the Stackelberg game, CET, dynamic carbon emission factor, carbon emission flow, CCS, and P2A. As shown in Table 1, most existing studies focus on only a subset of these mechanisms, and their integrated modeling within a unified low-carbon dispatch framework remains limited. In contrast, the present work comprehensively integrates all six key elements into a single framework, enabling a more complete low-carbon dispatch solution that achieves simultaneous improvements in both environmental and economic performance.

Table 1: Classification statistics of relevant literature.

Reference	Stackelberg Game	CET	Dynamic Carbon Factor	Carbon Emission Flow	CCS	P2A	Model Benefits
[4]	×	×	×	×	✓	×	CCS-P2G coupling for multiple stakeholders, while also promoting energy conservation and reducing carbon emissions.
[5]	×	×	×	×	×	×	Multi-level thermal energy coupling with integrated demand response is employed for IES scheduling.
[7]	×	×	×	×	×	✓	Operational revenue is improved by 28.74%–67.26% via coordinated electricity-hydrogen-ammonia IES scheduling.
[9]	×	✓	×	×	×	×	Carbon trading cost is reduced, and renewable accommodation is improved via variable hydrogen blending and tiered carbon trading.
[6]	✓	✓	×	×	✓	×	Carbon emissions are reduced by 43.99% and total benefits are increased by 86.43% via Stackelberg-Nash bargaining.
[21]	✓	✓	✓	×	✓	×	Carbon emissions are reduced by 22.1%, and operational costs are reduced by 6.8% via dynamic carbon factors for hydrogen-enriched IES.
[15]	×	✓	×	✓	×	×	Carbon emissions are reduced by 53.97%, with a 16.10% increase in total cost, via carbon emission flow tracking and stepped carbon pricing.
This paper	✓	✓	✓	✓	✓	✓	Carbon emissions are reduced by 13.2% while IESO profit is improved by 18.0% via Stackelberg game with DCEFs.

Existing studies have advanced multi-energy coupling, carbon trading, DCEF, and game-based coordination, but several gaps remain: (1) insufficient modeling of the complete “carbon capture-P2A-methanation-urea synthesis” carbon cycle chain; (2) limited closed-loop integration of DCEF with dispatch optimization; and (3) insufficient transformation and solution methods for Stackelberg games with discrete variables and bilinear terms. To address these limitations, this paper proposes a low-carbon Stackelberg game-based optimal dispatch model for an IES considering DCEF and solves it using a hybrid CFO-CPLEX approach. The main contributions are as follows:

- (1) A bi-level Stackelberg game model for IES is constructed to coordinate the conflicting interests of IESO, energy supplier (ES), and load aggregator (LA), enabling interactive decision-making and interest balance among multiple agents;
- (2) DCEFs are introduced into IES low-carbon dispatch model to capture the time-varying carbon responsibility of grid power purchases. DCEFs enable real-time reflection of grid carbon intensity, providing precise carbon signals for guiding load shifting and energy storage optimization;

- (3) Branch carbon emission flows and nodal carbon potentials are calculated based on carbon emission flow theory. Electricity carbon factors and heat carbon factors are introduced in both the power and heating networks. Meanwhile, carbon trading, CCS, P2A, methanation, and urea synthesis are integrated into a unified dispatch framework, improving carbon accounting accuracy and carbon resource utilization;
- (4) A hybrid CFO-CPLEX solution method is proposed, where CFO performs outer-layer pricing strategy search, and CPLEX exactly solves the inner-layer dispatch problem. Compared with the best results among the four benchmark algorithms, the proposed method improves IESO profit by 18.0% and reduces carbon emissions by 13.2%, while its runtime is 15.2% longer than the fastest benchmark.

The remainder of this paper is organized as follows. [Section 2](#) presents the system architecture and Stackelberg framework. [Section 3](#) formulates IES multi-energy and carbon-flow coupling models. [Section 4](#) introduces DCEF and nodal carbon potential calculation. [Section 5](#) describes KKT and strong-duality-based transformation and CFO-CPLEX algorithm. [Section 6](#) presents the case study and multi-scenario result analysis. [Section 7](#) concludes the paper.

2 System Architecture and Stackelberg Game Framework

2.1 IES Architecture

The structure of IES investigated in this paper is shown in [Fig. 1](#). The system forms a deeply coupled collaborative network involving multiple energy flows, including electricity, heat, gas, hydrogen, and ammonia, as well as carbon-containing material flow. The main devices include combined heat and power (CHP), gas boiler (GB), wind turbines (WT), photovoltaic (PV) units, electrolyzer (EL), P2A unit, pressure swing adsorption (PSA) unit, (CCS) system, methanation reactor (MR), urea synthesis unit (USU), battery energy storage system (BESS), thermal energy storage (TES), and hydrogen tank (HT). These devices are operated by ES. IESO does not directly operate energy conversion devices; instead, it acts as the market coordinator between ES and LA. LA aggregates electricity and heat loads, which are supplied through multi-energy conversion and coupling.

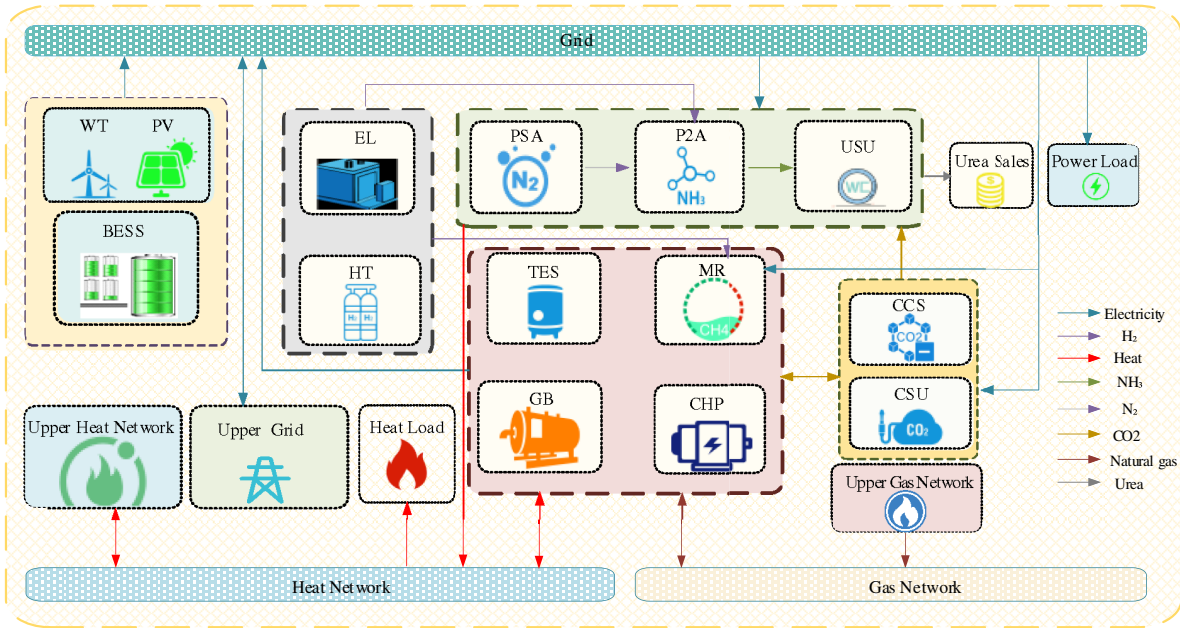


Figure 1: Schematic of IES.

2.2 Stackelberg Game Framework

As shown in Fig. 2, this paper constructs a bi-level Stackelberg game framework consisting of an IESO, an ES, and an LA. IESO acts as the leader, uses CFO algorithm to optimize 24 h electricity and heat prices, and sends price signals to ES and LA. IESO serves as the unified energy trading center in the park: all electricity and heat produced by ES are purchased by IESO, while the electricity and heat required by ES and LA are purchased through the IESO. Under this framework, direct energy transactions between ES and LA are not allowed, and neither ES nor LA can directly purchase energy from the upper grid.

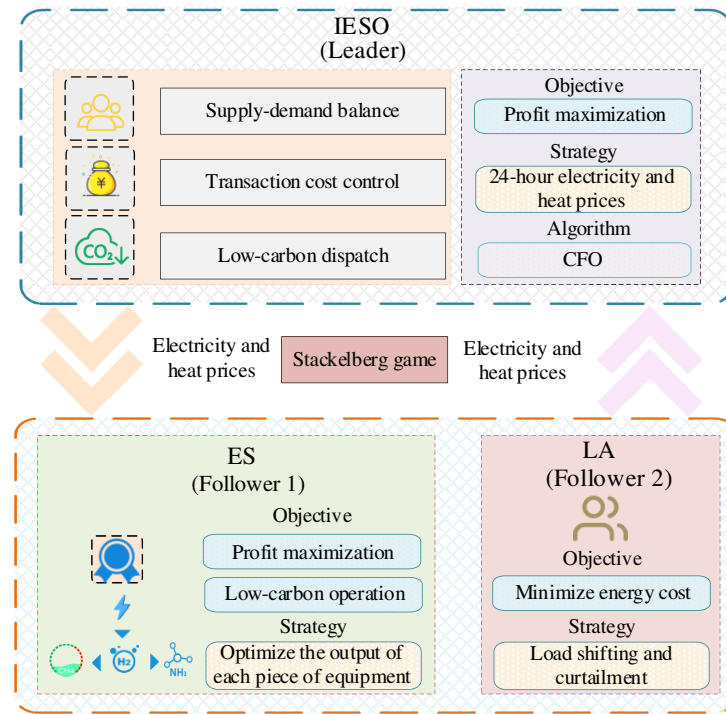


Figure 2: Framework of Stackelberg game.

ES and LA act as followers and respond to the price signals released by IESO. ES maximizes its own profit by optimizing the operating power and charging/discharging strategies of CHP, GB, WT, PV, EL, P2A, CCS, MR, USU, and energy storage units. LA minimizes its energy purchase cost while satisfying energy demand by adjusting electricity and heat demand response strategies. Specifically, Stackelberg game process consists of three stages. First, IESO optimizes and announces electricity and heat prices. Second, ES optimizes device outputs and energy purchase/sale strategies, while LA adjusts electricity and heat demand response. Finally, IESO evaluates its own profit based on the responses of ES and LA and coordinates electricity exchange with the upper grid. Through this leader-follower mechanism, IESO guides the supply-demand balance among the upper grid, ES, and LA through energy price optimization, while maximizing its own profit and promoting low-carbon park operation.

