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A Multi-Level Equivalent Driving Force Framework for Fatigue Life Prediction of Nickel-Based Single-Crystal Superalloys under Stress Ratio and Notch Effects

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ABSTRACT: Hot-section nickel-based single-crystal superalloy components under isothermal cyclic loading often exhibit systematic life shifts when datasets span different stress ratios and notch severities, making it difficult to maintain a globally consistent parameter set using conventional models. Because the effects of temperature, stress ratio, and stress concentration on cyclic response and damage evolution are typically nonlinear and coupled, this study proposes a multi-level equivalent driving force framework for fatigue life prediction, in which condition-induced life differences are represented as comparable shifts on a unified engineering driving-force scale. The proposed framework links the nominal cyclic response, the notch-root equivalent response, and crystallographic orientation weighting. Nominal responses from stress-controlled and strain-controlled tests are first unified using the cyclic Ramberg–Osgood relation, and the notch-root elastoplastic response is estimated using Neuber localization. The obtained notch-root equivalent stress response is then combined with the crystallographic Schmid factor to define an equivalent shear-type driving parameter for life correlation. Stress-ratio and notch corrections are introduced sequentially on a baseline master-curve framework to construct an equivalent driving force and identify one parameter set for each alloy–temperature combination. The validation results demonstrate that, at a fixed temperature, the proposed approach provides a unified description across multiple stress ratios and notch severities and maintains stable predictive consistency for combined conditions not included in calibration, with controlled prediction scatter.

KEYWORDS: Nickel-based superalloys; fatigue life prediction; temperature; stress ratio; notch

1 Introduction

Nickel-based superalloys, benefiting from a multi-mechanism strengthening architecture dominated by the γ/γ' system, can retain excellent high-temperature strength over the service window of 650°C–1100°C [1–4]. They therefore constitute key structural materials for load-bearing hot-section components in aero-engines and gas turbines [5–8]. Under realistic service conditions, such components are typically subjected to coupled thermal–mechanical cyclic loading, where fatigue life is governed by pronounced interactions among temperature (T), stress ratio (R_σ), and geometric discontinuities characterized by the stress concentration factor (K_t) [9–11]. Temperature can shift the dominant damage mechanisms through its influence on cyclic deformation, time-dependent creep, microstructural evolution, and environmental effects [12–14]; stress ratio modifies the cyclic mean stress level and tension–compression asymmetry [15,16];

meanwhile, notches, fillets, and holes introduce multiaxial constraint and stress redistribution near the notch root, accelerating damage accumulation and complicating unified life assessment.

Across the effects of temperature, stress ratio, material defects, and stress concentration, prior studies have established substantial experimental evidence and mechanistic understanding [17–19]. Regarding temperature effects, He et al. reported a characteristic life maximum and turning point in the intermediate-temperature range for a directionally solidified nickel-based alloy tested from 550°C to 850°C, accompanied by systematic changes in fracture morphology and crack-growth characteristics [20]. For single-crystal systems, Wang et al. further showed that cyclic deformation mechanisms and crack-growth tendencies are strongly temperature-sensitive, highlighting the role of temperature in altering the dominant slip and damage paths [21]. Regarding stress-ratio effects, Gupta et al. demonstrated that increasing R_σ from -1 to higher positive values reduces the fatigue strength at 10^7 cycles and shifts the S–N curves downward overall for directionally solidified CM247LC at 650°C, reflecting the systematic influence of mean stress at elevated temperatures [22]. The coupling between mean stress and high-temperature oxidation may further deteriorate fatigue performance, potentially amplifying the apparent R_σ dependence through oxidation and stress-relaxation processes [23,24]. R_σ effects are commonly treated using the Walker model and the Smith–Watson–Topper (SWT) parameter and their variants, although the associated correction parameters are often material- and life-regime-dependent and require calibration for target conditions [25]. For notch effects, Shi et al. showed via *in situ* observations at 850°C that K_t markedly alters crack initiation location and life partitioning in U-shaped notched DZ125 specimens, closely related to the local principal stress and the position of peak equivalent stress in the notch region [26]. Notch-life conversion based on the critical distance theory has also been applied to high-temperature notched low-cycle fatigue (LCF), demonstrating the feasibility of linking local fields to life through a characteristic length [27,28], while Neuber’s rule remains widely used for engineering approximations of notch-root elastoplastic response and preprocessing in life assessment [29].

From a modeling perspective, fatigue life prediction for nickel-based superalloys can be broadly divided into macroscopic parameter-based approaches and mechanistic approaches. At the macroscopic level, classical stress–life and strain–life relations form the basis, including the Basquin law for high-cycle fatigue (HCF) [30], the Coffin relation for plastic strain-controlled fatigue [31], and the Manson total-strain framework bridging different life regimes [32]. Mean-stress effects are typically incorporated via SWT [33], and notch effects are often addressed through localization schemes such as Neuber’s rule [34]. In contrast, mechanistic approaches link life to crystal plasticity and micro-deformation mechanisms by formulating fatigue indicator parameters on favorably oriented slip planes, thereby improving interpretability of orientation anisotropy and life scatter for single-crystal and directionally solidified superalloys [35]. Further coupling of damage evolution with crystal viscoplasticity, including slip-system-level damage accumulation and applications to notched specimens, has also been explored, for example in the framework proposed by Levkovitch et al. [36]. Comparative studies, such as that by Arakere et al., have shown that classical macroscopic parameters including Brown–Miller, Fatemi–Socie, and SWT can exhibit markedly different predictive capabilities for single-crystal superalloys, with accuracy varying strongly with loading conditions [37–39].

Although existing critical-plane and energy-based approaches can improve the physical interpretation of fatigue damage, they often still require recalibration or grouped fitting when stress ratio and notch severity vary simultaneously. This makes it difficult to describe multi- R_σ and multi- K_t fatigue data using one consistent parameter set within a fixed alloy–temperature condition. For nickel-based single-crystal superalloys, crystallographic orientation affects the fatigue driving response, while the connection among nominal loading, notch-root equivalent response, and crystallographic-orientation-weighted equivalent

shear parameters remains insufficiently established in current engineering life-prediction models. To address this issue, this study proposes an equivalent driving force framework that maps stress-ratio effects, notch effects, and crystallographic orientation weighting into a unified engineering fatigue indicator parameter. A stepwise correction strategy is then developed to separately identify and subsequently combine the stress-ratio and notch corrections for life prediction under multi- R_σ and multi- K_t conditions at a fixed temperature. The nomenclature and symbols used throughout this manuscript are summarized in [Table A1](#).

2 Experimental Methods and Data Sources

2.1 Materials and Fatigue Tests

The material used in the strain-controlled LCF tests was the second-generation nickel-based single-crystal superalloy DD6. Its nominal chemical composition was Ni–6.4Cr–9.1Co–0.6Mo–6.4W–1.0Ti–6.5Ta–3.0Re–0.1Hf–5.6Al, in wt.%, with Ni as the balance. The material was subjected to a standard heat-treatment procedure consisting of solution treatment at 1320°C for 3 h followed by air cooling, aging at 1140°C for 4 h followed by air cooling, and final aging at 870°C for 16 h followed by air cooling. The specimen loading axes were nominally parallel to the [001] crystallographic direction, and the orientation deviations were within 8°.

Smooth cylindrical specimens with a gauge length of 15 mm and a gauge diameter of 5 mm were used. The LCF tests were conducted using an Instron hydraulic servo fatigue testing system under total-strain control. An extensometer attached to the gauge section was used to measure and control the axial strain. The specimens were heated using a resistance furnace, and the temperature was monitored using thermocouples. The temperature fluctuation during testing was maintained within $\pm 2^\circ\text{C}$. All tests were conducted in laboratory air.

A fully reversed triangular waveform was applied under total-strain control with a strain ratio of $R_\epsilon = -1$ and a fixed loading frequency of 0.3 Hz. The test temperatures were room temperature, 700°C, 800°C, 900°C, and 1000°C. At each temperature, total strain amplitudes, $\Delta\epsilon_t/2$, of 0.50%, 0.65%, 0.75%, and 0.90% were considered. Fatigue life, N_f , was defined as the number of cycles at which the specimen completely fractured and separated.

Two valid tests were obtained for each temperature–strain–amplitude condition. Each data point in [Fig. 1a](#) represents the mean fatigue life of the two valid specimens and is used only to illustrate the overall temperature-dependent fatigue-life trends. The scatter between the two replicate tests was characterized using the maximum-to-minimum fatigue-life ratio and is discussed in [Section 3.1](#).

Stress-controlled HCF tests were also conducted using smooth cylindrical DD6 single-crystal superalloy specimens. The material and heat-treatment condition were the same as those described above for the LCF tests. The loading axes were nominally parallel to the [001] crystallographic direction. The specimens had a nominal gauge diameter of approximately 4.5 mm, with the measured minimum diameters ranging from approximately 4.32 to 4.54 mm. The tests were performed at room temperature, 700°C, 800°C, and 900°C under fully reversed loading with a stress ratio of $R_\sigma = -1$. The measured loading frequencies were approximately 87–111 Hz, with most tests conducted within the range of 90–100 Hz. Depending on temperature, the investigated maximum stresses ranged from 285 to 705 MPa.

Fatigue life was defined as the number of cycles to fracture within the gauge section. Tests that did not fracture after 10^7 cycles were treated as runout observations. Specimens that fractured at the threaded ends or exhibited other test abnormalities were excluded from the valid fatigue dataset.

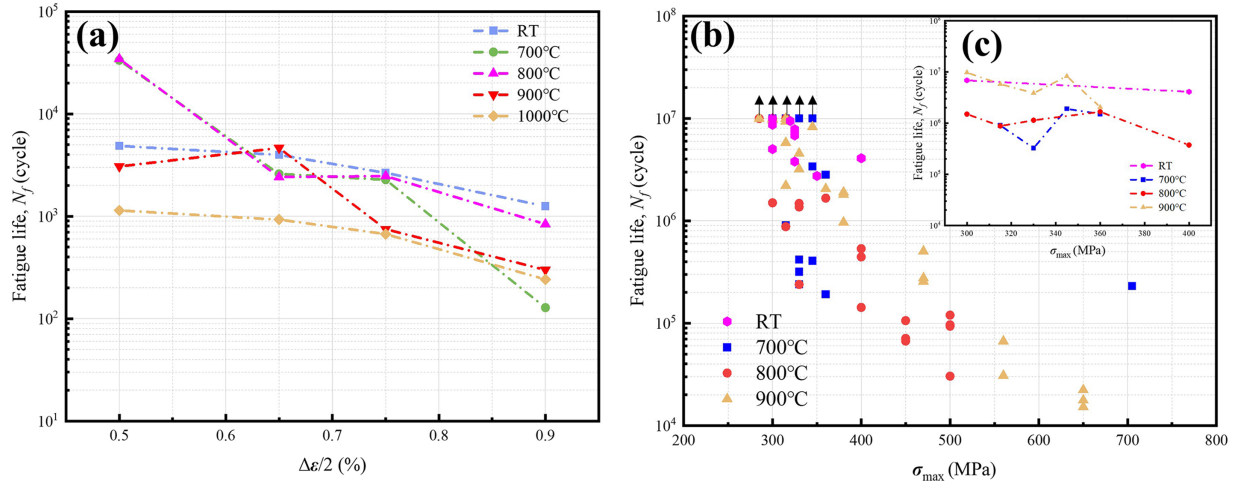


Figure 1: Fatigue life at different temperatures: (a) LCF, (b) HCF, (c) N_f at the same σ_{max} .

