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The Effect of the Geometrical Non-Linearity on the Stress Distribution in the Infinite Elastic Body with a Periodically Curved Row of Fibers

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Abstract: In the framework of the piecewise homogeneous body model with the use of the three-dimensional geometrically non-linear exact equations of the theory of elasticity, the method for determination of the stress-strain state in the infinite body containing periodically located row of periodically curved fibers is developed. It is assumed that the midlines of the fibers are in the same plane. With respect to the location of the fibers according to each other the sinphase and antiphase curving cases are considered. Numerical results on the effect of the geometrical non-linearity to the values of the self balanced shear and normal stresses are presented. In particular, it is established that this effect causes to increase (to decrease) the absolute values of these stresses in compression (in tension) along the fibers.

Keywords: Row of fibers, Self balanced stresses, geometrical non-linear, periodical curving, unidirectional fibrous composites.

1 Introduction

It is known that in the structure of the unidirectional fibrous composites the fibers have an initial curving or bending. These curvatures may be due to design features (see: Akbarov and Guz (2000), Chou, Cullough and Pipes (1986), Feng, Allen and Moy (1998), Ganesh and Naik (1996), Tarnopolsky, Jigun and Polyakov (1987)) or to various technological processes resulting from the action of various factors (see: Corten (1967), Tomashevskii and Yakovlev (2004)). Normally, the curvature caused by design features is modeled as a periodical, whereas the curvature caused

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by the technological process is modeled as a local one. The successful practical use of artificially created composite materials are associated, to a considerable extend, with the determination of the stress-strain state in these materials, taking the above mentioned curvature (or distortion) into account. As a result of the curvature of the fibers, the self-balanced stresses arise and these stresses can lead to the separation of the fibers from matrix under uniaxial tension or compression along the fibers (see: Akbarov and Guz (2000), Corten (1967), Guz (1990), Tarnopolsky and Rose (1969)). This separation causes the formation of macrocracks that is the accumulation of which can change significantly the stiffness and strength properties of the composites (Kashtalyan (2005)). Besides these, the initial insignificant curving of the reinforcing fibers is taken as a model for the investigation of the various fracture or stability loss problems of unidirectional composite materials (see: Akbarov and Kosker (2004), Akbarov, Cilli and Guz (1999), Akbarov, Sisman and Yahnioglu (1997), Akbarov and Mamedov (2009)). Consequently, establishing the mechanics of composite materials with curved structures is urgent both from the viewpoint of fundamental developments in the mechanics of a solid deformable body and from the viewpoint of applications to specific composite material components used in modern engineering.

There are two basic approaches to the study of the aforementioned problems. The first one is the continuum approach that may be used to calculate the components of the stress-strain state for the areas considerably greater in size than the curving; the influence of fibers curving in the structure is taken into account by means of quantitative variation in the normalized mechanical characteristics. Note that the investigations carried out in Bazhant (1968), Bolotin (1966), Mansfied and Purslow (1974), Swift (1975), Witney (1966) and many others relate to the first type approaches.

The second approach, which can be named the "local approach", was developed considerably later than the first one; it enables one to take into account the influence of reinforcing fibers curving in calculating the components of the stress-strain state in areas comparable to or smaller than that of the curving. This approach was developed both in the framework of the continuum theories and in that of a piecewise homogeneous body model. These methods are essential for estimating the influence of curving on the local distribution of the stress-strain state, which may determine local failure.

The systematic consideration of the results of the investigations regarding to the second approach was made in the monograph Akbarov and Guz (2000), and the review of those was given in the paper Akbarov and Guz (2004). It follows from this review that the considerable part of these investigations was made for layered composites. Up to recent years the investigations of the stress-strain state in the unidirectional fibrous composites with curved fibers are carried out for only small concentration of fibers and in this case composite material is modeled as an infinite elastic body containing a single periodical curved fiber. For investigation of such a problem in the framework of the piecewise homogeneous body model with the use of three-dimensional linear theory of elasticity, a method is developed and the corresponding numerical results are analyzed in Akbarov and Guz (1985a, 1985b).

It is evident that the investigation on how the stress distribution mentioned above is effected by the reciprocal effect between fibers, as volume ratio of fibers gets bigger in composites, is very important. For this purpose in Akbarov and Kosker (2003a, 2003b), Kosker and Akbarov (2003) the aforementioned method and investigations were extended to the corresponding problem for two neighboring fibers in an infinite elastic matrix and numerical results obtained are analyzed. But, in fact, the inquiry into the interaction of such fibers requires a more complicated model. Therefore, in Akbarov, Kosker and Ucan (2004, 2006) the foregoing approach was developed for a periodically located row of fibers in an infinite matrix and corresponding numerical results were presented. In the paper Akbarov, Kosker and Ucan (2004) (Akbarov, Kosker and Ucan (2006)) it was assumed that the curving of the fibers relative to each other is sinphase (antiphase) one and the investigations were made within the framework of the linear theory of elasticity.