3 Low-Carbon Dispatch Model Formulation

3.1 Upper-Level Model

As the leader, IESO aims to maximize its own profit. The profit is derived from selling energy to LA and ES, minus the cost of purchasing energy from ES and the interaction cost with the upper power grid. The objective function F_{IESO} is expressed as:

$$F_{\text{IESO}} = \max \sum_{t=1}^T \left[p_t^{e,s} (P_t^{\text{LA},e} + P_t^{\text{ES,buy},e}) + p_t^{h,s} (Q_t^{\text{LA},h} + Q_t^{\text{ES,buy},h}) - p_t^{e,b} P_t^{\text{ES,sell},e} - p_t^{h,b} Q_t^{\text{ES,sell},h} - C_t^{\text{grid}} - C_t^{\text{IESO,CET}} \right] \quad (1)$$

where $p_t^{e,s}$ and $p_t^{h,s}$ are the electricity and heat selling prices set by IESO at time t (to LA and ES); $p_t^{e,b}$ and $p_t^{h,b}$ are the electricity and heat purchasing prices paid by IESO to ES; $P_t^{\text{LA},e}$ and $Q_t^{\text{LA},h}$ denote the electric and heat loads of LA; $P_t^{\text{ES,buy},e}$ and $Q_t^{\text{ES,buy},h}$ are the electricity and heat purchased by ES from IESO; $P_t^{\text{ES,sell},e}$ and $Q_t^{\text{ES,sell},h}$ are the electricity and heat sold by ES to IESO; C_t^{grid} is the interaction cost between IESO and the upper grid; and $C_t^{\text{IESO,CET}}$ is the stepped carbon trading cost for the electricity purchased from the upper grid.

The grid interaction cost C_t^{grid} and the carbon cost $C_t^{\text{IESO,CET}}$ for IESO are expressed as:

$$\begin{cases} C_t^{\text{grid}} = p_t^{\text{grid,buy}} P_t^{\text{grid,buy}} - p_t^{\text{grid,sell}} P_t^{\text{grid,sell}} \\ m_{\text{CO}_2,t}^{\text{grid}} = e_t^{\text{grid}} P_t^{\text{grid,buy}} \\ C_t^{\text{IESO,CET}} = f^{\text{CET}}(m_{\text{CO}_2,t}^{\text{grid}}) \end{cases} \quad (2)$$

where $p_t^{\text{grid,buy}}$ and $p_t^{\text{grid,sell}}$ are the known on-grid purchase and sale prices of electricity; and $P_t^{\text{grid,sell}}$ are the corresponding purchased and sold electricity quantities; $m_{\text{CO}_2,t}^{\text{grid}}$ is Carbon emission from grid electricity purchase; e_t^{grid} is the dynamic carbon emission factor obtained from [Section 4](#); f^{CET} denotes the stepped carbon trading cost function (see literature [\[36\]](#) for details).

The price constraints are as follows:

- (1) Price constraints for IESO interacting with the upper power grid:

$$\begin{cases} p_t^{e,b} < p_t^{\text{grid,buy}} < p_t^{e,s} \\ p_t^{e,\min} < p_t^{e,b} < p_t^{\text{grid,sell}} < p_t^{e,\max} \end{cases} \quad (3)$$

where $p_t^{e,\min}$ and $p_t^{e,\max}$ are the minimum and maximum allowed electricity prices set by the upper grid, respectively.

- (2) Price constraints for IESO purchasing and selling heat:

$$\begin{cases} p_t^{h,b} < p_t^{h,s} \\ p_t^{h,\min} < p_t^{h,b} \\ p_t^{h,s} < p_t^{h,\max} \end{cases} \quad (4)$$

where $p_t^{h,\min}$ and $p_t^{h,\max}$ are the minimum allowed heat purchase price and maximum allowed heat selling price, respectively.

The no-direct-transaction rule between ES and LA is enforced by the price constraints [\(3\)](#) and [\(4\)](#), which create a price clamp that makes direct bilateral trading unprofitable. Compared to a fully decentralized market, this restriction simplifies market clearing and ensures a unique Stackelberg equilibrium, although it may lead to a loss of Pareto efficiency because IESO maximizes its own profit rather than social welfare.

All constraints are linear inequalities, ensuring that the strategy set is a convex polyhedron.

3.2 Lower-Level Model: Responses of ES and LA

The lower-level model consists of two followers: ES and LA. Both followers independently optimize their own strategies after receiving the price signals issued by IESO and feed the optimization results back to the upper-level.

3.2.1 Objective Function of ES

ES aims to maximize its profit, which comes from selling electricity and heat to IESO, selling urea, minus the cost of purchasing electricity from IESO, fuel cost, operation and maintenance (O&M) cost, and carbon trading cost. The objective function F_{ES} is expressed as:

$$F_{ES} = \max \sum_{t=1}^T \left[p_t^{e,b} P_t^{ES,sell,e} + p_t^{h,b} Q_t^{ES,sell,h} + r^{urea} m_t^{urea} - p_t^{e,s} P_t^{ES,buy,e} - c^{gas} V_t^{gas,buy} - C_t^{OM} - C_t^{ES,CET} \right] \quad (5)$$

$$C_t^{OM} = \sum_d c_d^{OM} \cdot P_{d,t} \quad (6)$$

where r^{urea} is the unit revenue of urea; m_t^{urea} is the urea production mass; c^{gas} is the natural gas price; $V_t^{gas,buy}$ is the volume of natural gas purchased from the external gas network; C_t^{OM} is the total O&M cost at time t ; c_d^{OM} is O&M cost coefficient of device d ; $P_{d,t}$ is its output power; $C_t^{ES,CET}$ is the stepped carbon trading cost.

The stepped carbon trading cost $C_t^{ES,CET}$ is a convex piecewise linear function of the actual carbon emission. Actually, its calculation method can be found in the literature. This function can be linearized by introducing auxiliary variables, which do not affect the linearity of the lower-level problem.

3.2.2 Objective Function of LA

LA minimizes its total cost, which includes the cost of purchasing electricity and heat from IESO, as well as carbon fees proportional to the carbon intensities of electricity and heat consumption at each node. The objective function F_{LA} is:

$$F_{LA} = \min \sum_{t=1}^T \sum_{i=1}^N \left(p_t^{e,s} P_{i,t}^{LA,e} + p_t^{h,s} Q_{i,t}^{LA,h} + \beta_e \cdot e_{i,t} \cdot P_{i,t}^{LA,e} + \beta_h \cdot h_{i,t} \cdot Q_{i,t}^{LA,h} \right) \quad (7)$$

where N is the total number of load nodes; $P_{i,t}^{LA,e}$ and $Q_{i,t}^{LA,h}$ are the electric and heat loads of LA at node i after demand response; β_e and β_h are the conversion coefficients from carbon intensity to price for electricity and heat, respectively; $e_{i,t}$ is the nodal carbon potential at node i ; and $h_{i,t}$ is the carbon intensity of heat consumed at node i .

3.3 Constraints and Device Models

The decision variables of ES and LA must satisfy the following linear constraints:

3.3.1 Electric Power Balance

$$P_t^{CHP} + P_t^{WT} + P_t^{PV} + P_{BESS,dis,t} + P_t^{ES,buy,e} = P_t^{ES,sell,e} + P_{BESS,ch,t} + P_t^{EL} + P_t^{CCS} + P_t^{PSA} + P_t^{P2A} \quad (8)$$

where P_t^{CHP} , P_t^{WT} and P_t^{PV} are the power outputs of CHP, WT and PV, respectively; $P_{BESS,dis,t}$ and $P_{BESS,ch,t}$ are the discharging and charging power of BESS; P_t^{EL} , P_t^{CCS} , P_t^{PSA} and P_t^{P2A} are the power consumptions of EL, CCS, PSA, and P2A, respectively.

3.3.2 Heat Power Balance

$$\begin{cases} Q_t^{\text{CHP}} + Q_t^{\text{GB}} + P_{\text{TES},\text{dis},t} + Q_t^{\text{P2A}} = Q_t^{\text{ES},\text{sell},h} + P_{\text{TES},\text{ch},t} + Q_t^{\text{loss}} \\ Q_t^{\text{P2A},h} = \beta \cdot m_t^{\text{P2A}} \end{cases} \quad (9)$$

where Q_t^{CHP} and Q_t^{GB} are the heat outputs of CHP and GB; $P_{\text{TES},\text{dis},t}$ and $P_{\text{TES},\text{ch},t}$ are the discharging and charging power of TES; $Q_t^{\text{P2A},h}$ is the waste heat from ammonia synthesis; β is the heat generation coefficient per unit ammonia mass; Q_t^{loss} is the heat network loss (constant); m_t^{P2A} is the ammonia production mass.

Because the heating network operates under the control-by-temperature mode, where the mass flow rate is nearly constant, and the network has only 6 nodes, the variation of thermal loss is negligible; therefore, Q_t^{loss} is assumed constant in this paper.

3.3.3 Hydrogen Balance

$$H_t^{\text{EL}} = H_t^{\text{MR}} + H_t^{\text{P2A}} + (P_{\text{H}_2,\text{dis},t} - P_{\text{H}_2,\text{ch},t}) \quad (10)$$

where H_t^{EL} is the hydrogen produced by EL; H_t^{MR} is the hydrogen consumed by MR; H_t^{P2A} is the hydrogen consumed by P2A; $P_{\text{H}_2,\text{ch},t}$ and $P_{\text{H}_2,\text{dis},t}$ are the charging and discharging power (mass flow rate) of hydrogen storage [37,38].

3.3.4 CO₂ Balance

$$\begin{cases} m_{\text{CO}_2,t}^{\text{CHP}} + m_{\text{CO}_2,t}^{\text{GB}} = m_{\text{CO}_2,t}^{\text{CCS}} + m_{\text{CO}_2,t}^{\text{actually}} \\ m_{\text{CO}_2,t}^{\text{CCS}} = m_{\text{CO}_2,t}^{\text{CSU}} + m_{\text{CO}_2,t}^{\text{MR}} + m_{\text{CO}_2,t}^{\text{urea}} \\ m_{\text{CO}_2,t}^{\text{CCS}} = \eta_{\text{CCS}} \cdot (m_{\text{CO}_2,t}^{\text{CHP}} + m_{\text{CO}_2,t}^{\text{GB}}) \end{cases} \quad (11)$$

where $m_{\text{CO}_2,t}^{\text{CHP}}$ and $m_{\text{CO}_2,t}^{\text{GB}}$ are the CO₂ mass emitted by CHP and GB; $m_{\text{CO}_2,t}^{\text{CCS}}$ is the CO₂ mass captured by CCS; $m_{\text{CO}_2,t}^{\text{actually}}$ is the actual residual emission; $m_{\text{CO}_2,t}^{\text{CSU}}$ is the CO₂ mass sequestered by CSU; $m_{\text{CO}_2,t}^{\text{MR}}$ and $m_{\text{CO}_2,t}^{\text{urea}}$ are the CO₂ masses consumed by methanation and urea synthesis, respectively; η_{CCS} is the capture efficiency of CCS.

3.3.5 Methanation and Natural Gas Balance

$$\begin{cases} V_t^{\text{CH}_4} = \eta_{\text{MR}} \cdot H_t^{\text{MR}} \\ V_t^{\text{CHP}} + V_t^{\text{GB}} = V_t^{\text{CH}_4} + V_t^{\text{gas},\text{buy}} \end{cases} \quad (12)$$

where $V_t^{\text{CH}_4}$ is the volume of natural gas produced by methanation; η_{MR} is the methanation efficiency; V_t^{CHP} and V_t^{GB} are the natural gas volumes consumed by CHP and GB.

3.3.6 Urea Synthesis (CO₂ Abundant)

$$m_t^{\text{urea}} = \eta_{\text{urea}} \cdot m_t^{\text{NH}_3} \quad (13)$$

where η_{urea} is the urea synthesis efficiency.

