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ARTICLE

Computational Assessment of Energy Supply Sustainability Using Picture Fuzzy Choquet Integral Decision Support System

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ABSTRACT: For any country, the availability of electricity is crucial to the development of the national economy and society. As a result, decision-makers and policy-makers can improve the sustainability and security of the energy supply by implementing a variety of actions by using the evaluation of these factors as an early warning system. This research aims to provide a multi-criterion decision-making (MCDM) method for assessing the sustainability and security of the electrical supply. The weights of criteria, which indicate their relative relevance in the assessment of the sustainability and security of the energy supply, the MCDM method allow users to express their opinions. To overcome the impact of uncertainty and vagueness of expert opinion, we explore the notion of picture fuzzy theory, which is a more efficient and dominant mathematical model. Recently, the theory of Aczel-Alsina operations has attained a lot of attraction and has an extensive capability to acquire smooth approximated results during the aggregation process. However, Choquet integral operators are more flexible and are used to express correlation among different attributes. This article diagnoses an innovative theory of picture fuzzy set to derive robust mathematical methodologies of picture fuzzy Choquet Integral Aczel-Alsina aggregation operators. To prove the intensity and validity of invented approaches, some dominant properties and special cases are also discussed. An intelligent decision algorithm for the MCDM problem is designed to resolve complicated real-life applications under multiple conflicting criteria. Additionally, we discussed a numerical example to investigate a suitable electric transformer under consideration of different beneficial key criteria. A comparative study is established to capture the superiority and effectiveness of pioneered mathematical approaches with existing methodologies.

KEYWORDS: Picture fuzzy information; Choquet integrals; electric supply; Aczel-Alsina operations; multi-criteria decision-making

1 Introduction

One of the most important energy carriers, electricity, is crucial for maintaining economic growth [1]. However, manufacturing electricity has become extremely reliant on fossil fuels, resulting in several serious issues, including air pollution, significant greenhouse gas emissions, imported energy, and inadequate energy security [2,3]. To improve the security of power, numerous nations implemented a variety of policies. For example, China established the renewable energy development plan with the aim of reducing harmful emissions, reducing reliance on energy imports, and improving air and water quality [4]. In order to reduce its reliance on fossil fuels for electricity generation, improve environmental performance, and potentially increase the security of the electricity supply, Brazil has implemented or planned a number of biofuel projects. These include the production of electricity from sugarcane bagasse [5], hybrid concentrated solar power (CSP)–biomass plants [6], and the production of rice husk electricity [7]. The goal of each study was to improve the reliability of the electrical supply. The basis for recommending practical policies and initiatives to improve a nation's electrical supply sustainability and security (ESSS) is the examination of ESSS and its evolving trend [8]. Nonetheless, the quantity of research works that concentrate on creating the techniques for ESSS state analysis is constrained. The creation of the ESSS evaluation framework is, therefore, a necessary and crucial step. Multiple dimensions are typically taken into account when evaluating ESSS. Power system reliability analysis and power market analysis were linked by Kjølle and Gjerde [9] to create an integrated framework for the security of energy supply analysis. Numerous studies have also taken into account more than three variables for the analysis and evaluation of ESSS.

Especially in the field of energy planning, multi-criteria decision-making techniques have proven to be a very helpful instrument for resolving conflicts between disparate criteria. Nevertheless, social and environmental factors frequently conflict with technical and financial factors in the design of rural electrification plans. To choose the best option for remote rural locations' electrical supply while taking into account social, economic, environmental, and technical factors, we suggest advanced multi-attribute decision-making (MADM) technique. The MADM problem has been applied to various complex issues. We assess different decision-making problems in daily life and important goods, and we can learn how to make the best decisions. We saw several research scholars working on the decision-making problems under different mathematical models [10–12].

To handle unpredictable situations of crisp theory, an innovative theory of fuzzy set (FS) was developed by Zadeh [13] with a certain membership degree (MD) of an object. The concept of intuitionistic fuzzy sets (IFSs) was introduced by Atanassov [14], who modified a version of fuzzy theory. The IFS contains two different aspects of an object, like membership degree (MD) and non-membership degree (NMD). The sum of MD and NMD lies in a close interval $[0, 1]$ with subject to the condition $0 \leq \mu(x) + \nu(x) \leq 1$. Yager and Basson [15] utilized the theory of an IFS to derive new operations and mathematical approaches. Yager [16] introduced the concept of the pythagorean fuzzy set (PyFS) by taking the sum of the squares of MD and NMD with the mathematical shape $0 \leq \mu(x)^2 + \nu(x)^2 \leq 1$. The enlargement of PyFS in the shape of complex numbers was introduced by Yager and Abbasov [17]. Some important results for PyFS improved by Peng and Yang [18]. Yager presented the concept of q-rung orthopair fuzzy sets (q-ROFSs), which is an effective method to explain the vagueness and ambiguous information in the MADM problem. The uniqueness of q-ROFS is to handle uncertainty and vagueness more effectively by incorporating multiple degrees to represent the MD and NMD of an element. Decision makers face some crucial challenges due to incomplete information and a lack of information provided by experts. To overcome such a scenario, Cường [19] presented the idea of a picture fuzzy set (PFS) with four possible terms, such as MD, abstinence degree (AD), NMD and refusal degree (RD). The mathematical expression of PFS is given by $0 \leq \mu(x) + \alpha(x) + \nu(x) \leq 1$. A PFS is more effective and advanced framework of FS, IFS, PyFS and q-ROFSs. The PFS is a relatively recent extension of fuzzy set theory that aims to handle uncertainties with a spatial context, particularly in scenarios involving image processing, computer vision, and spatial data analysis. The application of picture fuzzy sets in pattern recognition and classification in scenarios where imprecision is inherent in the image data.

1.1 Literature Review

Yager [20,21] explored the aggregation operators (AOs) made use of in view of humans under the consideration of picture fuzzy informing by different researchers from different states. Cuong et al. [22] explored the idea of picture triangular norm (t-corm) and picture triangular co-norms (t-conorms) and discussed the properties of PFS. Hussain et al. [23] discussed the significance of vendor management enterprises by exploring the characteristics of the Hamy mean model with advanced picture fuzzy theory. Wang et al. [24] suggested a possible plan for the Choquet integral and proposed operator. Hussain et al. [25] classified diverse construction materials, taking into account Hamy mean models and Aczel-Alsina operators in the decision-making process. Sahoo et al. [26] resolved energy energy-related real-life problem using multi-criteria and mathematical aggregation operators. Petchimuthu et al. [27] constructed decision-making models to investigate a suitable optimal option using unknown weights of criteria.

Ali et al. [28] described some methods by using the basic operation of the Aczel-Alsina aggressions operator and the system of PFSs. Mahmood et al. [29,30] explored the PFAAWG operator bases on the Azcel Alsina operation to find effective and reliable proposed aggression operators. Choquet integral is a useful technique for computing rank. Hussain et al. [31] discussed mathematical results of Aczel Alsina Heronian mean operators. Ahmmad et al. [32] initiated a list of Aczel-Alsina averaging operators to resolve a real-life application related to medical diagnosis with decision analysis process. Hussain et al. [33] elaborated on Dombi Bonferroni mean operators for choosing reliable recycling machines to reduce the impact of waste materials.

In order to construct the intuitionistic fuzzy aggregation operators previously stated, the criteria are expected to be independent. It should be noted that correlated criteria, as opposed to independent ones, are used in most MCDM problems. In these cases, the evaluations cannot be aggregated using weighted aggregation operators, and the criterion's relevance levels are described as fuzzy measures as opposed to weights. Fuzzy measures can effectively capture the correlative relationships between criterion sets, including independent, redundant, and complementary ones. Since the total of all fuzzy measures of a criterion set may be greater than one, fuzzy measures are really extensions of weights because the fuzzy measure of the complete set is only one. Thus, the fuzzy measures have greater versatility. Demirel et al. [34] employed Choquet integral mathematical models to investigate a suitable location for a warehouse under different key criteria. Tan and Chen [35] generalized the concepts of Choquet integral operators to acquire an authentic results from decision analysis process under different criteria. Furthermore, a family of Sugeno-Weber aggregation operators also utilized to investigate unknown degree of weights to assess a suitable security techniques [36]. The intuitionistic fuzzy Choquet averaging and geometric operators based on Einstein were created by Yu [37]. Yu and Xu [38] presented the intuitionistic fuzzy interaction aggregation operators based on the Choquet integral and created a two-sided matching decision-making model based on aggregation operators.

Menger [39] introduced the theoretical concepts of triangular norms with prominent characteristics. Algebraic t-norm and t-conorm are flexible and efficient generalizations of triangular norms. A diverse generalization and extension of triangular norms developed by different scholars, such as Dombi aggregation operators [40], Frank t-norm and t-conorm [41], and Hamacher aggregation operators [42]. Numerous mathematicians employed these aggregation models to acquire smooth aggregated results during the decision analysis process. Aczél and Alsina [43] deduced a family of robust aggregation models by exploring the characteristics of triangular norms. Babu and Ahmed [44] characterized various properties to derive effective mathematical strategies for designing and developing to resolve different applications such as diagnostics, article intelligence, forecasting and pattern recognition. Farahbod and Eftekhari [45] compared various classifications of t-norms and t-conorms to evaluate the best aggregation model. After the evaluation and aggregation process, they concluded that Aczel-Alsina is the best one and Dombi aggregation is the second one. Hussain et al. [46] deliberated a list of Aczel-Alsina aggregation models to handle awkward and incomplete information of human opinion. Senapati et al. [47] proposed Aczel-Alsina aggregation models, taking into account an intuitionistic fuzzy context. Senapati et al. [48] derived innovative mathematical strategies by incorporating the theory of an interval values intuitionistic fuzzy theory. Mu et al. [49] investigated a family of efficient and effective models of Maclaurin symmetric operators with a decision support system. Hussain et al. [50] derived new methodologies of Aczel-Alsina operators to evaluate an electric motor car taking into account the complex spherical fuzzy circumstances. De et al. [51] developed a decision support system with a doubt-fuzzy environment.

1.2 Research Gap

Several mathematical approaches and terminologies were established to aggregate expert's opinions or judgments. Most of them failed to integrate multiple conflicting criteria without any external weights. To investigate the weight of criteria or attributes, Choquet integral operators are authentic mathematical approaches under the system of picture fuzzy environment and Aczel-Alsina triangular norms. We examined that no one worked on this robust mathematical framework under the consideration of a picture fuzzy situation and Choquet integral operators.

1.3 Novelty and Contributions

The PFS has extended the capabilities of traditional FSs and IFSs by introducing a third dimension: neutrality. In many decision-making scenarios, it's common to encounter not only positive (supportive) and negative (opposing) opinions but also neutral stances where no clear judgment is made. Traditional fuzzy sets and even intuitionistic fuzzy sets, which include membership and non-membership degrees, fall short in accurately capturing this neutral perspective. For instance, voter express their opinion about any candidate in a specific way, vote to favor (MD), against (NMD), neutral (AD) and refuse to vote (RD). The FSs and IFSs unable to handle such a scenario, which has more than three components of human opinion. However, the PFS is a more abundant and flexible fuzzy framework that contains four aspects of human opinion. Furthermore, the PFSs address the limitation by incorporating a degree of neutrality alongside the traditional membership and non-membership degrees. This three-dimensional approach allows for a more nuanced representation of complex scenarios where neutrality plays a significant role, enabling better modelling and decision-making in situations with uncertain, hesitant, or indeterminate responses.