According to the mechanical consideration and to the investigations carried out for layered composites with curved structures and analyzed in the monograph Akbarov and Guz (2000), there are some combinations of geometric and curvature parameters of the fibers and of the values of external force intensities under which it is necessary to investigate of the considered problems within the framework of the geometrical non-linear statement. Using the results of such investigations, it can be determined the limit of the intensity of the external forces for which the results obtained in the linear statement are acceptable. Furthermore, using these results it can be determined to the character of the influence of the geometrical non-linearity on the mechanical behaviour of the unidirectional fibrous composites with curved structure. In connection with this in the present paper the investigation carried out in Akbarov, Kosker and Ucan (2004, 2006) is developed for the geometrical non-linear statement. In this case the stress distribution is studied when the body is loaded at infinity by uniformly distributed normal forces with intensity p acting in the direction of the fibers. The investigation is carried out in the scope of the three-dimensional geometrical non-linear exact equations of the theory of elasticity.

Throughout the investigations, repeated indices are summed over their ranges; however, underlined repeated indices are not summed. Furthermore, to simplify the consideration we will use the tensor notation and physical components of tensors simultaneously.

2 Formulation of the problem

We consider the infinite body (matrix) containing periodically located row of fibers, which have initial periodically curving. With respect to the location of the fibers according to each other the following two cases are considered: (I) sinphase curving in plane and (II) antiphase curving in plane. Here under “in plane” it is understood that the midlines of the fibers lie in the same plane. With the middle line of each fiber we associate Lagrangian rectilinear $O_k x_{1k} x_{2k} x_{3k}$ and cylindrical $O_k r_k \theta_k z_k$ system of coordinates (Fig. 1) where k $(= -\infty, ..., -2, -1, 0, 1, 2, ..., +\infty)$ shows the fiber number. According to Fig.1,

$$x_{2k} = x_{20}, \quad x_{3k} = x_{30} = x_3, \quad x_{1k} = kR_{12} + x_{10}$$

$$r_0 e^{i\theta_0} = kR_{12} + r_k e^{i\theta_k}, \quad z_k = z_0 = z. \tag{1}$$

The midlines of the fibres are given by the equations
for the sinphase curving

$$x_{1k} = L\sin(\frac{2\pi}{\ell} x_{3k}), \quad x_{2k} = 0 \tag{2}$$

for the antiphase curving.

$$x_{1k} = (-1)^k L\sin(\frac{2\pi}{\ell} x_{3k}), \quad x_{2k} = 0 \tag{3}$$

We assume that the cross-section of each fibre perpendicular to its midline is a circle with constant radius R along the entire length of the fibres. Moreover we assume that, the curving amplitude L is smaller than the curving period ℓ and introduce a small parameter

$$\varepsilon = \frac{L}{\ell}, \ (0 < \varepsilon \ll 1) \tag{4}$$

In what follows, the values related to the fibres will be denoted by the superscripts (2k), but those related to the matrix by the superscript (1). The materials of the fibers and matrix are isotropic and homogeneous. We investigate the stress-strain state in the considered body in the case where the body loaded, at infinity, by uniformly distributed normal forces with an intensity p acting in the direction of the fibers. For this purpose, within the fibers and infinite matrix in the geometrical non-linear statement and in the cylindrical system of coordinates, we write governing

