



Dynamic S-Box Generation Using Novel Chaotic Map with Nonlinearity Tweaking

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Abstract: A substitution box (S-Box) is a crucial component of contemporary cryptosystems that provide data protection in block ciphers. At the moment, chaotic maps are being created and extensively used to generate these S-Boxes as a chaotic map assists in providing disorder and resistance to combat cryptanalytical attempts. In this paper, the construction of a dynamic S-Box using a cipher key is proposed using a novel chaotic map and an innovative tweaking approach. The projected chaotic map and the proposed tweak approach are presented for the first time and the use of parameters in their working makes both of these dynamic in nature. The tweak approach employs cubic polynomials while permuting the values of an initial S-Box to enhance its cryptographic fort. Values of the parameters are provided using the cipher key and a small variation in values of these parameters results in a completely different unique S-Box. Comparative analysis and exploration confirmed that the projected chaotic map exhibits a significant amount of chaotic complexity. The security assessment in terms of bijectivity, nonlinearity, bits independence, strict avalanche, linear approximation probability, and differential probability criteria are utilized to critically investigate the effectiveness of the proposed S-Box against several assaults. The proposed S-Box's cryptographic performance is comparable to those of recently projected S-Boxes for its adaption in real-world security applications. The comparative scrutiny pacifies the genuine potential of the proposed S-Box in terms of its applicability for data security.

Keywords: Substitution-box; chaotic map; data security; tweaking



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1 Introduction

It is evident that the role of information in every aspect of our everyday lives and the modern industrial revolution are changing the planet. Reliance on Internet technology is increasing as it grows more and more integrated into many facets of our lives. Organizations need to store the data and share it with different users at different levels via the Internet or other networking protocols. During this interchange of data and information, there is a very real risk of data loss and misuse caused by malware or other harmful software. An organization that maintains digital data or financial information needs to keep it well-protected from intruders. Protecting data and information resources from illegitimate access and usage has become crucial. When data is kept on public networks, the need to shield data and systems is more demanding [1]. Consequently, an increasing number of researchers are investigating cryptographic techniques to enhance the sanctuary of information being communicated over the Internet [2]. These techniques assist users in transmitting data and information in a protected way over an apprehensive network. Assailants use cryptanalytical practices to enervate this protection of data, and therefore diverse cryptosystems have been contrived for the fortification of the data to defy these illegitimate practices [3].

Along with other arithmetic operations, permutation and substitution processes are extensively used in different phases of modern block ciphers. The permutation process is used to change the locations of bits/bytes in the original message to make it disordered and achieves the diffusion principle of cryptography. A substitution process replaces plaintext bits/bytes randomly with other bits/bytes to produce visually meaningless and futile data called ciphertext. This substitution of the original data into meaningless data is carried out in a nonlinear fashion and the process achieves the confusion principle of cryptography. A substitution box plays a crucial part in the encryption of data and is a fundamental part of modern block encryption algorithms (block ciphers). It facilitates an encryption algorithm to accomplish the confusion activity by carrying out a nonlinear transfiguration between the plaintext and ciphertext bits/bytes. The encryption algorithms using static S-Boxes always employ the same S-Box for the plaintext-ciphertext transformation. Such S-Boxes have weaknesses that make it possible for intruders to get the clue of plaintext from the ciphertext's characteristics [4]. In comparison to static S-Boxes, dynamic S-Boxes are mostly reliant on the cipher key and are sturdy, robust, and more efficacious in accomplishing the confusion of data. Researchers in the cryptography domain over time have ornated numerous S-Box generation methods using various techniques like linear fractional transformation [5,6], DNA computing [7,8], elliptic curves [9–12], compressive sensing [13–17], cellular automata [18], optimization techniques [19,20].

Recently, chaos theory has been a major focus and gained the attention of many S-Box designers to create this nonlinear component of modern block ciphers [21,22]. In order to build robust S-Boxes, authors in [23] developed a novel technique based on Logistic chaotic map that employed the idea of matrix rotation and affine transformation to generate key-dependent S-Boxes. Four steps that make up the suggested process for S-Box construction include the calculation of the Galois Field inverse, creating keys using Logistic map, computing the rotational matrix, and ultimately creating a new S-Box. Alghafis et al. [24] presented an S-Box construction technique based on a three-dimensional Liu chaotic system. The resultant S-Box is utilized along with other chaotic systems for image encryption and the outcomes reflect the robustness of the substitution process. Lu et al. [25] projected a new compound chaotic map that is a composition of two chaotic maps (Sine map and Logistic map) and employed this compound map for the creation of an S-Box. The compound chaotic map offers more chaotic behaviour, improved chaotic map features, and computational efficiency. However, the nonlinearity score of the resultant S-Box is not an encouraging one. A method for producing S-Box using a 1-D chaotic map was proposed by Tanyildizi et al. [26]. Liu et al. [27] created an improved

coupling quadratic map (ICQM). Using this map, a key-dependent strong S-Box was designed. The outcomes of the experiments demonstrated the viability of the suggested S-Box construction strategy. Riaz et al. [28] suggested a compound chaotic map that is based on the Tent map and Chebyshev map for constructing an effectual S-Box that is employed for the encryption of images.