The Choquet integral operators are used to overcome the limitations of traditional aggregation methods, particularly in contexts where criteria are interdependent. Traditional aggregation operators, like weighted averages, typically assume that criteria are independent, which often oversimplifies the decision-making process. In reality, criteria frequently interact with one another, with some being complementary (synergistic) or substitutive (antagonistic). The Choquet integral was designed to handle such interactions effectively using a fuzzy measure that captures the relationship between criteria. This allows for a more accurate and nuanced aggregation of information, particularly in multi-criteria decision-making situations where the interplay between factors significantly influences the outcome. By accounting for these interactions, the Choquet integral provides a more sophisticated tool for decision-makers, leading to better-informed and more reliable decisions. The primary features of this presentation are given by:

(a) To explore the picture fuzzy theory for handling awkward and unpredictable situations of human opinion. The PFS uniquely captures extensive information, enabling smoother approximations during decision analysis.

(b) Choquet integral operators are more efficient and effective, used to aggregate approximated information with fuzzy measures of criteria and the decision analysis process.
(c) We derive a list of new aggregation operators by incorporating the picture fuzzy theory and Choquet integral operators, namely PFCIAAA and PFCIAAG operators with prominent properties and special cases.
(d) To resolve complicated, genuine real-life dilemmas, an algorithm of the MADM problem is utilized by incorporating derived mathematical methodologies.
(e) To validate the robustness and compatibility of the diagnosed mathematical methodologies, we gave a numerical example to evaluate a suitable electric transformer for electricity supply.

1.4 Layout of the Manuscript

In order to maintain research work, divide the remaining parts as follows: Section 2 briefly discusses fundamental concepts and basic preliminaries of triangular norms, Aczel-Alsina operations, and PFS with dominant rules. Section 3 illustrates a family of averaging Choquet integral operators. Moreover, we also derive mathematical methodologies of geometric Choquet integral operators in the light of picture fuzzy information. The robust technique of the MCDM problem is applied to resolve genuine real-life applications with derived mathematical methodologies in Section 5. An experimental case study is also adopted to reveal the feasibility of the invented approaches. Section 6 conducted an extensive comparative study to mitigate the flexibility of pioneered approaches with previously developed approaches. Section 7 enclosed some remarkable remarks related to the developed research work. Additionally, we also portrayed the main aspects of this article in Fig. 1.

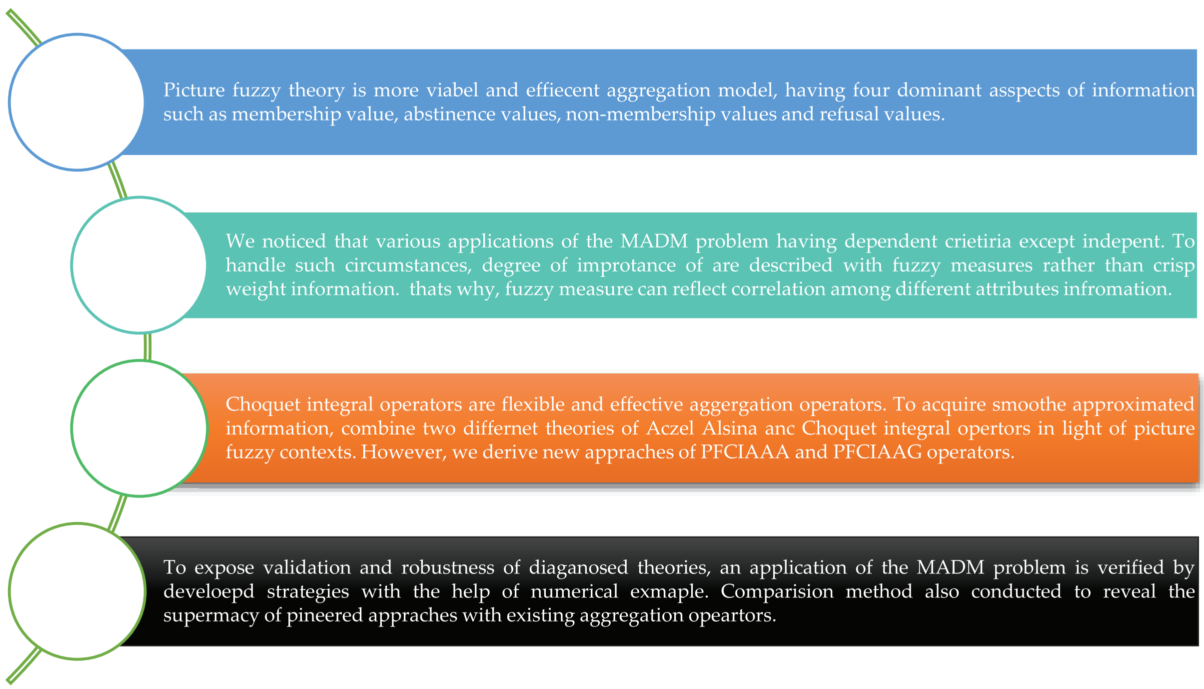

Figure 1: The primary features of the article

2 Preliminaries

This section presents basic and necessary notions of triangular norms, Aczel-Alsina operations, and PFSs with dominant rules.

Definition 1 [52]**:** A mapping $T: [0,1]^2 \rightarrow [0,1]$ is known as t-norm if underlying the following axioms:

(a) $T(\varkappa, \sigma) = T(\sigma, \varkappa)$
(b) $T(\varkappa, \sigma) \leq T(\varkappa, r)$ if $\sigma \leq \mathbb{c}$
(c) $T(\varkappa, T(\sigma, \mathbb{c})) = T(T(\varkappa, \sigma), \mathbb{c})$
(d) $T(\varkappa, 1) = \varkappa$

For all $\varkappa, \sigma, \mathbb{c} \in [0,1]$.

Example 1: The following expressions are examples of Definition 1:

(a) $T_M(\kappa, \sigma) = \min(\kappa, \sigma)$
(b) $T_p(\varkappa, \sigma) = \kappa.\sigma$
(c) $T_L(\varkappa, \sigma) = \max(\varkappa + \sigma - 1, 0)$
(d) $T_D(\varkappa, \sigma) = \begin{cases} \varkappa, if \sigma = 1 \\ \sigma, if \varkappa = 1 \\ 0, otherwise \end{cases}$

For any $\varkappa, \sigma \in [0,1]$.

Definition 2 [52]**:** A mapping $S: [0,1]^2 \rightarrow [0,1]$ is known as t-norm if underlying the following axioms:

(a) $S(\varkappa, \sigma) = S(\sigma, \varkappa)$
(b) $S(\varkappa, \sigma) \leq S(\varkappa, \mathbb{c})$ if $\sigma \leq \mathbb{c}$
(c) $S(\varkappa, S(\sigma, \mathbb{c})) = S(S(\varkappa, \sigma), \mathbb{c})$
(d) $S(\varkappa, 0) = \varkappa$

For any $\varkappa, \sigma, \mathbb{c} \in [0,1]$.

Example 2: The following expressions are examples of Definition 2:

(a) $S_M(\varkappa, \sigma) = \max(\varkappa, \sigma)$
(b) $S_p(\varkappa, \sigma) = \varkappa + \sigma - \varkappa.\sigma$
(c) $S_L(\varkappa, \sigma) = \min(\varkappa + \sigma, 1)$
(d) $S_D(\varkappa, \sigma) = \begin{cases} \varkappa, if\ \sigma = 0 \\ \sigma, if\ \varkappa = 0 \\ 1, otherwise \end{cases}$

For any $\varkappa, \sigma \in [0,1]$

Definition 3 [43]**:** The theory of the Aczel-Alsina t-nom $(S_A^{\aleph})$ and t-conorm $S_A^{\aleph}(\varkappa, \sigma)$ is given by:

$$T_A^{\aleph}(\varkappa, \sigma) = \left\{ \begin{array}{c} T_D(\varkappa, \sigma) \quad if\ \aleph = 0 \\ \min(\varkappa, \sigma) \quad if\ \aleph = \infty \\ e^{-\left(\left(-\mathcal{L}n\varkappa\right)^{\aleph}\right) + \left(-\mathcal{L}n\sigma\right)^{\aleph}\right)^{\frac{1}{\aleph}}} otherwise \end{array} \right\}$$

and

$$S_A^{\aleph}(\varkappa, \sigma) = \left\{ \begin{array}{c} S_D(\varkappa, \sigma) \quad if\ \aleph = 0 \\ \max(\varkappa, \sigma) \quad if\ \aleph = \infty \\ 1 - e^{-\left(\left(-\mathcal{L}\mathrm{n}(1-\varkappa)\right)^{\aleph} + \left(-\mathcal{L}\mathrm{n}(1-\sigma)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}\ otherwise \end{array} \right\}$$

for each $\aleph \in [0, \infty]$.

Definition 4 [14]: An IFS K on X is defined as $K = \{\langle ℸ, \mu(ℸ), \nu(ℸ)\rangle | ℸ \in X\}$. Where $\mu(ℸ): X \to [0,1]$ and $\nu(ℸ): X \to [0,1]$ represent MD and NMD with subject to the condition:

$0 \le \mu(ℸ) + \nu(ℸ) \le 1$

Additionally, a hesitancy degree of ℸ in K is denoted by $\pi(ℸ) = 1 - (\mu(ℸ) + \nu(ℸ))$ and a pair $(\mu(ℸ), \nu(ℸ))$ is known as an intuitionistic fuzzy value (IFV).

Definition 5 [53]: A PFS K on X is defined as $K = \{\langle ℸ, \mu(ℸ), \alpha(ℸ), \nu(ℸ)\rangle | ℸ \in X\}$. Where $\mu(ℸ): X \to [0,1]$, $\alpha(ℸ): X \to [0,1]$ and $\nu(ℸ): X \to [0,1]$ indicate MD, AD and NMD, respectively, subject to the condition:

$0 \le \mu(ℸ) + \alpha(ℸ) + \nu(ℸ) \le 1$

Additionally, the RD of ℸ in K is denoted by $\pi(ℸ) = 1 - (\mu(ℸ) + \alpha(ℸ) + \nu(ℸ))$ and a triplet $\mathfrak{g} = (\mu(ℸ), \alpha(ℸ), \nu(ℸ))$ is known as a picture fuzzy value (PFV).

Definition 6 [33]: Consider any three PFVs $\mathfrak{g} = (\mu, \alpha, \nu)$, $\mathfrak{g}_1 = (\mu_1, \alpha_1, \nu_1)$ and $\mathfrak{g}_2 = (\mu_2, \alpha_2, \nu_2)$. Then some prominent rules are given bellow:

(a) $\mathfrak{g}_1 \subseteq \mathfrak{g}_2$, if $\mu_1 \le \mu_2, \alpha_1 \le \alpha_2$ and $\nu_1 \le \nu_2$
(b) $\mathfrak{g}_1 = \mathfrak{g}_2$ iff $\mathfrak{g}_1 \subseteq \mathfrak{g}_2$ and $\mathfrak{g}_2 \subseteq \mathfrak{g}_1$
(c) $\mathfrak{g}_1 \cup \mathfrak{g}_2 = (\max\{\mu_1, \mu_2\}, \min\{\alpha_1, \alpha_2\}, \min\{\nu_1, \nu_2\})$
(d) $\mathfrak{g}_1 \cap \mathfrak{g}_2 = (\min\{\mu_1, \mu_2\}, \min\{\alpha_1, \alpha_2\}, \max\{\nu_1, \nu_2\})$
(e) $\overline{\mathfrak{g}} = (\mu, \alpha, \nu)$
(f) $\mathfrak{g}_1 \bigoplus \mathfrak{g}_2 = (\mu_1 + \mu_2 - \mu_1\mu_2, \alpha_1\alpha_2, \nu_1\nu_2)$
(g) $\mathfrak{g}_1 \otimes \mathfrak{g}_2 = (\mu_1\mu_2, \alpha_1\alpha_2, \nu_1 + \nu_2 - \nu_1\nu_2)$
(h) $\tau\mathfrak{g} = \left(1 - (1 - \mu_1)^\tau, \alpha_{\mathfrak{g}}^\tau, \nu_{\mathfrak{g}}^\tau\right)$
(i) $\mathfrak{g}^\tau = \left(\mu_{\mathfrak{g}}^\tau, \alpha_{\mathfrak{g}}^\tau, 1 - (1 - \nu)^\tau\right)$