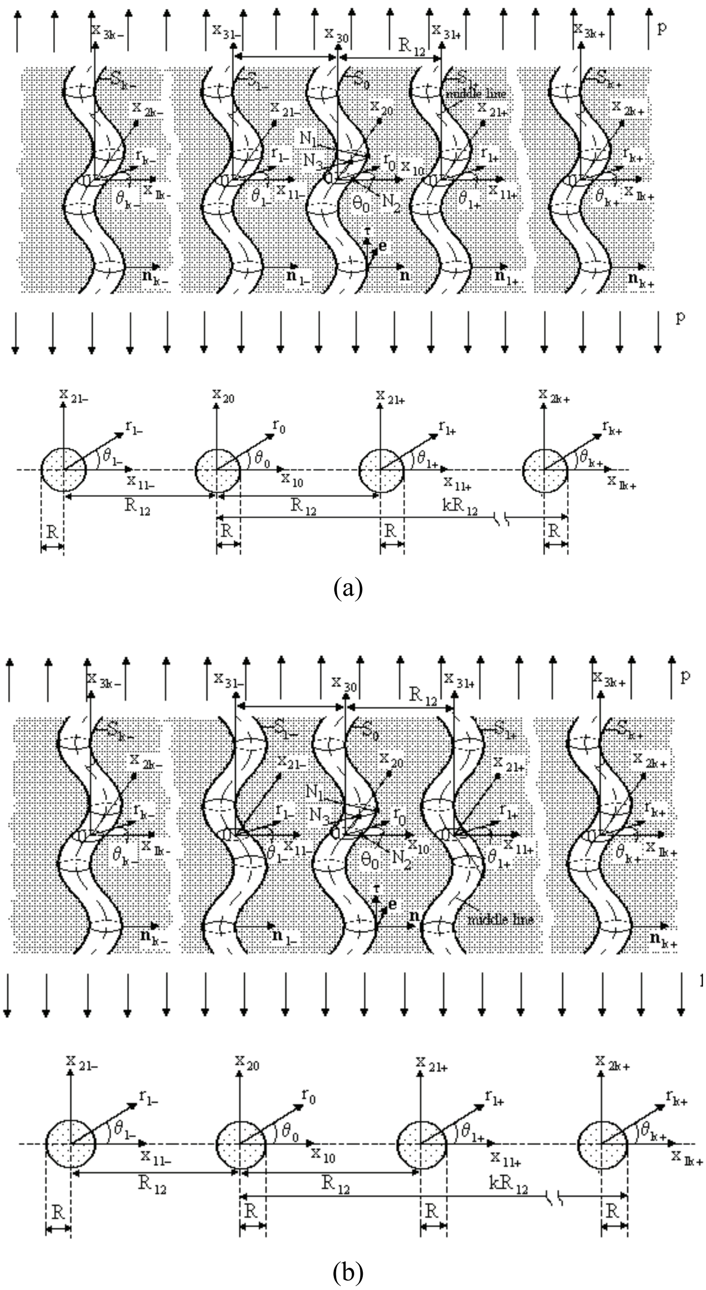

Figure 1: The geometry of material structure and chosen coordinates: (a) sinphase curving; (b) antiphase curving.

field equations.

$$
\begin{aligned}
&\nabla_i\left[\sigma^{(\underline{m})in}\left(g_n^j+\nabla_n u^{(\underline{m})j}\right)\right]=0,\\
&2\varepsilon_{jn}^{(m)}=\nabla_j u_n^{(m)}+\nabla_n u_j^{(m)}+\nabla_j u^{(\underline{m})i}\nabla_n u_i^{(\underline{m})},\\
&\sigma_{(ij)}^{(m)}=\lambda^{(\underline{m})}(e^{(\underline{m})}\delta_i^j)+2\mu^{(\underline{m})}\varepsilon_{(ij)}^{(\underline{m})},\\
&e^{(m)}=\varepsilon_{rr}^{(m)}+\varepsilon_{\theta\theta}^{(m)}+\varepsilon_{zz}^{(m)}.
\end{aligned}
\tag{5}
$$

It is assumed that on the interfaces between the fibers and matrix (denote them by S_q, where $q=-\infty,...,-1,0,1,...,+\infty$) the completely cohesion conditions are satisfied.

$$
\begin{aligned}
&\sigma^{(2\underline{q})in}\left(g_n^j+\nabla_n u^{(2\underline{q})j}\right)\Big|_{S_{\underline{q}}} n_{\underline{q}(j)}=\sigma^{(1)in}\left(g_n^j+\nabla_n u^{(1)j}\right)\Big|_{S_{\underline{q}}} n_{\underline{q}(j)},\\
&u_j^{(2\underline{q})}\Big|_{S_{\underline{q}}}=u_j^{(1)}\Big|_{S_{\underline{q}}}
\end{aligned}
\tag{6}
$$

where n_{qj} are the components of the unit normal vector to the surfaces S_q.

In the considered case it is also assumed that the conditions

$$
\left|\sigma_{(ij)}^{(2k)}\right|<\infty,\quad \left|u_{(i)}^{(2k)}\right|<\infty,\quad \sigma_{zz}^{(1)}\xrightarrow[|x_{20}|\to\infty]{} p,\quad \sigma_{(ij)}^{(1)}\xrightarrow[|x_{20}|\to\infty]{}0 \text{ for } (ij)\neq zz \tag{7}
$$

are satisfied.

According to the geometry of the structure of the considered body we can also write the following symmetry conditions with respect to the $x_{20}=0$ plane (Fig. 1).

$$
\begin{aligned}
&\sigma_{(ij)}^{(m)}(x_{1k},x_{2k},x_{3k})=\sigma_{(ij)}^{(m)}(x_{1k},-x_{2k},x_{3k}),\\
&u_{(i)}^{(m)}(x_{1k},x_{2k},x_{3k})=u_{(i)}^{(m)}(x_{1k},-x_{2k},x_{3k}).
\end{aligned}
\tag{8}
$$

Moreover, the periodical location of the row of fibers along the Ox_1 axis requires the satisfaction of the following conditions in the case where the materials of the fibers are the same.

