Non-Deterministic Structural Response and Reliability Analysis Using a Hybrid Perturbation-Based Stochastic Finite Element and Quasi-Monte Carlo Method

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Abstract: The random interval response and probabilistic interval reliability of structures with a mixture of random and interval properties are studied in this paper. Structural stiffness matrix is a random interval matrix if some structural parameters and loads are modeled as random variables and the others are considered as interval variables. The perturbation-based stochastic finite element method and random interval moment method are employed to develop the expressions for the mean value and standard deviation of random interval structural displacement and stress responses. The lower bound and upper bound of the mean value and standard deviation of random interval structural responses are then determined by the quasi-Monte Carlo method. The structural reliability is not a deterministic value but an interval as the structural stress responses are random interval variables. Using a combination of the first order reliability method and interval approach, the lower and upper bounds of reliability for structural elements, series, parallel, parallel-series and series-parallel systems are investigated. Three numerical examples are used to demonstrate the effectiveness and efficiency of the proposed method.

Keywords: Hybrid probabilistic interval analysis, perturbation-based stochastic finite element method, quasi-Monte Carlo method, random interval response, probabilistic interval reliability.

1   Introduction

Non-deterministic analysis of structures with uncertainty has been received considerable attention over the past two decades [Zhang, Chen, Liu, and Liu (1996); Rao and Berke (1997); Chen and Yang (2000); Chen, Lian, and Yang (2003); Ma, Chen, and Gao (2006); Gao and Kessissoglou (2007); Hua, Ni, Chen, and Ko (2008); Zhai and Stewart (2010); Zhang, Chandrangsu and Rasmussen (2010)]. Generally, uncertainties are involved in most of structural analysis and design. For

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example, the material properties of a real structure may vary considerably from the design values due to construction or manufacturing tolerances. Over the lifetime of a structure, the damaging effects associated with attacks from aggressive environmental agents, such as progressive deterioration of concrete and corrosion of steel, usually lead to significant variations of system parameters. Stochastic methods are most popular for analysis of structures with uncertainties. In stochastic methods, uncertainties in system parameters and inputs are quantified by random functions (process/fields). These methods can provide numerical characteristics including mean value and standard deviation, even probability density for structural responses. Monte-Carlo method [Papadrakakis and Kotsopulos (1999); Spanos and Kontsos (2008); Pellissetti and Schueller (2009)], perturbation-based stochastic finite element method [Zhang, Chen, Liu, and Liu (1996); Hua, Ni, Chen, and Ko (2008); Papadrakakis and Kotsopulos (1999); Pandit, Singh, and Sheikh (2009); Cavdar, Bayraktar, Cavdar, and Adanur (2008)], spectral stochastic finite element method [Verhoosel, Gutierrez, and Hulshoff (2006); Nagh and Young (2007); Chen and Soares (2008); Nouy (2008)] and other stochastic methods [Gao and Kessissoglou (2007); Chen and Li (2007, 2009); Fraccone, Volovoi, and Ruzzene (2010)] have been widely used to analyze structures and engineering systems having random parameters.

For probabilistic approaches, however, it is important to obtain appropriate measurement data and sufficient statistical information to justify the probability densities of the random variables. In some instances, however, the available data of such measurements and statistical information are too limited and scarce to permit a probabilistic analysis. Alternatively, a discipline, called interval analysis, has been developed for structural analyses and for applied mechanics problems to account for these uncertainties. In the interval analysis, the uncertain input variables, vectors and matrices are defined in closed bounded intervals. The bounds on system response are sought through various interval analytical and numerical approaches. Over the past decade, significant progresses in interval analysis of structures with bounded parameters have been achieved. Combined with interval operations, anti-optimization method [Qiu and Elishakoff (1998)], evolutionary optimization method [Yuan, He, and Leng (2008)], perturbation method [Chen, Lian, and Yang (2003)], convex method [Elishakoff, Elisseeff, and Glegg (1994); Hu and Qiu (2010)], Monte-Carlo simulation method [Gao, Di, Song, Tin-Loi, and Li (2011)], interval finite element method [Degrauwe, Lombaert, and De Roeck (2010); Lee, Park, and Shin (2008)] and affine arithmetic [Zhu and Chen (2009); Degrauwe, Lombaert, and De Roeck (2010)] have been adopted to analyze structures with interval parameters.

Structural reliability analysis assesses the structural safety considering possible un-

certainties in structural parameters and loads. It is vital in the analysis and design of structures because a structural designer must verify, within a prescribed safety level, the serviceability and ultimate conditions. The first-order reliability method (FORM) is considered to be one of the most reliable computational methods. Numerous studies have contributed to the development of reliability methods based on FORM [Zhao and Ono (1999); Yang, Gang, and Cheng (2006); Low and Tang (2007)]. Consequently, FORM becomes a basic method for analysis of structural reliability. Second-order reliability theory [Zhao, Ono, and Kato (2002)], higher order moment method [Zhao and Lu (2007)] and response surface method [Gavin and Yau (2008)], Monte-Carlo simulation method [Melchers and Ahammed (2004); Puatatsananon and Saouma (2006)] and other methods have been also used for reliability analysis. For a structural system with pure random properties, its failure probability and reliability index are deterministic values. However, structural reliability becomes an interval number having the lower and upper bounds if both random variables and interval variables are included in the structural system. Recently, a few of researchers have conducted research on probabilistic interval reliability analysis [Du and Guo (2009); Guo and Du (2010)].

In a structural system, it is desirable to model structural parameters/loads as random variables if sufficient statistical data of them can be obtained, but some of them are best considered as interval variables if only their change ranges can be determined. To the authors' knowledge, there is very little research has been done for analysis of structural systems with a mixture of random and interval structural parameters. Recently, the authors have proposed the random interval perturbation method [Gao, Song, and Tin-Loi (2010)] and mixed perturbation Monte Carlo method [Gao, Di, Song, Tin-Loi, and Li (2011)] to analyze structures with random and interval properties. However, the random interval perturbation method may yield conservative intervals of the mean value and standard deviation of the structural responses if an interval parameter is used more than the necessary times or multiplication/division is involved. The mixed perturbation Monte Carlo method requires huge computational work to determine the exact bounds of random interval structural responses because the Monte Carlo method is used as a random search method to solve interval problems. In addition, only the expressions for reliability of series and parallel systems have been developed, other typical systems such as series-parallel and parallel-series are not investigated.

In this paper, a hybrid perturbation-based stochastic finite element method and quasi-Monte Carlo method is proposed to investigate the static response of structures with a mixture of random and interval properties. In contrast to traditional Monte Carlo methods using pseudo random numbers, the quasi-Monte Carlo method produces deterministic sequences of well-chosen points that provide the best-possible

spread in the change ranges of variables [Lei (2002)]. These deterministic sequences are often referred to as low discrepancy sequences filling the sample area efficiently and uniformly [Morokoff and Caflisch (1995)]. Using the same number of simulations, therefore, quasi-Monte Carlo methods has better accuracy than Monte Carlo methods in uncertain analysis [Singhee and Rutenbar (2010)]. Quasi-Monte Carlo methods have been also successfully used to solve optimal problems [Lei (2002); Wang (2002); Alessandri, Cervellera, Maccio, and Sanguineti (2010)] as they can generate samplings from the feasible solution areas effectively. In this study, the expressions for the first- and second-order moments of random interval structural response are developed by using the perturbation-based stochastic finite element method and random interval moment method. The lower bound (minimum value) and upper bound (maximum value) of the mean value and standard deviation of structural response are determined by the quasi-Monte Carlo method which is considered as an optimization method. The probabilistic reliability analysis for a structural element and different types of structural systems is then implemented by using the first order reliability method and interval operations.