Definition 7 [54]: Let $\mathfrak{g} = (\mu, \alpha, \nu)$, $\mathfrak{g}_1 = (\mu_1, \alpha_1, \nu_1)$ and $\mathfrak{g}_2 = (\mu_2, \alpha_2, \nu_2)$ be three PFVs with $\tau, \tau_1, \tau_2 > 0$. Then:

(a) $\mathfrak{g}_1 \bigoplus \mathfrak{g}_2 = \mathfrak{g}_2 \bigoplus \mathfrak{g}_1$
(b) $\mathfrak{g}_1 \otimes \mathfrak{g}_2 = \mathfrak{g}_2 \otimes \mathfrak{g}_1$
(c) $\tau(\mathfrak{g}_1 \bigoplus \mathfrak{g}_2) = \tau\mathfrak{g}_1 \bigoplus \tau\mathfrak{g}_2$
(d) $(\mathfrak{g}_1 \otimes \mathfrak{g}_2)^\tau = \mathfrak{g}_1^\tau \bigoplus \mathfrak{g}_2^\tau$
(e) $\tau_1\mathfrak{g} \bigoplus \tau_2\mathfrak{g}) = (\tau_1 + \tau_2)\mathfrak{g}$
(f) $\mathfrak{g}^{\tau_1} \otimes \mathfrak{g}^{\tau_2} = \mathfrak{g}^{(\tau_1+\tau_2)}$
(g) $(\mathfrak{g}^{\tau_1})^{\tau_2} = \mathfrak{g}^{\tau_1\tau_2}$

Definition 8 [33]: Let $\mathfrak{g} = (\mu, \alpha, \nu)$ be a PFV, the score function $S(\mathfrak{g})$ and accuracy function $H(\mathfrak{g})$ are defined as follows:

$$Æ(\mathfrak{g}) = \left(\frac{1}{3}\right)(2 + \mu - \alpha - \nu), S(\mathfrak{g}) \in [0,1] \tag{1}$$

and

$$H(\mathfrak{g}) = \mu + \alpha + \nu, H(\mathfrak{g}) \in [0,1] \tag{2}$$

Consider two PFVs $\mathfrak{g}_1$ and $\mathfrak{g}_2$. We make a comparison by using the following steps:

(a) If $S(\mathfrak{g}_1) > S(\mathfrak{g}_2)$, then $\mathfrak{g}_1$ is superior to $\mathfrak{g}_2$.
(b) If $S(\mathfrak{g}_1) = S(\mathfrak{g}_2)$. Then:

$H(\mathfrak{g}_1) = H(\mathfrak{g}_2)$ Implies that $\mathfrak{g}_1$ is equivalent to $\mathfrak{g}_2$.

$H(\mathfrak{g}_1) > H(\mathfrak{g}_2)$ Implies that $\mathfrak{g}_1$ is superior to $\mathfrak{g}_2$.

Example 1: Let $\mathfrak{g}_1 = (0.12, 0.32, 0.27)$ and $\mathfrak{g}_2 = (0.21, 0.42, 0.19)$ are two PFVs. Then, the score and accuracy functions are illustrated as follows:

$$\text{Æ}(\mathfrak{g}_1) = \left(\frac{1}{3}\right)(2 + 0.12 - 0.32 - 0.27), S(\mathfrak{g}_1) = 0.51 \in [0,1]$$

$$\text{Æ}(\mathfrak{g}_2) = \left(\frac{1}{3}\right)(2 + 0.21 - 0.42 - 0.19), S(\mathfrak{g}_2) = 0.53 \in [0,1]$$

$$H(\mathfrak{g}) = 0.12 + 0.32 + 0.27, H(\mathfrak{g}) = 0.71 \in [0,1]$$

$$H(\mathfrak{g}) = 0.21 + 0.42 + 0.19, H(\mathfrak{g}) = 0.82 \in [0,1]$$

Definition 9 [35]**:** A mapping $\chi: P(X) \to [0,1]$ is known a fuzzy measure based on the following properties:

(a) $\chi(\phi) = 0$ and $\chi(X) = 1$.
(b) If $P \subseteq Q$ then $\chi(P) \leq \chi(Q)$. For any $P, Q \in P(X)$.

We see above investigation process for finding fuzzy measures is quite complex. Therefore, Sugeno proposed a λ-fuzzy measure, which can be defined as follows:

$$\chi(P \cup Q) = \chi(P) + \chi(Q) + \lambda\chi(P) \cdot \chi(Q)$$

where $\chi(P \cup Q) = \phi$ with parameter $\lambda \in [-1, \infty)$ denote correlation among attributes. If $\lambda = 0$, if $\chi(P \cup Q) = \chi(P) + \chi(Q)$, $P \cup Q = \phi$.

Definition 10 [35]**:** Let g and χ be the functions of non-negative real numbers and fuzzy measure on X. The Choquet integral operator of g is expressed as follows:

$$C_\mu(g) = \sum_{i=1}^{n} g(i)\left[\chi(P_{(i)} - \chi\left(P_{(i+1)}\right)\right] \tag{3}$$

here, the permutation on X is indicated by $g_{(i)} \leq g_{(i+1)}$, $i = 1, 2, \ldots, n-1$ and $P_{(i)} = \left\{g_{(i)}, \ldots, g_{(n)}\right\}$ with $P_{(n+1)} = \phi$.

Definition 11 [54]**:** Let $\mathfrak{g} = (\mu, \alpha, \nu)$, $\mathfrak{g}_1 = (\mu_1, \alpha_1, \nu_1)$ and $\mathfrak{g}_2 = (\mu_2, \alpha_2, \nu_2)$ be the three PFVs with, $\aleph \geq 1$ and $\tau > 0$. A list of fundamental operations of Aczel-Alsina t-norm and t-conorm are expressed as follows:

(a) $$\mathfrak{g}_1 \oplus \mathfrak{g}_2 = \begin{pmatrix} 1 - e^{-\left((-\mathcal{L}n(1-\mu_1))^{\aleph} + (-\mathcal{L}n(1-\mu_2))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left((-\mathcal{L}n(\alpha_1))^{\aleph} + (-\mathcal{L}n(\alpha_2))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left((-\mathcal{L}n(\nu_1))^{\aleph} + (-\mathcal{L}n(\nu_2))^{\aleph}\right)^{\frac{1}{\aleph}}} \end{pmatrix}$$

(b) $$\mathfrak{g}_1 \otimes \mathfrak{g}_2 = \begin{pmatrix} e^{-\left((-\mathcal{L}n(\mu_1))^{\aleph} + (-\mathcal{L}n(\mu_2))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left((-\mathcal{L}n(\alpha_1))^{\aleph} + (-\mathcal{L}n(\alpha_2))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ 1 - e^{-\left((-\mathcal{L}n(1-\nu_1))^{\aleph} + (-\mathcal{L}n(1-\nu_2))^{\aleph}\right)^{\frac{1}{\aleph}}} \end{pmatrix}$$

(c) $$\tau\mathfrak{g} = \begin{pmatrix} 1 - e^{-\left(\tau(-\mathcal{L}n(1-\mu))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\tau(-\mathcal{L}n(\alpha))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\tau(-\mathcal{L}n(\nu))^{\aleph}\right)^{\frac{1}{\aleph}}} \end{pmatrix}$$

(d) $\mathfrak{g}^{\tau} = \left(\begin{array}{c} e^{-\left(\tau(-\mathcal{L}n(1-\mu))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\tau(-\mathcal{L}n(1-\alpha))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ 1-e^{-\left(\tau(-\mathcal{L}n(1-\nu))^{\aleph}\right)^{\frac{1}{\aleph}}} \end{array} \right)$

Remark: The above-discussed operations failed, if $\mathfrak{g}_1 = (1, 0, 0)$ and $\mathfrak{g}_2 = (0, 0, 1)$ are two PFVs. Finally, we can say the above-discussed operations of Aczel Alsian t-norms and t-conorms cannot handle extreme MD, AD, and NMD values in a triplet.

Theorem 1 [54]: *Let $\mathfrak{g} = (\mu, \alpha, \nu)$, $\mathfrak{g}_1 = (\mu_1, \alpha_1, \nu_1)$ and $\mathfrak{g}_2 = (\mu_2, \alpha_2, \nu_2)$ be the three PFVs with $\aleph \geq 1$ and $\tau > 0$. Then, a few fundamental OLs are defined as follows:*

(a) $\mathfrak{g}_1 \oplus \mathfrak{g}_2 = \mathfrak{g}_2 \oplus \mathfrak{g}_1$
(b) $\mathfrak{g}_1 \otimes \mathfrak{g}_2 = \mathfrak{g}_2 \otimes \mathfrak{g}_1$
(c) $\tau\left(\mathfrak{g}_1 \oplus \mathfrak{g}_2\right) = \tau\mathfrak{g}_1 \oplus \tau\mathfrak{g}_2, \tau > 0$
(d) $\left(\tau_1 + \tau_2\right)\mathfrak{g} = \tau_1\mathfrak{g} + \tau_2\mathfrak{g}, \tau_1, \tau_2 > 0$
(e) $\left(\mathfrak{g}_1 \otimes \mathfrak{g}_2\right)^{\tau} = \mathfrak{g}_1{}^{\tau} \otimes \mathfrak{g}_2{}^{\tau}, \tau > 0$
(f) $\mathfrak{g}^{\tau_1} \otimes \mathfrak{g}^{\tau_2} = \mathfrak{g}^{\left(\tau_1+\tau_2\right)}, \tau_1, \tau_2 > 0$

3 Picture Fuzzy Choquet Integral Aczel-Alsina Averaging Operators

This section developed a family of robust mathematical approaches of Choquet integral operators and Aczel-Alsina operations such as PFCIAAA and PFCIAAOA operators.