For the sinphase curving case

$$
\begin{aligned}
&\sigma_{(ij)}^{(20)}(x_{10},x_{20},x_{30})=\sigma_{(ij)}^{(2k)}(x_{1k},x_{2k},x_{3k})=\sigma_{(ij)}^{(2k)}(x_{10}+kR_{12},x_{2k},x_{3k}),\\
&u_{(i)}^{(20)}(x_{10},x_{20},x_{30})=u_{(i)}^{(2k)}(x_{1k},x_{2k},x_{3k})=u_{(i)}^{(2k)}(x_{10}+kR_{12},x_{2k},x_{3k}),\\
&\sigma_{(ij)}^{(1)}(x_{10},x_{20},x_{30})=\sigma_{(ij)}^{(1)}(x_{10}+kR_{12},x_{20},x_{30}),
\end{aligned}
$$

$$u_{(i)}^{(1)}(x_{10},x_{20},x_{30}) = u_{(i)}^{(1)}(x_{10}+kR_{12},x_{20},x_{30}). \tag{9}$$

For the antiphase curving case

$$\sigma_{(ij)}^{(20)}(x_{10},x_{20},x_{30}) = (-1)^k\sigma_{(ij)}^{(2k)}(x_{1k},x_{2k},x_{3k}) = (-1)^k\sigma_{(ij)}^{(2k)}(x_{10}+kR_{12},x_{2k},x_{3k}),$$

$$u_{(i)}^{(20)}(x_{10},x_{20},x_{30}) = (-1)^k u_{(i)}^{(2k)}(x_{1k},x_{2k},x_{3k}) = (-1)^k u_{(i)}^{(2k)}(x_{10}+kR_{12},x_{2k},x_{3k}),$$

$$\sigma_{(ij)}^{(1)}(x_{10},x_{20},x_{30}) = (-1)^k\sigma_{(ij)}^{(1)}(x_{10}+kR_{12},x_{20},x_{30}),$$

$$u_{(i)}^{(1)}(x_{10},x_{20},x_{30}) = (-1)^k u_{(i)}^{(1)}(x_{10}+kR_{12},x_{20},x_{30}). \tag{10}$$

In equations (5)-(10) the conventional tensor notation is used and subscripts in parantheses show the physical components of the corresponding tensors. It is known that

$$\sigma_{(ij)} = \sigma^{\underline{ij}}H_{\underline{i}}H_{\underline{j}} = \sigma_{\underline{ij}}\frac{1}{H_{\underline{i}}H_{\underline{j}}}, \quad \varepsilon_{(ij)} = \varepsilon^{\underline{ij}}H_{\underline{i}}H_{\underline{j}} = \varepsilon_{\underline{ij}}\frac{1}{H_{\underline{i}}H_{\underline{j}}},$$

$$u_{(i)} = u^{\underline{i}}H_{\underline{i}} = u_{\underline{i}}\frac{1}{H_{\underline{i}}}, \tag{11}$$

where $(ij) = rr, \theta\theta, zz, r\theta, rz, z\theta$; $(i) = r, \theta, z$. Here and in the previous equations the contravariant (covariant) components of corresponding tensors or vectors are indicated by upper (lower) indices. Also, in equation (11) the Lamé's coefficients are denoted by H_i. Writing the expression of the Lamé's coefficients in the cylindrical coordinate system and after some rearrangements we can obtain the expression of the equations (5)-(10) in the cylindrical system of coordinates in explicit form.

Thus, with the above-stated, the formulation of the considered problem is exhausted.

3 Method of solution

For investigation of this problem we use the version of the boundary shape perturbation method developed in Akbarov and Guz (2000). In this case, using equations of midlines of the fibers and the condition of fiber cross-section, the equations of the interfaces S_1 and S_2 are derived as follows:

$$r_k = (1+\varepsilon^2(\delta'_{\underline{k}}(t_3))^2\sin^2\theta_{\underline{k}})^{-1}.$$

$$\left\{\begin{array}{c} (\varepsilon\delta_{\underline{k}}(t_3)+\varepsilon^3\delta_{\underline{k}}(t_3)(\delta'_{\underline{k}}(t_3))^2)\sin\theta_{\underline{k}}+ \\ \left[\begin{array}{c} R^2-\varepsilon^2(\delta_{\underline{k}}(t_3))^2- \\ \varepsilon^4(\delta'_{\underline{k}}(t_3))^2(\delta_{\underline{k}}(t_3))^2(1+\varepsilon^2(\delta'_{\underline{k}}(t_3))^2)\sin^2\theta_{\underline{k}} \end{array}\right]^{1/2} \end{array}\right\}$$

$$z_k = t_3 - \varepsilon\delta'_{\underline{k}}(t_3)r_{\underline{k}}(t_3,\theta_{\underline{k}})\sin\theta_{\underline{k}} + \varepsilon^2\delta_{\underline{k}}(t_3)\delta'_{\underline{k}}(t_3),$$

$$\delta'_k(t_3) = \frac{d\delta_k(t_3)}{dt_3} \tag{12}$$

where

$$\delta_k(t_3) = \ell \sin(\frac{2\pi}{\ell} t_3) \text{ for sinphase curving} \tag{13}$$

$$\delta_k(t_3) = (-1)^k \ell \sin(\frac{2\pi}{\ell} t_3) \text{ for antiphase curving} \tag{14}$$

and t_3 is a parameter and $t_3 \in (-\infty, +\infty)$.