2   Probabilistic interval static analysis

2.1   *Mean value and standard deviation of random interval structural response*

Consider a structural system having random and interval parameters and loads. Let random vector $\vec{a}^R = (a_1^R, a_2^R, \cdots, a_n^R)$ represent all random variables of the structural system and $\vec{b}^I = (b_1^I, b_2^I, \cdots, b_m^I)$ represent all interval variables. The structural stiffness matrix $[K]$ and load vector $\{f\}$ are a random interval matrix and vector respectively as they are functions of $\vec{a}^R$ and $\vec{b}^I$. Apparently structural displacement vector $\{U\}$ is also a random interval vector. Then, the finite element governing equations of the structural system can be expressed as

$$\left[K(\vec{a}^R, \vec{b}^I) \right] \left\{ U(\vec{a}^R, \vec{b}^I) \right\} = \left\{ f(\vec{a}^R, \vec{b}^I) \right\} \tag{1}$$

Using Taylor expansion, structural stiffness matrix and load vector can be expressed as

$$\left[K(\vec{a}^R, \vec{b}^I) \right] = \left[K(\bar{\vec{a}}, \vec{b}^c) \right] +$$

$$\sum_{j=1}^{m} \frac{\partial \left[K(\bar{\vec{a}}, \vec{b}^c) \right]}{\partial b_j^I} \Delta b_j^I + \sum_{i=1}^{n} \left\{ \frac{\partial \left[K(\bar{\vec{a}}, \vec{b}^c) \right]}{\partial a_i^R} + \sum_{j=1}^{m} \frac{\partial^2 \left[K(\bar{\vec{a}}, \vec{b}^c) \right]}{\partial a_i^R \partial b_j^I} \Delta b_j^I \right\} (a_i^R - \bar{a}_i)$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{l=1}^{n} \left\{ \frac{\partial^2 \left[K(\bar{\vec{a}}, \vec{b}^c) \right]}{\partial a_i^R \partial a_l^R} + \sum_{j=1}^{m} \frac{\partial^3 \left[K(\bar{\vec{a}}, \vec{b}^c) \right]}{\partial a_i^R \partial a_l^R \partial b_j^I} \Delta b_j^I \right\} (a_i^R - \bar{a}_i)(a_l^R - \bar{a}_l) + \cdots \tag{2}$$

$$\left\{ f(\vec{a}^R, \vec{b}^I) \right\} = \left\{ f(\bar{\vec{a}}, \vec{b}^c) \right\}$$

$$+ \sum_{j=1}^{m} \frac{\partial \left\{ f(\bar{\vec{a}}, \vec{b}^c) \right\}}{\partial b_j^I} \Delta b_j^I + \sum_{i=1}^{n} \left\{ \frac{\partial \left\{ f(\bar{\vec{a}}, \vec{b}^c) \right\}}{\partial a_i^R} + \sum_{j=1}^{m} \frac{\partial^2 \left\{ f(\bar{\vec{a}}, \vec{b}^c) \right\}}{\partial a_i^R \partial b_j^I} \Delta b_j^I \right\} (a_i^R - \bar{a}_i)$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{l=1}^{n} \left\{ \frac{\partial^2 \left\{ f(\bar{\vec{a}}, \vec{b}^c) \right\}}{\partial a_i^R \partial a_l^R} + \sum_{j=1}^{m} \frac{\partial^3 \left\{ f(\bar{\vec{a}}, \vec{b}^c) \right\}}{\partial a_i^R \partial a_l^R \partial b_j^I} \Delta b_j^I \right\} (a_i^R - \bar{a}_i)(a_l^R - \bar{a}_l) + \cdots \quad (3)$$

where $\bar{\vec{a}} = (\bar{a}_1, \bar{a}_2, \cdots, \bar{a}_n)$ and $\vec{b}^c = (b_1^c, b_2^c, \cdots, b_m^c)$ are the deterministic value of $\vec{a}^R$ and $\vec{b}^I$, respectively. Δb_j^I is also an interval variable which represent the interval width of b_j^I.

Substituting Eqs. (2) and (3) into Eq. (1), we can get the first three equations corresponding to zero, first and second-order perturbations.

The zero-order perturbation equation is

$$\left[K(\vec{a}^R, \vec{b}^I) \right]^0 \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^0 = \left\{ f(\vec{a}^R, \vec{b}^I) \right\}^0 \quad (4)$$

where

$$\left[K(\vec{a}^R, \vec{b}^I) \right]^0 = \left[K(\bar{\vec{a}}, \vec{b}^c) \right] = K_D \quad (5)$$

$$\left\{ f(\vec{a}^R, \vec{b}^I) \right\}^0 = \left\{ f(\bar{\vec{a}}, \vec{b}^c) \right\} = f_D \quad (6)$$

The first-order perturbation equation is

$$\left[K(\vec{a}^R, \vec{b}^I) \right]^0 \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^1 + \left[K(\vec{a}^R, \vec{b}^I) \right]^1 \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^0 = \left\{ f(\vec{a}^R, \vec{b}^I) \right\}^1 \quad (7)$$

where

$$\left[K(\vec{a}^R, \vec{b}^I) \right]^1 =$$

$$\sum_{j=1}^{m} \frac{\partial \left[K(\bar{\vec{a}}, \vec{b}^c) \right]}{\partial b_j^I} \Delta b_j^I + \sum_{i=1}^{n} \left\{ \frac{\partial \left[K(\bar{\vec{a}}, \vec{b}^c) \right]}{\partial a_i^R} + \sum_{j=1}^{m} \frac{\partial^2 \left[K(\bar{\vec{a}}, \vec{b}^c) \right]}{\partial a_i^R \partial b_j^I} \Delta b_j^I \right\} (a_i^R - \bar{a}_i)$$

$$= K_b^I + K_{a;b}^I (a_i^R - \bar{a}_i) \quad (8)$$

$$\left\{ f(\vec{a}^R, \vec{b}^I) \right\}^1 =$$

$$\sum_{j=1}^{m} \frac{\partial \left\{ f(\vec{\bar{a}}, \vec{b}^c) \right\}}{\partial b_j^I} \Delta b_j^I + \sum_{i=1}^{n} \left\{ \frac{\partial \left\{ f(\vec{\bar{a}}, \vec{b}^c) \right\}}{\partial a_i^R} + \sum_{j=1}^{m} \frac{\partial^2 \left\{ f(\vec{\bar{a}}, \vec{b}^c) \right\}}{\partial a_i^R \partial b_j^I} \Delta b_j^I \right\} (a_i^R - \bar{a}_i)$$

$$= f_b^I + f_{a_i b}^I (a_i^R - \bar{a}_i) \tag{9}$$

The second-order perturbation equation is

$$\left[K(\vec{a}^R, \vec{b}^I) \right]^0 \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^2 + \left[K(\vec{a}^R, \vec{b}^I) \right]^1 \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^1$$

$$+ \left[K(\vec{a}^R, \vec{b}^I) \right]^2 \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^0 = \left\{ f(\vec{a}^R, \vec{b}^I) \right\}^2 \tag{10}$$

where

$$\left[K(\vec{a}^R, \vec{b}^I) \right]^2 =$$

$$\frac{1}{2} \sum_{i=1}^{n} \sum_{l=1}^{n} \left\{ \frac{\partial^2 \left[K(\vec{\bar{a}}, \vec{b}^c) \right]}{\partial a_i^R \partial a_l^R} + \sum_{j=1}^{m} \frac{\partial^3 \left[K(\vec{\bar{a}}, \vec{b}^c) \right]}{\partial a_i^R \partial a_l^R \partial b_j^I} \Delta b_j^I \right\} (a_i^R - \bar{a}_i)(a_l^R - \bar{a}_l)$$

$$= K_{a_i a_l b}^I (a_i^R - \bar{a}_i)(a_l^R - \bar{a}_l) \tag{11}$$

$$\left\{ f(\vec{a}^R, \vec{b}^I) \right\}^2 =$$

$$\frac{1}{2} \sum_{i=1}^{n} \sum_{l=1}^{n} \left\{ \frac{\partial^2 \left\{ f(\vec{\bar{a}}, \vec{b}^c) \right\}}{\partial a_i^R \partial a_l^R} + \sum_{j=1}^{m} \frac{\partial^3 \left\{ f(\vec{\bar{a}}, \vec{b}^c) \right\}}{\partial a_i^R \partial a_l^R \partial b_j^I} \Delta b_j^I \right\} (a_i^R - \bar{a}_i)(a_l^R - \bar{a}_l)$$

$$= f_{a_i a_l b}^I (a_i^R - \bar{a}_i)(a_l^R - \bar{a}_l) \tag{12}$$

The solutions of Eqs. (4), (7) and (10) are respectively given as follows

$$\left\{ U(\vec{a}^R, \vec{b}^I) \right\}^0 = \left(\left[K(\vec{a}^R, \vec{b}^I) \right]^0 \right)^{-1} \left\{ f(\vec{a}^R, \vec{b}^I) \right\}^0 \tag{13}$$

$$\left\{ U(\vec{a}^R, \vec{b}^I) \right\}^1 =$$

$$\left(\left[K(\vec{a}^R, \vec{b}^I) \right]^0 \right)^{-1} \left(\left\{ f(\vec{a}^R, \vec{b}^I) \right\}^1 - \left[K(\vec{a}^R, \vec{b}^I) \right]^1 \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^0 \right) \tag{14}$$

$$\left\{ U(\vec{a}^R, \vec{b}^I) \right\}^2 = \left(\left[K(\vec{a}^R, \vec{b}^I) \right]^0 \right)^{-1}$$

$$\left(\left\{ f(\vec{a}^R, \vec{b}^I) \right\}^2 - \left[K(\vec{a}^R, \vec{b}^I) \right]^1 \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^1 - \left[K(\vec{a}^R, \vec{b}^I) \right]^2 \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^0 \right)$$