The urea synthesis efficiency η_{urea} directly governs the carbon utilization effectiveness of the low-carbon chain. A higher η_{urea} leads to more captured CO_2 being permanently fixed into urea, thereby reducing net carbon emissions and carbon trading costs. Based on typical industrial data, η_{urea} is set to 0.85 in this study.

3.3.7 Energy Storage Constraints

$$\begin{cases} 0 \leq P_{s,\text{ch},t} \leq P_{s,\text{ch},\text{max}} \\ 0 \leq P_{s,\text{dis},t} \leq P_{s,\text{dis},\text{max}} \\ P_{s,\text{ch},t} + P_{s,\text{dis},t} \leq P_{s,\text{max}} \\ E_{s,t+1} = E_{s,t} + \eta_{s,\text{ch}} P_{s,\text{ch},t} - \frac{1}{\eta_{s,\text{dis}}} P_{s,\text{dis},t} \\ E_{s,\text{min}} \leq E_{s,t} \leq E_{s,\text{max}} \\ E_{s,1} = E_{s,T+1} \end{cases} \quad (14)$$

where the subscript $s \in \{\text{BESS, TES, H}_2\}$ denotes the storage type (electric, heat, hydrogen). $P_{s,\text{ch},t}$ and $P_{s,\text{dis},t}$ are the charging and discharging power; $P_{s,\text{ch},\text{max}}$ and $P_{s,\text{dis},\text{max}}$ are the power limits; $P_{s,\text{max}}$ is the total power limit for simultaneous charging and discharging; $E_{s,t}$ is the stored energy (or hydrogen mass); $\eta_{s,\text{ch}}$ and $\eta_{s,\text{dis}}$ are the charging and discharging efficiencies; $E_{s,\text{min}}$ and $E_{s,\text{max}}$ are the capacity limits; $E_{s,1} = E_{s,T+1}$ ensures the state of charge returns to its initial value at the end of the cycle.

3.3.8 CHP and GB Models

The power and heat conversion relationships of CHP and GB are:

$$\begin{cases} P_t^{\text{CHP}} = \eta_e V_t^{\text{CHP}} \\ Q_t^{\text{CHP}} = \eta_h V_t^{\text{CHP}} \\ Q_t^{\text{GB}} = \eta_{\text{GB}} V_t^{\text{GB}} \end{cases} \quad (15)$$

where η_e and η_h are the electrical and heat efficiencies of CHP (including natural gas calorific value conversion); η_{GB} is the heat efficiency of GB.

In addition, the following output limits and ramping constraints apply:

$$\begin{cases} P_{\text{min}} \leq P_t^{\text{CHP}} \leq P_{\text{max}} \\ Q_{\text{min}} \leq Q_t^{\text{CHP}} \leq Q_{\text{max}} \\ Q_{\text{min}} \leq Q_t^{\text{GB}} \leq Q_{\text{max}} \\ -R_{\text{CHP}}^{\text{down}} \leq P_t^{\text{CHP}} - P_{t-1}^{\text{CHP}} \leq R_{\text{CHP}}^{\text{up}} \\ -R_{\text{GB}}^{\text{down}} \leq Q_t^{\text{GB}} - Q_{t-1}^{\text{GB}} \leq R_{\text{GB}}^{\text{up}} \end{cases} \quad (16)$$

where P_{min} and P_{max} are the lower and upper limits of CHP electric power; Q_{min} and Q_{max} are the lower and upper limits of CHP heat power; Q_{min} and Q_{max} are the lower and upper limits of GB heat power; $R_{\text{CHP}}^{\text{down}}$ and $R_{\text{CHP}}^{\text{up}}$ are the downward and upward ramping rates of CHP electric power; $R_{\text{GB}}^{\text{down}}$ and $R_{\text{GB}}^{\text{up}}$ are the downward and upward ramping rates of GB heat power.

3.3.9 LA Model

The electric load $P_t^{LA,e}$ and heat load $Q_t^{LA,h}$ consist of fixed parts and flexible parts:

$$\begin{cases} P_t^{LA,e} = P_t^{\text{fixed},e} + P_t^{\text{shift}} \\ Q_t^{LA,h} = Q_t^{\text{fixed},h} - Q_t^{\text{cut}} \end{cases} \quad (17)$$

where $P_t^{\text{fixed},e}$ and $Q_t^{\text{fixed},h}$ are the fixed electric and heat loads; P_t^{shift} is the shiftable electric load; Q_t^{cut} is the curtailable heat load. These flexible loads satisfy:

$$\begin{cases} 0 \leq P_t^{\text{shift}} \leq P_{\text{shift,max}} \\ \sum_{t=1}^T P_t^{\text{shift}} = 0 \\ 0 \leq Q_t^{\text{cut}} \leq Q_{\text{cut,max}} \end{cases} \quad (18)$$

where $P_{\text{shift,max}}$ and $Q_{\text{cut,max}}$ are the maximum allowable values of shiftable electric load and curtailable heat load, respectively.

4 Carbon Emission Factor Modeling

4.1 Dynamic Carbon Emission Factor Modeling

Conventional carbon accounting uses static average emission factors, which fail to reflect the variations in carbon intensity caused by changes in generation mix and renewable output. Therefore, the dynamic carbon emission factor e_t^{grid} is introduced, defined as the real-time carbon intensity of one unit of electricity purchased from the upper grid at time t . Specifically, e_t^{grid} is not a fixed exogenous carbon factor or an optimization decision variable, but a time-varying parameter calculated from carbon emission flow tracing and the nodal carbon potential of the grid-connection node at each period to evaluate the carbon emissions and carbon-related costs of grid power purchases.

Based on the carbon emission flow theory of power systems, the nodal carbon potential $e_{i,t}$ is the average carbon intensity of electricity consumed at node i at time t . Using the proportional sharing principle for power flow tracing, the matrix form of the nodal carbon potential vector is:

$$\mathbf{e}_N = (\mathbf{P}_N - \mathbf{P}_B^\top)^{-1} \mathbf{P}_G^\top \mathbf{e}_G \quad (19)$$

where $\mathbf{e}_N$ is the carbon potential vector of node set N ; $\mathbf{P}_N$ is the active power flux matrix of node set N ; $\mathbf{P}_B$ is the branch active power distribution matrix of branch set B ; $\mathbf{P}_G$ is the generator injection distribution matrix of generator set G ; $\mathbf{e}_G$ is the carbon emission intensity vector of generator set G .

In practice, the matrix $(\mathbf{P}_N - \mathbf{P}_B^\top)$ may be singular when isolated nodes with zero net injection exist. In this paper, the IEEE 69-bus system is connected and has no isolated nodes (every node either has a generator or connects to the upper grid), which guarantees non-singularity.

$$e_t^{\text{grid}} = e_{i,t} \quad (20)$$

The electricity carbon factor $e_{i,t}$ is embedded into the optimization model of [Section 3](#), making the carbon emission of grid power purchases vary with the real-time grid carbon intensity and guiding LA's electricity purchase behavior via carbon flow.

4.2 Carbon Emission Flow Calculation and Nodal Carbon Potential Analysis

Carbon emission flow is the virtual carbon flux carried by power flows in the network. The carbon flow density of a branch equals the carbon potential of its sending node. For a branch l from node i to node j with active power $P_{l,t}$, the carbon flow density is:

$$\rho_{l,t} = e_{i,t} \quad (21)$$

where $\rho_{l,t}$ is the carbon flow density of branch l at time t .

The carbon potential of node k equals the sum of all branch carbon flows and generator carbon flows entering the node divided by the total incoming power:

$$e_{i,t} = \frac{\sum_{l \in L_i^{\text{in}}} P_{l,t} \rho_{l,t} + \sum_{m \in M_i} P_{m,t} e_m}{\sum_{l \in L_i^{\text{in}}} P_{l,t} + \sum_{m \in M_i} P_{m,t}} \quad (22)$$

where L_i^{in} is the set of branches feeding node i ; M_i is the set of generators connected to node i ; $P_{m,t}$ is the active power output of generator m at time t ; and e_m is the carbon intensity of generator m .

To capture the spatiotemporal distribution of carbon responsibility in the heating network, a heat carbon flow model considering pipeline losses and varying heat source outputs is established, replacing the simplified average model.

(1) Carbon potential of heat source nodes:

$$h_{g,t} = \frac{m_{\text{CO}_2,t}^{\text{CHP}} + m_{\text{CO}_2,t}^{\text{GB}}}{Q_t^{\text{CHP}} + Q_t^{\text{GB}} + Q_t^{\text{P2A,h}}} \quad (23)$$

where $h_{g,t}$ is the carbon potential of heat source node set g .

(2) Thermal efficiency of a pipeline:

$$\eta_{ij}^h = \exp\left(-\frac{\lambda_{ij} L_{ij}}{C_p \dot{m}_{ij}}\right) \quad (24)$$

where η_{ij}^h is the thermal efficiency of pipeline i to j ; λ_{ij} is the heat loss coefficient per unit length; L_{ij} is the pipe length; C_p is the specific heat capacity of water; $\dot{m}_{ij}$ is the mass flow rate.

(3) Single-branch node carbon potential:

$$h_{i,t} = \frac{h_{j,t}}{\eta_{ji}^h} \quad (25)$$

(4) Mixing node carbon potential:

$$h_{i,t} = \frac{\sum_{j \in \Omega_i^{\text{in}}} h_{j,t} Q_{ji,t}}{\sum_{j \in \Omega_i^{\text{in}}} Q_{ji,t}} \quad (26)$$

where Ω_i^{in} is the set of upstream heat nodes directly connected to node i ; $Q_{ji,t}$ is the heat power transferred from node j to node i at time t .

The resulting nodal carbon potentials $h_{i,t}$ are directly used in the heat carbon cost term $\beta_h h_{i,t} Q_{i,t}^{\text{LA,h}}$ of the LA's objective function, enabling spatiotemporally refined heat-side carbon responsibility allocation.

5 Model Solution

Sections 3 and 4 established a Stackelberg game model with DCEF. The upper-level objective contains bilinear terms of price and power, while the lower-level is a linear program (LP). This section uses KKT conditions and the strong duality theorem to equivalently transform the original bilevel problem into a single-level LP, rigorously proving the existence of a Stackelberg equilibrium. A hybrid CFO-CPLEX solution algorithm is also proposed, providing the theoretical foundation and an efficient tool for the case studies in the next section.

5.1 Linearity of the Lower-Level Problem

After linear relaxation, the optimization problems of ES and LA are both LPs. Eqs. (5) and (7) are linear, and constraints (8)–(18) are all linear equalities or inequalities; the feasible set is a convex polyhedron. For a given price vector p , the lower-level LP has an optimal solution, and the solution set is convex. An optimal solution is attained at an extreme point of the feasible set; if the solution is unique, the reaction function $x^*(p)$ is single-valued and continuous in p .