After certain transformations, we obtain the following expressions from equation (12) for the components of the unit normal vector to the surfaces S_k:

$$n_{kr} = r_{\underline{k}}(\theta_{\underline{k}}, t_3) \frac{\partial z_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial t_3} \left[A_{\underline{k}}(\theta_{\underline{k}}, t_3)\right]^{-1},$$

$$n_{k\theta} = \left[\frac{\partial z_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial \theta_{\underline{k}}} \frac{\partial r_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial t_3} - \frac{\partial r_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial \theta_{\underline{k}}} \frac{\partial z_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial t_3}\right] \cdot \left[A_{\underline{k}}(\theta_{\underline{k}}, t_3)\right]^{-1},$$

$$n_{kz} = -r_{\underline{k}}(\theta_{\underline{k}}, t_3) \frac{\partial r_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial t_3} \left[A_{\underline{k}}(\theta_{\underline{k}}, t_3)\right]^{-1}, \tag{15}$$

where

$$A_{\underline{k}}(\theta_{\underline{k}}, t_3) = \left[\left(r_{\underline{k}}(\theta_{\underline{k}}, t_3) \frac{\partial z_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial t_3}\right)^2 + \left(\frac{\partial z_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial \theta_{\underline{k}}} \frac{\partial r_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial t_3} - \frac{\partial z_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial t_3} \frac{\partial r_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial \theta_{\underline{k}}}\right)^2 + \left(r_{\underline{k}}(\theta_{\underline{k}}, t_3) \frac{\partial r_{\underline{k}}(\theta_{\underline{k}}, t_3)}{\partial t_3}\right)^2\right]^{1/2} \tag{16}$$

According to Akbarov and Guz (2000), the unknowns are presented in series form in ε

$$\left\{\sigma_{(\underline{ij})}^{(\underline{m})}; \varepsilon_{(\underline{ij})}^{(\underline{m})}; u_{(\underline{i})}^{(\underline{m})}\right\} = \sum_{q=0}^{\infty} \varepsilon^q \left\{\sigma_{(\underline{ij})}^{(\underline{m}),q}; \varepsilon_{(\underline{ij})}^{(\underline{m}),q}; u_{(\underline{i})}^{(\underline{m}),q}\right\}, \tag{17}$$

Moreover, the expressions (15), (16) are also presented in the series form in ε as follows:

$$r_q = R + \sum_{k=1}^{\infty} \varepsilon^k a_{\underline{q}k}(\theta_{\underline{q}}, t_3), \quad z_q = t_3 + \sum_{k=1}^{\infty} \varepsilon^k b_{\underline{q}k}(\theta_{\underline{q}}, t_3),$$

$$n_{qr} = 1 + \sum_{k=1}^{\infty} \varepsilon^k c_{\underline{q}k}(\theta_{\underline{q}}, t_3), \quad n_{q\theta} = \sum_{k=1}^{\infty} \varepsilon^k d_{\underline{q}k}(\theta_{\underline{q}}, t_3),$$

$$n_{qz} = \sum_{k=1}^{\infty} \varepsilon^k g_{\underline{q}k}(\theta_{\underline{q}}, t_3). \tag{18}$$

The expressions of the functions $a_{\underline{q}k}(\theta_{\underline{q}}, t_3), ..., g_{\underline{q}k}(\theta_{\underline{q}}, t_3)$ in equation (18) can easily be obtained from equations (15) and (16); therefore these expressions are not given here.

Substituting equation (17) in equation (5), we obtain a set of equations for each approximation in equation (17). Using expressions (18) we expand the values of each approximation (17) in the series form in the vicinity of ($r_k = R$, $z_q = t_3$). Substituting these last expressions in contact condition (6) and using the expressions of n_{qr}, $n_{q\theta}$ and n_{qz} given in (12), after some manipulations we obtain contact conditions satisfied in $r_k = R$, $z_q = t_3$ for each approach in equation (17).