Many other researchers [29–48] have proposed chaos-based S-Box construction techniques using other concepts. Despite the fact that chaotic maps are frequently utilized to construct S-Boxes, these maps also have accompanying shortcomings [49]. The number of potential S-Boxes and the associated recital are improvised by using heuristics, optimization algorithms, and transformation techniques. For the creation of robust S-Boxes, Zahid et al. [50] offered an innovative permutation method and inventive polynomial algorithm. This permutation-based method is incredibly simple and effective. As cryptanalytic efforts are increasing day by day, new substitution boxes need to be designed all the time with improved and more reliable performance. This paper proposes to present the construction technique of a new S-Box to protect data to resist security assaults with the help of a novel chaotic map and a tweaking approach to improvise the nonlinearity feature of the S-Box.

The major contributions of this paper are summarized as follows:

- A novel 1-D discrete chaotic map is proposed which has frail free dynamics and performance.
- Dynamic S-Box generation method is developed to get a strong final S-Box.
- A tweaking approach is presented that improvises the nonlinearity feature of the S-Box.
- The performance analysis and comparative study are carried out to validate the effective performance of the proposed method.

Descriptions of the remaining parts of this research paper are organized as follows. A novel chaotic map-based method for the creation of an S-Box is elaborated in Section 2. The performance and comparison analysis of S-Boxes based on various parameters and pertinent security analyses is described in Section 3. Constraints associated with the projected chaotic map are described in Section 4. The conclusion of our work is reported in Section 5.

2 Proposed Approach for S-Box Design

In recent times, many researchers have utilized chaotic maps for the construction of new strong S-Boxes. Because these maps have the capability to offer decent cryptographic-suited features. A chaotic map is very sensitive to initial conditions, possesses the competence to generate randomness in outcomes, and is non-periodic in nature. These assets of chaotic maps help to achieve confusion and diffusion aspects of cryptographic systems. These benefits motivated us to design a novel chaotic map to generate dynamic S-Boxes which further can be utilized in the structure of new cryptosystems. The process for engendering the key-dependent and dynamic S-Boxes comprises of subsequent three steps:

- Novel Chaotic map design
- Initial S-Box construction
- Tweaking approach to improvise the nonlinearity of the final S-Box

2.1 Novel 1-D Chaotic Map

For the construction of the proposed S-Box, a novel chaotic map (named as AZ Map) as specified mathematically in Eq. (1) is proposed.

$$X_{n+1} = \begin{cases} 1.1 * R * X_n & 0.0 < X_n < 0.5 \\ 0.55 * R * (X_n)^3 & 0.5 \leq X_n < 1.0 \end{cases} \quad (1)$$

where, $R = (2 + A)^{0.5}$ and $A, X_n \in (0.0, 1.0)$. The values of the variables A and X_n as stated in Eq. (1) are provided using the cipher key. The proposed 1-D chaotic map is very delicate to the initial values of the variables, set conditions, and provides assistance for the augmentation of the cryptographic strength of S-Box to limit security assaults. Authors in [51] described the bifurcation diagram and Lyapunov exponent as the analytical metrics/methods to show the chaos present in a system. The recital of the proposed chaotic map is compared to that of the classical Logistic map and Tent map [45] in terms of the bifurcation diagram and Lyapunov exponent in Figs. 1 and 2, respectively. The presented diagrams and comparison analysis confirmed that the new chaotic map embraces a significant amount of chaotic intricacy as it spans greater spatial regions than those of the other two chaotic maps.