(15)

Eqs. (13) to (15) consist of a system of recursive equations. Substituting Eqs. (5) and (6) into Eq. (13) yields

$$\left\{ U(\vec{a}^R, \vec{b}^I) \right\}^0 = K_D^{-1} f_D$$

(16)

From Eqs. (8), (9), (14) and (16), we get

$$\left\{ U(\vec{a}^R, \vec{b}^I) \right\}^1 = K_D^{-1} \left(f_b^I - K_b^I K_D^{-1} f_D + f_{a_ib}^I \left(a_i^R - \bar{a}_i \right) - K_{a_ib}^I \left(a_i^R - \bar{a}_i \right) K_D^{-1} f_D \right)$$

(17)

Substituting Eqs. (5), (8), (11), (12), (16) and (17) into Eq. (15), we have

$$\left\{ U(\vec{a}^R, \vec{b}^I) \right\}^2 = K_D^{-1} \left(f_{a_ia_lb}^I (a_i^R - \bar{a}_i)(a_l^R - \bar{a}_l) - \left(K_b^I + K_{a_ib}^I \left(a_i^R - \bar{a}_i \right) \right) \right.$$

$$\cdot \left(K_D^{-1} \left(f_b^I - K_b^I K_D^{-1} f_D + f_{a_ib}^I \left(a_i^R - \bar{a}_i \right) - K_{a_ib}^I \left(a_i^R - \bar{a}_i \right) K_D^{-1} f_D \right) \right)$$

$$- K_{a_ia_lb}^I (a_i^R - \bar{a}_i)(a_l^R - \bar{a}_l) K_D^{-1} f_D)$$

(18)

The random interval structural displacement based on the first-order perturbation can be obtained as

$$U^{RI-1} = \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^0 + \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^1$$

(19)

Substituting Eqs. (16) and (17) into Eq. (19) yields

$$U^{RI-1} = K_D^{-1} f_D + K_D^{-1} \left(f_b^I - K_b^I K_D^{-1} f_D + f_{a_ib}^I \left(a_i^R - \bar{a}_i \right) - K_{a_ib}^I \left(a_i^R - \bar{a}_i \right) K_D^{-1} f_D \right)$$

(20)

The structural displacement based on the second-order perturbation can be calculated by

$$U^{RI-2} = \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^0 + \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^1 + \left\{ U(\vec{a}^R, \vec{b}^I) \right\}^2$$

(21)

Substituting Eqs. (16) - (18) into Eq. (21), we obtain

$$U^{RI-2} = K_D^{-1} f_D + K_D^{-1} \left(f_b^I - K_b^I K_D^{-1} f_D + f_{a_ib}^I \left(a_i^R - \bar{a}_i \right) - K_{a_ib}^I \left(a_i^R - \bar{a}_i \right) K_D^{-1} f_D \right)$$

$$+K_D^{-1}\left(f_{a_ia_lb}^I(a_i^R-\bar{a}_i)(a_l^R-\bar{a}_l)-\left(K_b^I+K_{a_ib}^I\left(a_i^R-\bar{a}_i\right)\right)\right.$$
$$\cdot\left(K_D^{-1}\left(f_b^I-K_b^IK_D^{-1}f_D+f_{a_ib}^I\left(a_i^R-\bar{a}_i\right)-K_{a_ib}^I\left(a_i^R-\bar{a}_i\right)K_D^{-1}f_D\right)\right)$$
$$-K_{a_ia_lb}^I(a_i^R-\bar{a}_i)(a_l^R-\bar{a}_l)\left.K_D^{-1}f_D\right) \tag{22}$$

From Eq. (20), using the random interval moment method [Gao, Di, Song, Tin-Loi, and Li (2011)], the mean value and variance of the random interval structural displacement based on the first-order perturbation can be obtained as

$$E(U^{RI-1})=K_D^{-1}f_D+K_D^{-1}\left(f_b^I-K_b^IK_D^{-1}f_D\right) \tag{23}$$

$$Var(U^{RI-1})=E\left(U^{RI-1}-E(U^{RI-1})\right)^2$$
$$=(K_D^{-1}f_{a_ib}^I-K_D^{-1}K_{a_ib}^IK_D^{-1}f_D)(K_D^{-1}f_{a_lb}^I-K_D^{-1}K_{a_lb}^IK_D^{-1}f_D)Cov\left(a_i^R,a_l^R\right) \tag{24}$$

From Eq. (24), the standard deviation of the random interval displacement can be expressed as

$$Std(U^{RI-1})=$$

$$\left\{(K_D^{-1}f_{a_ib}^I-K_D^{-1}K_{a_ib}^IK_D^{-1}f_D)(K_D^{-1}f_{a_lb}^I-K_D^{-1}K_{a_lb}^IK_D^{-1}f_D)Cov\left(a_i^R,a_l^R\right)\right\}^{1/2} \tag{25}$$

From Eqs. (23) and (25), it can be observed that the mean value and standard deviation of random interval structural displacement response are not deterministic values but interval variables.

Similarly, from Eq. (22), the mean value and standard deviation of the random interval structural displacement based on the second-order perturbation can be computed from

$$E(U^{RI-2})=K_D^{-1}f_D+K_D^{-1}A_b^I-K_D^{-1}K_b^IK_D^{-1}A_b^I+B_{a_ia_lb}^ICov\left(a_i^R,a_l^R\right) \tag{26}$$

$$Std(U^{RI-2})=\left\{C_{a_ib}^IC_{a_lb}^ICov\left(a_i^R,a_l^R\right)+2C_{a_ib}^IB_{a_la_kb}^IE\left((a_i^R-\bar{a}_i)(a_l^R-\bar{a}_l)(a_k^R-\bar{a}_k)\right)\right.$$

$$\left.+B_{a_ia_lb}^IB_{a_ka_mb}^IE\left((a_i^R-\bar{a}_i)(a_l^R-\bar{a}_l)(a_k^R-\bar{a}_k)(a_m^R-\bar{a}_m)\right)-\left(B_{a_ia_lb}^ICov\left(a_i^R,a_l^R\right)\right)^2\right\}^{1/2} \tag{27}$$

where

$$A_b^I=f_b^I-K_b^IK_D^{-1}f_D \tag{28}$$

$$B_{a_ia_lb}^I=K_D^{-1}\left(f_{a_ia_lb}^I-K_{a_ia_lb}^IK_D^{-1}f_D-K_{a_lb}^IK_D^{-1}\left(f_{a_ib}^I-K_{a_ib}^IK_D^{-1}f_D\right)\right) \tag{29}$$

$$C_{a_ib}^I = K_D^{-1}\left(f_{a_ib}^I - K_{a_ib}^I K_D^{-1} f_D - K_b^I K_D^{-1}\left(f_{a_ib}^I - K_{a_ib}^I K_D^{-1} f_D\right) - K_{a_ib}^I K_D^{-1} A_b^I\right) \tag{30}$$

Obviously, the second-order perturbation method constitutes much more complex computation than the first-order perturbation method. The third and fourth-order probabilistic information of random variables are required in addition to the mean values, variances and covariance of them. For analysis of structures with uncertainty, the first and second-order moments (mean value, variance and covariance) of the random system parameters are much more important than higher-order moments. Furthermore, in most of engineering problems, only the first and second-order moments of structural responses are of interest. In this paper, only the mean value and standard deviation of random interval structural responses are investigated.