It is evident that, for the zeroth approximation the equation (5) is valid and the conditions (6) are replaced by the same ones satisfied in $r_k = R$, $z_q = t_3$. We assume that $\nabla_n u^{(k)j,0} << 1$ and therefore we replace the terms $g_n^j + \nabla_n u^{(k)j,0}$ by δ_n^j where δ_n^j are Kronecker symbols. According to this assumption, for the zeroth approximation we obtain the following system of equations.

$$\nabla_i \sigma^{(k)ij,0} = 0, \quad 2\varepsilon_{ij}^{(k),0} = \nabla_j u_i^{(k),0} + \nabla_i u_j^{(k),0} \tag{19}$$

and contact conditions

$$\sigma_{(ij)}^{(2q),0}\Big|_{r_q=R} = \sigma_{(ij)}^{(1),0}\Big|_{r_q=R}, \quad u_{(i)}^{(2q),0}\Big|_{r_q=R} = u_{(i)}^{(1),0}\Big|_{r_q=R}; \tag{20}$$

$$(ij) = rr, r\theta, rz; \quad (i) = r, \theta, z$$

The conditions (7) are valid for zeroth approximation. Moreover, the conditions (8)-(10) are valid for each approximation separately.

Taking aforementioned assumption into account, for the subsequent approximations we obtain the following system of equations:

$$\nabla_i \left[\sigma^{(m)ij,q} + \sigma^{(\underline{m})in,0} \nabla_n u^{(\underline{m})j,q}\right] = \underline{-\sum_{k=1}^{q-1} \nabla_i \left(\sigma^{(\underline{m})in,q-k} \nabla_n u^{(\underline{m})j,k}\right)},$$

$$2\varepsilon_{ij}^{(m),q} = \nabla_j u_i^{(m),q} + \nabla_i u_j^{(m),q} + \underline{\sum_{k=1}^{q-1} \nabla_j u^{(\underline{m})n,q-k} \nabla_i u_n^{(\underline{m}),k}}. \tag{21}$$

Note that the underlined terms in equations (21) are equal to zero for the first approximation. Moreover note that the conditions (7) for the subsequent approximations are replaced by the following ones

$$\left|\sigma_{(ij)}^{(2k),q}\right| < \infty, \quad \left|u_{(i)}^{(2k),q}\right| < \infty, \quad \sigma_{(ij)}^{(1),q} \xrightarrow[|x_2|\to\infty]{} 0 \text{ for } (ij) \neq zz \tag{22}$$

It is necessary to add the mechanical relations to these equations

$$\sigma_{(in)}^{(m),q} = \lambda^{(\underline{m})} e^{(\underline{m}),q} \delta_i^n + 2\mu^{(\underline{m})} \varepsilon_{(in)}^{(\underline{m}),q},$$

$$e^{(m),q} = \varepsilon_{rr}^{(m),q} + \varepsilon_{\theta\theta}^{(m),q} + \varepsilon_{zz}^{(m),q} \tag{23}$$

which are satisfied for each approximation separately.

Now we write the contact conditions for the first approximation by physical components of the stress tensor and displacement vector.

$$\left[\sigma_{(i)r}\right]_{1,1}^{2k,1} + f_{1\underline{k}} \left[\frac{\partial \sigma_{(i)r}}{\partial r}\right]_{1,0}^{2\underline{k},0} + \varphi_{1\underline{k}} \left[\frac{\partial \sigma_{(i)r}}{\partial z}\right]_{1,0}^{2\underline{k},0} + \gamma_{r\underline{k}} \left[\sigma_{(i)r}\right]_{1,0}^{2\underline{k},0} +$$

$$\gamma_{\theta\underline{k}} \left[\sigma_{(i)\theta}\right]_{1,0}^{2\underline{k},0} + \gamma_{z\underline{k}} \left[\sigma_{(i)z}\right]_{1,0}^{2\underline{k},0} = 0$$

$$\left[u_{(i)}\right]_{1,1}^{2k,1} + f_{1\underline{k}} \left[\frac{\partial u_{(i)}}{\partial r}\right]_{1,0}^{2\underline{k},0} + \varphi_{1\underline{k}} \left[\frac{\partial u_{(i)}}{\partial z}\right]_{1,0}^{2\underline{k},0} = 0 \tag{24}$$

where $(i) = r, \theta, z$.

In equation (24) replacing (i) with r, θ and z we obtain the explicit form of the corresponding contact conditions (24). Moreover, in equation (24) the following notation is used:

$$[\varphi]_{1,s}^{2k,s} = \varphi^{(2k),s} - \varphi^{(1),s}; \quad f_{1k} = \delta_{\underline{k}}(t_3)\cos\theta_{\underline{k}}; \quad \varphi_{1k} = -R\delta_{\underline{k}}'(t_3)\cos\theta_{\underline{k}},$$

$$\gamma_{rk} = \left(\frac{\delta_{\underline{k}}(t_3)}{R} - \delta_{\underline{k}}''(t_3)R\right)\cos\theta_{\underline{k}};$$

$$\gamma_{\theta k} = -\frac{\delta_{\underline{k}}(t_3)}{R}\sin\theta_{\underline{k}};$$

$$\gamma_{zk} = -\delta_{\underline{k}}'(t_3)\cos\theta_{\underline{k}}; \quad \delta_k'(t_3) = \frac{d\delta_k(t_3)}{dt_3}. \tag{25}$$

Note that, similar contact conditions are obtained for the second and subsequent approximations. According to the regarding investigations reviewed in Akbarov and

Guz (2004), the main effect of the fibres curving on the stress distribution arises within the framework of the first approximation. The second and subsequent approximations give only some insignificant quantitative corrections to these results. Since, the determinations of these approximations is very complicated and cumbersome, the investigations in the present paper are made only within the framework of the zeroth and the first approximations.