5.2 Equivalent Optimality Conditions of the Lower-Level

For a linear program, KKT conditions are both necessary and sufficient for global optimality. Write ES problem in compact form:

$$\min_{\mathbf{x}_{ES}} \mathbf{c}_{ES}(\mathbf{p})^\top \mathbf{x}_{ES} \text{ s.t. } \mathbf{A}_{ES} \mathbf{x}_{ES} \leq \mathbf{b}_{ES}, \mathbf{x}_{ES} \geq 0 \quad (27)$$

where $\mathbf{x}_{ES}$ is the vector of all ES decision variables; $\mathbf{A}_{ES}$ and $\mathbf{b}_{ES}$ are the coefficient matrix and constant vector, respectively; $\mathbf{c}_{ES}(\mathbf{p})$ is the cost coefficient vector that depends linearly on the price vector $\mathbf{p}$.

Introduce dual variables $\lambda_{ES} \geq 0$ associated with the inequality constraints $\mathbf{A}_{ES} \mathbf{x}_{ES} \leq \mathbf{b}_{ES}$. Then primal feasibility, dual feasibility, and strong duality together form the necessary and sufficient optimality conditions:

$$\begin{cases} \mathbf{A}_{ES} \mathbf{x}_{ES} \leq \mathbf{b}_{ES}, \mathbf{x}_{ES} \geq 0 \\ \lambda_{ES} \geq 0, \mathbf{c}_{ES}(\mathbf{p}) + \mathbf{A}_{ES}^\top \lambda_{ES} \geq 0 \\ \mathbf{c}_{ES}(\mathbf{p})^\top \mathbf{x}_{ES} = \lambda_{ES}^\top \mathbf{b}_{ES} \end{cases} \quad (28)$$

The third equality is the strong duality condition, which is equivalent to the complementary slackness conditions but avoids nonlinear terms and does not require binary variables.

Similarly, write the LA problem in compact form:

$$\min_{\mathbf{x}_{LA}} \mathbf{c}_{LA}(\mathbf{p})^\top \mathbf{x}_{LA} \text{ s.t. } \mathbf{A}_{LA} \mathbf{x}_{LA} \leq \mathbf{b}_{LA}, \mathbf{x}_{LA} \geq 0 \quad (29)$$

where $\mathbf{x}_{LA}$ is the vector of LA decision variables (including the electric and heat loads at all nodes); $\mathbf{A}_{LA}$ and $\mathbf{b}_{LA}$ are the coefficient matrix and constant vector, respectively; $\mathbf{c}_{LA}(\mathbf{p})$ is the cost coefficient vector linear in $\mathbf{p}$.

Introduce dual variables $\lambda_{LA} \geq 0$. The optimality conditions are:

$$\begin{cases} \mathbf{A}_{LA} \mathbf{x}_{LA} \leq \mathbf{b}_{LA}, \mathbf{x}_{LA} \geq 0 \\ \lambda_{LA} \geq 0, \mathbf{c}_{LA}(\mathbf{p}) + \mathbf{A}_{LA}^\top \lambda_{LA} \geq 0 \\ \mathbf{c}_{LA}(\mathbf{p})^\top \mathbf{x}_{LA} = \lambda_{LA}^\top \mathbf{b}_{LA} \end{cases} \quad (30)$$

Thus, the optimality of the lower-level ES and LA can be represented by a set of linear constraints and linear equalities, containing no integer variables.

5.3 Linearization of Upper-Level Bilinear Terms

The upper-level objective (1) contains bilinear terms such as $p_t^{e,s} P_t^{LA,e}$, where $P_t^{LA,e}$ is part of the lower-level solution. Since the lower-level is an LP, strong duality holds. For LA, the strong duality condition gives:

$$\mathbf{c}_{LA}(\mathbf{p})^\top \mathbf{x}_{LA}^* = \boldsymbol{\lambda}_{LA}^{*\top} \mathbf{b}_{LA} \quad (31)$$

where $\mathbf{x}_{LA}^*$ and $\boldsymbol{\lambda}_{LA}^*$ are the optimal primal and dual solutions of LA problem, respectively, and $\mathbf{b}_{LA}$ is the constant vector of constraints.

Because $\mathbf{c}_{LA}(\mathbf{p})$ is linear in the price vector $\mathbf{p}$, the product $\mathbf{p} \cdot \mathbf{x}_{LA}^*$ can be expressed linearly in terms of the dual variables $\boldsymbol{\lambda}_{LA}^*$ and constants. Specifically, the electricity purchase cost term $p_t^{e,s} P_{k,t}^{LA,e}$ in the LA's objective is part of $\mathbf{c}_{LA}(\mathbf{p})$; using the above equality, it is replaced by a dual expression, eliminating the bilinearity. Substituting this into IESO's objective removes all bilinear terms, making the upper-level objective linear in $\mathbf{p}$ and $\boldsymbol{\lambda}_{LA}$.

5.4 Single-Level Equivalent Linear Program

With the bilinear terms eliminated via strong duality (Section 5.3) and the piecewise linear carbon costs linearized via auxiliary variables (Section 3.2.1), all nonlinearities in the original bilevel problem are transformed into linear forms. Similar transformations using KKT conditions and strong duality have been widely adopted in Stackelberg game-based energy system optimization [39]. Consequently, the resulting single-level problem is an LP.

Adding the lower-level optimality conditions (primal feasibility, dual feasibility, strong duality) as constraints to the upper-level and replacing the bilinear terms with the linear expressions derived from strong duality yields the following single-level LP, where $F_{IESO}^{linear}(\mathbf{p}, \boldsymbol{\lambda}_{LA})$ is the linearized IESO profit:

$$\left\{ \begin{array}{l} \max F_{IESO}^{linear}(\mathbf{p}, \boldsymbol{\lambda}_{LA}) \\ \text{s.t. } p_t^{e,b} < p_t^{grid,buy} < p_t^{e,s}, p_t^{e,min} < p_t^{e,b} < p_t^{grid,sell} < p_t^{e,max} \\ p_t^{h,b} < p_t^{h,s}, p_t^{h,min} < p_t^{h,b}, p_t^{h,s} < p_t^{h,max} \\ \mathbf{A}_{ES} \mathbf{x}_{ES} \leq \mathbf{b}_{ES}, \mathbf{x}_{ES} \geq 0, \boldsymbol{\lambda}_{ES} \geq 0, \mathbf{c}_{ES}(\mathbf{p}) + \mathbf{A}_{ES}^\top \boldsymbol{\lambda}_{ES} \geq 0 \\ \mathbf{c}_{ES}(\mathbf{p})^\top \mathbf{x}_{ES} = \boldsymbol{\lambda}_{ES}^\top \mathbf{b}_{ES} \\ \mathbf{A}_{LA} \mathbf{x}_{LA} \leq \mathbf{b}_{LA}, \mathbf{x}_{LA} \geq 0, \boldsymbol{\lambda}_{LA} \geq 0, \mathbf{c}_{LA}(\mathbf{p}) + \mathbf{A}_{LA}^\top \boldsymbol{\lambda}_{LA} \geq 0 \\ \mathbf{c}_{LA}(\mathbf{p})^\top \mathbf{x}_{LA} = \boldsymbol{\lambda}_{LA}^\top \mathbf{b}_{LA} \\ \text{equations (8)–(18)} \end{array} \right. \quad (32)$$

where $F_{IESO}^{linear}(\mathbf{p}, \boldsymbol{\lambda}_{LA})$ is the linearized profit of IESO obtained from Eq. (1) after the strong duality substitution; all variables and matrices are defined as before.

This single-level LP has a non-empty, bounded feasible set (guaranteed by the system's physical capacities and energy balances) and a linear objective, so an optimal solution exists. Let $(\mathbf{p}^*, \mathbf{x}_{ES}^*, \mathbf{x}_{LA}^*, \boldsymbol{\lambda}_{ES}^*, \boldsymbol{\lambda}_{LA}^*)$ be an optimal solution. Then:

- (1) Given ρ^* , $(\mathbf{x}_{ES}^*, \mathbf{x}_{LA}^*)$ satisfies the lower-level optimality conditions, i.e., it is the followers' optimal response;
- (2) Strong duality ensures that ρ^* maximizes IESO's profit.

Hence $(\rho^*, \mathbf{x}_{ES}^*, \mathbf{x}_{LA}^*)$ is a Stackelberg equilibrium. The original Stackelberg game possesses a Stackelberg equilibrium.

5.5 Solution Algorithm

5.5.1 CFO Algorithm

The upper-level determination of time-of-use electricity and heat prices is a high-dimensional continuous optimization problem. Within the Stackelberg framework, pricing strategies affect lower-level responses and carbon responsibility allocation, resulting in strong coupling among pricing, dispatch, and carbon flow. Therefore, CFO is used for upper-layer price search. Each particle represents a candidate electricity and heat price set within the prescribed bounds. For each candidate, CPLEX solves the lower-level dispatch problem, and the fitness value is evaluated based on system profit, user cost, and carbon responsibility. CFO then updates the population iteratively. Thus, CFO only optimizes upper-level prices, while lower-level responses and carbon flow calculations are solved exactly by CPLEX, balancing global search capability and solution accuracy.

5.5.2 Hybrid CFO-CPLEX Solution Procedure

The proposed hybrid CFO-CPLEX solution framework is shown in Fig. 3. Its core idea is to embed carbon emission flow, nodal carbon potential, and DCEF into fitness evaluation, combining upper-level price search with lower-level exact optimization. The detailed steps are as follows:

- (1) Input electric and heat loads, renewable output, device/network parameters, carbon trading parameters, and CFO settings.
- (2) Initialize the CFO population, where each particle represents a candidate time-of-use electricity and heat price set.
- (3) Pass candidate prices to the lower level and solve the equivalent single-level linear optimization problem transformed by KKT conditions and strong duality to obtain device outputs, storage operation, P2A, CCS, methanation, urea synthesis, and load responses.
- (4) Calculate power flows, branch carbon emission flows, power-network nodal carbon potentials, and DCEF for grid purchases.
- (5) Calculate heating-network nodal carbon potentials based on heat source outputs, network topology, and pipeline losses. Then compute LA electricity/heat carbon costs and IESO grid-purchase carbon cost for fitness evaluation.
- (6) Update the population using CFO rules. If convergence is not satisfied, return to step (3); otherwise, output the optimal prices, dispatch results, carbon flows, and nodal carbon potentials.

This hybrid approach combines the global search capability of CFO with the exact lower-level optimization of CPLEX, supporting the solution of the bilevel Stackelberg equilibrium.

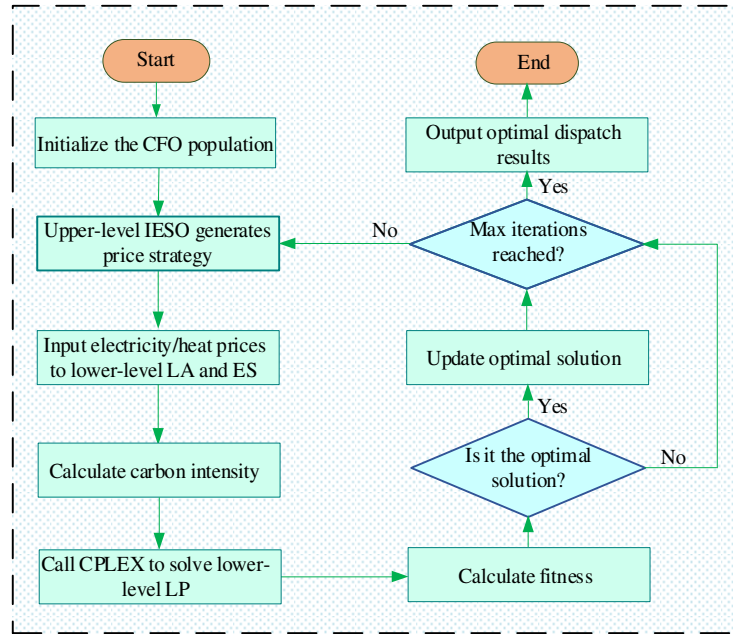


Figure 3: Flowchart of the hybrid CFO-CPLEX solution procedure.