Using the relationship between the node displacement and element stress, the stress response of the i-th element in the structure $\left\{\sigma_i\left(\vec{a}^R, \vec{b}^I\right)\right\}$ can be expressed as

$$\left\{\sigma_i\left(\vec{a}^R, \vec{b}^I\right)\right\} = \left[D\left(\vec{a}^R, \vec{b}^I\right)\right]\left\{U\left(\vec{a}^R, \vec{b}^I\right)\right\} \tag{31}$$

where $\left\{U\left(\vec{a}^R, \vec{b}^I\right)\right\}$ is the displacement of the nodal points of the i-th element and $\left[D\left(\vec{a}^R, \vec{b}^I\right)\right]$ is the elastic matrix.

In the following, only the first-order perturbation displacements are used to demonstrate how to calculate the numerical characteristics of the random interval stress response. The first-order Taylor series of the random interval matrix $\left[D\left(\vec{a}^R, \vec{b}^I\right)\right]$ can be expressed as

$$\left[D\left(\vec{a}^R, \vec{b}^I\right)\right] = \left[D\left(\vec{\bar{a}}^R, \vec{b}^c\right)\right] + \sum_{j=1}^{m}\frac{\partial\left[D(\vec{\bar{a}},\vec{b}^c)\right]}{\partial b_j^I}\Delta b_j^I$$

$$+ \sum_{i=1}^{n}\left\{\frac{\partial\left[D(\vec{\bar{a}},\vec{b}^c)\right]}{\partial a_i^R} + \sum_{j=1}^{m}\frac{\partial^2\left[D(\vec{\bar{a}},\vec{b}^c)\right]}{\partial a_i^R \partial b_j^I}\Delta b_j^I\right\}\left(a_i^R - \bar{a}_i\right)$$

$$= D_D + D_b^I + D_{a_ib}^I\left(a_i^R - \bar{a}_i\right) \tag{32}$$

Substituting Eqs. (20) and (32) into Eq.(31), we have

$$\sigma^{RI-1} = \left(D_D + D_b^I + D_{a_ib}^I\left(a_i^R - \bar{a}_i\right)\right)$$

$$\cdot\left(K_D^{-1}f_D + K_D^{-1}\left(f_b^I - K_b^I K_D^{-1}f_D + f_{a_ib}^I\left(a_i^R - \bar{a}_i\right) - K_{a_ib}^I\left(a_i^R - \bar{a}_i\right)K_D^{-1}f_D\right)\right) \tag{33}$$

The mean value and standard deviation of σ^{RI-1} are obtained as

$$E(\sigma^{RI-1}) = \left(D_D + D_b^I\right)\left(K_D^{-1} f_D + K_D^{-1}\left(f_b^I - K_b^I K_D^{-1} f_D\right)\right) + G_{a_i a_l b}^I Cov\left(a_i^R, a_l^R\right) \tag{34}$$

$$Std(U^{RI-2}) = \left\{ H_{a_i b}^I H_{a_l b}^I Cov\left(a_i^R, a_l^R\right) + 2 H_{a_i b}^I G_{a_l a_k b}^I E\left((a_i^R - \bar{a}_i)(a_l^R - \bar{a}_l)(a_k^R - \bar{a}_k)\right) \right.$$

$$\left. + G_{a_i a_l b}^I G_{a_k a_m b}^I E\left((a_i^R - \bar{a}_i)(a_l^R - \bar{a}_l)(a_k^R - \bar{a}_k)(a_m^R - \bar{a}_m)\right) - \left(G_{a_i a_l b}^I Cov\left(a_i^R, a_l^R\right)\right)^2 \right\}^{1/2} \tag{35}$$

where

$$G_{a_i a_l b}^I = D_{a_i b}^I K_D^{-1}\left(f_{a_l b}^I - K_{a_l b}^I K_D^{-1} f_D\right) \tag{36}$$

$$H_{a_i b}^I = \left(D_D + D_b^I\right) K_D^{-1}\left(f_{a_l b}^I - K_{a_l b}^I K_D^{-1} f_D\right) + D_{a_i b}^I\left(K_D^{-1} f_D + K_D^{-1}\left(f_b^I - K_b^I K_D^{-1} f_D\right)\right) \tag{37}$$

Eq. (35) is concisely rewritten as

$$Std(U^{RI-2}) = \left\{ H_{a_i b}^I H_{a_l b}^I Cov\left(a_{i,l}^R\right) + 2 H_{a_i b}^I G_{a_l a_k b}^I E\left(a_{i,l,k}^R\right) + G_{a_i a_l b}^I G_{a_k a_m b}^I E\left(a_{i,l,k,m}^R\right) \right.$$

$$\left. - \left(G_{a_i a_l b}^I Cov\left(a_{i,l}^R\right)\right)^2 \right\}^{1/2} \tag{38}$$

2.2   *Bounded numerical characteristics of random interval structural responses*

The lower and upper bounds of the mean value and standard deviation of the random interval displacement response can be computed by using optimization methods. The lower and upper bounds of the mean value and standard deviation of the displacement response based on the first-order perturbation can be expressed in the following optimization form

$$\underline{E(U^{RI-1})} = \min\left\{ K_D^{-1} f_D + K_D^{-1}\left(f_b^I - K_b^I K_D^{-1} f_D\right)\right\} \tag{39}$$

$$\overline{E(U^{RI-1})} = \max\left\{ K_D^{-1} f_D + K_D^{-1}\left(f_b^I - K_b^I K_D^{-1} f_D\right)\right\} \tag{40}$$

$$\underline{Std(U^{RI-1})} =$$

$$\min \left\{ \left\{ (K_D^{-1} f_{a_ib}^I - K_D^{-1} K_{a_ib}^I K_D^{-1} f_D)(K_D^{-1} f_{a_lb}^I - K_D^{-1} K_{a_lb}^I K_D^{-1} f_D) Cov\left(a_i^R, a_l^R\right) \right\}^{1/2} \right\} \tag{41}$$

$$\overline{Std(U^{RI-1})} =$$

$$\max \left\{ \left\{ (K_D^{-1} f_{a_ib}^I - K_D^{-1} K_{a_ib}^I K_D^{-1} f_D)(K_D^{-1} f_{a_lb}^I - K_D^{-1} K_{a_lb}^I K_D^{-1} f_D) Cov\left(a_i^R, a_l^R\right) \right\}^{1/2} \right\} \tag{42}$$

Similarly, the lower and upper bounds of mean value and standard deviation of structural stress response can be computed by

$$\underline{E(\sigma^{RI-1})} =$$

$$\min \left\{ \left(D_D + D_b^I\right) \left(K_D^{-1} f_D + K_D^{-1} \left(f_b^I - K_b^I K_D^{-1} f_D\right)\right) + G_{a_ia_lb}^I Cov\left(a_i^R, a_l^R\right) \right\} \tag{43}$$

$$\overline{E(\sigma^{RI-1})} =$$

$$\max \left\{ \left(D_D + D_b^I\right) \left(K_D^{-1} f_D + K_D^{-1} \left(f_b^I - K_b^I K_D^{-1} f_D\right)\right) + G_{a_ia_lb}^I Cov\left(a_i^R, a_l^R\right) \right\} \tag{44}$$

$$\underline{Std(\sigma^{RI-1})} =$$

$$\min\{\{H_{a_ib}^I H_{a_lb}^I Cov\left(a_{i,l}^R\right) + 2H_{a_ib}^I G_{a_la_kb}^I E\left(a_{i,l,k}^R\right) + G_{a_ia_lb}^I G_{a_ka_mb}^I E\left(a_{i,l,k,m}^R\right)$$

$$- \left(G_{a_ia_lb}^I Cov\left(a_{i,l}^R\right)\right)^2\}^{1/2}\} \tag{45}$$

$$\overline{Std(\sigma^{RI-1})} =$$

$$\max\{\{H_{a_ib}^I H_{a_lb}^I Cov\left(a_{i,l}^R\right) + 2H_{a_ib}^I G_{a_la_kb}^I E\left(a_{i,l,k}^R\right) + G_{a_ia_lb}^I G_{a_ka_mb}^I E\left(a_{i,l,k,m}^R\right)$$

$$- \left(G_{a_ia_lb}^I Cov\left(a_{i,l}^R\right)\right)^2\}^{1/2}\} \tag{46}$$

In this study, the quasi-Monte Carlo method is adopted as a so-called random search method to determine the lower and upper bounds of the mean value and standard deviation of structural responses. As the quasi-Monte Carlo method has the capability to generate values distributed within the given ranges uniformly, this method is suitable for interval analysis.