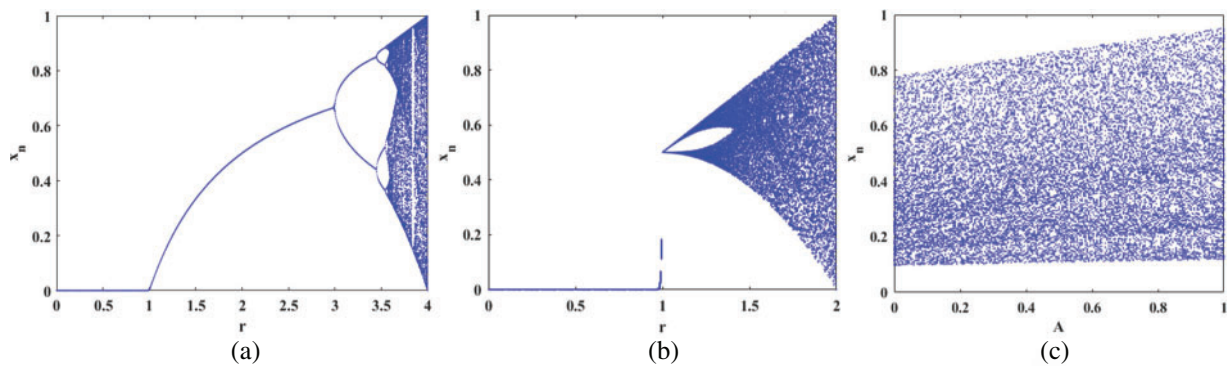


Figure 1: Bifurcation diagrams of (a) logistic map, (b) tent map, and (c) proposed AZ chaotic map

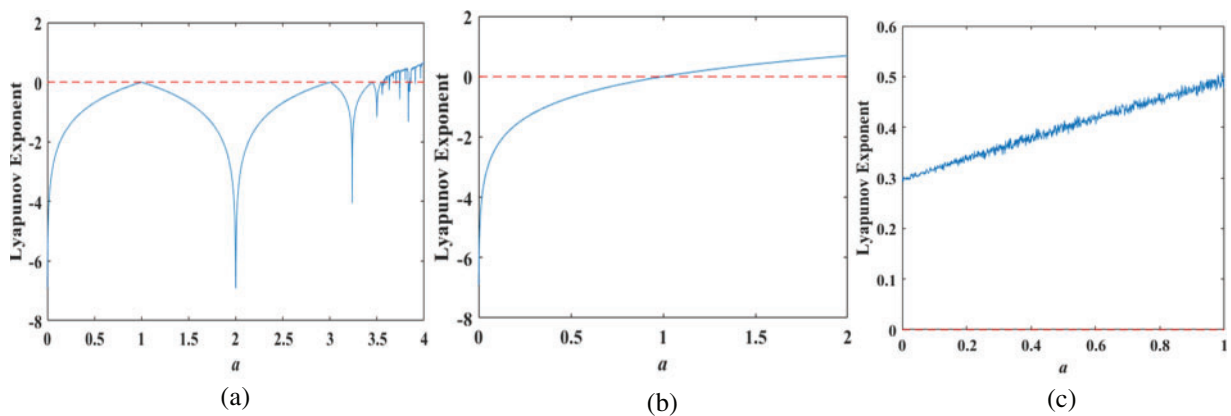


Figure 2: Lyapunov exponents of (a) logistic map, (b) tent map, and (c) proposed AZ chaotic map

2.2 Initial S-Box Creation

Using Eq. (1) and flowchart shown in Fig. 3, an initial S-Box is generated. Table 1 provides an illustration of a preliminary S-Box obtained.