5.5.3 Computational Complexity Analysis

The total time complexity of the hybrid CFO-CPLEX framework is:

$$O(n_{\text{pop}} \cdot n_{\text{iter}} \cdot n_{\text{var}}^3) \quad (33)$$

where n_{pop} is the population size; n_{iter} is the maximum number of iterations; and n_{var} is the number of decision variables in the lower-level LP. In this study, $n_{\text{pop}} = 50$, $n_{\text{iter}} = 200$, and $n_{\text{var}} \approx 10^3$. In contrast, directly solving the equivalent single-level MILP via branch-and-bound has worst-case exponential complexity in the number of binary variables, making it computationally prohibitive for large-scale problems. Therefore, the proposed hybrid framework achieves a favorable balance between solution quality and computational efficiency.

6 Case Studies

Case studies based on a typical electric-heat coupled IES are conducted to evaluate the proposed bilevel low-carbon optimization model with carbon emission flow and DCEF and the hybrid CFO-CPLEX method. Algorithm performance, multi-scenario comparison, system operation, and carbon responsibility allocation are analyzed.

6.1 Case Study System and Parameter Settings

A typical daily case study of an electric-heat coupled IES is constructed to evaluate the proposed model and solution method. The power network adopts the IEEE 69-bus distribution system, and the heating network adopts a 6-node system, as shown in Fig. 4. Key parameters are listed in Table 2. WT/PV outputs and electric/heat loads are based on typical daily forecasts, with time-series curves shown in Fig. 5. In all scenario comparisons, load profiles, renewable generation, equipment parameters, and network topology remain unchanged; only the game mechanism and carbon responsibility modeling method are varied.

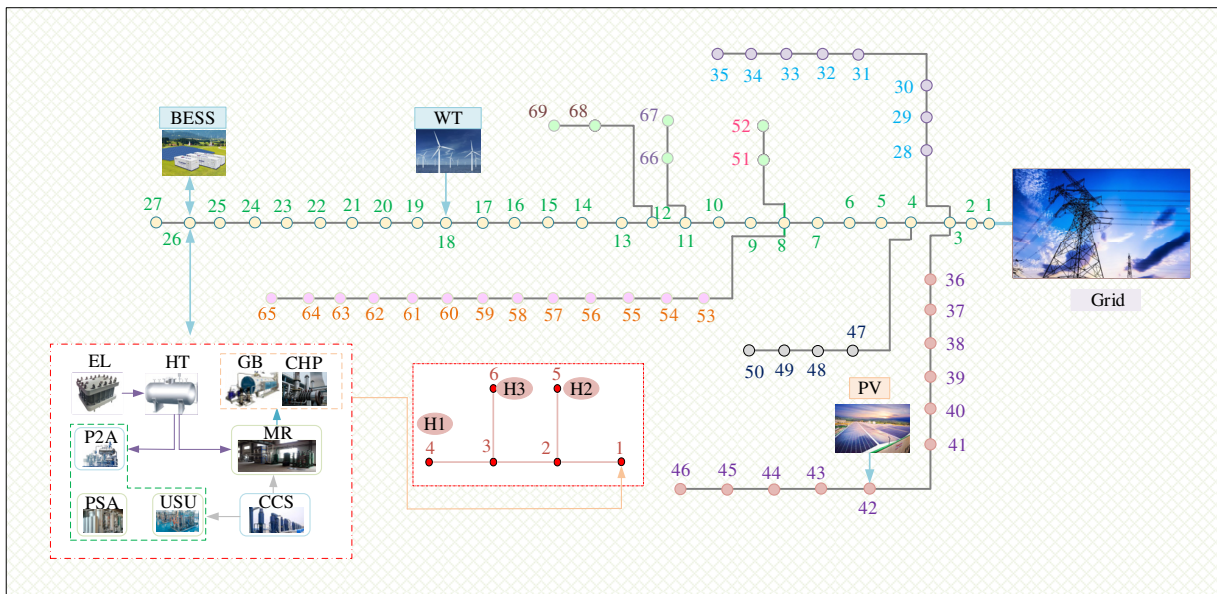


Figure 4: Topology of the electric-heat coupled IES.

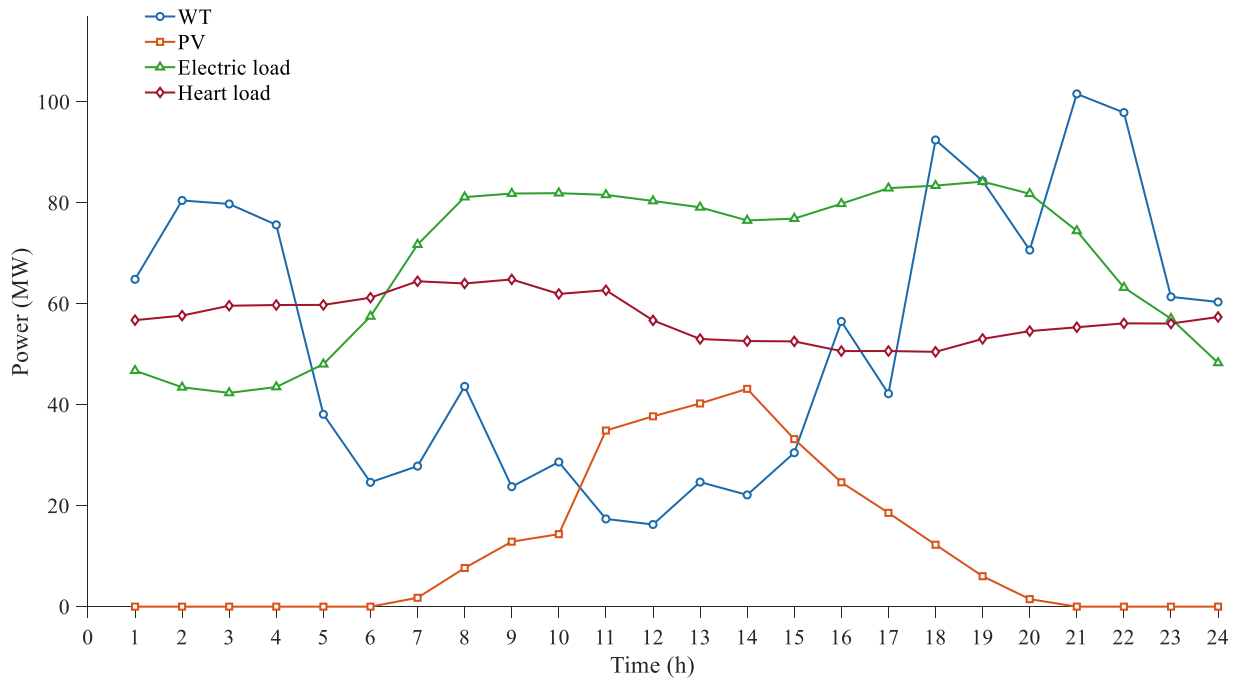
Table 2: System parameters.

Device	Parameter	Value	Device	Parameter	Value
CHP	Electric power upper/lower limit (MW)	50/0	TES	TES capacity upper/lower limit (MWh)	20/0
	Heat power upper/lower limit (MW)	50/0		Charging/discharging power upper/lower limit (MW)	10/0
	Ramp up/down rate (MW/h)	12.5/−12.5		Charging/discharging efficiency	0.95
	Electric/heat efficiency	0.4/0.3		Heat loss coefficient	0.01
GB	Heat power upper/lower limit (MW)	50/0	HT	Hydrogen storage capacity upper/lower limit (kg)	2000/0
	Ramp up/down limit (MW/h)	12.5/−12.5		Charging/discharging efficiency	0.95
	Heat efficiency	0.92		Initial capacity (kg)	1000
EL	Hydrogen production efficiency	0.8	Gas price	Unit price (CNY/m ³)	3
	Power consumption upper/lower limit (MW)	80/0	Carbon emission base price	Unit price (CNY/t)	400

(Continued)

Table 2 (continued)

Device	Parameter	Value	Device	Parameter	Value
MR	Methanation efficiency	0.8	Reference time-of-use electricity price	Peak price (CNY/kWh) (13:00–16:00, 21:00–24:00)	1.2
BESS	Storage capacity upper/lower limit (MWh)	20/0		Valley price (CNY/kWh) (1:00–9:00)	0.4
	Charging/discharging power upper/lower limit (MW)	20/0		Flat price (CNY/kWh) (10:00–12:00, 17:00–20:00)	0.8
	Charging/discharging efficiency	0.9		Urea unit price (CNY/t)	2500
CCS	Maximum capture power (MW)	90	Heat price	Upper limit (CNY/kWh)	0.5
	Carbon sequestration cost (CNY/t)	250		Lower limit (CNY/kWh)	0.15

**Figure 5:** Forecasted curves of WT, PV output, electric load and heat load on a typical day.

These profiles are derived from one-year historical data of an industrial park in southern China using the same typical-day extraction method as in [8]; they represent a typical daily operation scenario, while extreme seasonal or fault scenarios are not considered in this study.

6.2 Comparative Analysis of Algorithm Performance

To validate the CFO-CPLEX hybrid algorithm, this case study compares four meta-heuristic algorithms that have demonstrated excellent performance in complex optimization problems: dung beetle optimizer (DBO), equilibrium optimizer (EO), golden sine algorithm (Gold-SA), and gorilla troops optimizer (GTO). All algorithms use the same population size of 50, maximum iterations of 200, and 96 decision variables. Parameter settings are listed in Table 3. The algorithm terminates after 200 iterations, and convergence is confirmed when the difference of optimal objective values between successive iterations is below 10^{-4} .

Table 3: Algorithm parameters.

Algorithm	Parameters	Value
Shared parameters	Maximum number of iterations	200
	Population size	50
	Number of decision variables	96
	Number of objectives	3
CFO	Mutation factor	0.3
	Crossover probability	0.7
DBO	Initial step factor	0.5
	Random factor coefficient	0.99
EO	Equilibrium pool size	5
	Generation rate parameter 1	2
Gold-SA	Generation rate parameter 2	1
	Golden ratio coefficient	0.618
GTO	Migration probability	0.03
	Stage-2 selection threshold	0.5

Table 4 compares different algorithms using multiple indicators. Since the upper-level problem focuses on IESO pricing optimization, IESO objective value is used as the primary criterion, while carbon emissions, carbon trading cost, convergence stability, and runtime are considered auxiliary criteria. CFO obtains the highest IESO objective value, the lowest carbon emissions, and the smallest carbon trading cost, with emissions 13.2% lower than those of DBO. Its runtime is 6.28 h, slightly longer than DBO but shorter than EO, Gold-SA, and GTO. However, CFO is not superior in all indicators: its ES objective value is lower than those of DBO, EO, and Gold-SA, and its LA objective value is higher than those of the other algorithms due to carbon cost transfer to the user side. This reflects a trade-off among IESO profit, ES profit, LA cost, and emission reduction. Therefore, CFO is selected not because it is universally optimal, but because it best matches the upper-level pricing objective and low-carbon dispatch requirements of this study.