3   Probabilistic interval reliability analysis

Structural reliability analysis is to estimate the probability of exceeding the structural limit states imposed on structural components. The structural reliability problem is defined by the integral

$$P_f = \int_{g(X) \leq 0} f_X(X) dX \tag{47}$$

where P_f is the probability of failure, X is the vector of random variables, $f_X(X)$ is the joint probability density function, and $g(X)$ is the limit state function such that $g(X) \leq 0$ defines the failure domain.

Normally, a structure can be considered as a series, parallel, series-parallel or parallel-series system in reliability analysis. In the following reliability analysis, we assume that each typical system and its individual elements have only two states: working or failure.

3.1   Elements/components

The limit state function of the i-th element of a structure is defined as

$$g_i(W) = R_i - \sigma_i \tag{48}$$

where R_i and σ_i are the resistance (strength) and stress response of the i-th element, respectively.

Using the first-order second-moment reliability method, the element (component) failure probability can be expressed as

$$P_{f_i} = \phi(-\beta_i) \tag{49}$$

where ϕ is the standard normal cumulative distribution function. β_i is the reliability index and can be calculated by

$$\beta_i = \frac{\mu_{R_i} - \mu_{\sigma_i}}{\sqrt{\sigma_{R_i}^2 + \sigma_{\sigma_i}^2}} \tag{50}$$

where μ_{R_i} and μ_{σ_i} are the mean values of R_i and σ_i, σ_{R_i} and σ_{σ_i} are standard deviations of R_i and σ_i, respectively.

The reliability index of the i-th element β_i is an interval variable because the structural stress response σ_i is a random interval variable and μ_{σ_i} and σ_{σ_i} are interval variables, even if μ_{R_i} and σ_{R_i} are deterministic values. Let us assume that μ_{R_i} and

σ_{R_i} are also interval variables, the lower and upper bounds of the interval reliability index β_i^I can be obtained as

$$\underline{\beta_i} = \frac{\underline{\mu_{R_i}} - \overline{\mu_{\sigma_i}}}{\sqrt{(\overline{\sigma_{R_i}})^2 + (\overline{\sigma_{\sigma_i}})^2}}, \quad \overline{\beta_i} = \frac{\overline{\mu_{R_i}} - \underline{\mu_{\sigma_i}}}{\sqrt{(\underline{\sigma_{R_i}})^2 + (\underline{\sigma_{\sigma_i}})^2}} \tag{51}$$

The upper and lower bounds of the failure probability can be computed by

$$\overline{P_{f_i}} = \phi(-\underline{\beta_i}) = 1 - \phi\left(\frac{\underline{\mu_{R_i}} - \overline{\mu_{\sigma_i}}}{\sqrt{(\overline{\sigma_{R_i}})^2 + (\overline{\sigma_{\sigma_i}})^2}}\right),$$

$$\underline{P_{f_i}} = \phi(-\overline{\beta_i}) = 1 - \phi\left(\frac{\overline{\mu_{R_i}} - \underline{\mu_{\sigma_i}}}{\sqrt{(\underline{\sigma_{R_i}})^2 + (\underline{\sigma_{\sigma_i}})^2}}\right) \tag{52}$$

The midpoint and maximum width of the failure probability can be easily obtained as

$$P_{f_i}^c = \frac{\overline{P_{f_i}} + \underline{P_{f_i}}}{2}, \quad \Delta P_{f_i} = \frac{\overline{P_{f_i}} - \underline{P_{f_i}}}{2} \tag{53}$$

As $P_{r_i} = 1 - P_{f_i}$ denote the reliability (probability of survival) of the i-th element, the lower bound (worst possible value) and upper bound (best possible value) of the reliability can be expressed as

$$\underline{P_{r_i}} = 1 - \overline{P_{f_i}} = \phi(\underline{\beta_i}) = \phi\left(\frac{\underline{\mu_{R_i}} - \overline{\mu_{\sigma_i}}}{\sqrt{(\overline{\sigma_{R_i}})^2 + (\overline{\sigma_{\sigma_i}})^2}}\right), \tag{54}$$

$$\overline{P_{r_i}} = 1 - \underline{P_{f_i}} = \phi(\overline{\beta_i}) = \phi\left(\frac{\overline{\mu_{R_i}} - \underline{\mu_{\sigma_i}}}{\sqrt{(\underline{\sigma_{R_i}})^2 + (\underline{\sigma_{\sigma_i}})^2}}\right)$$

The midpoint and maximum width of the reliability are

$$P_{r_i}^c = \frac{\overline{P_{r_i}} + \underline{P_{r_i}}}{2}, \quad \Delta P_{r_i} = \frac{\overline{P_{r_i}} - \underline{P_{r_i}}}{2} \tag{55}$$

3.2 Series system

If the structure is considered as a series system shown in Figure 1, the reliability of this structure is

$$P_r = 1 - P_f = \prod_{i=1}^{n}(1 - P_{f_i}) = \prod_{i=1}^{n} P_{r_i} \tag{56}$$

The lower and upper bounds of the structural reliability (series system) can be expressed as

$$\underline{P_r} = \prod_{i=1}^{n} \underline{P_{r_i}}, \quad \overline{P_r} = \prod_{i=1}^{n} \overline{P_{r_i}} \tag{57}$$
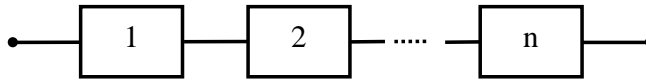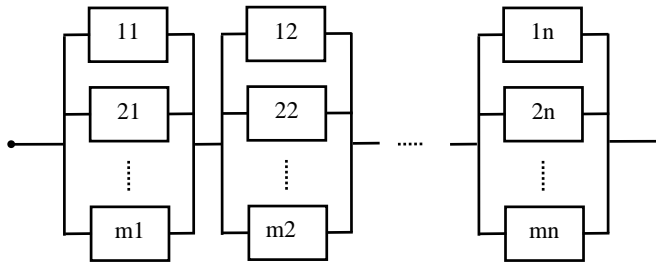

Figure 1: Series system

3.3 *Parallel system*

If the structure is considered as a parallel system shown in Figure 2, the reliability of this structure is given by

$$P_r = 1 - P_f = 1 - \prod_{i=1}^{n}(1 - P_{r_i}) \tag{58}$$

The lower and upper bounds of the structural reliability (parallel system) can be expressed as

$$\underline{P_r} = 1 - \prod_{i=1}^{n}(1 - \underline{P_{r_i}}), \quad \overline{P_r} = 1 - \prod_{i=1}^{n}(1 - \overline{P_{r_i}}) \tag{59}$$
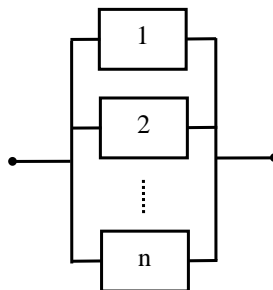
Figure 2: Parallel system

3.4 Series-parallel system

For the series parallel system shown in Figure 3, its reliability is calculated by

$$P_r = 1 - \prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n} P_{r_{ij}}\right) \tag{60}$$

The lower and upper bounds of the structural reliability (series-parallel system) can be expressed as

$$\underline{P_r} = 1 - \prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n} \underline{P_{r_{ij}}}\right), \quad \overline{P_r} = 1 - \prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n} \overline{P_{r_{ij}}}\right) \tag{61}$$
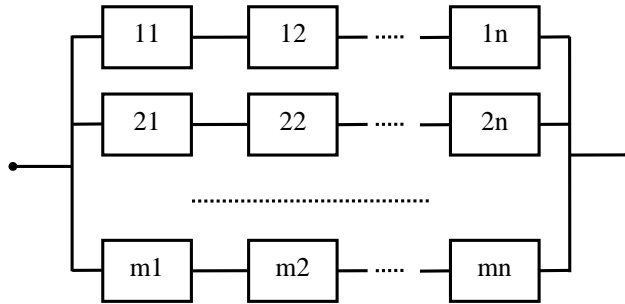
Figure 3: Series-parallel system