Definition 12: For any PFVs $\mathfrak{g}_{\mathfrak{s}} = \left(\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}\right), \mathfrak{s} = 1, 2, 3, \ldots, n$ and $\left[\mu(P_{(\mathfrak{s})} - \mu\left(P_{(\mathfrak{s}+1)}\right)\right]$ be the fuzzy measure of $\mathfrak{g}_{\mathfrak{s}}$. Then, the PFCIAAA operator is characterized as follows:

$$\begin{aligned} PFCIAAA\left(\mathfrak{g}_1, \mathfrak{g}_2, \ldots, \mathfrak{g}_n\right) = \oplus_{\mathfrak{s}=1}^{n}\left(\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_{\mathfrak{s}}\right), PFCIAAA\left(\mathfrak{g}_1, \mathfrak{g}_2, \ldots, \mathfrak{g}_n\right) \\ = \left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_1 \oplus \left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_2 \oplus \ldots \oplus \\ \left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_n \end{aligned} \tag{4}$$

Theorem 2: *For any PFVs $\mathfrak{g}_{\mathfrak{s}} = \left(\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}\right), \mathfrak{s} = 1, 2, 3, \ldots, n$ and $\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]$ be the fuzzy measure of $\mathfrak{g}_{\mathfrak{s}}$. Then, the aggregated outcome of the PFCIAAA operator is still a PFV, so we can express as follows:*

$$PFCIAAA = \sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_{\mathfrak{s}} = \left(\begin{array}{c} 1-e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(1-\mu_{\mathfrak{s}}))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\alpha_{\mathfrak{s}}))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\nu_{\mathfrak{s}}))^{\aleph}\right)^{\frac{1}{\aleph}}} \end{array} \right) \tag{5}$$

Proof: To prove the aforementioned expression, follow the induction method by taking the value of $p = 2$, and we have:

$$\left[\chi(P_1 - \chi\left(P_2\right)\right]\mathfrak{g}_1 = \left(\begin{array}{c} 1-e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_1-\chi(P_2)\right](-\mathcal{L}n(1-\mu_1))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_1-\chi(P_2)\right](-\mathcal{L}n(\alpha_1))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_1-\chi(P_2)\right](-\mathcal{L}n(\nu_1))^{\aleph}\right)^{\frac{1}{\aleph}}} \end{array} \right)$$

$$\left[\chi(P_2 - \chi\left(P_3\right)\right]\mathfrak{g}_2 = \left(\begin{array}{c} 1-e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_2-\chi(P_3)\right](-\mathcal{L}n(1-\mu_2))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_2-\chi(P_3)\right](-\mathcal{L}n(\alpha_2))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_2-\chi(P_3)\right](-\mathcal{L}n(\nu_2))^{\aleph}\right)^{\frac{1}{\aleph}}} \end{array} \right)$$

By using the above Definition 12, we have:

$$PFCIAAA\left(\mathfrak{g}_1,\mathfrak{g}_2\right)=\left(\begin{array}{c}\left(\begin{array}{c}1-e^{-\left(\sum_{\mathfrak{s}=1}^{n}[\chi(P_1-\chi(P_2)](-\mathcal{L}n(1-\mu_1))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}[\chi(P_1-\chi(P_2)](-\mathcal{L}n(\alpha_1))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}[\chi(P_1-\chi(P_2)](-\mathcal{L}n(\nu_1))^{\aleph}\right)^{\frac{1}{\aleph}}}\end{array}\right),\\ \left(\begin{array}{c}1-e^{-\left(\sum_{\mathfrak{s}=1}^{n}[\chi(P_2-\chi(P_3)](-\mathcal{L}n(1-\mu_2))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}[\chi(P_2-\chi(P_3)](-\mathcal{L}n(\alpha_2))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}[\chi(P_2-\chi(P_3)](-\mathcal{L}n(\nu_2))^{\aleph}\right)^{\frac{1}{\aleph}}}\end{array}\right)\end{array}\right)$$

$$=\left(\begin{array}{c}1-e^{-\left((\sum_{\mathfrak{s}=1}^{n}[\chi(P_1-\chi(P_2)])(-\mathcal{L}n(1-\mu_1))^{\aleph}\right)\oplus\left((\sum_{\mathfrak{s}=1}^{n}[\chi(P_2-\chi(P_3)])(-\mathcal{L}n(1-\mu_2))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left((\sum_{\mathfrak{s}=1}^{n}[\chi(P_1-\chi(P_2)])(-\mathcal{L}n(\alpha_1))^{\aleph}\right)\oplus\left((\sum_{\mathfrak{s}=1}^{n}[\chi(P_2-\chi(P_3)])(-\mathcal{L}n(\alpha_2))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left((\sum_{\mathfrak{s}=1}^{n}[\chi(P_1-\chi(P_2)])(-\mathcal{L}n(\nu_1))^{\aleph}\right)\oplus\left((\sum_{\mathfrak{s}=1}^{n}[\chi(P_2-\chi(P_3)])(-\mathcal{L}n(\nu_2))^{\aleph}\right)^{\frac{1}{\aleph}}}\end{array}\right)$$

$$=\left(\begin{array}{c}1-e^{-\left(\sum_{\mathfrak{s}=1}^{2}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(1-\mu_1))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{2}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\alpha_1))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{2}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\nu_1))^{\aleph}\right)^{\frac{1}{\aleph}}}\end{array}\right)$$

Hence, this is true for $\mathfrak{s}=2$:

i. Now, suppose that this is true for $\mathfrak{s}=k$. Then, we have:

$$PFCIAAA\left(\mathfrak{g}_1,\mathfrak{g}_2,\ldots,\mathfrak{g}_n\right)=\sum_{\mathfrak{s}=1}^{k}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_k=\left(\begin{array}{c}1-e^{-\left(\sum_{\mathfrak{s}=1}^{k}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(1-\mu_k))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{k}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\alpha_k))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{k}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\nu_k))^{\aleph}\right)^{\frac{1}{\aleph}}}\end{array}\right)$$

Now, for $\mathfrak{s}=k+1$. We get:

$$PFCIAAA\left(\mathfrak{g}_1,\mathfrak{g}_2,\ldots,\mathfrak{g}_k,\mathfrak{g}_{k+1}\right)=\oplus_{\mathfrak{s}=1}^{k}\left(\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_k\oplus\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_{k+1}\right)$$

$$=\left(\begin{array}{c}\left(\begin{array}{c}1-e^{-\left(\sum_{\mathfrak{s}=1}^{k}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(1-\mu_k))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{k}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\alpha_k))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{k}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\nu_k))^{\aleph}\right)^{\frac{1}{\aleph}}}\end{array}\right)\oplus\\ \left(\begin{array}{c}1-e^{-\left(\sum_{\mathfrak{s}=1}^{k}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(1-\mu_{k+1}))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{k}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\alpha_{k+1}))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{k}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\nu_{k+1}))^{\aleph}\right)^{\frac{1}{\aleph}}}\end{array}\right)\end{array}\right)$$

$$=\left(\begin{array}{c}1-e^{-\left(\sum_{\mathfrak{s}=1}^{k+1}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(1-\mu_{k+1}))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{k+1}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\alpha_{k+1}))^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{\mathfrak{s}=1}^{k+1}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\nu_{k+1}))^{\aleph}\right)^{\frac{1}{\aleph}}}\end{array}\right)$$

□

Theorem 3: *For any* $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}})$, $\mathfrak{s} = 1, 2, 3, \ldots, n$ *are equal, that is,* $\mathfrak{g}_{\mathfrak{s}} = \mathfrak{g}$ *for all* $\mathfrak{g}$. *Then, we have:*

$PFCIAAA(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n) = \mathfrak{g}$

Proof: Since $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}})$, $\mathfrak{s} = 1, 2, 3, \ldots, n$. Then:

$$PFCIAAA(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n) = \sum_{\mathfrak{s}=1}^{n} \left(\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right] \mathfrak{g}_{\mathfrak{s}} \right)$$

$$= \begin{pmatrix} 1 - e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(1-\mu_{\mathfrak{s}}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\alpha_{\mathfrak{s}}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\nu_{\mathfrak{s}}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}} \end{pmatrix}$$

$$= \begin{pmatrix} 1 - e^{-\left(-\mathcal{L}n\left(1-(\mu_{\mathfrak{s}})\right)^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\left(-\mathcal{L}n(\alpha_{\mathfrak{s}})\right)^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\left(-\mathcal{L}n(\nu_{\mathfrak{s}})\right)^{\aleph}\right)^{\frac{1}{\aleph}}} \end{pmatrix} = \mathfrak{g}$$

□

Theorem 4: *Let* $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}})$, $\mathfrak{s} = 1, 2, 3, \ldots, n$ *be the collection of* $PFVs$, *with PA* $\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]$ *operator for each* $\mathfrak{g}_{\mathfrak{s}}$. *Let* $\mathfrak{g}^- = \min(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots, \mathfrak{g}_n)$ *and* $\mathfrak{g}^+ = \max(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots, \mathfrak{g}_n)$. *So,*

$\mathfrak{g}^- \leq PFCIAAA(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots, \mathfrak{g}_n) \leq \mathfrak{g}^+$

Proof: Let $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}})$, $\mathfrak{s} = 1, 2, 3, \ldots, n$ be the collection of PFVs, with PA $\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)$ operator for each $\mathfrak{g}_{\mathfrak{s}}$. Let $\mathfrak{g}^- = \min(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots, \mathfrak{g}_n) = (\mu^-, \alpha^-, \nu^-)$ and $\mathfrak{g}^+ = \max(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n) = (\mu^+, \alpha^+, \nu^+)$. We have, $\mu^- = \min_{\mathfrak{s}}\{\mu_{\mathfrak{s}}\}, \alpha^- = \max_{\mathfrak{s}}\{\alpha_{\mathfrak{s}}\}$ *and* $\nu^- = \max_{\mathfrak{s}}\{\nu_{\mathfrak{s}}\}$ and $\mu^+ = \max_{\mathfrak{s}}\{\alpha_{\mathfrak{s}}\}, \alpha^+ = \min_{\mathfrak{s}}\{\alpha_{\mathfrak{s}}\}, \nu^+ = \min_{\mathfrak{s}}\{\alpha_{\mathfrak{s}}\}$.

$$1 - e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(1-\mu^-\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}} \leq 1 - e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(1-\mu_{\mathfrak{s}}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}$$
$$\leq 1 - e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(1-\mu^+\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}$$

$$e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\alpha^-\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}} \leq e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\alpha_{\mathfrak{s}}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}$$
$$\leq e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\alpha^+\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}$$

and

$$e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\nu^-\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}} \leq e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\nu_{\mathfrak{s}}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}$$
$$\leq e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})}-\chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\nu^+\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}$$

So,

$\mathfrak{g}^- \leq PFCIAAA(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n) \leq \mathfrak{g}^+$

□

Theorem 5: *Let* $\mathfrak{g}_{\mathfrak{s}}$ *and* $\mathfrak{g}_{\mathfrak{s}}^{'}$, $\mathfrak{s} = 1, 2, 3, \ldots, n$ *be two sets of PFVs, if* $\mathfrak{g}_{\mathfrak{s}} \leq \mathfrak{g}_{\mathfrak{s}}^{'}$ *for all* $\mathfrak{g}$. *So,*

$$PFCIAAA\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n\right) \leq PFCIAAA\left(\mathfrak{g}_1^{'}, \mathfrak{g}_2^{'}, \mathfrak{g}_3^{'}, \ldots \mathfrak{g}_n^{'}\right)$$

Proof: Let $\mathfrak{g}_{\mathfrak{s}}$ and $\mathfrak{g}_{\mathfrak{s}}^{'}$, $\mathfrak{s} = 1, 2, 3, \ldots, n$ be two sets of PFVs, we can write in the following method:

$$1 - e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(1-\mu_{\mathfrak{s}}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}} \leq 1 - e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(1-\mu_{\mathfrak{s}}^{\prime}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}$$

$$e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\alpha_{\mathfrak{s}}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}} \geq e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\alpha_{\mathfrak{s}}^{\prime}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}$$

$$e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\nu_{\mathfrak{s}}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}} \geq e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\nu_{\mathfrak{s}}^{\prime}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}$$

Hence, it is proved that $PFCIAAA\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots, \mathfrak{g}_n\right) \leq PFCIAAA\left(\mathfrak{g}_1^{'}, \mathfrak{g}_2^{'}, \mathfrak{g}_3^{'}, \ldots, \mathfrak{g}_n^{'}\right)$. □

Definition 13: For any PFVs $\mathfrak{g}_{\mathfrak{s}} = \left(\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}\right), \mathfrak{s} = 1, 2, 3, \ldots, n$ and $\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]$ be the fuzzy measure of $\mathfrak{g}_{\mathfrak{s}}$. Then, the PFCIAAOA operator is characterized as follows:

$$PFCIAAOA\left(\mathfrak{g}_1, \mathfrak{g}_2, \ldots, \mathfrak{g}_n\right) = \oplus_{\mathfrak{s}=1}^{n}\left(\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_{\mathfrak{p}(\mathfrak{s})}\right)$$

$$\begin{aligned} &PFCIAAOA\left(\mathfrak{g}_1, \mathfrak{g}_2, \ldots, \mathfrak{g}_n\right) \\ &\quad = \left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_{\mathfrak{p}(1)} \oplus \left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_{\mathfrak{p}(2)} \oplus \ldots \oplus \left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\mathfrak{g}_{\mathfrak{p}(n)} \end{aligned} \tag{6}$$

where $\mathfrak{p}\left(\mathfrak{s}\right) \leq \mathfrak{p}\left(\mathfrak{s}-1\right)$ be a class of finite set of permutations $\left(\mathfrak{p}\left(1\right), \mathfrak{p}\left(2\right), \ldots, \mathfrak{p}\left(n\right)\right)$ of $\mathfrak{g}_{\mathfrak{s}}$.