2.2 Literature Fatigue Datasets

The strain-controlled LCF and stress-controlled HCF datasets described in Section 2.1 were generated in the present study. These experimental datasets were used to examine the effects of temperature, strain or stress level, and loading mode on fatigue-life trends. They were not used for parameter calibration or quantitative evaluation of the proposed stress-ratio and notch-correction framework.

The datasets selected for model development and evaluation cover DD6, DZ125, IC10, and SRR99 single-crystal superalloys with nominal [001] orientations. Table 1 summarizes the conditions directly relevant to the present model, including alloy, temperature, stress ratio R_σ , stress concentration factor K_t , correction target, and data source. Further details of the specimens and test procedures can be found in the cited references.

Table 1: Validation datasets and their R_σ - K_t coverage.

Correction Target	Alloy	Temperature	R_σ	K_t	Source
R_σ	DZ125	700°C	-1, 0.1, 0.5	1	Ref. [40]
	DD6	1070°C	-1, -0.33, 0.1, 0.5		Ref. [41]
K_t	DZ125	760°C, 850°C, and 900°C	0.1	1, 3, 1.7	Ref. [40]
	IC10	700°C, 800°C, and 900°C		1, 3	Ref. [40]
	SRR99	700°C, 760°C, 850°C, and 900°C		1, 3	Ref. [42]
R_σ & K_t	DZ125	700°C, 760°C, and 850°C	-1, 0.1, 0.5	1, 3	Ref. [40]

Data from different alloy-temperature combinations were treated separately during parameter identification. Runout data, where available, were excluded from ordinary failure-data calibration.

3 Observed Effects and Motivation for the Proposed Framework

3.1 Effects of Temperature on Nickel-Based Superalloys

Fig. 1a shows the dependence of LCF life on the total strain amplitude at different temperatures. Each data point represents the mean fatigue life obtained from two valid specimens tested under the same

condition. Some scatter was observed between the replicate specimens. Except for the condition at 800°C and a total strain amplitude of 0.65%, the maximum-to-minimum fatigue-life ratios ranged from 1.06 to 1.69. A larger ratio of 3.40 was observed under this specific condition. Despite this scatter, the overall effects of temperature and strain amplitude remained consistent. At all temperatures, fatigue life generally decreased with increasing total strain amplitude. However, pronounced intersections occurred among the temperature-dependent curves within the investigated strain-amplitude range, indicating that the relative life ranking varied with the strain level. At lower strain amplitudes, the fatigue life at some intermediate temperatures exceeded that at room temperature, whereas the relative ordering among temperatures changed at higher strain amplitudes. These results demonstrate that, under strain-controlled loading, the temperature dependence of LCF life cannot be described by a fixed monotonic relationship.

Fig. 1b presents the relationship between HCF life and the maximum stress at different temperatures. The fatigue-life data at different temperatures overlap over certain stress intervals and exhibit mutual surpassing, without a consistent monotonic trend. It should be noted that Fig. 1b shows the full HCF datasets, whereas Fig. 1c extracts the fatigue lives at identical maximum-stress levels from the overlapping stress range for a more direct comparison of temperature effects. The comparison indicates that, even at the same $\sigma_{\max}$, the life differences among temperatures still show curve crossings and local fluctuations, and the life ranking associated with temperature changes with the stress level. This more clearly highlights the nonlinear and non-monotonic nature of the temperature effect.

In summary, the influence of temperature on both LCF and HCF life is strongly load-level dependent, as evidenced by curve intersections and changes in life ranking. Such behavior cannot be adequately captured by a single linear or monotonic relationship, underscoring the nonlinear and non-monotonic effects of temperature on fatigue life.

The nonlinear temperature dependence observed in fatigue life is also reflected in the tensile strength parameters. Fig. 2a,b shows the variations of the ultimate tensile strength σ_b and the 0.2% proof (yield) strength $\sigma_{0.2}$ with temperature for four nickel-based alloys, namely DD6, DD3, DZ125, and IC10 [40]. Overall, from room temperature to the intermediate-temperature range, both σ_b and $\sigma_{0.2}$ remain at relatively high levels for all alloys, exhibiting predominantly plateau-like trends with moderate fluctuations. In particular, σ_b of DD3 and DD6 shows a noticeable upturn around 650°C–850°C, forming a local peak, whereas σ_b of DZ125 and IC10 varies more smoothly with increasing temperature. Correspondingly, the fluctuations of $\sigma_{0.2}$ in the intermediate-temperature regime are generally smaller than those of σ_b , showing near-stable behavior or only slight variations.

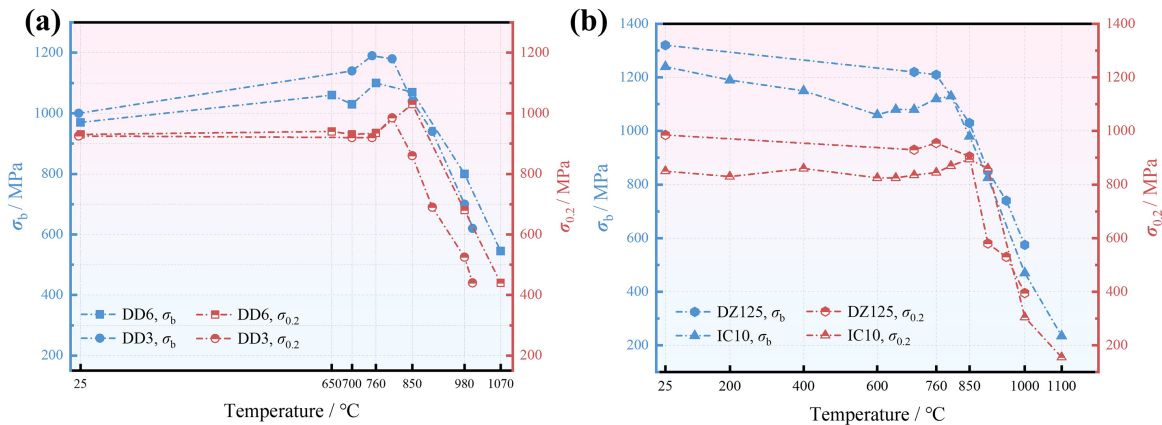


Figure 2: σ_b and $\sigma_{0.2}$ vs. temperature for nickel-based single-crystal superalloys: (a) DD6/DD3; (b) DZ125/IC10.

At higher temperatures, both σ_b and $\sigma_{0.2}$ decrease markedly for all four alloys, dropping to low levels within 980°C–1070°C or approaching 1100°C. Taken together, the strength–temperature response exhibits a clear segmented characteristic: a plateau with local undulations at intermediate temperatures, followed by a rapid degradation at high temperatures, indicating an overall nonlinear dependence on temperature.

3.2 Effect of Loading Frequency on the Fatigue Behavior of Nickel-Based Superalloys

Fig. 3a–d compares the stress–life distributions of the second-generation single-crystal superalloys PWA1484 and CMSX-4 under different stress ratios, loading frequencies, and stress levels, with the data taken from the literature [43,44]. Fig. 3a,c expresses life in terms of cycles to failure, N_f , whereas Fig. 3b,d uses time to failure, t_f . Specifically, Fig. 3a,b corresponds to PWA1484, and Fig. 3c,d corresponds to CMSX-4.

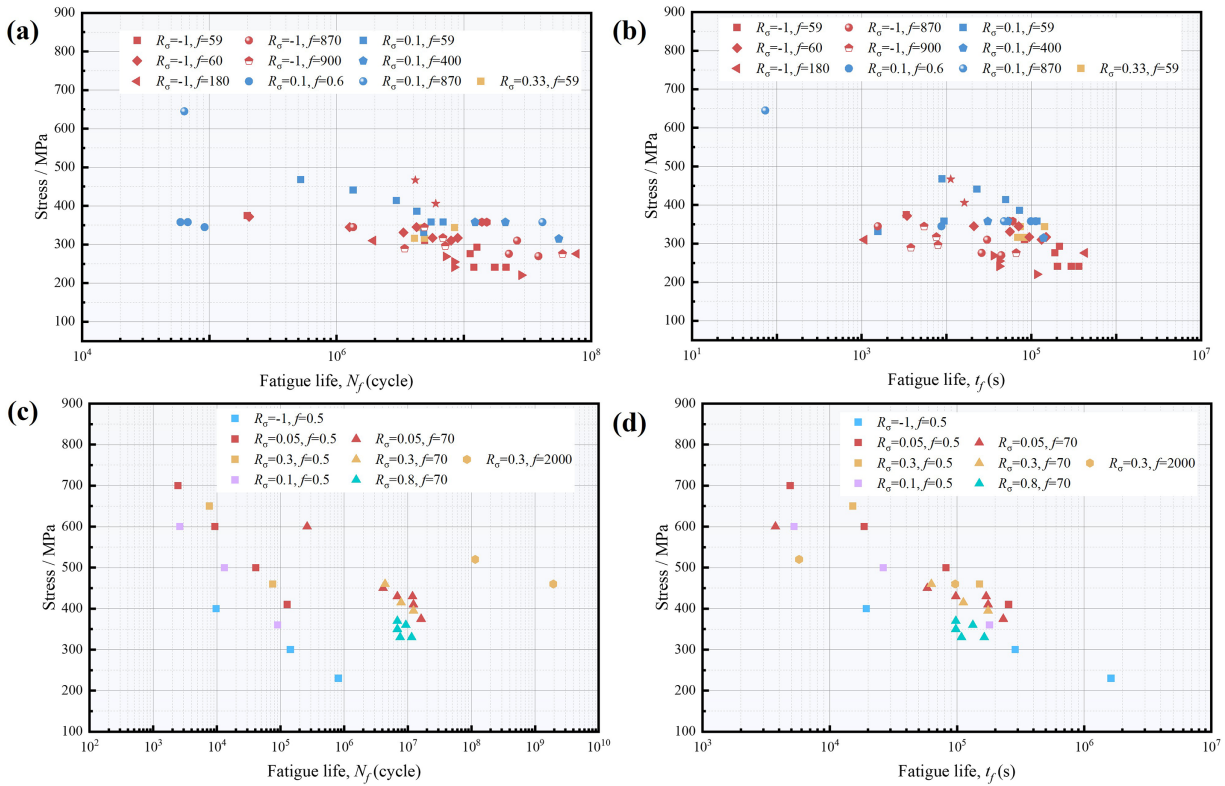


Figure 3: Stress–life data of PWA1484 and CMSX-4 at different R_σ and f : (a,c) N_f (cycles); (b,d) t_f (s).