Figure 4: Parallel-series system

3.5 *Parallel-series system*

If the structure is considered as a parallel series system shown in Figure 4, the reliability of this structure is

$$P_r = \prod_{j=1}^{n} \left(1 - \prod_{i=1}^{m} (1 - P_{r_{ij}}) \right) \tag{62}$$

The lower and upper bounds of the structural reliability (parallel-series system) can be expressed as

$$\underline{P_r} = \prod_{j=1}^{n} \left(1 - \prod_{i=1}^{m} (1 - \underline{P_{r_{ij}}}) \right), \quad \overline{P_r} = \prod_{j=1}^{n} \left(1 - \prod_{i=1}^{m} (1 - \overline{P_{r_{ij}}}) \right) \tag{63}$$

4 Examples

4.1 *Portal frame*

To demonstrate the efficiency of the approach presented in this paper for static response analysis, a planar portal frame shown in Figure 5 is used as an example. Suppose that there are no preload stresses in the structure and all elements have a circle cross section. The deterministic values of structural parameters for all members are Young's modulus$E = 2.1 \times 10^{11} N/m^2$ and the diameter$d = 4.0 \times 10^{-2} m$, respectively. A load acts on the node 15 along the negative Y-direction with the deterministic value$f = 6 \times 10^4 N$. Consider that the Young's modulus and load are random variables, and the diameter is an interval variable. These uncertain values of structural parameters and load are taken as $\sigma_E = 4.2 \times 10^9 N/m^2$, $\sigma_F = 600N$ and $d^I = [3.9, 4.1] \times 10^{-2} m$. In the following, 20000 times traditional Monte Carlo (MC) simulations and the Hybrid Perturbation-based Stochastic Finite Element and Quasi-Monte Carlo method (HPSFE-QMC) presented in this paper are used to calculate the lower and upper bounds of mean value and standard deviation of structural displacement response.

In every traditional Monte-Carlo simulation, the first step is to arbitrarily generate values within the given intervals for all interval variables by assuming all interval variables obey uniform distributions in their own intervals. The second step is using 20000 times Monte-Carlo simulations to determine the mean value and standard deviation of structural response. After repeat the whole procedure 20000 times, 20000 mean values and 20000 standard deviations can be obtained. Then the lower and upper bounds of them can be determined respectively. More simulation times can be used for the two steps to improve the accuracy of the results. In HPSFE-QMC, 3000, 5000, 10000 and 20000 simulations are used respectively
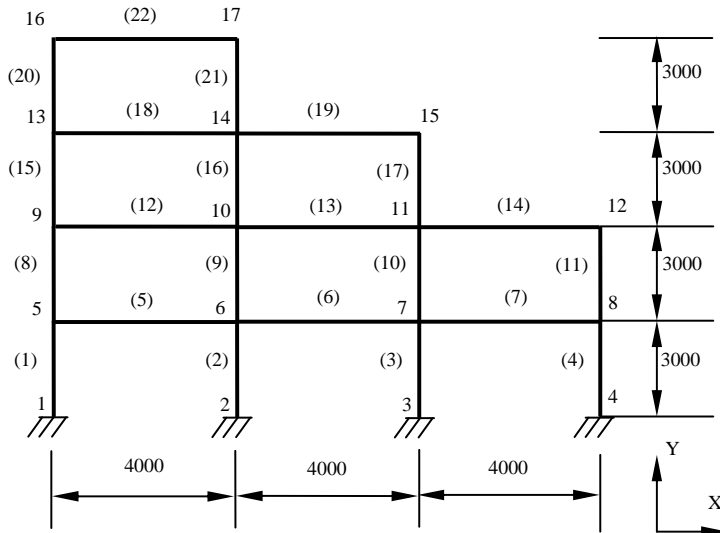
Figure 5: 22 elements portal frame structure (unit: mm)

to get the lower and upper bounds of the mean value (LBMV and UBMV) and standard deviation (LBSTD and UBSTD) of the structural displacement response. The computational results of the displacement response of node 15 in the negative Y-direction are listed in Table 1.

Generally, computational results obtained by HPSFE-QMC are in very good agreement with those computed by the traditional Monte-Carlo method. The intervals of mean value calculated by these two methods are quite close to each other. The accuracy of results can be improved if the second-order perturbation method is adopted. However, it requires more computational work. From Table 1, it can be also observed that the intervals of numerical characteristics of structural responses obtained by 3000, 5000, 10000 and 20000 quasi-Monte Carlo simulations are almost same. In other words, 3000 quasi-Monte Carlo simulations can provide reliable/accurate results in this example. In interval analysis, quasi-Monte Carlo method can greatly reduce the computational effort compared with the traditional Monte Carlo method. The traditional Monte-Carlo simulation method is time-consuming as the two different types of variables, random variables and interval variables must be treated separately. The total simulation times are $20000 \times 20000 = 4 \times 10^8$. The HPSFE-QMC is much more time saving due to much lesser simulation times. It should be noted that the accuracy of the results of these two methods can be improved if more simulations are used.

Table 1: Mean value and standard deviation of vertical displacement at node 15 (unit: 10^{-3} mm)

	LBMV	UBMV	LBSTD	UBSTD
HPSFE-QMC (3000 simulations)	216.4188	239.1734	4.8393	5.3481
HPSFE-QMC (5000 simulations)	216.4137	239.1734	4.8392	5.3481
HPSFE-QMC (10000 simulations)	216.4111	239.1734	4.8391	5.3481
HPSFE-QMC (20000 simulations)	216.4098	239.1734	4.8391	5.3481
MC (20000 simulations)	216.4265	239.2763	4.8311	5.3543
Relative error (20000 simulations) $\lvert (HPSFEQMC - MC)/MC \rvert$	0.0077%	0.0430%	0.8000%	0.1158%
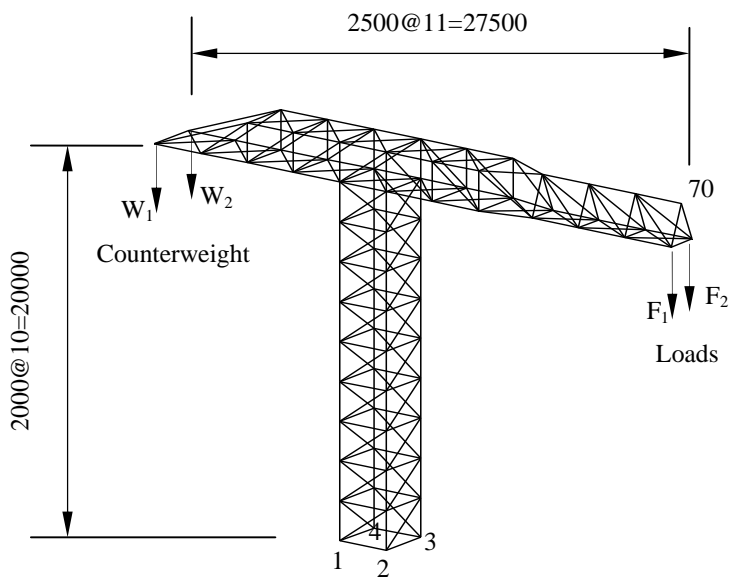
Figure 6: Crane Truss Tower (unit: mm)

4.2 Crane tower

In the following, a space crane tower truss structure shown in Figure 6 is under consideration.

This crane tower has 76 nodes and 237 elements. The material of this structure is steel. The deterministic values of the structural parameters and loads are Young's modulus $E = 2.1 \times 10^{11} N/m^2$, cross-sectional area $A = 1.0 \times 10^{-2} m^2$, counterweight $W_1 = W_2 = 2.5 \times 10^4 N$ and loads $f = F_1 = F_2 = 5.0 \times 10^4 N$, respectively. The cross-sectional areas are interval variables, Young's modulus and loads are random variables. Here, we introduce the coefficient of variation v_{a^R} for random variable a^R and the interval change ratio Δb_F for interval variable $b^I = [\underline{b}, \overline{b}]$ as follows

$$v_{a^R} = \frac{\sigma_{a^R}}{\bar{a}}, \Delta b_F = \frac{\Delta b}{b^c} \tag{64}$$

The dispersal degree of a random variable or an interval variable can be better reflected by the coefficient of variation or interval change ratio.