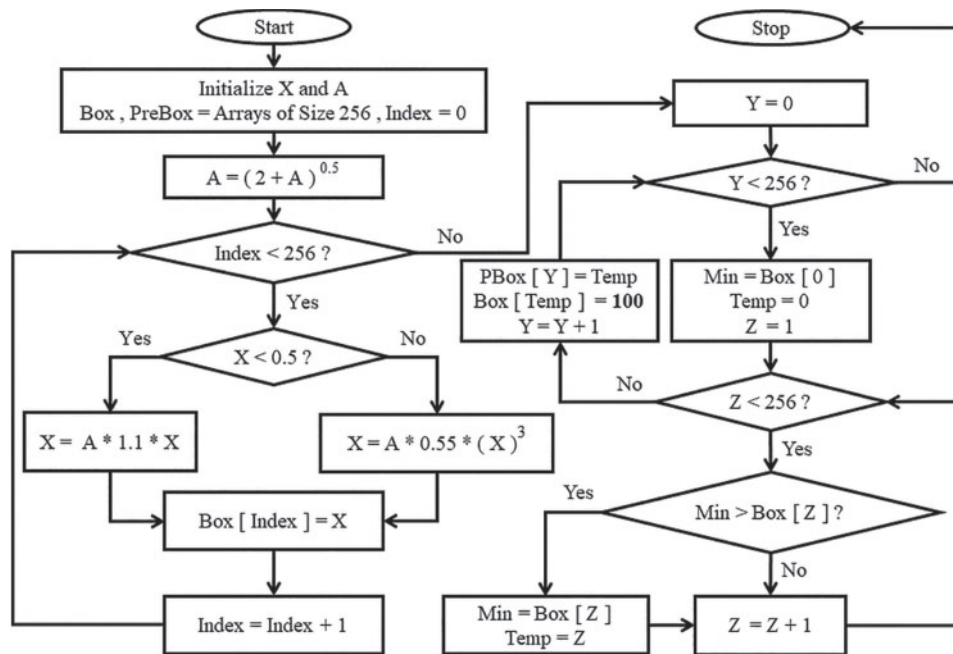


Figure 3: Preliminary S-Box creation procedure

Table 1: An initial S-Box for $X = 0.808625336927225$ and $A = 0.804718934433372$

38	154	55	28	82	48	95	193	107	125	102	170	175	200	63	180
73	190	205	208	218	252	211	149	23	68	185	247	242	136	39	155
114	56	29	221	9	83	49	96	164	194	255	108	126	141	234	78
44	91	103	171	176	201	214	19	64	181	132	160	74	119	3	36
152	26	191	123	61	71	188	206	250	209	147	245	240	219	7	253
139	232	42	212	158	117	34	150	24	59	69	186	248	243	137	230
40	156	115	32	57	228	30	226	224	222	10	84	12	50	97	165
195	109	86	14	127	142	235	52	79	45	92	104	99	167	172	197
177	202	215	20	65	182	133	111	161	75	88	16	129	0	120	144
237	4	37	153	54	27	81	47	94	192	106	124	101	169	174	199
62	179	72	189	204	207	217	251	210	148	22	67	184	246	241	135
113	220	8	163	254	140	233	77	43	90	213	18	131	159	118	2
35	151	25	122	60	70	187	249	146	244	239	6	138	231	41	157
116	33	58	229	31	227	225	223	11	85	13	51	98	166	196	110
87	15	128	143	236	53	80	46	93	105	100	168	173	198	178	203
216	21	66	183	134	112	162	76	89	17	130	1	121	145	238	5

2.3 Tweaking Approach for Final S-Box Generation

The final S-Box is created by processing an initial S-Box created by Fig. 3 with a novel tweak approach depicted in Fig. 4. The proposed tweak approach uses cubic polynomials in its working and assists in enhancing the nonlinearity of the initial S-Box by permuting the S-Box elements. The suggested tweak method is dynamic and is reliant on the values of the parameters supplied by the cipher key. Values, $P = 13071$, $Q = 94513$, $R = 4096$, and $S = 1957$ are selected for computation and demonstration purposes. Novel tweak approach permutes values of an initial S-Box through Fig. 4 and yields the final S-Box as presented in Table 2.