Table 4: Algorithm result comparison table.

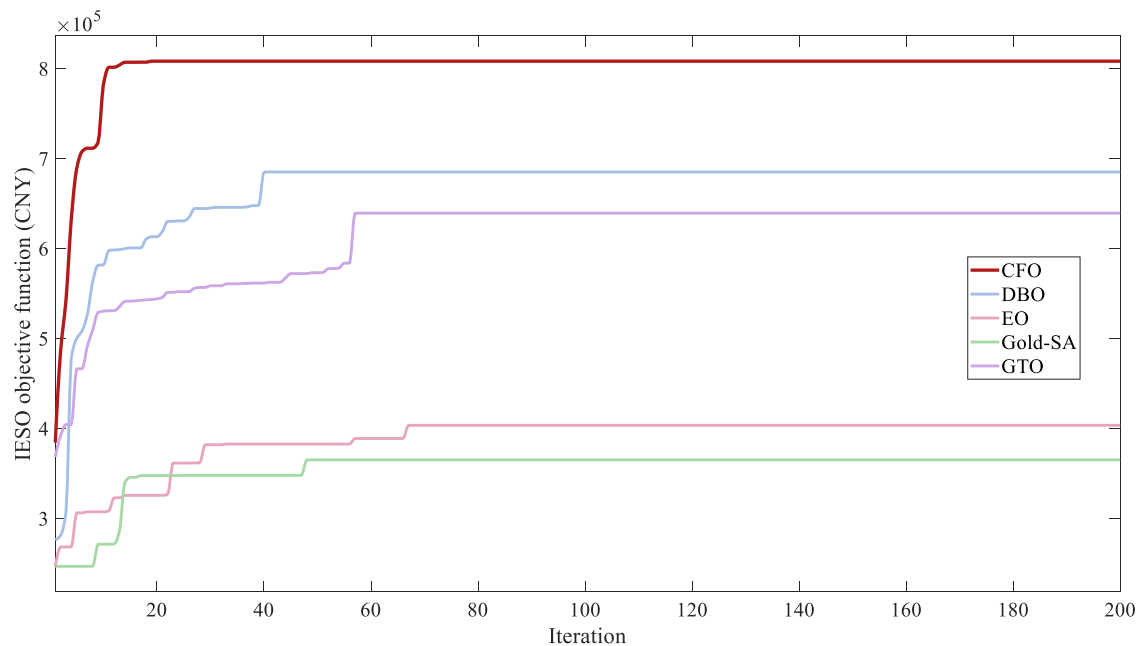
Algorithm	F_{IESO} (10^4 CNY)	F_{ES} (10^4 CNY)	Carbon Trading Cost (10^4 CNY)	F_{LA} (10^4 CNY)	Carbon Emission (t)	Total Runtime (h)
CFO	80.83	46.93	0.42	145.77	55.68	6.28
DBO	68.52	56.49	0.48	122.71	64.10	6.18
EO	40.36	78.33	0.48	137.88	63.67	5.45

(Continued)

Table 4 (continued)

Algorithm	F_{IESO} (10^4 CNY)	F_{ES} (10^4 CNY)	Carbon Trading Cost (10^4 CNY)	F_{LA} (10^4 CNY)	Carbon Emission (t)	Total Runtime (h)
Gold-SA	36.53	69.51	0.44	123.66	59.21	5.48
GTO	63.94	42.42	0.45	121.85	59.63	5.42

Figs. 6–8 show the convergence curves of IESO, ES, and LA objective functions, respectively. CFO converges rapidly for all three objectives and maintains stable convergence in the later iterations. In contrast, DBO and GTO show noticeable mid-stage fluctuations, while EO and Gold-SA converge more slowly. These results suggest that CFO-CPLEX algorithm provides better convergence stability and solution quality, supporting its use in subsequent scenario analyses. The fast convergence of CFO is mainly due to its adaptive re-parasitic mechanism, which effectively balances exploration and exploitation, and its elite retention strategy that ensures steady improvement.

**Figure 6:** IESO objective function convergence plot.

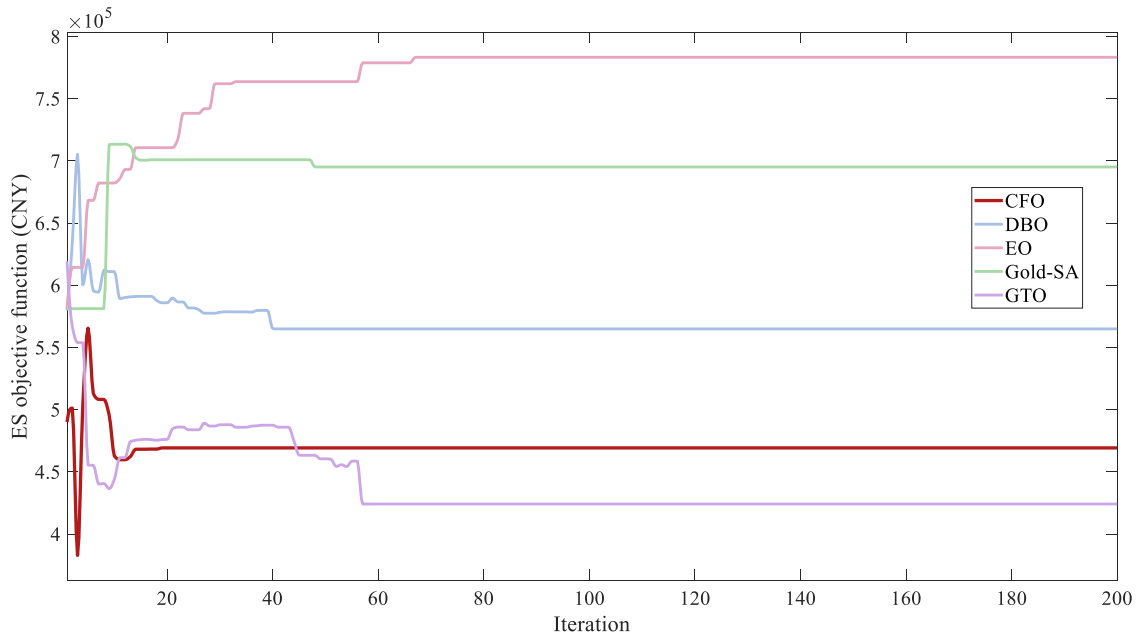


Figure 7: ES objective function convergence plot.

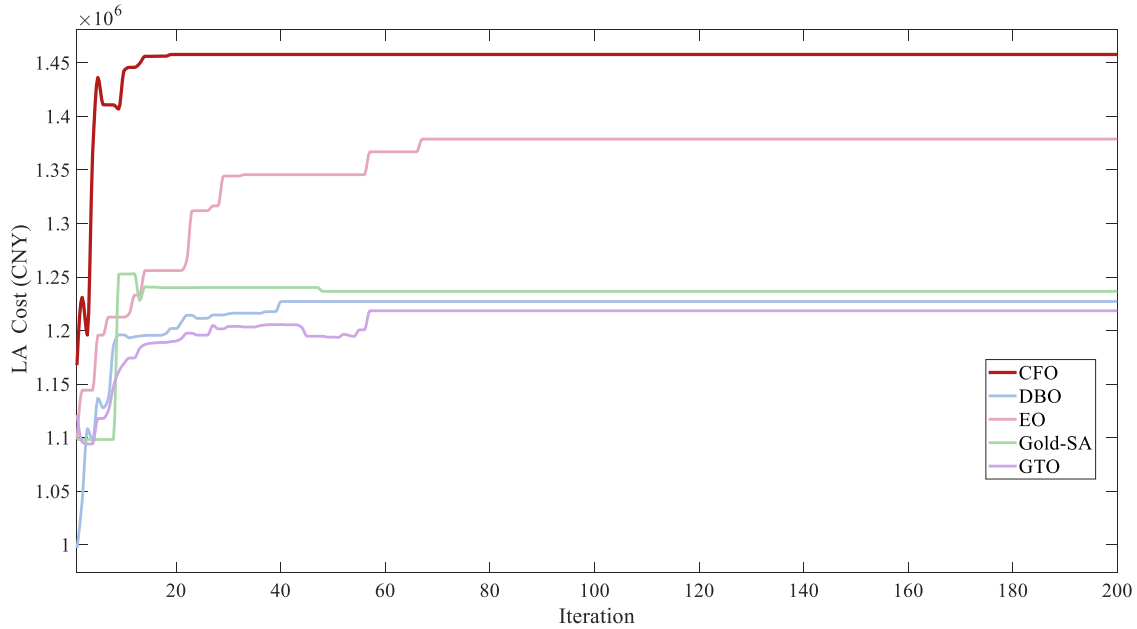


Figure 8: LA objective function convergence plot.

6.3 Multi-Scenario Comparison and Mechanism Validation

6.3.1 Scenario Settings

Five comparative scenarios are established to evaluate the contribution of each mechanism, as shown in Table 5. Scenario 1 is the baseline; scenario 2 introduces the Stackelberg pricing mechanism; scenario 3 adds CET; scenario 4 further considers DCEF and power-network carbon emission flow; and

scenario 5 incorporates heating-network nodal carbon potential, forming the complete electric-heat carbon responsibility allocation model.

Table 5: Scenario settings table.

Scenario	Stackelberg	CET	e_t^{grid}	$e_{k,t}$	$h_{k,t}$	Scenario Description
1	×	×	×	×	×	Basic economic dispatch, no game, and no carbon responsibility
2	✓	×	×	×	×	Only a bi-level game pricing mechanism
3	✓	✓	×	×	×	Introduce a carbon trading mechanism
4	✓	✓	✓	✓	×	Refined grid carbon responsibility
5	✓	✓	✓	✓	✓	Complete model, dynamic update of electric heat, integrated carbon responsibility

6.3.2 Comparison of Economic and Carbon Emission Results

Based on the five comparison scenarios, [Table 6](#) summarizes the economic and carbon emission results. IESO profit increases from 148,500 CNY in scenario 1 to 808,300 CNY in scenario 5, showing the economic benefit of progressively introducing Stackelberg game, CET, DCEF, and electric-heat carbon responsibility allocation. Compared with scenario 3 (Stackelberg and CET only), the complete model increases IESO profit by 72.6%, decreases ES profit by 15.2%, and increases the total LA cost by 24.7%. ES profit decreases from 649,200 to 469,300 CNY because ES undertakes more low-carbon constraints and operates additional low-carbon devices, while its CET cost increases from zero to 4200 CNY. LA energy purchase cost rises from 979,500 to 1,356,700 CNY, and LA carbon cost increases from 99,300 CNY in scenario 4 to 101,100 CNY in scenario 5 after heating-network nodal carbon potential is considered, indicating further carbon responsibility allocation to the load side.

Table 6: Results of different scenarios.