When life is represented by N_f , the datasets at different frequencies exhibit a more pronounced separation. In particular, CMSX-4 shows an overall shift of data points toward higher N_f with increasing frequency, while PWA1484 also displays noticeable frequency-dependent differences. In contrast, after converting life to the time domain, the separation between different-frequency datasets is substantially reduced and the distributions become more overlapped. Overall, frequency grouping is more evident on the cycle-based scale, whereas it largely converges on the time-based scale, indicating that the apparent frequency effect depends on whether life is characterized by N_f or by t_f .

It should be emphasized that the conversion from cycles to failure to time to failure using $t_f = N_f/f$ is a mathematical transformation and does not, by itself, represent a physical normalization of the loading-frequency effect. The closer grouping observed in the time domain is therefore interpreted only as a qualitative indication that time-dependent deformation or environmental damage may contribute to fatigue

failure. The frequency-related datasets in Fig. 3 are not used for parameter calibration or quantitative model evaluation. The present framework focuses on stress-ratio and notch effects and does not include an explicit frequency or dwell-time correction; therefore, it should not be directly applied to predictions across different loading frequencies or under dwell-fatigue conditions without further model development.

3.3 Effect of R_σ on the Fatigue Behavior of Nickel-Based Superalloys

Fig. 4 illustrates the stress-ratio effect under iso-life conditions reported in the literature [40]. Fig. 4a,b presents the iso-life stress–stress-ratio relationships obtained from different literature datasets and temperature ranges. In both subfigures, the curves correspond to fixed fatigue-life levels of $N_f = 10^6$ or $N_f = 10^7$ cycles at different temperatures. For each curve, the applied stress required to reach the same fatigue life is plotted as a function of stress ratio, enabling a direct comparison of how the required stress changes as the stress ratio increases from negative to positive values.

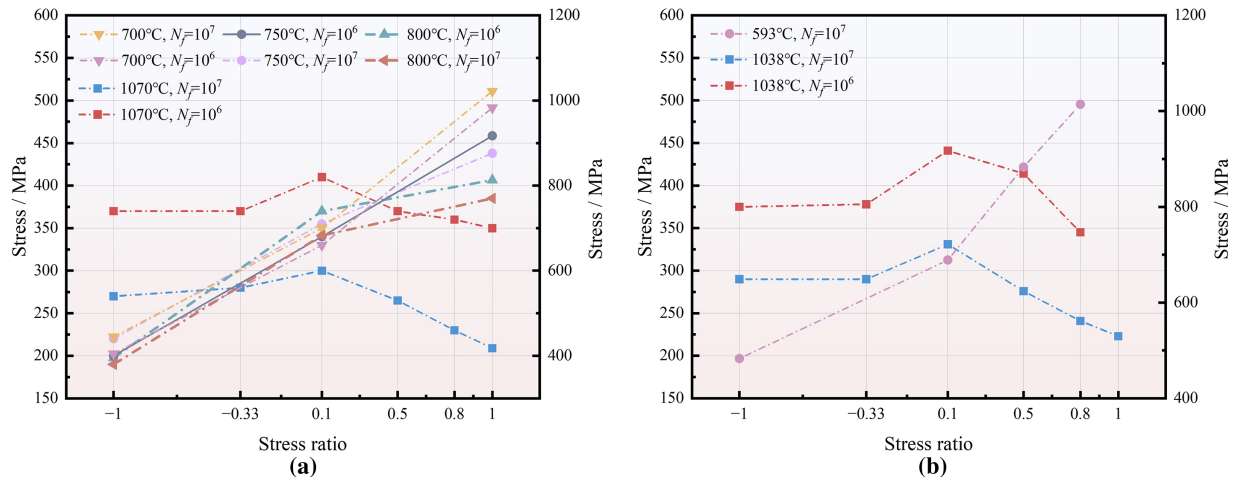


Figure 4: Literature iso-life stress as a function of stress ratio R_σ at different temperatures for $N_f = 10^6$ and 10^7 cycles: (a) dataset covering 700°C–1070°C; (b) dataset covering 593°C and 1038°C. Data replotted from Ref. [40].

Overall, the iso-life stress increases with increasing stress ratio under most temperature conditions, whereas the curve shapes are not uniformly linear. In the low-to-moderate stress-ratio range, some curves exhibit a more pronounced increase; as the stress ratio further increases, the rise becomes less significant and tends to level off. Meanwhile, for certain high-temperature curves, a drop or an inflection point appears at high stress ratios, indicating a local non-monotonic behavior. In addition, both the slope and the stress-ratio range where the inflection occurs vary with temperature, suggesting that the iso-life stress response to stress ratio is temperature dependent.

3.4 Effect of Loading K_t on the Fatigue Behavior of Nickel-Based Superalloys

Fig. 5 shows the variation of fatigue life N_f with stress ratio R_σ under a complex stress state for different stress concentration factors K_t [24]. Distinct N_f - R_σ trends are observed for different K_t values, and curve intersections occur over certain R_σ ranges, indicating that the stress-ratio effect is modulated by the degree of stress concentration.

For $K_t = 1$, the fatigue life generally decreases with increasing R_σ and remains at a low level in the intermediate-to-high R_σ range. For $K_t = 2$, N_f exhibits a non-monotonic trend, increasing first and then decreasing, with relatively higher life in the moderate-to-high R_σ range. For $K_t = 3$, the fatigue life overall

increases with increasing R_σ and continues to rise toward the high- R_σ end. Overall, the life differences among K_t values are pronounced at the same R_σ , and the R_σ -dependence of N_f is not consistent across K_t , featuring curve crossings and non-monotonic behavior, which further reflects the nonlinear nature of the stress-ratio effect.

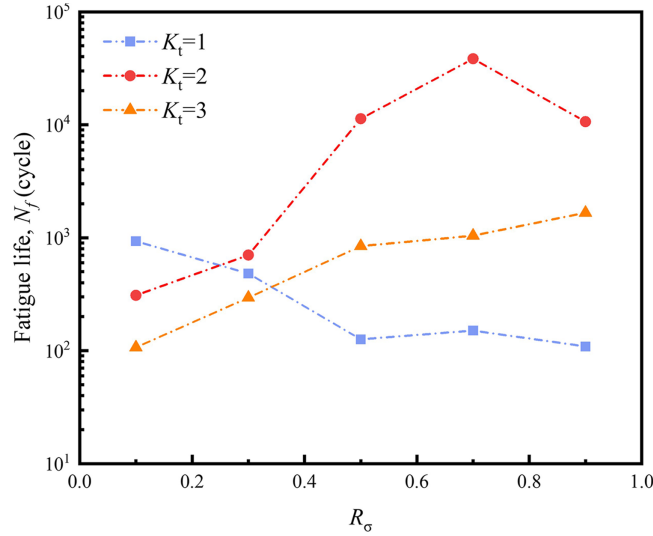


Figure 5: Fatigue life vs. stress ratio for $K_t = 1, 2$, and 3 . Data replotted from Ref. [24].

4 Life Prediction Procedure

Section 3 shows that the effects of temperature, stress ratio, and notch severity on fatigue life are strongly nonlinear, featuring data crossings and reversals in life ranking. This indicates that temperature not only changes the nominal mechanical level but also alters the life sensitivity to R_σ and K_t . As a result, a single set of global constant parameters over the full temperature range would typically fit well at some temperatures but produce systematic bias at others.

The proposed framework links three engineering characterization levels: the nominal loading level, the notch-root equivalent response level, and the crystallographic-orientation-weighted equivalent shear level. The nominal loading level describes the overall cyclic loading amplitude, stress ratio, and mean stress state. The notch-root equivalent response level estimates the local elastoplastic response associated with stress concentration by using Neuber localization. The crystallographic-orientation-weighted level introduces the Schmid factor to convert the notch-root equivalent stress response into an equivalent shear-type driving parameter. Through this mapping, the effects of loading condition, notch severity, and crystallographic orientation are incorporated into a unified engineering equivalent driving force.

Based on the above considerations, baseline life model parameters are calibrated independently at each temperature. A stepwise variable separation scheme is then applied at the same temperature to correct the mean-stress effect and the notch effect separately, and to decouple and identify the influences of R_σ and K_t , enabling unified life prediction for multiple R_σ and multiple K_t conditions at a given temperature.

4.1 Unified Fatigue Life Prediction Framework

For stress-controlled fatigue tests, the nominal minimum stress can be obtained from the nominal maximum stress and the stress ratio R_σ as

$$\sigma_{\min} = R_\sigma \sigma_{\max} \quad (1)$$

For strain-controlled fatigue tests, the nominal minimum strain can be obtained from the nominal maximum strain and the strain ratio R_ϵ as

$$\epsilon_{\min} = R_\epsilon \epsilon_{\max} \quad (2)$$

The nominal stress extrema are back-calculated using the cyclic Ramberg–Osgood relation [45],

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'} \right)^{1/n'} \quad (3)$$

where E is the elastic modulus at the test temperature, K' is the cyclic strength coefficient, and n' is the cyclic strain-hardening exponent. The values of $\epsilon = \epsilon_{\max}$ and $\epsilon = \epsilon_{\min}$ are determined by numerical fitting, and $\sigma_{\max}$ and $\sigma_{\min}$ are then solved accordingly.

$$\Delta\sigma_{\text{nom}} = \sigma_{\max} - \sigma_{\min} \quad (4)$$

The two loading modes are finally unified in terms of the nominal stress range $\Delta\sigma$ and the nominal mean stress, defined as

$$\sigma_{\text{m,nom}} = \frac{\sigma_{\max} + \sigma_{\min}}{2} \quad (5)$$

For specimens with stress concentration, the local stress–strain state at the notch root governs fatigue crack initiation. In this study, the Neuber energy equivalence principle is adopted to relate the nominal elastic stress field to the elastoplastic response at the notch root.