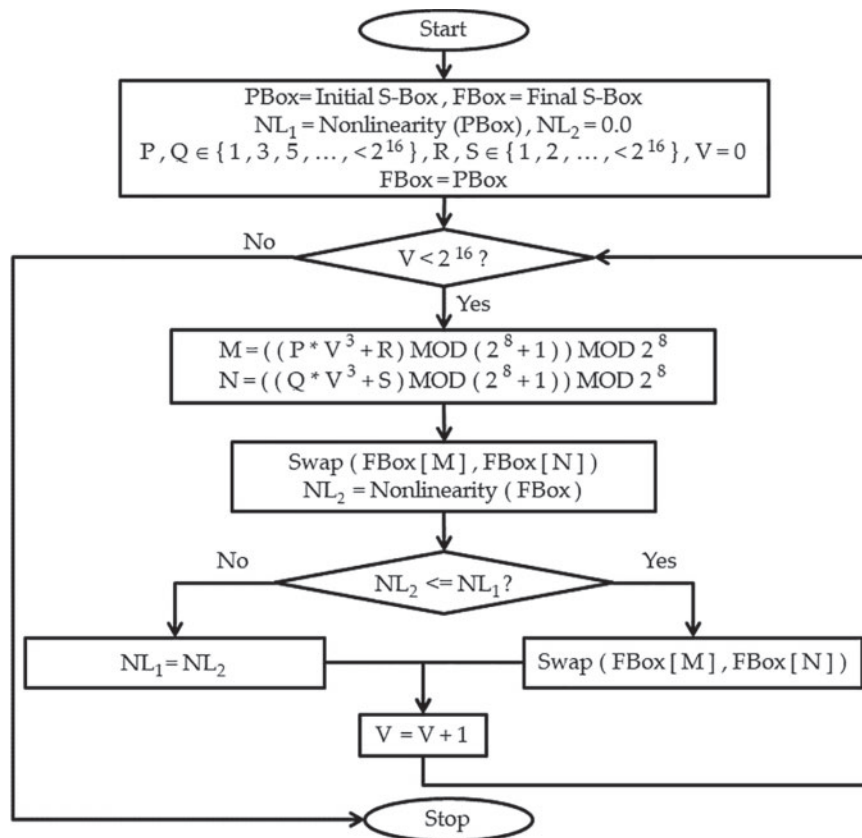


Figure 4: Tweaking approach for final S-Box generation

Table 2: Final S-Box through tweaking approach for $P = 13071$, $Q = 94513$, $R = 4096$, and $S = 1957$

38	170	55	146	96	224	95	183	201	40	118	90	71	75	63	91
180	190	138	22	161	250	211	29	23	152	185	247	242	67	115	240
237	56	149	129	10	251	49	82	164	66	199	108	126	141	234	78
205	8	212	171	176	107	39	19	100	181	132	160	74	119	3	60
233	26	117	139	61	175	46	206	83	209	147	21	155	219	7	213
94	232	42	114	73	203	34	70	24	59	105	186	248	243	89	230

(Continued)

Table 2: Continued

197	156	214	218	150	228	30	226	48	222	9	84	12	25	97	52
195	109	191	14	127	163	235	172	85	45	92	204	99	167	165	125
177	202	215	20	77	27	133	33	32	200	88	16	221	0	57	144
103	4	37	153	192	182	244	47	123	216	41	124	101	169	174	255
62	179	72	189	104	151	217	252	210	148	208	120	184	110	241	135
113	220	158	142	154	140	246	65	43	254	253	18	131	159	102	2
80	207	50	122	173	136	121	249	28	81	239	157	44	231	11	225
116	111	58	229	31	227	196	187	106	79	13	51	98	166	6	68
87	15	53	143	236	128	35	188	93	69	64	168	112	198	178	86
54	245	194	193	134	36	162	76	137	17	130	1	223	145	238	5

3 Security Assessment of Proposed S-Box

The development of new S-Boxes is a significant research contribution in the realm of information security. After the creation of an S-Box, it is examined to determine its strength against various linear and differential assaults. The strength of the projected S-Box has been evaluated through a critical analysis using the following predetermined criteria:

- FPA–Fixed Points Analysis
- Bijectivity Test
- NL–Nonlinearity
- SAC–Strict Avalanche Criterion
- BIC–Bit Independence Criterion
- LP–Linear Approximation Probability
- DP–Differential Approximation Probability

3.1 Fixed Points Analysis (FPA)

If there is any fixed point (FP) in an S-Box, an attacker may be able to decipher the original data from the seized ciphertext. Therefore, the presence of any number of fixed points causes the weakness of the final S-Box. This criterion was applied to the projected S-Box of [Table 2](#), and no fixed points were discovered throughout the whole table.

3.2 Bijectivity Test

For an $m \times n$ S-Box, this attribute must convert a distinct input of m bits to a distinct output of n bits. An S-Box must reflect this input-output mapping as one-to-one [51]. Proposed 8×8 S-Box shown in [Table 2](#) has 256 distinct values ranging from 0 to 255. The projected S-Box satisfies this bijectivity criterion very well as every possible unique input has a unique output associated with it.

3.3 Nonlinearity (NL)

A crucial feature in assessing the effectiveness of a substitution box is nonlinearity [52]. An S-Box is the only nonlinear element of today's block ciphers in particular. The strength of an S-Box against various linear and differential assaults is feeble if it is built in such a way that the conversion between original data and scrambled data is linear. A large value of nonlinearity is required for an effective defense against these malevolent efforts [53].

With the help of Eq. (2) as follows, the value of nonlinearity of a Boolean function T is determined.

$$N_L(T) = 2^{n-1} - \frac{1}{2} (S_{max}(T)) \quad (2)$$

where, $S_{max}(T)$ = Walsh-Hadamard Spectrum of a Boolean function T having n bits.

Table 3 lists the nonlinearity values of different Boolean functions of the projected S-Box.