Scenario	F_{IESO} (10^4 CNY)	F_{ES} (10^4 CNY)	LA Energy Purchase Cost (10^4 CNY)	LA Carbon Cost (10^4 CNY)	ES Carbon Trading Cost (10^4 CNY)	Actual Carbon Emission (t)	CCS Cost (10^4 CNY)	O&M Cost (10^4 CNY)
1	14.85	64.92	97.95	0.00	0.00	63.10	−9.82	61.64
2	25.20	60.95	99.70	0.00	0.00	58.74	−9.14	60.13
3	46.83	55.34	116.90	0.00	0.41	54.86	−8.54	58.96
4	69.80	55.50	132.56	9.93	0.40	52.95	−8.24	59.59
5	80.83	46.93	135.67	10.11	0.42	55.68	−8.67	65.56

Carbon emissions decrease from 63.10 t in scenario 1 to the minimum value of 52.95 t in scenario 4, and then slightly increase to 55.68 t in scenario 5. Compared to scenario 3, the comprehensive model results in a 1.5% increase in carbon emissions. However, this slight increase is accompanied by a significant rise in IESO profits and provides a comprehensive reflection of the carbon footprint associated with CHP model. Although scenario 5 does not produce the lowest emissions, it achieves a more complete characterization of electric-heat integrated carbon responsibility. The negative CCS cost reflects the cost benefit of CCS operation, while O&M cost increases from 616,400 to 655,600 CNY due to additional low-carbon device

operation. Overall, the results show that the proposed mechanisms can coordinate economic performance and low-carbon responsibility.

Figs. 9–11 present the convergence processes of the IESO, ES, and LA objective functions under the five scenarios. As the scenario complexity increases, more iterations are required to reach convergence.

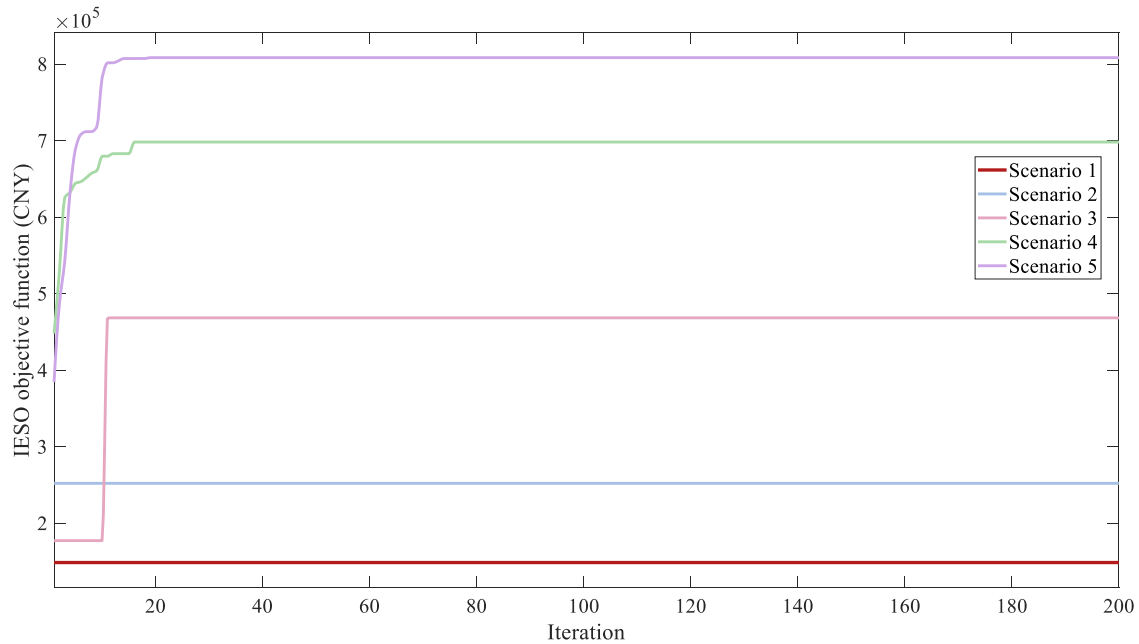


Figure 9: Convergence of IESO objective function for five scenarios.

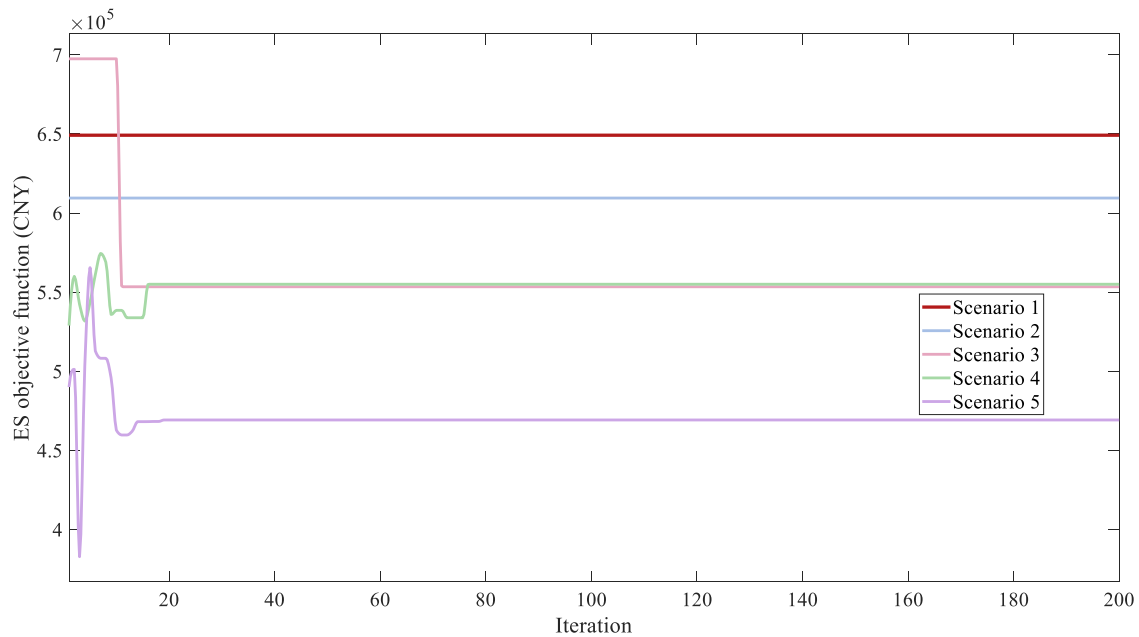


Figure 10: Convergence of ES objective function for five scenarios.

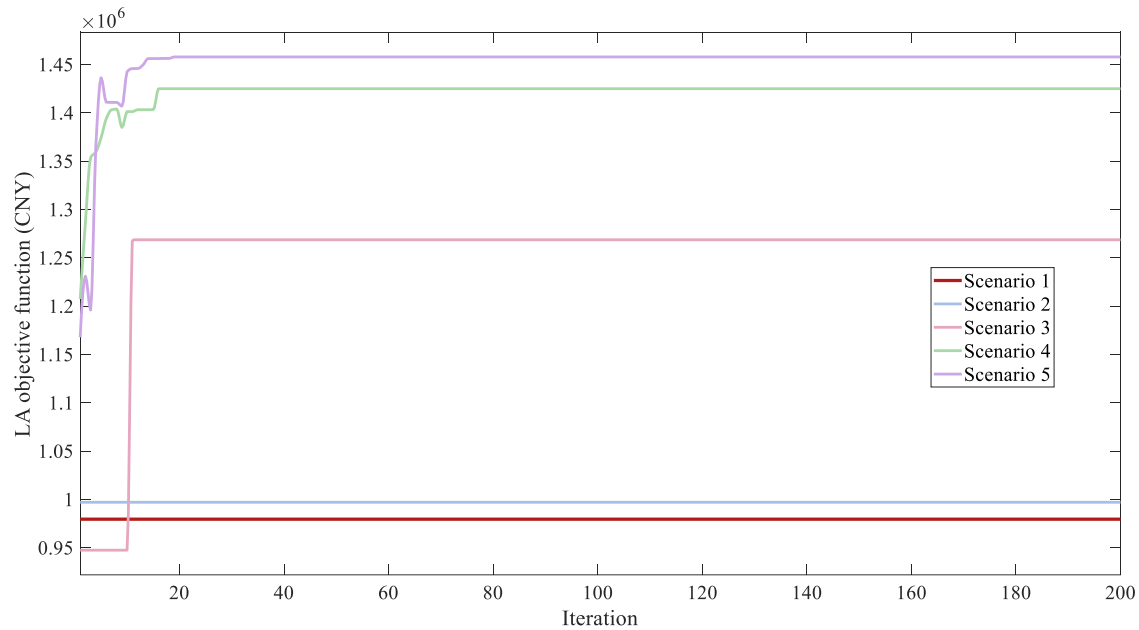


Figure 11: Convergence of LA objective function for five scenarios.

6.3.3 Analysis of Device Output and Low-Carbon Response

Comparative results of the electric balance, heat balance, and main device outputs under different scenarios are presented in Figs. S1–S3 of Supplementary Materials. In scenario 1, loads are mainly supplied by grid purchases, CHP, and GB, with storage responding only to time-of-use prices. Scenario 2 introduces Stackelberg pricing, which adjusts CHP, GB, and TES outputs according to price signals. Scenario 3 adds CET, restraining carbon-intensive CHP and GB operation, and promoting low-carbon coupling devices. Scenario 4 further uses DCEF and power-network nodal carbon potential to guide grid purchases, CCS, and P2A-related operations toward low-carbon periods. In scenario 5, heating-network nodal carbon potential enables smoother CHP/GB outputs and coordinated operation of TES, CCS, and P2A. These results show that the proposed mechanisms improve electric-heat coordination and reduce carbon-intensive energy supply.

6.3.4 Low-Carbon Response Analysis

The CO₂ balance results for five scenarios are shown in Fig. S4 of Supplementary Materials. In scenario 1, carbon emissions come entirely from CHP and GB. Scenario 2 slightly reduces the emissions. In scenario 3, CET mechanism activates CCS, and part of the captured CO₂ is used for MR and P2A. In scenario 4, the grid DCEF increases CCS capture amount and accounts for the embodied carbon in power purchases. In the complete model (scenario 5), the carbon factor of the heating network further lowers the emissions of CHP/GB, and the captured CO₂ is preferentially used for MR and P2A. Compared with other scenarios, scenario 5 operates in a low-carbon manner obviously during low-load periods while maintaining economic operation during peak-load periods, thereby achieving low-carbon economic operation.

Optimized electricity and heat prices under different scenarios are given in Figs. S5 and S6 of the Supplementary Materials. In scenario 1, prices mainly depend on load variations and basic supply costs. Scenario 2 introduces Stackelberg pricing, leading to clearer time-of-use differences through ES and LA responses. Scenario 3 adds CET, which increases prices during some high-emission periods by internalizing the carbon costs of CHP and GB. In scenario 4, DCEF transmits grid-side carbon responsibility through

electricity prices and indirectly affects heat prices via electric-heat coupling. In scenario 5, heating-network nodal carbon potential further enables heat prices to reflect carbon responsibility differences among heat sources and heat nodes. These results show that the proposed mechanisms provide refined low-carbon price signals for electric and heat loads.