The maximum elastic stress, $\sigma_{\max}^{\text{notch}}$, at the notch is represented by the elastic stress concentration factor K_t , namely,

$$\sigma_{\max}^{\text{notch}} = K_t \sigma_{\max} \quad (6)$$

If $\sigma_{\max}^{\text{notch}} < \sigma_{\text{yield}}$, the response remains elastic and the elastic stress range is used, with $\sigma_{\max}^{\text{notch}} = K_t \sigma_{\max}$ and $\sigma_{\min}^{\text{notch}} = K_t \sigma_{\min}$. If $\sigma_{\max}^{\text{notch}} \geq \sigma_{\text{yield}}$, the response enters the plastic regime. According to Neuber's rule, the nominal elastic strain energy density equals the local elastoplastic strain energy density at the notch root. In incremental form,

$$\Delta\sigma^{\text{notch}} \Delta\epsilon^{\text{notch}} = \frac{(K_t \Delta\sigma_{\text{nom}})^2}{E} \quad (7)$$

where $\Delta\sigma^{\text{notch}}$ and $\Delta\epsilon^{\text{notch}}$ are the local stress range and local strain range at the notch root, $K_t \Delta\sigma_{\text{nom}}$ is the pseudo-elastic notch-root stress range, not the actual elastoplastic notch-root stress range. In this work, the scalar Neuber relation is used as an engineering localization approximation to estimate the first-order notch-root elastoplastic response from the nominal cyclic stress range, rather than to fully resolve the anisotropic multiaxial stress field around the notch. The local stress–strain relation also follows the cyclic Ramberg–Osgood constitutive law,

$$\Delta\epsilon^{\text{notch}} = \frac{\Delta\sigma^{\text{notch}}}{E} + 2 \left(\frac{\Delta\sigma^{\text{notch}}}{2K'} \right)^{1/n'} \quad (8)$$

Substituting Eq. (8) into Eq. (7), and defining $x = \Delta\sigma^{\text{notch}}$, yields the following scalar nonlinear equation. In this equation, $K_t\Delta\sigma^{\text{nom}}$ is a prescribed pseudo-elastic reference stress range and is therefore treated as a constant during the Newton–Raphson iteration:

$$f(x) = \frac{(x)^2}{E} + 2x \left(\frac{x}{2K'} \right)^{1/n'} - \frac{(K_t\Delta\sigma^{\text{nom}})^2}{E} = 0 \quad (9)$$

here, $x = \Delta\sigma_{\text{notch}}$ is the only unknown variable, while $(K_t\Delta\sigma_{\text{nom}})^2/E$ is constant with respect to x . Eq. (9) is solved using the Newton–Raphson iteration,

$$\Delta\sigma_{(i+1)}^{\text{notch}} = \Delta\sigma_{(i)}^{\text{notch}} - \frac{f(\Delta\sigma_{(i)}^{\text{notch}})}{f'(\Delta\sigma_{(i)}^{\text{notch}})} \quad (10)$$

where the derivative $f'(x)$ is given by

$$f'(x) = \frac{2x}{E} + 2 \left(\frac{x}{2K'} \right)^{1/n'} + \frac{x}{n'K'} \left(\frac{x}{2K'} \right)^{(1-n')/n'} \quad (11)$$

The iteration is initialized with $\Delta\sigma_{(0)}^{\text{notch}} = K_t\Delta\sigma_{\text{nom}}$, and the convergence criterion is set to $|\Delta\sigma_{(i+1)}^{\text{notch}} - \Delta\sigma_{(i)}^{\text{notch}}|/\Delta\sigma_{(i+1)}^{\text{notch}} < 10^{-6}$.

The material parameters used in the Ramberg–Osgood relation and Neuber localization, including E , K' , n' , and σ_y , were obtained from the corresponding literature sources for each alloy and temperature [40–42,46]. After obtaining the local stress range $\Delta\sigma^{\text{notch}}$, the local maximum and minimum stresses, $\sigma_{\text{max}}^{\text{notch}}$ and $\sigma_{\text{min}}^{\text{notch}}$, can be determined.

After obtaining the local stress range $\Delta\sigma_{\text{notch}}$, the local stress extrema are determined by assuming that the stress ratio is preserved during scalar Neuber localization. Defining the nominal stress ratio after the stress or strain conversion as

$$R_{\text{loc}} = R_{\text{nom}} = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \quad (12)$$

The local maximum and minimum stresses are calculated as

$$\sigma_{\text{max}}^{\text{notch}} = \frac{\Delta\sigma_{\text{notch}}}{1 - R_{\text{nom}}} \cdot \sigma_{\text{min}}^{\text{notch}} = \frac{R_{\text{nom}}\Delta\sigma_{\text{notch}}}{1 - R_{\text{nom}}} \quad (13)$$

Equivalently, both nominal stress extrema are scaled by the same localization factor,

$$\lambda = \frac{\Delta\sigma_{\text{notch}}}{\Delta\sigma_{\text{nom}}}, \sigma_{\text{max}}^{\text{notch}} = \lambda\sigma_{\text{max}}, \sigma_{\text{min}}^{\text{notch}} = \lambda\sigma_{\text{min}}. \quad (14)$$

The cyclic Ramberg–Osgood relation in Eq. (8) represents a Masing-type range approximation. Separate loading and unloading branches, cyclic mean-stress relaxation, and the evolution of the local stress ratio are not explicitly considered. Therefore, $\sigma_{\text{max}}^{\text{notch}}$ and $\sigma_{\text{min}}^{\text{notch}}$ should be interpreted as engineering-equivalent stress extrema for life correlation rather than fully resolved notch-root stresses. For conditions involving pronounced local plasticity, mean-stress relaxation, dwell loading, or multiaxial constraint, elastoplastic finite-element analysis is required.

The stress quantity used for crystallographic weighting is expressed as an equivalent axial notch-root stress vector in Voigt notation,

$$\boldsymbol{\sigma} = [\sigma_{xx}, 0, 0, 0, 0, 0]^T \quad (15)$$

This vector should be interpreted as an engineering equivalent stress representation, rather than a complete multiaxial notch-root stress tensor. For the axial fatigue loading considered in this work, the scalar Neuber correction is used to estimate the dominant axial notch-root stress component. The transverse normal stresses and shear stresses are not independently solved in this simplified framework; therefore, only the axial component is retained in the slip-system projection, while the remaining components are taken as zero. The notch-root multiaxial constraint is considered phenomenologically through the notch correction term.

With the predefined Schmid factor matrix $\mathbf{M}^{(k)}$ for all slip systems, the local stress is projected onto each slip system to obtain the equivalent shear parameter vector [47],

$$\boldsymbol{\tau}^{(k)} = \mathbf{M}^{(k)} \boldsymbol{\sigma}_{axial}^{eq} \quad (16)$$

The resolved shear stresses on each slip system at the stress extrema are obtained as

$$\tau_{eq,max}^{(k)} = \mathbf{M}^{(k)} \boldsymbol{\sigma}_{axial,max}^{eq}, \tau_{eq,min}^{(k)} = \mathbf{M}^{(k)} \boldsymbol{\sigma}_{axial,min}^{eq} \quad (17)$$

here, $\tau_{eq,max}^{(k)}$ denotes the maximum equivalent shear parameter among the 30 slip systems at the maximum loading level, and $\tau_{eq,min}^{(k)}$ denotes the minimum equivalent shear parameter among the 30 slip systems at the minimum loading level.

Based on the Schmid-factor-weighted equivalent shear parameters obtained from Eq. (17), the equivalent shear-type driving parameter is defined as

$$\Delta\tau_{max} = \tau_{max} - \tau_{min} \quad (18)$$

here, τ_{max} and τ_{min} denote the maximum values of the Schmid-factor-weighted equivalent shear parameter among the considered slip systems at the maximum and minimum loading levels, respectively. Accordingly, $\Delta\tau_{max}$ is used as an engineering equivalent shear-type driving parameter for the subsequent life model.

4.2 Correction of the Equivalent Shear-Type Driving Parameter

In this study, the equivalent shear-type driving parameter $\Delta\tau_{max}$ is adopted as the unified driving parameter to establish the $\Delta\tau_{max}-N_f$ relationship. This parameter is derived from the notch-root equivalent stress response obtained by Neuber localization and the crystallographic Schmid-factor weighting. Therefore, it provides an engineering-scale measure for comparing fatigue data under different loading conditions, stress ratios, and notch severities. It should be noted that $\Delta\tau_{max}$ is used here as an equivalent parameter for life correlation, rather than a fully resolved slip-system shear stress calculated from a complete multiaxial notch-root stress tensor.

To extract the overall trend from scattered experimental data and to describe the systematic differences among different conditions, a Basquin-type power-law regression is applied to each data group,

$$\Delta\tau_{max} = A (N_f)^B \quad (19)$$

where A and B are obtained by least-squares fitting in the logarithmic domain. The dashed lines in the figures represent the corresponding main trend.

The literature data of DZ125 [40] are selected here to demonstrate and validate the proposed correction procedure, because this dataset covers two key variations within the same material system, namely the fatigue lives of smooth specimens under different stress ratios and those of specimens with different notch factors. Consequently, both the stress-ratio effect and the notch effect can be examined within a unified $\Delta\tau_{\max} - N_f$ framework using a reproducible reference case. As shown in Fig. 6a, the data for smooth specimens at $R_\sigma = -1$ and $R_\sigma = 0.5$ form two separated bands within a similar life range, indicating a stable shift in the $\Delta\tau_{\max}$ level. As shown in Fig. 6c, a clear stratification is also observed for different notch factors, where the $K_t = 3$ data correspond to a higher $\Delta\tau_{\max}$ level than the $K_t = 1$ data at comparable lives, reflecting a systematic shift induced by notch geometry. Furthermore, Fig. 6e combines three representative conditions in the same coordinates, namely $R_\sigma = -1$ with $K_t = 1$, $R_\sigma = 0.5$ with $K_t = 1$, and $R_\sigma = -1$ with $K_t = 3$ showing multiple layers and an overall misalignment caused by the combined influences of R_σ and K_t . These observations indicate that, at a fixed $\Delta\tau_{\max}$ level, N_f varies systematically with R_σ or K_t . Equivalently, for a given life range, $\Delta\tau_{\max}$ exhibits a stable offset among conditions, so that a single $\Delta\tau_{\max} - N_f$ relation is not sufficient for direct comparison and aggregation across multiple conditions.