Theorem 6: *For any PFVs* $\mathfrak{g}_{\mathfrak{s}} = \left(\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}\right), \mathfrak{s} = 1, 2, 3, \ldots, n$ *and* $\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]$ *be the fuzzy measure of* $\mathfrak{g}_{\mathfrak{s}}$. *Then, the aggregated outcome of the PFCIAAOA operator is still a PFV, so we can express as follows:*

$$PFCIAAOA = \begin{pmatrix} 1 - e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(1-\mu_{\mathfrak{p}(\mathfrak{s})}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\alpha_{\mathfrak{p}(\mathfrak{s})}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\nu_{\mathfrak{p}(\mathfrak{s})}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}} \end{pmatrix} \tag{7}$$

Theorem 7: *If all* $\mathfrak{g}_{\mathfrak{s}} = \left(\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}\right), \mathfrak{s} = 1, 2, 3, \ldots, n$ *are equal, that is,* $\mathfrak{g}_{\mathfrak{p}(\mathfrak{s})} = \mathfrak{g}$ *for all* $\mathfrak{g}$. *Then, we have:*

$$PFCIAAOA\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n\right) = \mathfrak{g}$$

Theorem 8: *Let* $\mathfrak{g}_{\mathfrak{s}} = \left(\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}\right), \mathfrak{s} = 1, 2, 3, \ldots, n$ *be the collection of* $PFVs$, *with PA* $\left[\chi\left(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]$ *operator for each* $\mathfrak{g}_{\mathfrak{s}}$. *Let* $\mathfrak{g}^{-} = \min\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n\right)$ *and* $\mathfrak{g}^{+} = \max\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n\right)$. *So,*

$$\mathfrak{g}^{-} \leq PFCIAAOA\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n\right) \leq \mathfrak{g}^{+}$$

Theorem 9: *Let* $\mathfrak{g}_{\mathfrak{s}}$ *and* $\mathfrak{g}_{\mathfrak{s}}^{'}$, $\mathfrak{s} = 1, 2, 3, \ldots, n$ *be two sets of PFVs, if* $\mathfrak{g}_{\mathfrak{s}} \leq \mathfrak{g}_{\mathfrak{s}}^{'}$ *and we have:*

$$PFCIAAOA\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots, \mathfrak{g}_n\right) \leq PFCIAAOA\left(\mathfrak{g}_1^{'}, \mathfrak{g}_2^{'}, \mathfrak{g}_3^{'}, \ldots \mathfrak{g}_n^{'}\right)$$

4 Picture Fuzzy Choquet Integral Aczel-Alsina Geometric Aggregation Operators

In order to achieve more flexible and effective mathematical methodologies, we derived a series of new approaches in the presence of picture fuzzy theory, such as PFCIAAG and PFCIAAOG operators.

Definition 14: Let $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}), \mathfrak{s} = 1, 2, 3, \ldots, n$ be the collection of PFVs, with PA $\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]$ operator for each $\mathfrak{g}_{\mathfrak{s}}$. Then, the PFCIAAG operator is particularized as:

$$PFCIAAG\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n\right) = \otimes_{\mathfrak{s}=1}^{n} \mathfrak{g}_{\mathfrak{s}}^{\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]}$$
$$PFCIAAG\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n\right) = \mathfrak{g}_1^{\left[\chi(P_{(1)} - \chi\left(P_{(2)}\right)\right]} \otimes \mathfrak{g}_2^{\left[\chi(P_{(2)} - \chi\left(P_{(3)}\right)\right]} \otimes \ldots \otimes \mathfrak{g}_n^{\left[\chi(P_{(n)} - \chi\left(P_{(n+1)}\right)\right]} \tag{8}$$

Theorem 10: *For any PFVs $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}), \mathfrak{s} = 1, 2, 3, \ldots, n$ and $\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]$ be the fuzzy measure of $\mathfrak{g}_{\mathfrak{s}}$. Then, the aggregated outcome of the PFCIAAG operator is still a PFV, so we can express as follows:*

$$PFCIAAG = \left(\begin{array}{c} e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\mu_{\mathfrak{s}}))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(\alpha_{\mathfrak{s}}))^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ 1 - e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right](-\mathcal{L}n(1-\nu_{\mathfrak{s}}))^{\aleph}\right)^{\frac{1}{\aleph}}} \end{array}\right) \tag{9}$$

Theorem 11: *If all $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}), \mathfrak{s} = 1, 2, 3, \ldots, n$ are equal, that is, $\mathfrak{g}_n = \mathfrak{g}$ for all $\mathfrak{g}$. Then:*

$$PFCIAAG\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n\right) = \mathfrak{g}$$

Theorem 12: *For any PFVs $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}), \mathfrak{s} = 1, 2, 3, \ldots, n$ and $\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]$ be the fuzzy measure of $\mathfrak{g}_{\mathfrak{s}}$. Let $\mathfrak{g}^{-} = \min\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n\right)$ and $\mathfrak{g}^{+} = \max\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n\right)$. Then*:

$$\mathfrak{g}^{-} \leq PFCIAAG\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots, \mathfrak{g}_n\right) \leq \mathfrak{g}^{+}$$

Theorem 13: *Let $\mathfrak{g}_{\mathfrak{s}}$ and $\mathfrak{g}_{\mathfrak{s}}^{'}, \mathfrak{s} = 1, 2, 3, \ldots, n$ be two sets of PFVs, if $\mathfrak{g}_{\mathfrak{s}} \leq \mathfrak{g}_{\mathfrak{s}}^{'}$ for all $\mathfrak{g}$. Then:*

$$PFCIAAG\left(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n\right) \leq PFCIAAG\left(\mathfrak{g}_1^{'}, \mathfrak{g}_2^{'}, \mathfrak{g}_3^{'}, \ldots \mathfrak{g}_n^{'}\right)$$

Definition 15: For any PFVs $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}), \mathfrak{s} = 1, 2, 3, \ldots, n$ and $\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]$ be the fuzzy measure of $\mathfrak{g}_{\mathfrak{p}(\mathfrak{s})}$. Then, the PFCIAAOG operator is expressed as follows:

$$PFCIAAOG\left(\mathfrak{g}_1, \mathfrak{g}_2, \ldots, \mathfrak{g}_n\right) = \otimes_{\mathfrak{s}=1}^{n} \mathfrak{g}_{\mathfrak{p}(\mathfrak{s})}^{\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]}$$
$$PFCIAAOG\left(\mathfrak{g}_1, \mathfrak{g}_2, \ldots, \mathfrak{g}_n\right) = \mathfrak{g}_{\mathfrak{p}(1)}^{\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]} \otimes \mathfrak{g}_{\mathfrak{p}(2)}^{\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]} \otimes \ldots \otimes \mathfrak{g}_{\mathfrak{p}(n)}^{\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]} \tag{10}$$

Theorem 14: *For any PFVs $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}}), \mathfrak{s} = 1, 2, 3, \ldots, n$ and $\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]$ be the fuzzy measure of $\mathfrak{g}_{\mathfrak{s}}$. Then, the aggregated outcome of the PFCIAAOG operator is still a PFV, so we can express as follows:*

$$PFCIAAOG = \left(\begin{array}{c} e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\mu_{\mathfrak{p}(\mathfrak{s})}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(\alpha_{\mathfrak{p}(\mathfrak{s})}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}, \\ 1 - e^{-\left(\sum_{\mathfrak{s}=1}^{n}\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]\left(-\mathcal{L}n\left(1-\nu_{\mathfrak{p}(\mathfrak{s})}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}} \end{array}, \right) \tag{11}$$

where $\mathfrak{p}(\mathfrak{s}) \leq \mathfrak{p}(\mathfrak{s}-1)$ be a class of a finite set of permutations $\left(\mathfrak{p}(1), \mathfrak{p}(2), \ldots, \mathfrak{p}(n)\right)$ of $\mathfrak{g}_{\mathfrak{s}}$.

Theorem 15: *If all* $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}})$, $\mathfrak{s} = 1, 2, 3, \ldots, n$ *are equal, that is,* $\mathfrak{g}_{\mathfrak{p}(\mathfrak{s})} = \mathfrak{g}$ *for all* $\mathfrak{g}$. *Then, we have:*

$$PFCIAAOG(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n) = \mathfrak{g}$$

Theorem 16: *Let* $\mathfrak{g}_{\mathfrak{s}}$ *and* $\mathfrak{g}'_{\mathfrak{s}}$, $\mathfrak{s} = 1, 2, 3, \ldots, n$ *be two sets of PFVs, if* $\mathfrak{g}_{\mathfrak{s}} \le \mathfrak{g}'_{\mathfrak{s}}$ *for all* $\mathfrak{g}$. *So,*

$$PFCIAAOG(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n) \le PFCIAAOG\left(\mathfrak{g}'_1, \mathfrak{g}'_2, \mathfrak{g}'_3, \ldots \mathfrak{g}'_n\right)$$

Theorem 17: *Let* $\mathfrak{g}_{\mathfrak{s}} = (\mu_{\mathfrak{s}}, \alpha_{\mathfrak{s}}, \nu_{\mathfrak{s}})$, $\mathfrak{s} = 1, 2, 3, \ldots, n$ *be the collection of* $PFVs$, *with PA* $\left[\chi(P_{(\mathfrak{s})} - \chi\left(P_{(\mathfrak{s}+1)}\right)\right]$ *operator for each* $\mathfrak{g}_{\mathfrak{s}}$. *Let* $\mathfrak{g}^- = \min(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n)$ *and* $\mathfrak{g}^+ = \max(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n)$. *So,*

$$\mathfrak{g}^- \le PFCIAAOG(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \ldots \mathfrak{g}_n) \le \mathfrak{g}^+$$

5 Multi-Criteria Decision-Making Problem Based on Picture Fuzzy Framework

To resolve the MADM problem, assume a family of alternative $\mathbb{A} = (\mathbb{A}_1, \mathbb{A}_2, \ldots, \mathbb{A}_m)$ and a set of criteria $\Pi\mathfrak{h} = (\Pi\mathfrak{h}_1, \Pi\mathfrak{h}_2, \ldots, \Pi\mathfrak{h}_n)$. By considering Choquet integral operators, some features are how to classify different types of attributes the aggregation process to evaluate a suitable optimal option with derived mathematical strategies. To serve this purpose, decision-makers adopted various types of information in the form of picture-fuzzy information such as $a_{ij} = \left(\mu_{ij}, \alpha_{ij}, \nu_{ij}\right), (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$, where $\mu_{ij} \in [0, 1], \alpha_{ij} \in [0, 1]$ and $\nu_{ij} \in [0, 1]$ with restricted condition $0 \le \mu_{ij} + \alpha_{ij} + \nu_{ij} \le 1$. The decision maker maintained picture fuzzy information into a decision matrix with alternatives and attributes. To make a decision about a suitable optimal option, aggregate accumulated information, taking into account the steps of an algorithm of the MADM problem. Fig. 2 also portrays the steps of an algorithm of the MADM problem in the light of PF information.