6.4 Analysis of Dynamic Carbon Responsibility Allocation Results

This section analyzes carbon responsibility allocation from temporal and spatial dimensions based on carbon emission flow and DCEF.

Fig. 12 shows the temporal variations of grid purchase power, DCEF, and grid-side CO₂ emissions. DCEF varies over time, reflecting the changing carbon intensity of purchased electricity at the corresponding grid node. Grid-side CO₂ emissions depend on both purchased power and DCEF: emissions increase when both are high, while low purchased power limits emissions even under a relatively high DCEF. This confirms the necessity of DCEF for grid-side carbon responsibility allocation.

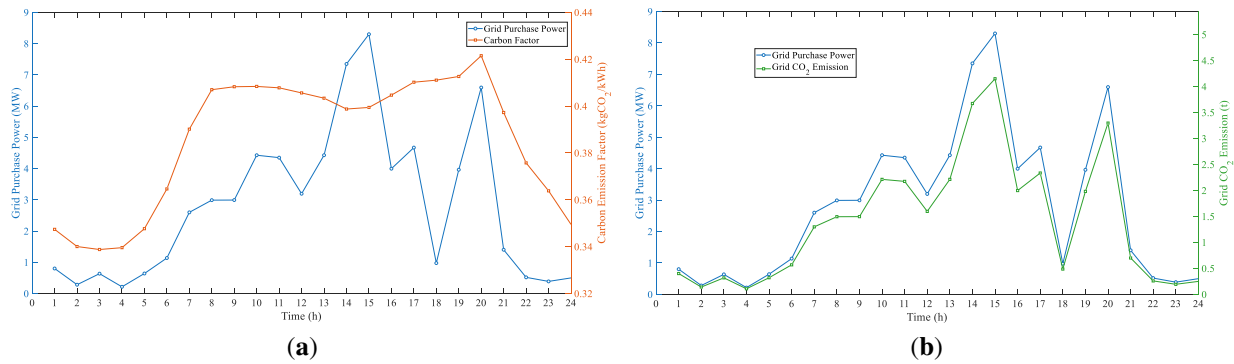


Figure 12: Temporal variation of grid purchase power, dynamic carbon factor, and associated carbon emissions. (a) Grid purchase power and dynamic carbon factor; (b) Grid CO₂ emission.

Detailed nodal carbon potential data of the power and heat network at $t = 7$ h are summarized in Table S1 and Table S2 of the Supplementary Materials, respectively.

Power-network nodal carbon potentials vary significantly. Node 1, connected to the upper grid, has a carbon potential of 0.35 kg CO₂/kWh, while WT and PV nodes have zero carbon potential because renewable generation has no direct emissions. Some nodes, such as nodes 52, 43, 53, 10, and 35, show high carbon potentials of 1.02, 0.94, 0.90, 0.76, and 0.73 kg CO₂/kWh, respectively. The comparison between node 61 and node 52 further shows that nodal carbon potential is not determined only by active power, but also by power flow paths and carbon emission flow distribution.

Table S2 shows that nodes 4, 5, and 6 bear the main heat loads, while nodes 1–3 have zero heat power at $t = 7$ h. Heat nodal carbon potentials range from 0.3365 to 0.3373 kg CO₂/kWh, indicating small spatial differences caused by heating network topology and pipeline losses. The limited variation is reasonable because the heating network is small and the heat sources are relatively concentrated. Overall, these results show that nodal carbon potential can characterize both grid-side and heat-side spatial carbon responsibility and support carbon cost allocation.

6.5 Sensitivity Analysis

To evaluate the impact of key parameters on system performance, a sensitivity analysis is conducted on carbon trading price, electricity price fluctuation, and renewable energy penetration. The results are shown in Fig. 13.

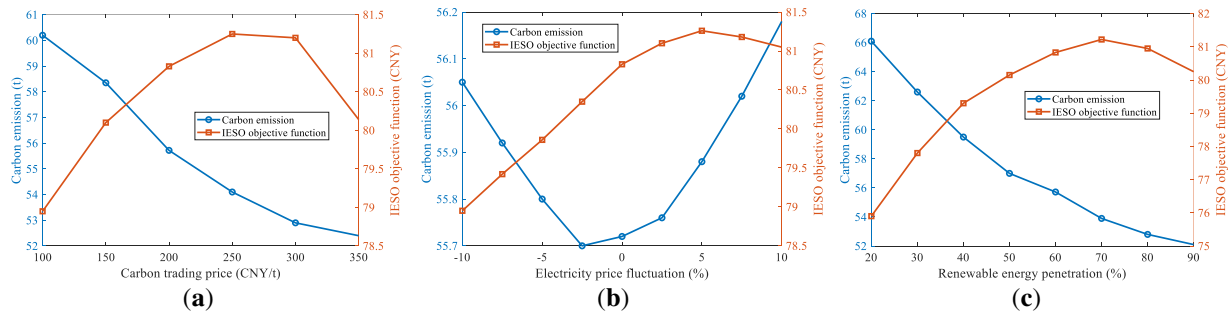


Figure 13: Sensitivity analysis results. (a) carbon trading price; (b) electricity price fluctuation; (c) renewable energy penetration.

When the carbon trading price increases from 100 to 350 CNY/t, carbon emissions are reduced by 13.2%, indicating that the carbon pricing mechanism effectively guides low-carbon dispatch; IESO objective function reaches its peak in the range of 200–250 CNY/t, reflecting the trade-off between emission reduction cost and economic benefit. Within a $\pm 10\%$ fluctuation of electricity price, the variation in carbon emissions does not exceed 0.5 t, and IESO objective function first increases and then stabilizes, demonstrating that the proposed strategy can adapt to electricity price disturbances. When the renewable energy penetration increases from 20% to 90%, carbon emissions decrease by 20.5%, confirming the emission reduction potential of clean energy; however, IESO objective function reaches its optimum around 60% penetration and then declines slightly, suggesting that higher penetration levels require enhanced energy storage to alleviate curtailment pressure.

7 Conclusion

This paper proposes a Stackelberg game-based low-carbon dispatch model for an industrial park IES integrating DCEF and full electric-heat carbon responsibility allocation. The main findings and lessons are summarized below.

- (1) The Stackelberg pricing mechanism improves IESO profit by 72.6% compared to scenario 3, confirming that the leader-follower pricing effectively coordinates conflicting interests among IESO, ES, and LA.
- (2) Stepped CET and DCEF reduce emissions from 58.74 t (scenario 2) to 52.95 t (scenario 4), demonstrating their effectiveness. But in the complete model (scenario 5), which adds heat-side nodal carbon potential, emissions rise slightly to 55.68 t. This indicates that while CET and DCEF successfully lower emissions, the introduction of full electric-heat carbon responsibility allocation leads to a modest emission increase in exchange for more refined fairness.
- (3) Nodal carbon potentials vary significantly, showing that carbon responsibility depends on load level, source location, and power flow. By introducing carbon emission flow in both power and heating networks, the model provides spatially refined carbon signals.
- (4) Compared with four benchmark algorithms, the hybrid CFO-CPLEX method improves IESO profit by 18.0% and reduces carbon emissions by 13.2%, demonstrating its superiority in balancing economic and environmental performance.

- (5) Beyond the deterministic framework, future research will incorporate uncertainties of renewable generation, load, and energy prices via robust or stochastic optimization. Additional flexible resources will be added. The carbon responsibility allocation method will be extended to larger-scale IES with meshed heating networks. Finally, instead of treating carbon costs as weighted terms in existing objectives, future work will explore making the carbon economy an independent game agent to better align economic and low-carbon incentives.

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Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

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Nomenclature

Abbreviations

BESS	Battery energy storage system
CCS	Carbon capture and storage
CET	Carbon emission trading
CFO	Caterpillar fungus optimizer
CHP	Combined heat and power
DCEF	Dynamic carbon emission factors
EL	Electrolyzer
ES	Energy supplier
GB	Gas boiler
HT	Hydrogen tank
IES	Integrated energy system
IESO	Integrated energy system operator
KKT	Karush-Kuhn-Tucker
LA	Load aggregator
MR	Methanation reactor
P2A	Power-to-ammonia
P2G	Power-to-gas
PSA	Pressure swing adsorption
PV	Photovoltaic
TES	Thermal energy storage

USU Urea synthesis unit
 WT Wind turbine

Variables

p_t^{es}	Electricity selling price set by IESO at time t
p_t^{hs}	Heat selling price set by IESO at time t
$p_t^{grid,buy}$	Electricity purchase price from the upper grid
$p_t^{grid,sell}$	Electricity selling price to the upper grid
$p^{e,min}, p^{e,max}$	Minimum and maximum allowed electricity prices by the upper grid
$p^{h,min}, p^{h,max}$	Minimum heat purchase price and maximum heat selling price
$p_t^{LA,e}$	Electric load of LA after demand response
$p_t^{ES,buy,e}$	Electricity purchased by ES from IESO
$p_t^{ES,sell,e}$	Electricity sold by ES to IESO
p_t^{CHP}	Electrical power output of CHP
p_t^{wind}	Power output of WT
p_t^{PV}	Power output of PV
$P_{BESS,dis,t}$	Discharging power of BESS
$P_{BESS,ch,t}$	Charging power of BESS
p_t^{EL}	Power consumption of EL
p_t^{CCS}	Power consumption of CCS
p_t^{PSA}	Power consumption of PSA
p_t^{P2A}	Power consumption of P2A unit
p_t^{CHP}	Electrical power output of CHP
p_t^{wind}	Power output of WT
p_t^{PV}	Power output of PV
$P_{TES,dis,t}$	Discharging power of TES
$P_{TES,ch,t}$	Charging power of TES
C_t^{grid}	Grid interaction cost of IESO
$C_t^{IESO,CET}$	Stepped carbon trading cost of IESO
C_t^{OM}	Total operation and maintenance cost
$C_t^{ES,CET}$	Stepped carbon trading cost of ES
c_{gas}	Natural gas price
$Q_t^{LA,h}$	Heat load of LA after demand response
$Q_t^{ES,sell,h}$	Heat sold by ES to IESO
$Q_t^{ES,buy,h}$	Heat purchased by ES from IESO
Q_t^{CHP}	Heat power output of CHP
Q_t^{GB}	Heat power output of GB
$Q_t^{P2A,h}$	Waste heat from P2A
Q_t^{loss}	Heat network loss (constant)
H_t^{EL}	Hydrogen produced by EL
H_t^{MR}	Hydrogen consumed by MR
$h_{i,t}$	Carbon intensity of heat consumed at node i
$e_{i,t}$	Nodal carbon potential of power node i at time t
e_t^{grid}	Dynamic carbon emission factor of grid electricity at time t
$m_{CO_2,t}^{grid}$	Carbon emission from grid electricity purchase
m_t^{P2A}	Ammonia production mass
m_t^{urea}	Urea production mass
$V_t^{gas,buy}$	Volume of natural gas purchased from external network
r^{urea}	Unit revenue of urea
β	Heat generation coefficient per unit ammonia mass

β_{cha}, β_{dis}	Charging and discharging efficiency
β_e	Conversion coefficient from carbon intensity to price for electricity
β_h	Conversion coefficient from carbon intensity to price for heat

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