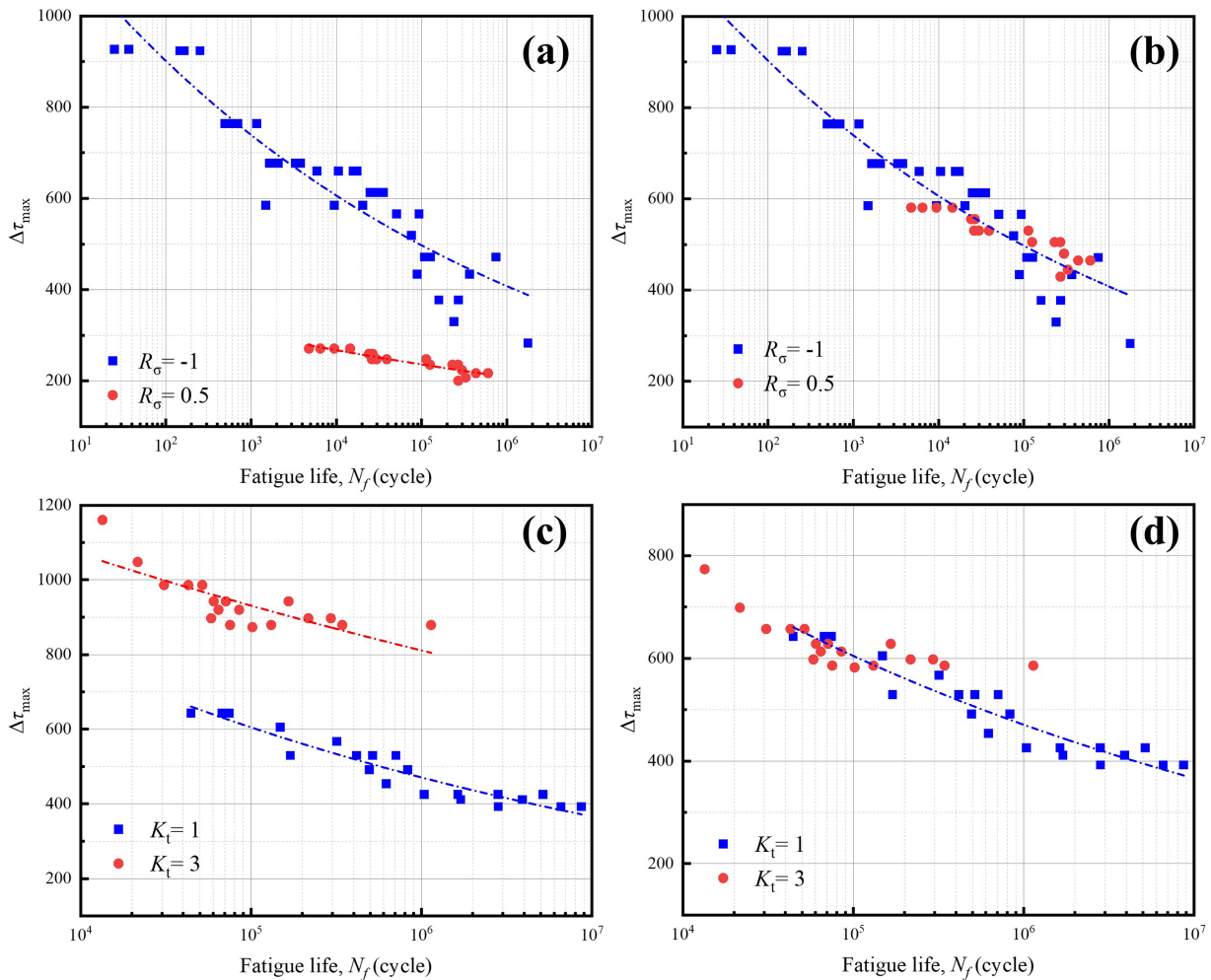


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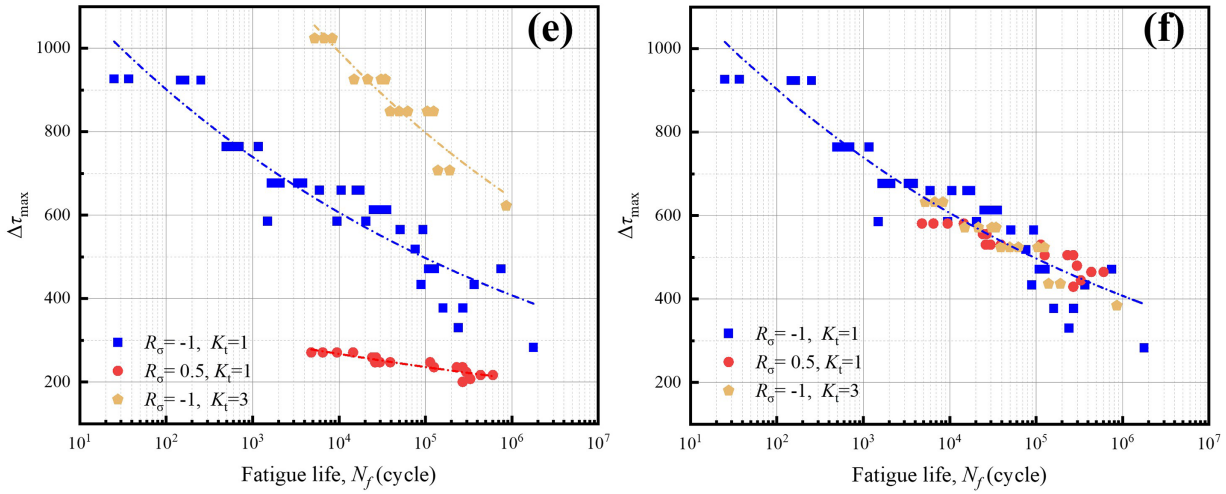


Figure 6: $\Delta\tau_{\max} - N_f$ results for DZ125 and the effect of corrections: (a) original data for different stress ratios. (b) data after the stress-ratio correction. (c) original data for different notch factors. (d) data after the notch correction. (e) original data under combined R_σ and K_t conditions. (f) data after the combined correction.

Based on the above features, correction factors are introduced to perform an equivalent rescaling of the driving parameter without changing the shape of the reference master curve, so that datasets from different conditions become comparable in a unified reference frame. First, a reference master curve is obtained by fitting the baseline condition,

$$\Delta\tau_{\max} = C_0 (N_f)^b \quad (20)$$

where C_0 and b are determined by least-squares fitting of the baseline fatigue data in the logarithmic domain and remain fixed in the subsequent correction and validation. The term correction used in this study refers to defining an equivalent driving parameter within the reference-master-curve framework to represent and compensate systematic shifts associated with changes in loading or geometry, rather than arbitrarily modifying the original experimental data.

The correction is implemented in a stepwise manner. The stress-ratio effect is considered first. Fig. 6a shows the original $\Delta\tau_{\max} - N_f$ relation for smooth specimens at two stress ratios, where the $R_\sigma = -1$ and $R_\sigma = 0.5$ data form separated bands. To equivalently represent the systematic shift caused by the stress ratio while keeping the baseline unchanged, a mean-stress correction factor is introduced as

$$f_R(R_\sigma) = \left(\frac{1 - R_\sigma}{1 - R_{\sigma 0}} \right)^y \quad (21)$$

where, $f_R(R_0) = 1$. The stress-ratio-corrected equivalent maximum shear stress amplitude is then defined as

$$\Delta\tau_R = \frac{\Delta\tau_{\max}}{f_R(R_\sigma)} \quad (22)$$

The corrected results are shown in Fig. 6b. Compared with Fig. 6a, the stratification associated with different stress ratios is reduced, and the data exhibit an increased overlap around the reference trend band.

The notch effect is then considered. Fig. 6c shows the original $\Delta\tau_{\max} - N_f$ relation for different notch factors, where the $K_t = 1$ and $K_t = 3$ datasets are clearly separated and the notched data shift

toward higher $\Delta\tau_{\max}$. To equivalently represent the systematic shift induced by notch geometry within the baseline-preserving framework, a notch correction factor is introduced as

$$f_{K_t}(K_t) = K_t^\delta \quad (23)$$

where $f_{K_t}(1) = 1$. The notch-corrected equivalent maximum shear stress amplitude is defined as

$$\Delta\tau_{K_t} = \frac{\Delta\tau_{\max}}{f_{K_t}(K_t)} = \frac{\Delta\tau_{\max}}{K_t^\delta} \quad (24)$$

The correction factors $f_R(R_\sigma)$ and $f_{K_t}(K_t)$ are introduced as semi-empirical factors with physical meanings. The term $(1 - R_\sigma)/(1 - R_{\sigma 0})$ represents the relative change in stress range with respect to the baseline stress ratio $R_{\sigma 0}$, while the exponent γ characterizes the sensitivity of the equivalent driving force to stress-ratio and mean-stress effects. Similarly, K_t is a direct measure of notch severity and local stress amplification, and the power-law form K_t^δ is used to describe the sensitivity of the equivalent driving force to notch effects. In the present framework, K_t is used to characterize the overall stress amplification caused by notch severity, while the detailed effects of notch shape, notch-root radius, and stress gradient are not independently separated. Therefore, γ and δ should be regarded as correction exponents identified from fatigue data, rather than universal material constants. Accordingly, the notch correction factor $f_{K_t}(K_t)$ should be understood as a semi-empirical equivalent correction for the systematic life shift associated with notch severity within the calibrated datasets. It is not intended to replace a detailed local multiaxial notch-root stress analysis.

The corrected results are shown in Fig. 6d. Compared with Fig. 6c, the separation between different K_t datasets is reduced, and the data become closer to sharing a common trend band.

The three representative conditions in Fig. 6e are used to illustrate the calibration logic of the combined correction. The condition with $R_\sigma = -1$ and $K_t = 1$ is taken as the baseline master curve. The comparison between $R_\sigma = -1, K_t = 1$ and $R_\sigma = 0.5, K_t = 1$ is used to isolate the stress-ratio effect, whereas the comparison between $R_\sigma = -1, K_t = 1$ and $R_\sigma = -1, K_t = 3$ is used to isolate the notch effect. Based on these single-variable comparisons, the correction exponents γ and δ in the combined model are jointly identified using all observations from the three designated calibration conditions by minimizing the objective function in Eq. (28). Once identified, the resulting combined parameter set is kept fixed and applied without further refitting to all withheld loading conditions within the same alloy–temperature combination.