Step 1: Construct a decision matrix of picture fuzzy information in the form of alternatives and attributes:

$$R = \begin{pmatrix} a_{11}, a_{12}, \ldots, a_{1n} \\ a_{21}, a_{22}, \ldots, a_{2n} \\ \cdots \quad \cdots \quad \cdots \\ a_{m1}, a_{m2}, \ldots, a_{mn} \end{pmatrix} \tag{12}$$

Step 2: To consider the same type of attributes or criteria, the following expression is used to normalize the standard decision matrix into a normalized decision matrix:

$$R^C = \left(a_{ij}\right)_{m \times n} = \begin{cases} \left(\mu_{ij}, \alpha_{ij}, \nu_{ij}\right) & beneficial attributes \\ \left(\nu_{ij}, \alpha_{ij}, \mu_{ij}\right) & non-beneficial attributes \end{cases} \tag{13}$$

If all discussed attributes are the same type, then no need to normalize the process.

Step 2: Compute score and accuracy values to rank the practical evaluation of different preferences by using Definition 8. So, a practical evaluation of a_{ij} and a_{ik} of alternative or individual provides re-ordering among different preferences such as $a_{i(j)} \le a_{i(j+1)}$.

Step 3: It is clear that the investigation of fuzzy measures is quite complex. However, we can compute fuzzy measures by using different expressions or formulas derived by several mathematicians, like linear methods [55], quadratic methods [56,57] and genetic algorithms [58], and many methods are available in the literature [35,59].

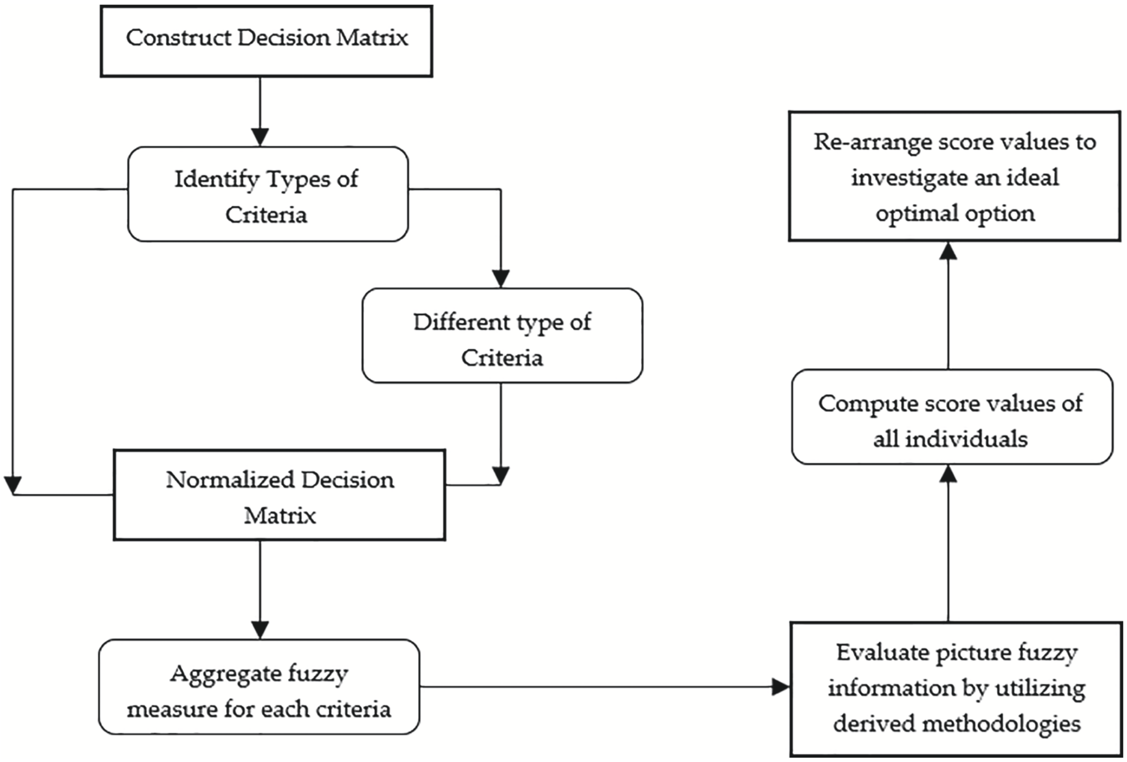

Figure 2: Flowcharts of the MADM problem

Step 4: Aggregate organized picture fuzzy information by utilizing the following expressions:

$$PFCIAAA\left(a_{i1},\ldots,a_{in}\right)=\begin{pmatrix}1-e^{-\left(\sum_{j=1}^{n}\left[\chi\left(P_{(j)}-\chi\left(P_{(j+1)}\right)\right]\left(-\mathcal{L}n\left(1-\mu_{ij}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{j=1}^{n}\left[\chi\left(P_{(j)}-\chi\left(P_{(j+1)}\right)\right]\left(-\mathcal{L}n\left(\alpha_{ij}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{j=1}^{n}\left[\chi\left(P_{(j)}-\chi\left(P_{(j+1)}\right)\right]\left(-\mathcal{L}n\left(\nu_{ij}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}\end{pmatrix}\tag{14}$$

and

$$PFCIAAG\left(a_{i1},\ldots,a_{in}\right)=\begin{pmatrix}e^{-\left(\sum_{j=1}^{n}\left[\chi\left(P_{(j)}-\chi\left(P_{(j+1)}\right)\right]\left(-\mathcal{L}n\left(\mu_{ij}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}},\\ e^{-\left(\sum_{j=1}^{n}\left[\chi\left(P_{(j)}-\chi\left(P_{(j+1)}\right)\right]\left(-\mathcal{L}n\left(\alpha_{ij}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}\\ 1-e^{-\left(\sum_{j=1}^{n}\left[\chi\left(P_{(j)}-\chi\left(P_{(j+1)}\right)\right]\left(-\mathcal{L}n\left(1-\nu_{ij}\right)\right)^{\aleph}\right)^{\frac{1}{\aleph}}}\end{pmatrix}\tag{15}$$

Step 5: Investigate the score or accuracy values of all alternatives according to their need by using the following expressions:

$$AE\left(\Gamma\mathfrak{h}_i\right)=\left(\frac{1}{3}\right)\left(2+\mu_{ij}-\alpha_{ij}-\nu_{ij}\right),AE\left(\Gamma\mathfrak{h}_i\right)\in[0,1]$$

and

$$H\left(\Gamma\mathfrak{h}_i\right)=\left(\mu_{ij}+\alpha_{ij}+\nu_{ij}\right),H\left(I\right)\in[0,1]$$

Step 6: To analyze a reliable option, we investigate the ranking of alternatives based on computed scores or accuracy values.

5.1 Experimental Case Study

The importance of a reliable electric supply cannot be overstated, as electricity powers nearly every aspect of modern life. It serves as the backbone of industries, homes, healthcare, education, and communication systems. In industries, electricity drives machinery, automation, and production processes, enabling large-scale manufacturing and economic growth. It powers computers, servers, and communication networks, ensuring businesses remain operational and connected to the global economy. Without electricity, modern economies would grind to a halt, with productivity and innovation severely limited. Electricity is also vital for critical infrastructure, such as water and transportation systems. It powers pumps for clean water distribution and sewage systems, ensuring public health and sanitation. Electric transportation, from trains to electric vehicles, is becoming increasingly important as societies aim for sustainability and reduced carbon emissions. A stable electricity supply is key to meeting the growing energy demands of our increasingly digital and connected world, supporting both everyday activities and long-term progress.

Electric transformers are essential components in the transmission and distribution of electrical power. They play a critical role in adjusting the voltage levels, allowing electricity generated at power plants to be efficiently transported over long distances. By stepping up the voltage for long-distance transmission, transformers reduce energy losses due to resistance in the wires. When the electricity reaches its destination, another transformer steps down the voltage to a safer, usable level for homes, businesses, and industries. Without transformers, the efficient and widespread distribution of electricity would be nearly impossible, as high-voltage transmission is crucial to minimizing energy loss. In this case study, we assess different types of electric transformers for improving the supply of electricity to spinning textile factories. To achieve this goal, there are five different alternatives are expressed as follows:

Step-up and step-down transformers $\mathbb{A}_1$

Single-phase and three-phase transformers $\mathbb{A}_2$

Distribution and power transformers $\mathbb{A}_3$

Oil-cooled and dry-type transformers $\mathbb{A}_4$

Core type and shell type transformers $\mathbb{A}_5$

Above above-discussed transformers are evaluated under the following four expressed criteria or attributes. The expert assigns fuzzy measures to each criterion or attribute, which are independent to express correlation among the expressed attributes. Some appropriate characteristics of electric transformers also demonstrated in Fig. 3.

Efficient Power Transmission $\Gamma\mathfrak{h}_1$: One of the most significant advantages of transformers is their ability to step up or step-down voltage levels. High voltage is required for long-distance power transmission to minimize energy loss due to resistance in transmission lines. By stepping up the voltage, transformers reduce the current, which in turn reduces losses.

Flexibility in Power Distribution $\Gamma\mathfrak{h}_2$: Transformers allow for the interconnection of power grids with varying voltage levels, enhancing the flexibility and resilience of electrical systems.

Grid Stability and Reliability $\Gamma\mathfrak{h}_3$: Transformers play a crucial role in maintaining the stability and reliability of power grids. They allow for the regulation and balancing of electricity flow, ensuring that supply meets demand even during peak periods.

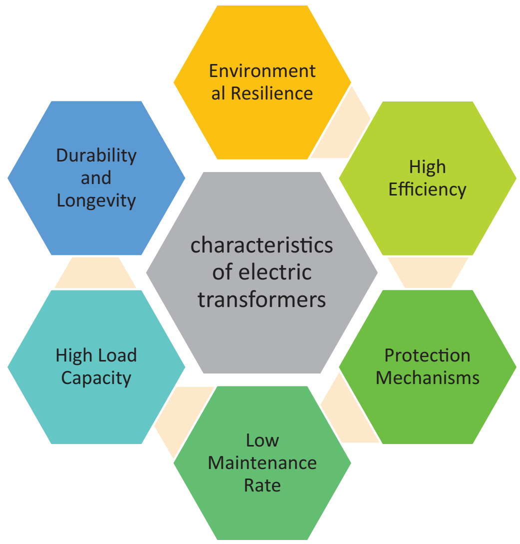

Figure 3: Characteristics of electric transformers

Durability and Low Maintenance Ђ_4: Electric transformers are highly durable and require relatively low maintenance. Most transformers can operate efficiently for decades, often with minimal upkeep, making them a reliable and long-lasting investment for power utilities and industries.

By considering an algorithm of the MADM problem and discovering mathematical methodologies, we evaluated a dominant electric transformer under the following steps of the aggregating procedure.

Step 1. Owning a decision matrix of picture fuzzy information in the form of diverse criteria that are associated with each alternative or individual. Table 1 maintains the picture fuzzy information about different types of transformers. The above-discussed experimental case study has only one type of criterion, such as beneficial types. So, there is no need to action on step 2. We can proceed with the evaluation process with the given information in Table 1.

Table 1: Decision matrix of picture fuzzy information

Alt.	Ђ_1	Ђ_2	Ђ_3	Ђ_4
$\mathbb{A}_1$	$(0.32, 0.44, 0.15)$	$(0.21, 0.64, 0.12)$	$(0.09, 0.61, 0.24)$	$(0.23, 0.09, 0.23)$
$\mathbb{A}_2$	$(0.26, 0.26, 0.45)$	$(0.42, 0.31, 0.24)$	$(0.51, 0.32, 0.08)$	$(0.45, 0.21, 0.07)$
$\mathbb{A}_3$	$(0.23, 0.21, 0.33)$	$(0.09, 0.32, 0.55)$	$(0.43, 0.16, 0.34)$	$(0.57, 0.23, 0.09)$
$\mathbb{A}_4$	$(0.45, 0.22, 0.26)$	$(0.13, 0.61, 0.21)$	$(0.28, 0.27, 0.12)$	$(0.72, 0.09, 0.17)$
$\mathbb{A}_5$	$(0.33, 0.55, 0.09)$	$(0.09, 0.15, 0.54)$	$(0.14, 0.32, 0.35)$	$(0.08, 0.23, 0.41)$

Step 2. The normalization process is meaningless because all discussed criteria are beneficial and same type.