To investigate the differences between the effects produced by random and interval variables on structural response, the values of coefficient of variation of random variables, and interval change ratio of interval variables are varied from 0 to 0.1. If *DD* denotes the dispersal degree of uncertain variables, then its value will vary from 0 to 0.1. The lower and upper bounds on the mean value and standard deviation of the random interval displacement (MOD and SOD) of node 70 in vertical direction are given in Figures 7 and 8, respectively.

Figures 7(b) and (c) show that the random variables do not affect the mean value of the random interval structural response. In other words, the mean value of structural response is not an interval but a deterministic value if all structural parameters and loads are random variables. However, the mean value of structural response is an interval if the structure has interval parameters or loads as shown in Figures 7(a) and (d). The interval width of structural response depends on the dispersal degree of the interval parameter.

Figure 8(a) shows that the standard deviation of structural response is zero if the structure has interval variables only. Structural response is an interval variable (not a random interval variable) if all structural parameters and loads are interval variables. Figures 8(b) and (c) show that standard deviation of structural response is a deterministic value rather than interval number if structural parameters and loads are random variables and its values depend on the randomness of random structural parameters/loads. Figure 8(d) shows that the standard deviation of the structural response is bigger, compared with figures 8(b) and 8(c), if more random variables are considered, and the standard deviation of structural response is an interval variable when the structural system has a mixture of random and interval parameters/loads. The interval width is dependent on both the dispersal degrees of random and interval variables.
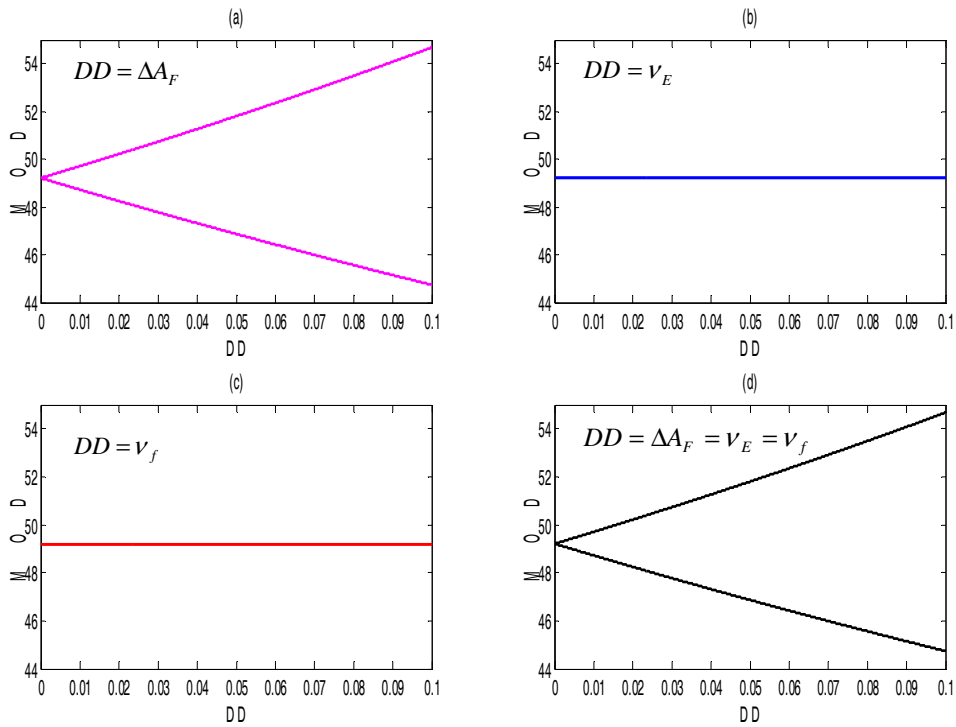
Figure 7: Mean value of random interval displacement of node 70 in vertical direction (unit: mm)

4.3 Planar truss

An 11-bar 2D statically determinate truss structure shown in Figure 9 is considered in this example. The Young's modulus and cross-sectional areas for all elements are same. Their deterministic values are $E = 2.1 \times 10^{11} N/m^2$ and $A = 4.0 \times 10^{-4} m^2$. The deterministic values of the forces are $\vec{f} = [f_1, f_2, f_3]^T = [1.2, 2.0, 1.5]^T \times 10^4 N$. Cross-sectional areas are interval variables, Young's modulus and loads are random variables. The mean value and standard deviation of the resistance (strength) R_i are taken as $\mu_{R_i} = 300\ MPa$ and $\sigma_{R_i} = 30\ MPa\ i = (1, 2, \cdots, 11)$, respectively. The failure probability (FP) and reliability (R) of element 11 are respectively shown in Figure 10 and Figure 11 while the dispersal degree of structural parameters, loads and resistance are varied from 0 to 0.1.

From Figures 10(a) and Figure 11(a), it can be observed that the system parameters produce different effects on both the failure probability and reliability of element 11. The failure probability and reliability are very sensitive to the change
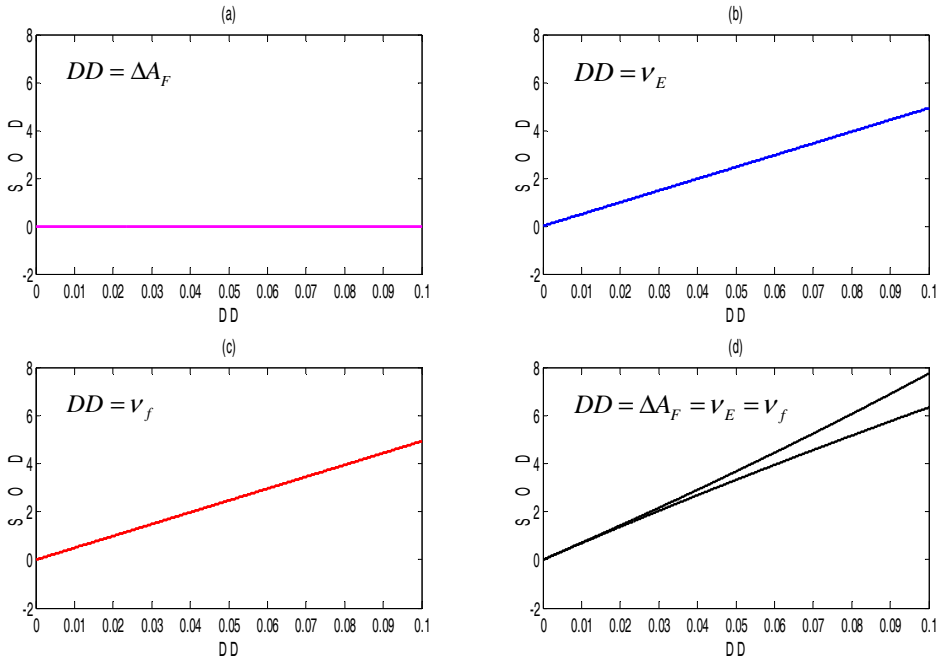
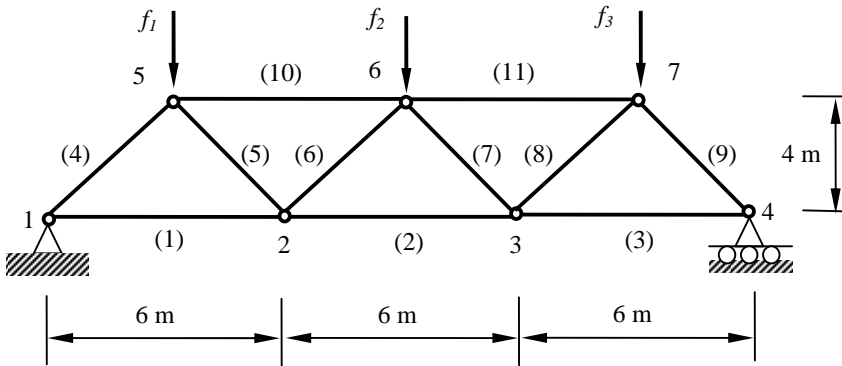Figure 8: Standard deviation of random interval displacement of node 70 in vertical direction (unit: mm)