Based on this logic, the two correction factors are combined for a unified correction. Fig. 6e shows the multi-layer distribution and overall misalignment among the three representative conditions in the original coordinates. To simultaneously represent both systematic shifts within a unified framework, the combined corrected equivalent driving parameter is defined as

$$\Delta\tau_{R,K_t} = \frac{\Delta\tau_{\max}}{f_R(R_\sigma) f_{K_t}(K_t)} = \frac{\Delta\tau_{\max}}{\left(\frac{1 - R_\sigma}{1 - R_{\sigma 0}}\right)^\gamma K_t^\delta} \quad (25)$$

The parameters C_0 , b , γ , and δ are identified independently for each alloy–temperature combination, and the calibrated parameter values are summarized in Table A2. Once calibrated, the same parameter set is retained for all stress-ratio and notch conditions within that combination. Temperature is not introduced as an explicit model variable; therefore, application to a new temperature requires separate parameter calibration.

With $R_{\sigma 0} = -1$, this expression becomes

$$\Delta \tau_{R_{\sigma}, K_t} = \frac{\Delta \tau_{\max}}{\left(\frac{1 - R_{\sigma}}{2}\right)^{\gamma} K_t^{\delta}} \quad (26)$$

The parameters γ and δ are identified by minimizing the mean squared error between the predicted life and the experimental life in the logarithmic domain. Specifically, by substituting $\Delta \tau_{R_{\sigma}, K_t}$ into the baseline master curve $\Delta \tau_{\max} = C_0 (N_f)^b$, the predicted life is obtained as

$$N_{\text{pred}} = \left(\frac{\Delta \tau_{\max}}{C_0 f_R(R_{\delta}) f_{K_t}(K_t)} \right)^{\frac{1}{b}} \quad (27)$$

and the objective function is defined as

$$J(\gamma, \delta) = \frac{1}{n} \sum_{i=1}^n [\log_{10} N_{\text{pred},i} - \log_{10} N_{f,i}]^2 \quad (28)$$

The optima γ and δ are determined by searching for the combination that minimizes J . Fig. 6f presents the results after applying both f_R and f_{K_t} . Compared with Fig. 6e, the overall misalignment among conditions is reduced and the datasets exhibit a more consistent distribution around the reference trend band, providing a unified driving-parameter input for subsequent life prediction across multiple conditions.

Fig. 6a,c,e demonstrates that, before correction, the datasets corresponding to different stress ratios and notch factors exhibit systematic stratification and condition-dependent offsets rather than random scatter around a single trend. Consequently, a single uncorrected $\Delta \tau_{\max} - N_f$ relation with one common set of parameters cannot simultaneously provide a satisfactory fit for all R_{σ} and K_t conditions. A direct global fit would either introduce systematic prediction bias for individual conditions or require separate parameter recalibration for each data group. The proposed corrections therefore rescale the driving parameter to compensate for the systematic shifts induced by stress ratio and notch severity, allowing the datasets to be represented within a common reference framework.

Established approaches, including Walker-type mean-stress corrections, SWT-based fatigue parameters, and Neuber-based local stress-strain estimation, provide alternative treatments of stress-ratio and notch effects. However, a consistent quantitative comparison requires additional information, such as stabilized cyclic stress-strain properties, local strain histories, detailed notch-root geometry, and stress-gradient data. These quantities were not consistently reported for the literature datasets used in the present study. Reconstructing them through additional assumptions would introduce uncontrolled uncertainty and could lead to an inequitable comparison. Therefore, the present study focuses on an internal comparison between the uncorrected baseline relation and the proposed correction framework. The results should not be interpreted as demonstrating the general superiority of the proposed method over established classical approaches.

5 Model Validation and Discussion

This section validates the proposed fatigue-life prediction framework under multiple operating variables. The focus is placed on evaluating the effectiveness of the stress-ratio correction and the notch-factor correction, and on assessing the consistency of the unified life prediction when the two corrections are combined. Ideally, the combined model should be calibrated and validated using a complete experimental matrix covering multiple stress ratios and multiple notch factors. However, due to limitations in test duration,

cost, and specimen availability, the current dataset exhibits clear coverage gaps in the R_σ - K_t space, which precludes a large-scale, full-range validation.

Given these constraints, a stepwise validation strategy is adopted to maximize the use of available data while keeping the validation process interpretable and reproducible. First, single-variable validations are conducted. The stress-ratio correction is examined by varying R_σ while keeping K_t fixed, and the notch-factor correction is examined by varying K_t while keeping R_σ fixed. Within each alloy-temperature combination, C_0 and b are first calibrated from the baseline master curve and subsequently kept fixed. For the final combined model, the correction exponents γ and δ are jointly identified using the designated calibration conditions. The resulting parameter set is then kept fixed and applied without further updating to the remaining R_σ - K_t conditions. The combined correction is then assessed by forward prediction for the limited multi- R_σ -multi- K_t conditions covered by the available data, in order to evaluate the condition-level forward-prediction capability under coupled effects. For clarity and traceability, the validation datasets and their corresponding R_σ - K_t coverage are summarized in Table 1, including the datasets used for stress-ratio correction validation, notch-factor correction validation, and combined correction validation [40–42,46]. All calibration and validation datasets used in this study correspond to [001]-oriented nickel-based single-crystal superalloys.

5.1 Validation of the Stress-Ratio Correction

Based on the stress-ratio correction model and the identified parameters, Fig. 7 compares the predicted life N_{pred} with the experimental life N_{exp} for DZ125 and DD6 under different stress ratios R_σ . As shown in Fig. 7a,c, the data points from different R_σ levels distribute around the identity line $N_{\text{pred}} = N_{\text{exp}}$, and no evident group-wise systematic shift is observed, indicating that the stress-ratio correction mitigates the systematic deviation induced by R_σ . Most points fall within the $\pm 3\times$ scatter band, and the overall scatter is largely bounded by the $\pm 5\times$ band.

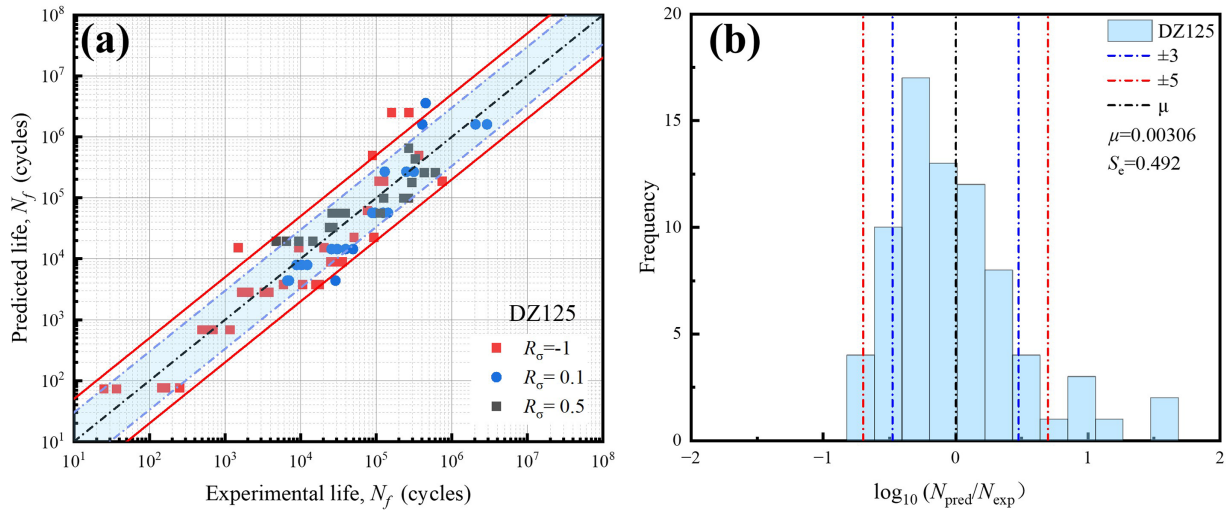


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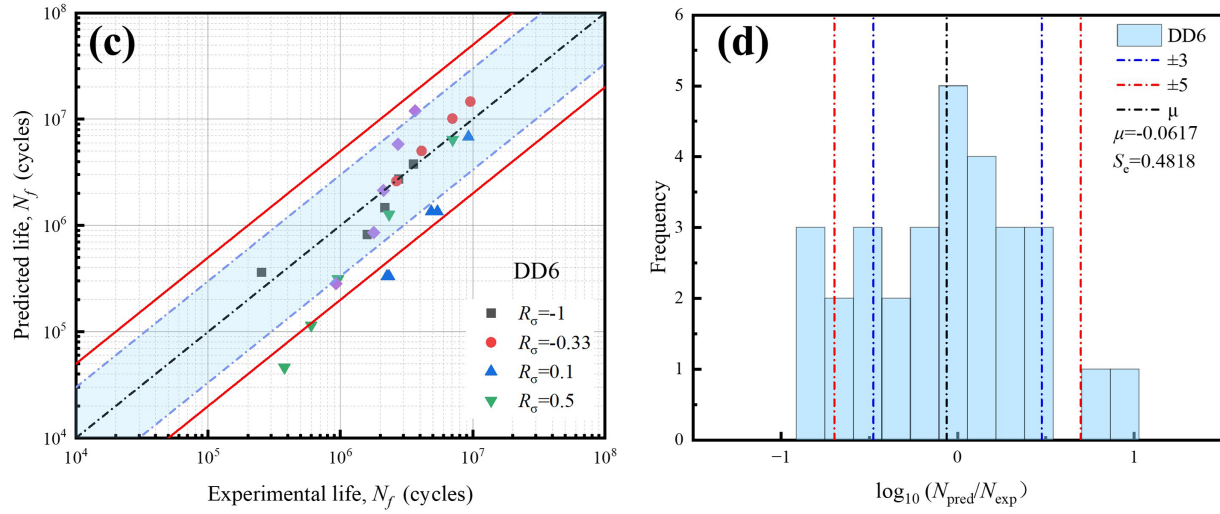


Figure 7: Stress-ratio correction validation. (a) DZ125 predicted and experimental lives; (b) DZ125 error histogram; (c) DD6 predicted and experimental lives; (d) DD6 error histogram.

To quantify the error distribution, the logarithmic error is defined as

$$e = \log_{10} \left(\frac{N_{\text{pred}}}{N_{\text{exp}}} \right) \quad (29)$$

here, μ denotes the mean value of e and reflects the overall prediction bias, whereas S_e denotes the standard deviation of e and characterizes the prediction scatter. The logarithmic limits corresponding to the factor-of-3 and factor-of-5 scatter bands are ± 0.477 and ± 0.699 , respectively. The error histograms are shown in Fig. 7b,d. For DZ125, 129 predictions were evaluated, with $\mu = 0.00306$, $S_e = 0.492$, and $RMSE_{\log} = 0.4901$. The 95% confidence interval of the mean logarithmic error ranged from -0.083 to 0.089 , while 74.4% and 89.1% of the predictions fell within factors of 3 and 5, respectively, indicating no evident systematic bias. For DD6, 24 predictions were evaluated, with $\mu = -0.0617$, $S_e = 0.4818$, and $RMSE_{\log} = 0.4757$. The corresponding 95% confidence interval ranged from -0.265 to 0.142 , while 62.5% and 83.3% of the predictions fell within factors of 3 and 5, respectively, indicating a slight conservative tendency.