Step 3. By using the score function, re-ordering the attribute information associated with each alternative or individual. So, Table 2 maintained a new ordering of attributes corresponding to each alternative.

Table 2: Arranged decision matrix of picture fuzzy information

Alt.	Ҧ_1	Ҧ_2	Ҧ_3	Ҧ_4
$\mathbb{A}_1$	$(0.23, 0.09, 0.23)$	$(0.32, 0.44, 0.15)$	$(0.21, 0.61, 0.12)$	$(0.09, 0.61, 0.24)$
$\mathbb{A}_2$	$(0.45, 0.21, 0.07)$	$(0.51, 0.32, 0.08)$	$(0.26, 0.26, 0.45)$	$(0.42, 0.31, 0.24)$
$\mathbb{A}_3$	$(0.57, 0.23, 0.09)$	$(0.43, 0.16, 0.34)$	$(0.23, 0.21, 0.33)$	$(0.09, 0.32, 0.55)$
$\mathbb{A}_4$	$(0.72, 0.09, 0.17)$	$(0.45, 0.22, 0.26)$	$(0.28, 0.27, 0.12)$	$(0.13, 0.61, 0.21)$
$\mathbb{A}_5$	$(0.33, 0.55, 0.09)$	$(0.14, 0.32, 0.35)$	$(0.08, 0.23, 0.41)$	$(0.09, 0.15, 0.54)$

Step 4. We see that the investigation process of fuzzy measures is quite complex. In order to avoid these complexities, we have adopted fuzzy measures of criteria and criterion sets from [35].

$$\chi(\text{Ҧ}_1) = 0.40, \chi(\text{Ҧ}_2) = 0.25, \chi(\text{Ҧ}_3) = 0.3, \chi(\text{Ҧ}_4) = 0.20, \chi(\text{Ҧ}_1, \text{Ҧ}_2) = 0.76, \chi(\text{Ҧ}_1, \text{Ҧ}_3) = 0.65,$$
$$\chi(\text{Ҧ}_1, \text{Ҧ}_4) = 0.50, \chi(\text{Ҧ}_2, \text{Ҧ}_3) = 0.34, \chi(\text{Ҧ}_3, \text{Ҧ}_4) = 0.42, \chi(\text{Ҧ}_1, \text{Ҧ}_2, \text{Ҧ}_3) = 0.85,$$
$$\chi(\text{Ҧ}_1, \text{Ҧ}_2, \text{Ҧ}_4) = 0.68, \chi(\text{Ҧ}_1, \text{Ҧ}_3, \text{Ҧ}_4) = 0.76, = 0.45, \chi(\text{Ҧ}_2, \text{Ҧ}_4)\ \chi(\text{Ҧ}_2, \text{Ҧ}_3, \text{Ҧ}_4) = 0.57,$$
$$\chi(\text{Ҧ}_1, \text{Ҧ}_2, \text{Ҧ}_3, \text{Ҧ}_4) = 1.0$$

Step 5. We utilized our proposed aggression operators of PFCIAAA and PFCIAAG operators to aggregate the given information of each attribute corresponding to each alternative that is shown in Table 2. However, Table 3 illustrates aggregated outcomes by the derived methodologies.

Table 3: Results of PFCIAAA and PFCIAAG operators

Alt.	PFCIAAA	PFCIAAG
$\mathbb{A}_1$	$(0.1953, 0.4288, 0.1788)$	$(0.1693, 0.4288, 0.1886)$
$\mathbb{A}_2$	$(0.4044, 0.2801, 0.1910)$	$(0.3857, 0.2801, 0.2566)$
$\mathbb{A}_3$	$(0.2933, 0.2356, 0.3300)$	$(0.2111, 0.2356, 0.3964)$
$\mathbb{A}_4$	$(0.3649, 0.2971, 0.1815)$	$(0.2671, 0.2971, 0.1904)$
$\mathbb{A}_5$	$(0.1380, 0.2391, 0.3504)$	$(0.1156, 0.2391, 0.4145)$

Step 6. Table 4 illustrates computed score values associated with each alternative using Definition 5. To analyze the optimal option for the electric transformer, rank the computed score values of all individuals or alternatives. Fore a better understanding of the aggregated information, we conceive a graphical representation of a bar chart of Fig. 4. This technique plays a significant role in the MADM problem and makes it easier for the readers.

Table 4: Computed score values of all individuals with their ranking

Alt.	$AE(\mathbb{A}_1)$	$AE(\mathbb{A}_2)$	$AE(\mathbb{A}_3)$	$AE(\mathbb{A}_4)$	$AE(\mathbb{A}_5)$	Ranking of alternatives
PFCIAAA	0.5292	0.6444	0.5759	0.6287	0.5162	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
PFCIAAG	0.5173	0.6163	0.5264	0.5932	0.4873	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$

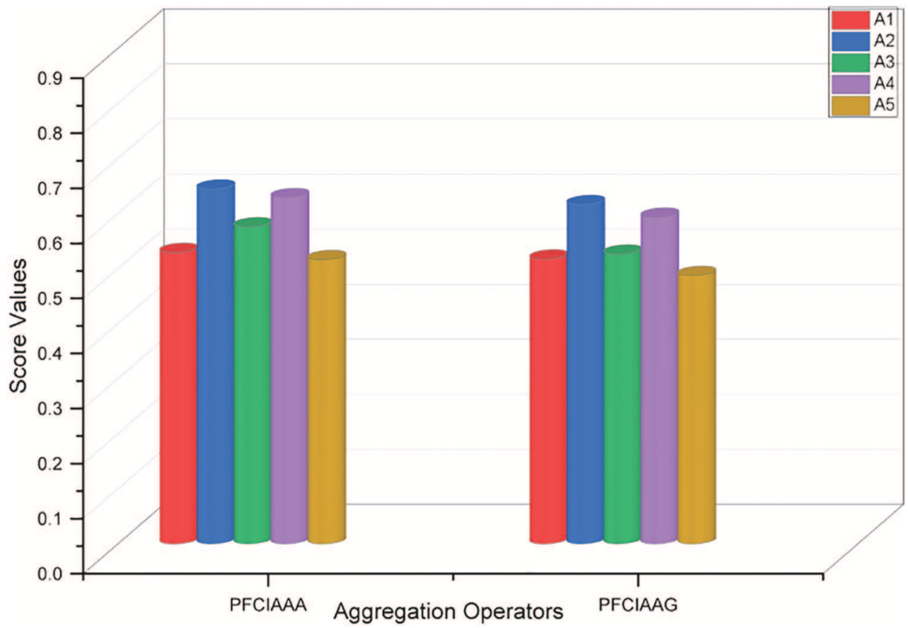

Figure 4: Obtained score values by the PFCIAAA and PFCIAAG operators

5.2 Influence Study

In order to maintain the reliability and consistency of derived mathematical approaches in the MADM problem. The decision maker faces diverse challenges during the decision analysis process due to redundant and incomplete information on human opinion. To investigate an ideal and efficient aggregated result, the parametric value of Aczel-Alsina operators can be changed in step 4 of an algorithm for the MADM problem.

By setting different parametric values from $\aleph = 1$ to $\aleph = 200$ in the PFCIAAA and PFCIAAG operators. Table 5 demonstrates aggregated results of score values associated with all individuals, with their ranking by the PFCIAAA operators. From Table 5, the analysis of the ranking of all individuals remains the same $\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_5$ with $\aleph \geq 1$. However, the results of score values gradually increase by increasing the parametric value of Aczel-Alsina operators.

Table 5: Influence of the parametric value of Aczel-Alsina t-norms on the MADM problem

Parametric values	$AE(\mathbb{A}_1)$	$AE(\mathbb{A}_2)$	$AE(\mathbb{A}_3)$	$AE(\mathbb{A}_4)$	$AE(\mathbb{A}_5)$	Ranking of alternatives
$\aleph = 1$	0.5292	0.6444	0.5759	0.6287	0.5162	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
$\aleph = 2$	0.5671	0.6583	0.6134	0.6727	0.5484	$\mathbb{A}_4 > \mathbb{A}_2 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
$\aleph = 20$	0.6839	0.7192	0.7469	0.8125	0.6763	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_5$
$\aleph = 35$	0.6922	0.7289	0.7582	0.8230	0.6851	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_5$
$\aleph = 50$	0.6955	0.7331	0.7628	0.8271	0.6886	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_5$
$\aleph = 65$	0.6974	0.7354	0.7652	0.8294	0.6905	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_5$
$\aleph = 85$	0.6988	0.7373	0.7671	0.8311	0.6919	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_5$
$\aleph = 100$	0.6995	0.7382	0.7681	0.8319	0.6926	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_5$
$\aleph = 125$	0.7002	0.7392	0.7691	0.8329	0.6935	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_5$
$\aleph = 140$	0.7006	0.7397	0.7696	0.8333	0.6938	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_5$
$\aleph = 185$	0.7012	0.7406	0.7705	0.8341	0.6945	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_5$

(Continued)

Table 5 (continued)

Parametric values	$AE(\mathbb{A}_1)$	$AE(\mathbb{A}_2)$	$AE(\mathbb{A}_3)$	$AE(\mathbb{A}_4)$	$AE(\mathbb{A}_5)$	Ranking of alternatives
$\aleph = 200$	0.7014	0.7408	0.7707	0.8343	0.6947	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_5$

Similarly, the above procedure was repeated for the PFCIAAG operator and the aggregated results are mentioned in Table 6. Ranking of alternatives $\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$ remains fixed when we change the parametric value of Aczel-Alsina operations in the PFCIAAG operator. The consistency in aggregated results shows the reliability and effectiveness of derived mathematical methodologies. That's why diagnosed research work and mathematical approaches are more efficient and superior to existing aggregation operators that exist in the literature.

Table 6: Influence of the parametric value of Aczel-Alsina t-norms on the MADM problem.

Parametric values	$AE(\mathbb{A}_1)$	$AE(\mathbb{A}_2)$	$AE(\mathbb{A}_3)$	$AE(\mathbb{A}_4)$	$AE(\mathbb{A}_5)$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
$\aleph = 1$	0.5173	0.6163	0.5264	0.5932	0.4873	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_5$
$\aleph = 2$	0.5403	0.5974	0.5065	0.5940	0.4816	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$
$\aleph = 20$	0.5860	0.5377	0.4618	0.5963	0.4674	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$
$\aleph = 35$	0.5865	0.5356	0.4609	0.5952	0.4660	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$
$\aleph = 50$	0.5866	0.5349	0.4606	0.5947	0.4654	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$
$\aleph = 65$	0.5867	0.5346	0.4605	0.5944	0.4650	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$
$\aleph = 85$	0.5867	0.5343	0.4604	0.5941	0.4646	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$
$\aleph = 100$	0.5867	0.5341	0.4603	0.5940	0.4644	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$
$\aleph = 125$	0.5867	0.5340	0.4603	0.5939	0.4642	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$
$\aleph = 140$	0.5867	0.5339	0.4602	0.5938	0.4641	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$
$\aleph = 185$	0.5867	0.5338	0.4602	0.5937	0.4639	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$
$\aleph = 200$	0.5867	0.5337	0.4602	0.5937	0.4639	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3$

6 Comparison Method

In this section, a comprehensive comparison technique is conducted to illustrate the advantages and effectiveness of pioneered mathematical methodologies with previously diagnosed aggregation operators by numerous research scholars. To serve this purpose, we employed different mathematical approaches to the given information in Table 1, which were invented by different scientists namely Wang et al. [24], Senapati [54], Garg [60], Seikh and Mandal [61], Jana et al. [62], Wei [63], Naeem et al. [64]. A brief discussion about these aggregation operators is also presented as picture fuzzy Aczel-Alsina weighted average and picture fuzzy Aczel-Alsina weighted geometric operators by Senapati [54] and Naeem et al. [64], picture fuzzy weighted average and picture fuzzy weighted geometric by Garg [60] and Wang et al. [24], respectively. Seikh and Mandal [61] deliberated the theory of Frank aggregation operators by incorporating picture fuzzy environments. Jana et al. [62] utilized the properties of Dombi aggregation operators to develop new weighted averaging and weighted geometric operators. An innovative approach of Hamacher aggregation operators with picture fuzzy information was developed by Wei [63]. Tables 7 and 8 considered the aggregated results by the existing mathematical approaches in the literature.