Table 3: Nonlinearity scores of the final S-Box

Boolean function	T ₁	T ₂	T ₃	T ₄	T ₅	T ₆	T ₇	T ₈
NL Score	110	110	112	110	112	112	110	110

Table 4 demonstrates that the minimum, maximum, and average nonlinearity scores of projected S-Box are 110, 112, and 110.75 respectively. The NL scores of the proposed S-Box are compared to those of designed in recent times in Table 4. It is clear from Table 4 that the proposed S-Box has a stronger resistance against assaults like linear cryptanalysis as its NL scores (min, max, and avg) are higher than the NL scores of the majority of the other S-Boxes.

Table 4: Recent S-Boxes and nonlinearity (NL) values

S-Box	Minimum NL	Maximum NL	Average NL
Proposed	110	112	110.75
[45]	98	106	102.75
[54]	104	108	106.75
[55]	112	110	111.5
[56]	104	110	106.25
[57]	104	110	106.5
[58]	112	114	112.25
[59]	106	108	106
[60]	104	110	107
[61]	106	108	106.5
[62]	106	108	106.8
[63]	106	110	108.5
[64]	106	108	106.8
[65]	106	110	108
[66]	108	110	109.75
[67]	98	106	103.75

3.4 Strict Avalanche Criterion (SAC)

Tavares and Webster were the first to present the Strict Avalanche Criterion (SAC) [68]. In order to comply with this criterion, 50% bits of the ciphertext due to the application of a cipher must alter for any change to one of the input bits. A dependency matrix is used to compute the SAC score of a

substitution box. Values of this matrix for the projected S-Box have been calculated and quantified in Table 5.

Table 5: SAC dependence values of projected S-Box

0.4844	0.5000	0.5000	0.5156	0.4688	0.5469	0.5469	0.5156
0.5000	0.4844	0.5469	0.4844	0.5313	0.5000	0.5000	0.4375
0.5000	0.4219	0.4844	0.5469	0.5156	0.4531	0.4844	0.4844
0.4844	0.5313	0.5313	0.5313	0.5156	0.4844	0.5781	0.5156
0.4531	0.4844	0.5469	0.4844	0.4844	0.4844	0.5625	0.5781
0.4531	0.4375	0.4688	0.4844	0.4844	0.5000	0.5156	0.4688
0.4688	0.4688	0.4375	0.4531	0.4531	0.4844	0.5469	0.5000
0.4688	0.4844	0.4688	0.5000	0.5000	0.4531	0.5313	0.5000

A SAC value of 0.5 for an S-Box is considered an ideal score. The projected S-Box has an average SAC score equal to 0.496 which is very close to the ideal value.

3.5 Bit Independence Criterion (BIC)

Tavares and Webster developed Bit Independence Criterion (BIC) which is another standard for assessing S-Box recitals [68]. This criterion states that if input bits change in any way, changes in output bits should be independent of each other. BIC-NL values of the projected S-Box are described in Table 6, whereas the average BIC-NL score is 102.9.

Table 6: BIC-NL scores of projected S-Box

-	100	102	106	104	102	104	104
100	-	104	104	104	96	104	98
102	104	-	106	104	106	102	104
106	104	106	-	100	104	96	104
104	104	104	100	-	106	100	98
102	96	106	104	106	-	106	108
104	104	102	96	100	106	-	106
104	98	104	104	98	108	106	-

3.6 Linear Approximation Probability (LP)

Matsui put forth linear cryptanalysis as a statistical attack to test the strengths and weaknesses of the Data Encryption Standard (DES) in 1993 [69]. Linear cryptanalysis crooks the probability of the existence of linear relationships between different inputs (key, plaintext) and output (ciphertext) of a cryptosystem. Today, linear cryptanalysis helps cryptanalysts to look into the feebleness of modern-day block ciphers. DES cipher showed severity and compromise against linear cryptanalysis and consequently, the National Institute of Standards and Technology (NIST) developed Advanced Encryption Standard (AES) to prevent such malicious efforts by attackers [70]. If the probability of

a linear relationship between inputs and output (called linear probability) for a substitution box is computed and emanated to be small, it shows that the S-Box under consideration is robust against linear cryptanalysis. Eq. (3) is used to calculate the value of Linear Probability (LP) for an S-Box.

$$LP = \max_{t_x, t_y \neq 0} \left| \frac{\# \{x \in V | x.t_x = S(x).t_y\}}{2^n} - \frac{1}{2} \right| \quad (3)$$

where, t_x = Input mask, t_y = Output mask, and $V = \{0, 1, \dots, 2^n - 1\}$.

The projected S-Box has a very low value of LP as 0.125, demonstrating its efficacy against linear cryptanalysis.