5.2 Validation of the Notch-Factor Correction

Based on the notch-factor correction model and the identified parameters, Fig. 8 presents the validation results for DZ125, IC10, and SRR99 under different temperatures and notch factors K_t . As shown in Fig. 8a,c,e, the data points generally cluster around the identity line, and no distinct group-wise shift is observed between different K_t levels, indicating that the notch-factor correction maps datasets with different notch severities onto a comparable scale. Most points lie within the $\pm 3\times$ band and the overall scatter remain largely within the $\pm 5\times$ band.

The corresponding error histograms are shown in Fig. 8b,d,f. For DZ125, 146 predictions were evaluated, with $\mu = -0.0223$, $S_e = 0.4666$, and $RMSE_{\log} = 0.4655$. The 95% confidence interval of the mean logarithmic error ranged from -0.099 to 0.054 , while 76.0% and 91.1% of the predictions fell within factors of 3 and 5, respectively. For IC10, 110 predictions were evaluated, with $\mu = 0.0613$, $S_e = 0.5504$, and $RMSE_{\log} = 0.5513$. The corresponding 95% confidence interval ranged from -0.043 to 0.166 , while 60.0% and 82.7% of the predictions fell within factors of 3 and 5, respectively. For SRR99, 79 predictions were evaluated, with

$\mu = -0.00289$, $S_e = 0.4649$, and $RMSE_{\log} = 0.4620$. The corresponding 95% confidence interval ranged from -0.108 to 0.102 , while 70.9% and 81.0% of the predictions fell within factors of 3 and 5, respectively. Thus, the predictions for DZ125 and SRR99 exhibit negligible overall bias, whereas the IC10 results show relatively larger scatter.

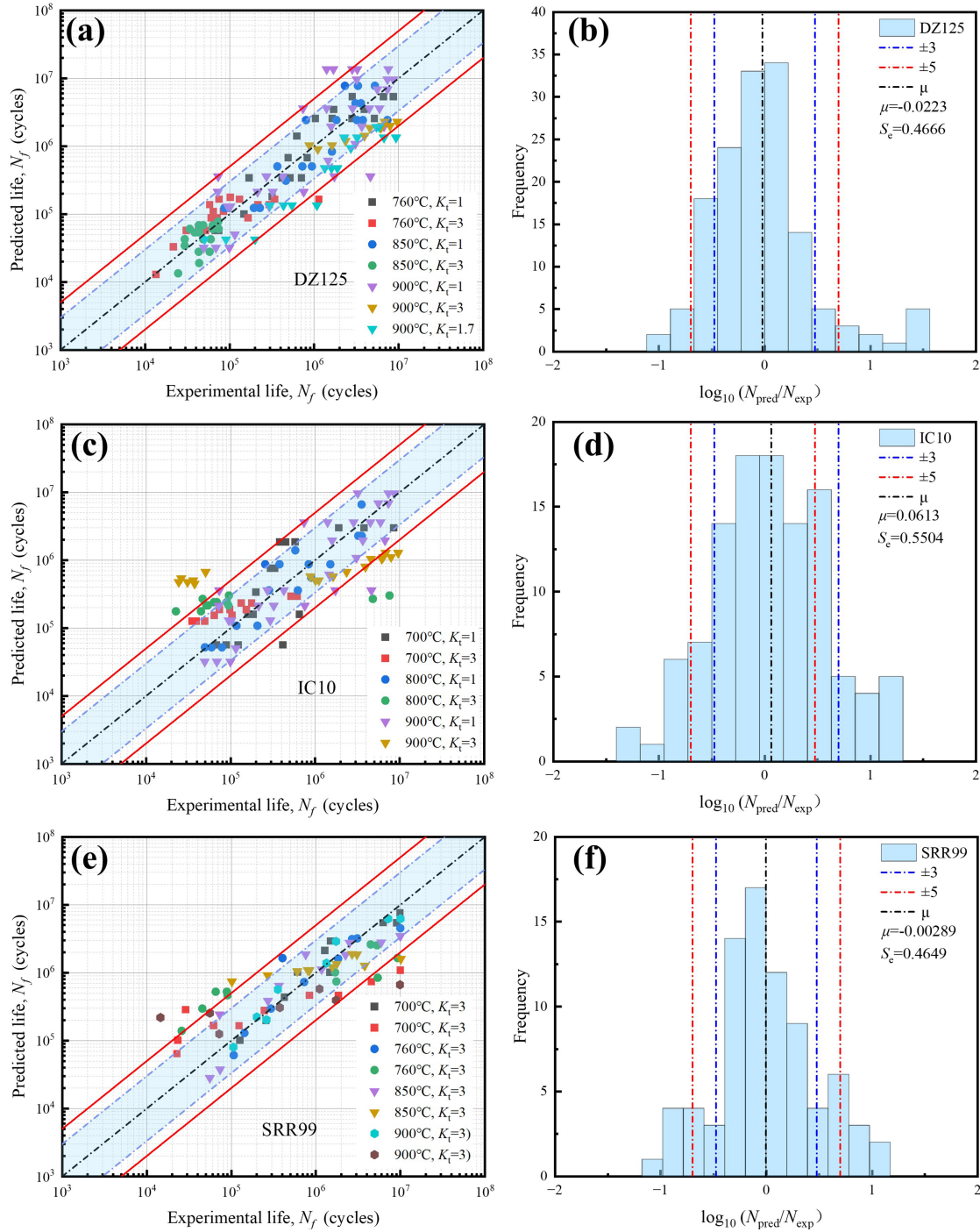


Figure 8: Notch-factor correction validation. (a) DZ125 predicted and experimental lives; (b) DZ125 error histogram; (c) IC10 predicted and experimental lives; (d) IC10 error histogram; (e) SRR99 predicted and experimental lives; (f) SRR99 error histogram.

5.3 Validation of the Combined Correction

After establishing the stress-ratio and notch-factor correction forms, the correction exponents γ and δ of the final combined model are jointly identified using the designated calibration conditions. The resulting combined parameter set is then kept fixed and evaluated for the withheld multi- R_σ –multi- K_t conditions at multiple temperatures. Fig. 9 shows the predicted-vs.-experimental comparisons and the error statistics after applying the combined correction. As shown in Fig. 9a,c,e, the data points at each temperature form a banded distribution around the identity line, and no evident group-wise shift is observed among different combinations of R_σ and K_t , suggesting that the combined correction mitigates the coupled influences of stress ratio and notch factor and enables cross-condition prediction within a unified framework.

At each temperature, the DZ125 dataset contained six loading conditions formed by three stress ratios, $R_\sigma = -1, 0.1$, and 0.5 , and two stress concentration factors, $K_t = 1$ and 3 . The conditions $R_\sigma = -1, K_t = 1, R_\sigma = 0.5, K_t = 1$, and $R_\sigma = -1, K_t = 3$ were designated as the calibration conditions. The comparison between $R_\sigma = -1, K_t = 1$ and $R_\sigma = 0.5, K_t = 1$ was used to isolate the stress-ratio effect, whereas the comparison between $R_\sigma = -1, K_t = 1$ and $R_\sigma = -1, K_t = 3$ was used to isolate the notch effect. Based on these single-variable comparisons, the correction exponents γ and δ in the final combined model were jointly identified using all observations from the three calibration conditions by minimizing the objective function in Eq. (28).

The remaining conditions $R_\sigma = 0.1, K_t = 1, R_\sigma = 0.1, K_t = 3$, and $R_\sigma = 0.5, K_t = 3$ were withheld from parameter identification and used for condition-level forward prediction. Once the combined parameter set had been identified, no parameter refitting or updating was performed for these withheld conditions.

The partition was performed at the loading-condition level rather than by randomly separating individual observations. All observations belonging to the same fatigue curve were assigned entirely to either the calibration subset or the withheld prediction subset. Therefore, no individual observation or fatigue curve was shared between the two subsets. Although the baseline condition $R_\sigma = -1, K_t = 1$ contributed to the joint identification of γ and δ , its observations were counted only once and were used exclusively for parameter calibration. The statistical indicators reported below were calculated using all failure observations shown in the corresponding panels of Fig. 9, including both the calibration conditions and the loading conditions withheld from parameter identification. They should therefore be interpreted as overall descriptive statistics rather than withheld-subset-only validation metrics.

The error histograms in Fig. 9b,d,f show that the logarithmic errors are generally distributed close to zero, although their scatter and bias vary among temperatures. At 700°C, 124 predictions were evaluated, with $\mu = 0.0194$, $S_e = 0.5008$, and $\text{RMSE}_{\log} = 0.499$. The corresponding 95% confidence interval ranged from -0.070 to 0.108 , while 59.4% and 77.4% of the predictions fell within factors of 3 and 5, respectively. Because the confidence interval includes zero, no statistically evident overall prediction bias was observed at 700°C. At 760°C, 117 predictions were evaluated, with $\mu = 0.0980$, $S_e = 0.5746$, and $\text{RMSE}_{\log} = 0.580$. The corresponding 95% confidence interval ranged from -0.007 to 0.203 , while 65.3% and 91.5% of the predictions fell within factors of 3 and 5, respectively. The positive mean logarithmic error indicates a mild tendency toward life overprediction; however, the overall bias is not statistically distinct from zero because the confidence interval includes zero. At 850°C, 133 predictions were evaluated, with $\mu = 0.0743$, $S_e = 0.4049$, and $\text{RMSE}_{\log} = 0.410$. The corresponding 95% confidence interval ranged from 0.005 to 0.144 , while 76.1% and 82.7% of the predictions fell within factors of 3 and 5, respectively. The relatively low S_e and $\text{RMSE}_{\log}$ indicate smaller prediction scatter than that obtained at 700°C and 760°C. However, because the confidence interval lies slightly above zero, the results indicate a small overall tendency toward life overprediction at 850°C. Taken together, the combined correction produces relatively small mean logarithmic errors at the three investigated temperatures, while the prediction scatter remains temperature dependent. The 850°C

results exhibit the lowest logarithmic scatter and the highest factor-of-3 coverage, whereas the 760°C results show the largest S_e and $RMSE_{\log}$.