Table 7: Explores the results of existing weighted averaging operators

	Score values	Ranking of alternatives
PFCIAAA	$AE(\mathbb{A}_1) = 0.5292$, $AE(\mathbb{A}_2) = 0.6444$, $AE(\mathbb{A}_3) = 0.5759$, $AE(\mathbb{A}_4) = 0.6287$, $AE(\mathbb{A}_5) = 0.5162$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
Senapati [54]	$\mathrm{AE}(\mathbb{A}_1) = 0.6057$, $AE(\mathbb{A}_2) = 0.6920$, $AE(\mathbb{A}_3) = 0.6821$, $AE(\mathbb{A}_4) = 0.7398$, $AE(\mathbb{A}_5) = 0.5444$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
Garg [60]	$AE(\mathbb{A}_1) = 0.5754$, $AE(\mathbb{A}_2) = 0.6836$, $AE(\mathbb{A}_3) = 0.6544$, $AE(\mathbb{A}_4) = 0.7056$, $AE(\mathbb{A}_5) = 0.5156$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
Seikh and Mandal [61]	$AE(\mathbb{A}_1) = 0.5715$, $AE(\mathbb{A}_2) = 0.6825$, $AE(\mathbb{A}_3) = 0.6510$, $AE(\mathbb{A}_4) = 0.7011$, $AE(\mathbb{A}_5) = 0.5130$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
Jana et al. [62]	$AE(\mathbb{A}_1) = 0.6369$, $AE(\mathbb{A}_2) = 0.7029$, $AE(\mathbb{A}_3) = 0.7110$, $AE(\mathbb{A}_4) = 0.7746$, $AE(\mathbb{A}_5) = 0.5873$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
Wei [63]	$AE(\mathbb{A}_1) = 0.5917$, $AE(\mathbb{A}_2) = 0.6061$, $AE(\mathbb{A}_3) = 0.6440$, $AE(\mathbb{A}_4) = 0.6087$, $AE(\mathbb{A}_5) = 0.5434$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$

Table 8: Explores the results of existing weighted geometric operators

	Score values	Ranking of alternatives
PFCIAAG	$AE(\mathbb{A}_1) = 0.5173$, $AE(\mathbb{A}_2) = 0.6163$, $AE(\mathbb{A}_3) = 0.5264$, $AE(\mathbb{A}_4) = 0.5932$, $\mathrm{AE}(\mathbb{A}_5) = 0.4873$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
Naeem et al. [64]	$AE(\mathbb{A}_1) = 0.4885$, $AE(\mathbb{A}_2) = 0.6337$, $AE(\mathbb{A}_3) = 0.5545$, $AE(\mathbb{A}_4) = 0.5817$, $AE(\mathbb{A}_5) = 0.4556$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
Wang et al. [24]	$AE(\mathbb{A}_1) = 0.5152$, $AE(\mathbb{A}_2) = 0.6588$, $AE(\mathbb{A}_3) = 0.5935$, $AE(\mathbb{A}_4) = 0.6309$, $AE(\mathbb{A}_5) = 0.4779$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
Seikh and Mandal [61]	$AE(\mathbb{A}_1) = 0.5184$, $AE(\mathbb{A}_2) = 0.6605$, $AE(\mathbb{A}_3) = 0.5976$, $AE(\mathbb{A}_4) = 0.6362$, $AE(\mathbb{A}_5) = 0.4800$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
Jana et al. [62]	$AE(\mathbb{A}_1) = 0.4680$, $AE(\mathbb{A}_2) = 0.6180$, $AE(\mathbb{A}_3) = 0.5144$, $AE(\mathbb{A}_4) = 0.5397$, $AE(\mathbb{A}_5) = 0.4380$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$
Wei [63]	$AE(\mathbb{A}_1) = 0.5118$, $AE(\mathbb{A}_2) = 0.5434$, $AE(\mathbb{A}_3) = 0.4806$, $AE(\mathbb{A}_4) = 0.5387$, $AE(\mathbb{A}_5) = 0.5299$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_5$

In decision analysis, combining the advantages of the Choquet integral and Aczel-Alsina operators provides a powerful approach for handling complex, uncertain, and interrelated criteria. The Choquet integral excels in managing interactions among criteria, including synergies and redundancies between factors, which is crucial when criteria are not independent. On the other hand, Aczel-Alsina operators bring flexibility by providing parametric control over the aggregation process, making them adept at handling fuzzy or imprecise information. Together, these methods enhance the decision-making process by offering both nuanced handling of criterion interactions and robust aggregation of uncertain data, making them highly effective for multi-criteria decision-making scenarios where precision and flexibility are needed. The consistency in ranking alternatives shows the supremacy and effectiveness of diagnosed theories with existing mathematical approaches. Figs. 5 and 6 illustrate the aggregated outcomes by the existing weighted averaging and weighted geometric operators that exist in the literature.

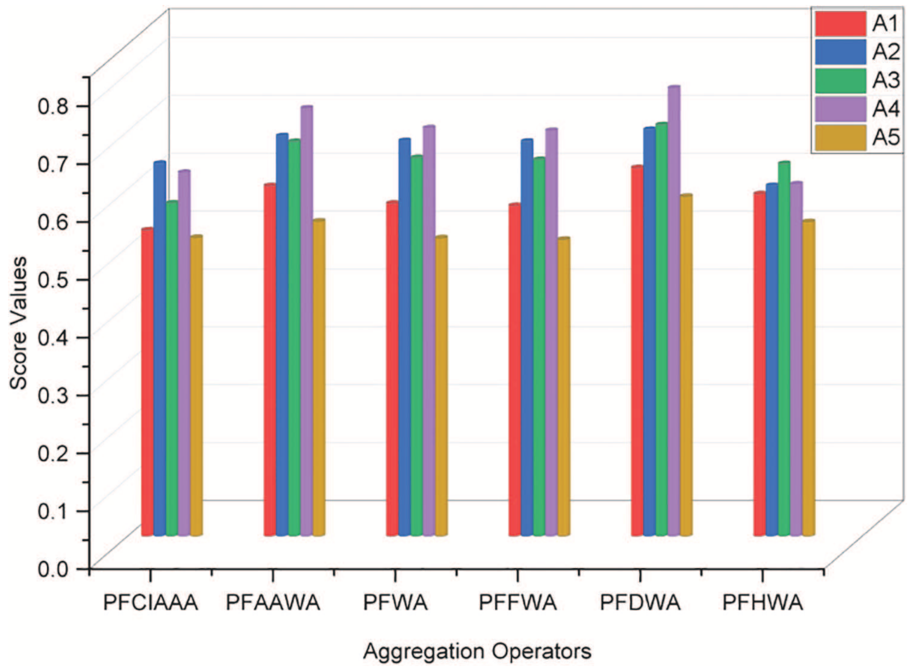

Figure 5: Displayed aggregated outcomes by the existing weighted averaging

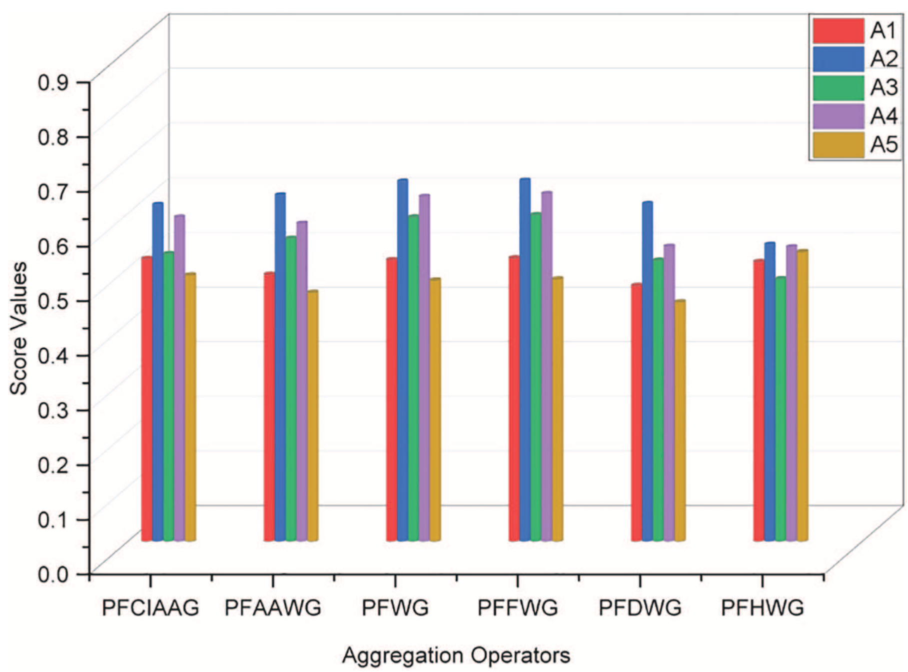

Figure 6: Displayed aggregated outcomes by the existing weighted geometric

7 Conclusion

This research study contributes to the expanding body of knowledge on aggregation operators by analyzing issues in which the valuations fall within the picture fuzzy environment. The capacity of PFS theory to accurately represent fuzzy information in the context of membership, abstinence, and non-membership functions makes it a classic and important fuzzy theory among those now in existence. Fuzzy measures in fuzzy information systems provide a good description of the correlative links between criterion sets, and the Choquet integral can handle these interactions even further. We have introduced aggregation operators, called the PFCIAAA and PFCIAAG operators, which are inspired by the Choquet integral. We have expanded the definitions of the PFCIAAA and PFCIAAG operators by merging the weights of the various points. Various characteristics of derived approaches, including monotonicity, boundedness, and idempotency, have been demonstrated. An algorithm of the MADM has been presented by considering invented mathematical approaches of the PFCIAAA and PFCIAAG operators. Furthermore, an illustrative numerical example is applied to demonstrate a suitable optimal option based on suggested aggregation operators. Finally, by examining the acquired decision-making outcomes, we have contrasted the suggested approaches with several other aggregation operators that exist in the literature. While different results from the ranking of alternatives show the benefits of the suggested technique, the same findings from the best alternative show that the suggested method is feasible. A comprehensive comparative study shows the flexibility and advantages of derived mathematical approaches by comparing the results of pioneered approaches with suggested aggregation operators that exist in the literature.

In the future, derived mathematical methodologies can be modified to accumulate various complicated frameworks such as interval-valued picture fuzzy domain, triangular picture fuzzy context, and spherical and t-spherical fuzzy theory. We can also apply to resolve various crucial challenges by using advanced decision-making techniques. Some robust optimization techniques may also be discussed to resolve real-life applications using EDAS [65], TOPSIS [66], VIKOR [67], and MARCOS [68] methods.

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List of Symbols

Symbol	Meaning
μ	Membership degree
α	Abstinence degree
v	Non-membership degree
ℸ	Element of non-empty set
AE	Score function
H	Accuracy function
T	t-norm
S	t-conorm
X	Non-empty set
Ꙧ	Attribute

$\mathbb{A}$ Alternative
χ Fuzzy measure

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