3.7 Differential Approximation Probability (DP)

Biham and Shamir introduced differential cryptanalysis as a brand-new method of assault against the Data Encryption Standard (DES) [71]. All cryptosystems that utilize substitution and permutation operations like those of DES, are vulnerable to this attack. Using differential cryptanalysis, an attacker attempts to identify dissimilarities between related scrambled plaintexts and tries to exploit the non-uniformity in existences of differences between plaintext and ciphertext. Original plaintexts may vary by one or more bits.

The strength of an S-Box against this attack is assessed using values of differential uniformity (DU) and differential probability (DP). For a given substitution box B, its differential uniformity (DU) is determined by Eq. (4).

$$DU = \text{Max}_{\Delta a \neq 0, \Delta b} [\# \{a \in P | B(a) \oplus B(a \oplus \Delta a) = \Delta b\}] \quad (4)$$

where, ‘P’ stands for all potential inputs. Table 7 shows DU score of projected S-Box as 0.039 that is very low and specifies that the projected S-Box has the power to resist against differential cryptanalysis. Table 8 compares SAC, SAC-Offset, BIC-NL, LP and DP values of projected S-Box with those of various recently designed S-Boxes.

Table 7: Differential uniformity table of proposed S-Box

6	8	8	8	8	6	8	6	4	8	6	8	6	4	6	6
6	6	8	6	6	8	6	6	8	8	6	6	6	8	8	8
10	6	6	8	8	8	6	6	10	6	6	8	6	4	6	8
8	6	8	6	6	6	8	6	6	6	8	6	6	6	6	8
6	8	8	6	6	8	6	6	8	8	6	6	8	8	6	6
8	6	6	8	6	8	6	6	8	6	8	6	6	10	8	6
6	6	6	6	6	8	8	6	6	6	8	6	6	8	6	6
6	6	6	6	6	6	6	6	6	6	8	6	6	6	6	8
6	6	8	6	8	6	8	6	6	8	6	8	6	6	6	6
6	6	6	8	8	6	6	6	6	10	8	10	8	8	6	8
6	6	6	6	8	8	8	8	6	6	6	6	6	8	8	6
6	6	8	6	8	6	8	6	6	10	6	8	8	6	6	10
6	6	6	6	6	6	6	8	6	6	8	6	6	8	6	8

(Continued)

Table 7: Continued

8	6	6	6	8	8	6	6	8	6	8	8	8	8	8	6
4	8	8	8	6	6	6	8	8	6	6	6	6	10	8	8
6	6	8	8	10	8	6	6	6	6	8	8	6	8	6	-

Table 8: SAC, BIC-NL, LP and DP scores of some recent S-Boxes

S-Box	SAC	SAC-Offset	BIC-NL	LP	DP
Proposed	0.496	0.004	102.9	0.125	0.039
[45]	0.4992	0.001	103.1	0.141	0.047
[54]	0.4976	0.002	102.85	0.132	0.039
[55]	0.506	0.006	104.2	0.125	0.039
[56]	0.4977	0.002	104.1	0.132	0.046
[57]	0.4995	0.001	104.57	0.117	0.039
[58]	0.4995	0.001	106.35	0.128	0.039
[59]	0.501	0.001	100	0.07	0.039
[60]	0.5101	0.010	106.25	0.105	0.039
[61]	0.4978	0.002	104.21	0.133	0.039
[62]	0.5034	0.003	103.8	0.133	0.039
[63]	0.4995	0.001	103.85	0.109	0.039
[64]	0.5034	0.003	103.79	0.133	0.039
[65]	0.499	0.001	104.29	0.125	0.039
[66]	0.5042	0.004	110.6	0.085	0.039
[67]	0.5022	0.002	112.4	0.156	0.039

Table 8 clearly demonstrates that 0.004 is the offset value of SAC of projected S-Box. This low value of SAC offset validates that the projected S-Box has a great potential for usage in real life applications where security is the need of time. Similarly, low values of LP and DP are evidence of its effectiveness against such assaults.

3.8 Efficiency Analysis

The computational efficiency of the method to generate the projected S-Box was evaluated using Visual C# and Windows 10 on an Intel-core i7 CPU (2.2 GHz) system with 8 GB RAM. The projected method's computational effectiveness was analyzed for both initial and final S-Boxes. The conception of the final S-Box is reliant on a novel tweak approach to improvise the cryptographic forte of the already generated S-Box. More than 10^6 diverse preliminary S-Boxes were engendered with the help of parameters through different initial values and computed their respective generation time. Similarly, overall times spent on the creation of final S-Boxes were measured. Table 9 enumerates the average times taken for the creation of these initial and final S-Boxes.