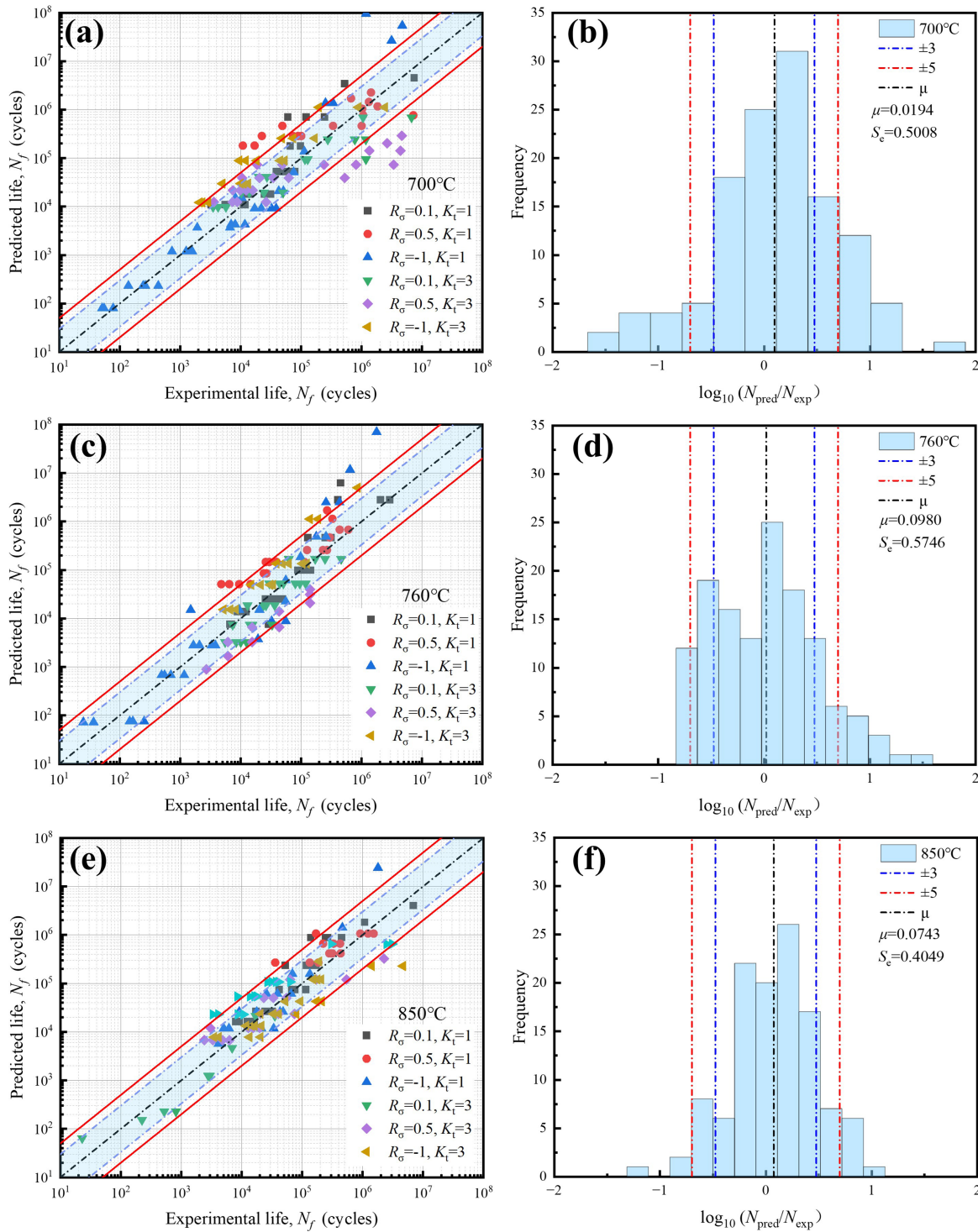


Figure 9: Combined correction validation for DZ125 at multiple temperatures. (a) 700°C predicted and experimental lives; (b) 700°C error histogram; (c) 760°C predicted and experimental lives; (d) 760°C error histogram; (e) 850°C predicted and experimental lives; (f) 850°C error histogram.

In summary, the results in this section demonstrate that the proposed method provides a unified description of isothermal fatigue data under different stress ratios and notch severities, while maintaining reasonable predictive consistency for loading conditions withheld from calibration. Most predictions fall within the prescribed scatter bands, indicating reasonable condition-level prediction capability within the investigated range. Within each alloy–temperature combination, the ability to achieve a single parameter set across multiple stress ratios and notch severities primarily arises from transforming condition dependence from “groupwise parameter recalibration” into “equivalent driving force mapping”. Specifically, stress-ratio effects are incorporated into a unified driving force through an equivalence correction that accounts for cyclic mean stress and tension–compression asymmetry, whereas notch effects are represented through the notch-root equivalent stress response and a semi-empirical notch correction factor. These terms describe the systematic life shift associated with notch severity within the calibrated datasets.

Compared with crystal plasticity finite element (CPFE)-based approaches, the proposed method does not require full-field crystal plasticity simulations, but constructs an engineering equivalent fatigue driving parameter through nominal loading, notch-root equivalent response, and crystallographic Schmid-factor weighting. Compared with the Critical Distance Theory, it does not introduce an additional characteristic distance parameter. Instead, the notch effect is incorporated through the notch-root equivalent response and the semi-empirical notch correction, while the crystallographic effect is introduced through Schmid-factor weighting. Therefore, the proposed framework provides a simple and practical way to achieve engineering-level life prediction without requiring complex full-field simulations or additional length-scale parameters. For applications where local multiaxial stress gradients, notch-root radius effects, and anisotropic stress redistribution dominate fatigue damage, anisotropic finite element analysis or CPFE analysis is still required to obtain a more physically rigorous fatigue indicator parameter. In addition, since the present calibration and validation are based on [001]-oriented single-crystal superalloys, the correction parameters γ and δ should not be regarded as universal for other crystallographic orientations. For [011], [111], or other orientations, the Schmid factor matrix should be recalculated, and further calibration and validation may be required.

6 Conclusion

This study develops a fatigue-life prediction framework for the unified treatment of stress-ratio and notch effects within fixed alloy–temperature conditions. By adopting an equivalent shear-type driving parameter based on notch-root equivalent response and crystallographic Schmid-factor weighting, and by introducing a stepwise, baseline-preserving correction strategy, multi-condition datasets are mapped into a consistent reference space for engineering life assessment. The main conclusions are as follows:

1. Strong nonlinear effects on fatigue life: Temperature, stress ratio, and notch severity exhibit coupled, non-separable influences on fatigue life, leading to nonlinear variations in the driving-parameter response and making a single uncorrected master curve inadequate for unified representation.
2. Unified engineering driving parameter via stepwise, baseline-preserving correction: The uncorrected $\Delta\tau_{\max}-N_f$ relation shows pronounced stratification and systematic offsets. By introducing f_R and f_{K_t} and defining the corrected equivalent driving parameter, multi- R_σ and multi- K_t datasets are collapsed into a common reference trend band without altering the baseline curve shape. In this framework, $\Delta\tau_{\max}$ should be interpreted as an equivalent shear-type parameter for life correlation rather than a fully resolved slip-system shear stress.
3. Model validation and condition-level forward-prediction capability: Validation results indicate stable predictive performance, with most predictions within the $\pm 3\times$ scatter band and generally constrained

within $\pm 5\%$. Once the baseline master curve and correction parameters are calibrated for a given alloy–temperature combination, the model can predict fatigue life for untested R_σ – K_t combinations within the investigated R_σ – K_t range. Prediction at a new temperature requires separate parameter calibration.

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Availability of Data and Materials: Data available on request from the authors.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Table A1: Nomenclature and symbols used in this manuscript.

Nomenclature	
Symbol	Description
E	Elastic modulus
σ_{yield}	Yield strength
K'	Cyclic strength coefficient
n'	Cyclic strain hardening exponent
σ_{max}	Nominal maximum stress
σ_{min}	Nominal minimum stress
R_σ	Stress ratio
R_ϵ	Strain ratio
$\Delta\sigma_{\text{nom}}$	Nominal stress range
$\Delta\epsilon/2$	Total strain amplitude
$\sigma_{0.2}$	0.2% offset yield strength
σ_b	Ultimate tensile strength
K_t	Theoretical stress concentration factor
$\sigma_{\text{max}}^{\text{notch}}, \sigma_{\text{min}}^{\text{notch}}$	Maximum and minimum stresses at notch root
$\Delta\sigma^{\text{notch}}, \Delta\epsilon^{\text{notch}}$	Stress and strain ranges at notch root
$\mathbf{M}^{(k)}$	Schmid factor matrix for slip system k
N_f	Cycles to failure
t_f	Time to failure
C_0, b	Parameters of the baseline master curve

(Continued)

Table A1 (continued)

Nomenclature	
γ, δ	Stress ratio and notch correction exponents
$f_R (R_\sigma)$	Stress ratio correction factor
$f_{K_t} (K_t)$	Notch correction factor
μ	Mean of logarithmic error
S_e	Standard deviation of logarithmic error
$J(\gamma, \delta)$	Objective function (mean squared logarithmic error)
σ	Stress tensor in Voigt notation
$\tau_{\max}, \tau_{\min}$	Maximum and minimum equivalent shear parameter on slip systems
$\Delta\tau_{\max}$	Equivalent maximum equivalent shear stress parameter stress range

Table A2: Alloy–temperature-specific calibrated parameters used in the prediction framework.

Validation Case	Alloy	Temperature	C_0	b	γ	δ
Stress-ratio correction	DZ125	700°C	1340.25	−0.086	0.55	–
Stress-ratio correction	DD6	1070°C	552.66	−0.0095	0.01	–
Notch-factor correction	DZ125	760°C	2119.62	−0.10926	–	0.39
Notch-factor correction	DZ125	850°C	1365.86	−0.07499	–	0.41
Notch-factor correction	DZ125	900°C	1240.49	−0.07546	–	0.62
Notch-factor correction	IC10	700°C	1735.56	−0.10224	–	0.01
Notch-factor correction	IC10	800°C	1579.24	−0.09433	–	0.02
Notch-factor correction	IC10	900°C	1240.49	−0.07548	–	0.58
Notch-factor correction	SRR99	700°C	1185.27	−0.12439	–	0.37
Notch-factor correction	SRR99	760°C	2157.26	−0.11614	–	0.05
Notch-factor correction	SRR99	850°C	925.13	−0.03689	–	1.57
Notch-factor correction	SRR99	900°C	832.26	−0.10312	–	0.57
Combined correction	DZ125	700°C	1340.25	−0.086	0.48	0.51
Combined correction	DZ125	760°C	2119.62	−0.10926	0.38	0.59
Combined correction	DZ125	850°C	1365.86	−0.07499	0.36	0.5

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