Now, we determine the unknown values belonging to these approximations. Assume that the materials of each fiber are the same. The Young's moduli and Poisson coefficient for fibers (matrix) material we denote as $E^{(2)}$ and $\nu^{(2)}$ ($E^{(1)}$ and $\nu^{(1)}$) respectively. We will suppose that $\nu^{(2)} = \nu^{(1)}$. Note that this supposition only slightly affects numerical results and is introduced to simplify the solution procedure. Because in this case the stress state regarding to the zeroth approximation is homogeneous state and is determined by the following relations:

$$\sigma_{zz}^{(1),0} = p; \quad \sigma_{zz}^{(2k),0} = \sigma_{zz}^{(2),0} = \frac{E^{(2)}}{E^{(1)}}p; \quad \varepsilon_{zz}^{(2k),0} = \varepsilon_{zz}^{(1),0} = \frac{p}{E^{(1)}},$$

$$u_z^{(2k),0} = u_z^{(1),0} = \varepsilon_{zz}^{(1),0} z, \quad \sigma_{(ij)}^{(2k),0} = \sigma_{(ij)}^{(1),0} = 0 \text{ for } (ij) = rr, \theta\theta, r\theta, \theta z, rz \qquad (26)$$

Now we consider the determination of the first approximation. According to the expression (26), we obtain the following equations from (21) for the first approximation:

$$\frac{\partial \sigma_{rr}^{(m),1}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}^{(m),1}}{\partial \theta} + \frac{\partial \sigma_{rz}^{(m),1}}{\partial z} + \frac{1}{r}\left(\sigma_{rr}^{(m),1} - \sigma_{\theta\theta}^{(m),1}\right) + \sigma_{zz}^{(\underline{m}),0}\frac{\partial^2 u_r^{(\underline{m}),1}}{\partial z^2} = 0$$

$$\frac{\partial \sigma_{r\theta}^{(m),1}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}^{(m),1}}{\partial \theta} + \frac{\partial \sigma_{\theta z}^{(m),1}}{\partial z} + \frac{2}{r}\sigma_{r\theta}^{(m),1} + \sigma_{zz}^{(\underline{m}),0}\frac{\partial^2 u_\theta^{(\underline{m}),1}}{\partial z^2} = 0,$$

$$\frac{\partial \sigma_{rz}^{(m),1}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta z}^{(m),1}}{\partial \theta} + \frac{\partial \sigma_{zz}^{(m),1}}{\partial z} + \frac{1}{r}\sigma_{rz}^{(m),1} + \sigma_{zz}^{(\underline{m}),0}\frac{\partial^2 u_z^{(\underline{m}),1}}{\partial z^2} = 0. \qquad (27)$$

The mechanical relations remain as in equation (23) for $q = 1$. Moreover, the geometrical relations have the following form:

$$\varepsilon_{rr}^{(m),1} = \frac{\partial u_r^{(m),1}}{\partial r}, \quad \varepsilon_{\theta\theta}^{(m),1} = \frac{\partial u_\theta^{(m),1}}{r\partial \theta} + \frac{u_r^{(m),1}}{r}, \quad \varepsilon_{zz}^{(m),1} = \frac{\partial u_z^{(m),1}}{\partial z},$$

$$\varepsilon_{r\theta}^{(m),1} = \frac{1}{2}\left(\frac{\partial u_r^{(m),1}}{r\partial \theta} + \frac{\partial u_\theta^{(m),1}}{\partial r} - \frac{u_\theta^{(m),1}}{r}\right),$$

$$\varepsilon_{\theta z}^{(m),1} = \frac{1}{2}\left(\frac{\partial u_\theta^{(m),1}}{\partial z} + \frac{\partial u_z^{(m),1}}{r\partial \theta}\right), \varepsilon_{zr}^{(m),1} = \frac{1}{2}\left(\frac{\partial u_z^{(m),1}}{\partial r} + \frac{\partial u_r^{(m),1}}{\partial z}\right). \qquad (28)$$

Taking the expression (26) into account the contact conditions (24) can be rewritten as follows:

$$[\sigma_{rr}]_{1,1}^{2k,1}=0, \quad [\sigma_{r\theta}]_{1,1}^{2k,1}=0,$$
$$[\sigma_{rz}]_{1,1}^{2k,1}=\delta'_{\underline{k}}(t_3)\left(\sigma_{zz}^{(1),0}-\sigma_{zz}^{(2),0}\right)\cos\theta_{\underline{k}},$$
$$[u_r]_{1,1}^{2k,1}=0, \quad [u_\theta]_{1,1}^{2k,1}=0, \quad [u_z]_{1,1}^{2k,1}=0. \tag{29}$$