Table 9 shows that the construction time for initial S-Boxes is very motivational. Nevertheless, the projected approach takes a little longer to produce the final S-box. Novel tweak approach used in the proposed method significantly increases the cryptographic fort of the final S-Box. The need to protect

one's data is very important, and this need should never be neglected keeping in mind the speeds offered by today's CPUs. Fig. 5 shows how the nonlinearity values of initial S-Boxes were enhanced while still being computationally efficient.

Table 9: Example S-box's creation time (seconds) using projected method

Preliminary S-box	Final S-box
0.029	12.397

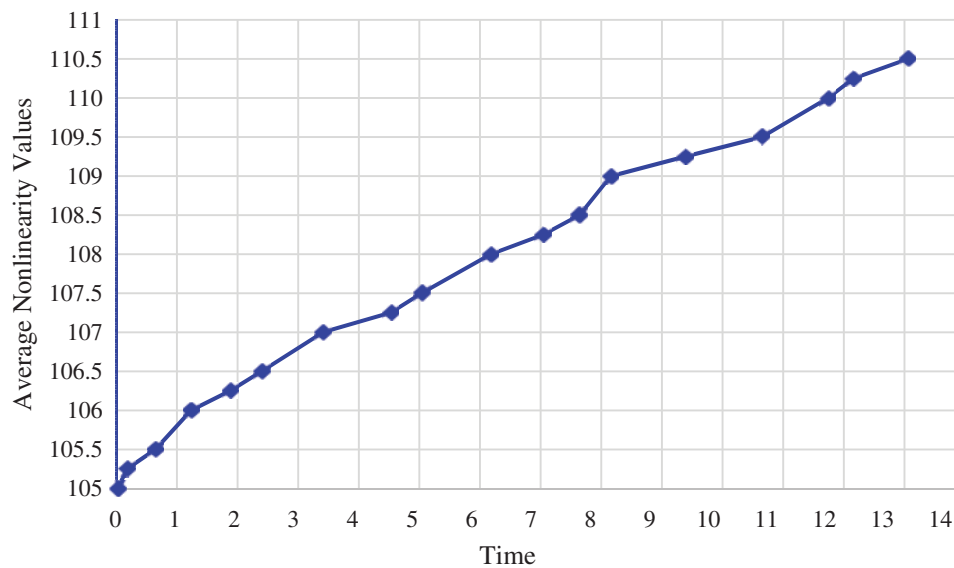


Figure 5: Nonlinearity evolution of initial S-box through tweaking approach w. r. t. time (secs)

3.9 Key Space Analysis

Being dynamic and key dependent in nature, the projected method with the selection of distinct preliminary values for the parameters aids in the development of a fresh S-Box every time. Table 10 lists the parameters employed in our proposed method together with their associated ranges and key spaces.

Table 10: Parameters and respective range and key space

Parameter	Range of parameter	Key space
X	$0 < X < 1.0$ (15 decimal digits)	10^{15}
A	$0 < A < 1.0$ (15 decimal digits)	10^{15}
P	$1, 3, 5, \dots, 2^{16} - 1$	$\sim 3.3 \times 10^4$
R	$1, 2, 3, \dots, 2^{16} - 1$	$\sim 6.6 \times 10^4$
Q	$1, 3, 5, \dots, 2^{16} - 1$	$\sim 3.3 \times 10^4$
S	$1, 2, 3, \dots, 2^{16} - 1$	$\sim 6.6 \times 10^4$

It can be seen that the projected method's entire key space is $\sim 4.7 \times 10^{48} \sim 2^{162}$ that is really a gigantic space for assailants. As a result, our suggested method is particularly resistant to intruders' use of brute force approach.

4 Conclusion

The construction method of an innovative, key-dependent, and dynamic substitution box to protect data is presented in this research article to defy security assaults using a novel chaotic map. A novel tweak approach, dynamic in nature, permutes the values of an initial S-Box to enhance its cryptographic fort. The usage of parameters in projected chaotic map and the proposed tweak approach makes the generated S-Box dynamic and key dependent. A small variation in values of these parameters results in a completely different unique S-Box. Further comparative analysis and exploration confirmed that the projected chaotic map exhibits a significant amount of chaotic complexity. Cryptographic fort evaluation of the final S-Box using well-defined criteria is performed. To check its potential for use in contemporary ciphers, its cryptographic strength is equated with those of the state-of-the-art S-Boxes. This analytical assessment certifies that the S-Boxes using proposed approach have profound aptness in cryptography domain.

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