Figure 9: Statically determinate truss structure

of the mean value of resistance and cross-sectional areas. The failure probability or reliability is not an interval variable when a structural system has pure random variables as shown in Figures 10(b) and 11(b). Figures 10(c) and 11(c) show that
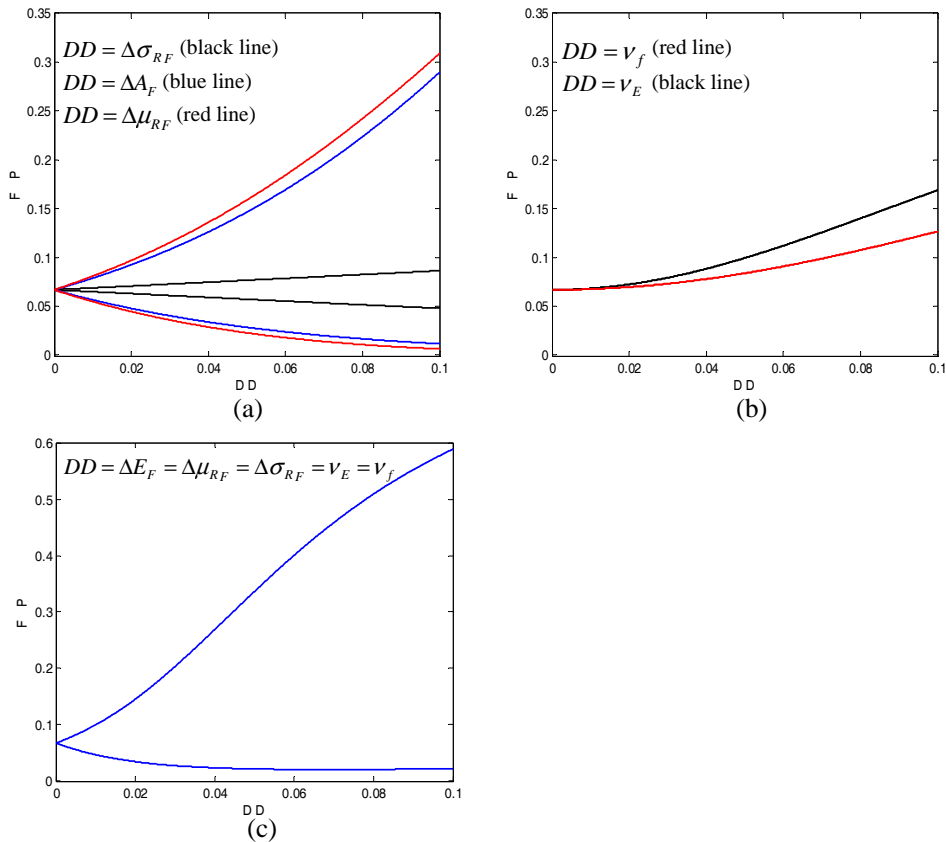
Figure 10: Failure probability of element 11

the maximum width of the reliability or failure probability is the biggest, when the uncertainties of all structural parameters and loads and resistance are considered simultaneously.

The statically determinate truss structure shown in Figure 9 can be considered as a series system. The structural reliability is given in Figure 12. It can be seen that the lower bound of the structural reliability, that is the worst possible value of structural reliability, decreases quickly as it is the product of the lower bounds of the reliability of all structural elements. For a series system, the reliability of all elements should be improved greatly if we want to improve the structural reliability. Figure 12 also shows that the midpoint of the structural reliability is not a constant and will drop down if the dispersal degree of structural parameters and load increases. Meanwhile, the interval width of structural reliability is getting bigger and bigger.
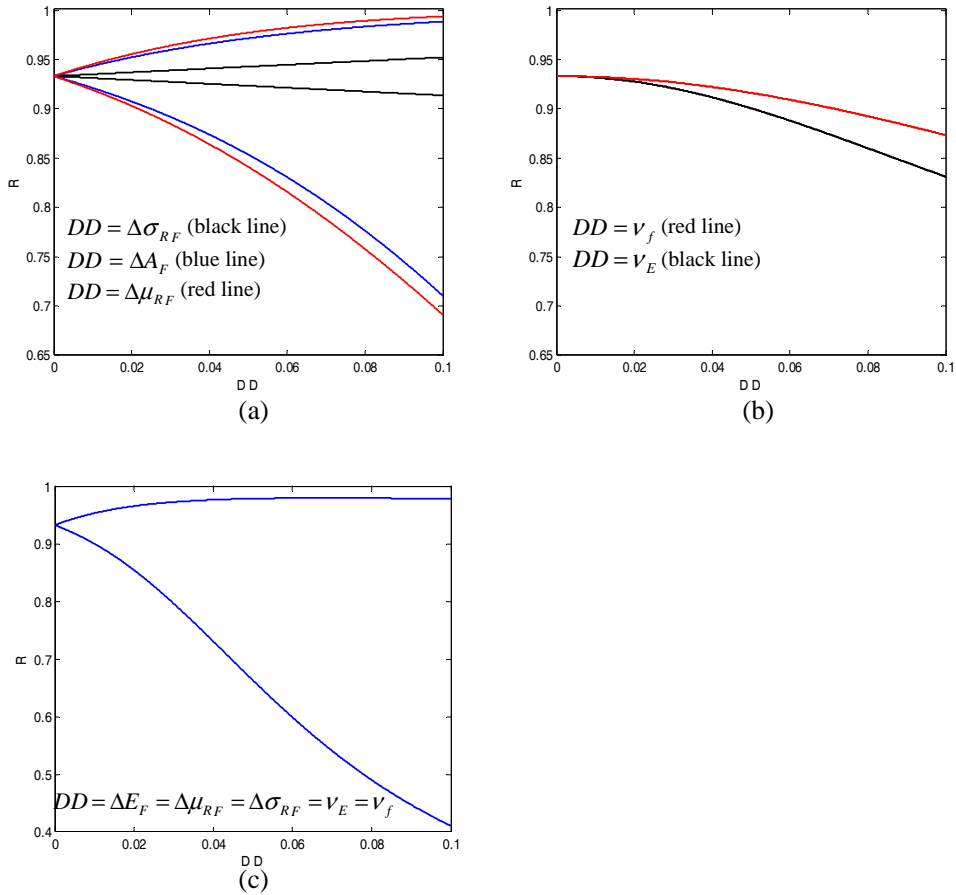
Figure 11: Reliability of element 11

5 Conclusions

A hybrid perturbation-based stochastic finite element and quasi-Monte Carlo method is proposed in this paper to investigate the static response of structures having both random and interval parameters/loads. Then the reliability analysis is implemented by a combination of first-order reliability method and interval approach. The expressions for calculating the mean value and standard deviation of random interval structural responses are developed. The effects of random and interval parameters/loads on structural response are also studied. The expressions of reliability of structural elements, and other typical systems, including series, parallel, series-parallel and parallel-series systems, are given in terms of intervals. The effective-
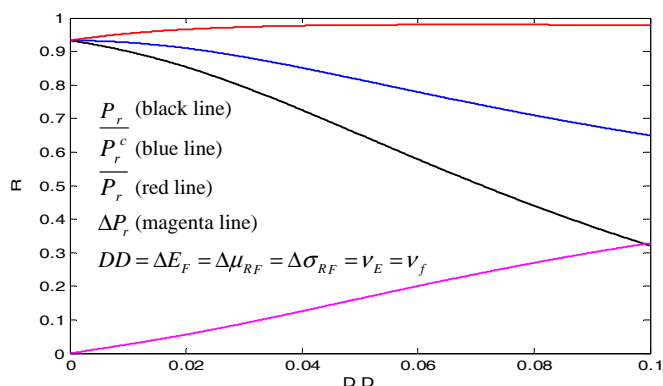
Figure 12: Reliability of the whole structure

ness and efficiency of the method has been demonstrated by the numerical examples. The proposed method can be applied to analyze complex structures.

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