By direct verification it is obtained that the equations (27) coincide with the equations of the Three-Dimensional Linearized Theory of Deformable Bodies (Biot (1965), Guz (1999)). Therefore, according to Guz (1999), we can use the following representations in the cylindrical system of coordinates to solve the equation systems (27), (28), (23).

$$u_r=\frac{1}{r}\frac{\partial}{\partial\theta}\psi-\frac{\partial^2}{\partial r\partial z}\chi, \quad u_\theta=-\frac{\partial}{\partial r}\psi-\frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}\chi,$$
$$u_3=(\lambda+\mu)^{-1}\left((\lambda+2\mu)\Delta_1+(\mu+\sigma_{zz}^0)\frac{\partial^2}{\partial z^2}\right)\chi,$$
$$\Delta_1=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}. \tag{30}$$

The functions ψ and χ are determined from the equations

$$\left(\Delta_1+\xi_1^2\frac{\partial^2}{\partial z^2}\right)\psi=0,$$
$$\left(\Delta_1+\xi_2^2\frac{\partial^2}{\partial z^2}\right)\left(\Delta_1+\xi_3^2\frac{\partial^2}{\partial z^2}\right)\chi=0 \tag{31}$$

where

$$\xi_1=\sqrt{\frac{\mu+\sigma_{zz}^0}{\mu}}, \quad \xi_2=\sqrt{\frac{\mu+\sigma_{zz}^0}{\mu}}, \quad \xi_3=\sqrt{\frac{\lambda+2\mu+\sigma_{zz}^0}{\lambda+2\mu}}. \tag{32}$$

In equations (30)-(32) we replace the quantities $u_{(i)}$, λ, μ, ζ_i (i=1,2,3) and σ_{zz}^0 with $u_{(i)}^{(1),1}$, $\lambda^{(1)}$, $\mu^{(1)}$, $\zeta_i^{(1)}$ and $\sigma_{zz}^{(1),0}$, respectively for the matrix and with $u_{(i)}^{(2k),1}$, $\lambda^{(2)}$, $\mu^{(2)}$, $\zeta_i^{(2k)}$ and $\sigma_{zz}^{(2k),0}$ respectively for the fiber. Taking the expression of the contact conditions (29) and the conditions (22) into account the solutions to equations (30) are found as follows:

$$\psi^{(2k)}=\alpha\sin\alpha z\sum_{n=-\infty}^{\infty}C_n^{(2k)}I_n(\xi_1^{(2k)}\alpha r_k)\exp(in\theta_k),$$

$$\chi^{(2k)} = \cos\alpha z \sum_{n=-\infty}^{\infty} \begin{bmatrix} A_n^{(2k)} I_n(\xi_2^{(2k)}\alpha r_k) \\ +B_n^{(2k)} I_n(\xi_3^{(2k)}\alpha r_k) \end{bmatrix} \exp(in\theta_k), \tag{33}$$

$$\psi^{(1)} = \alpha \sin\alpha z \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} C_n^{(1)m} K_n(\xi_1^{(1)}\alpha r_m) \exp(in\theta_m),$$

$$\chi^{(1)} = \cos\alpha z \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \begin{bmatrix} A_n^{(1)m} K_n(\xi_2^{(1)}\alpha r_m) \\ +B_n^{(1)m} K_n(\xi_3^{(1)}\alpha r_m) \end{bmatrix} \exp(in\theta_m), \tag{34}$$

where $\alpha = 2\pi/\ell$; $I_n(x)$ and $K_n(x)$ are the Bessel functions of a purely imaginary argument and the Macdonald functions, respectively. The unknowns $C_n^{(2k)}$, $A_n^{(2k)}$, $B_n^{(2k)}$, $C_n^{(1)m}$, $A_n^{(1)m}$ and $B_n^{(1)m}$ are the complex constants and satisfy the relations

$$A_n^{(2k)} = \overline{A_{-n}^{(2k)}},\ B_n^{(2k)} = \overline{B_{-n}^{(2k)}},\ C_n^{(2k)} = \overline{C_{-n}^{(2k)}},$$

$$Im\,A_0^{(2k)} = Im\,B_0^{(2k)} = Im\,C_0^{(2k)} = 0,$$

$$A_n^{(1)m} = \overline{A_{-n}^{(1)m}},\ B_n^{(1)m} = \overline{B_{-n}^{(1)m}},\ C_n^{(1)} = \overline{C_{-n}^{(1)m}},$$

$$Im\,A_0^{(1)m} = Im\,B_0^{(1)m} = Im\,C_0^{(1)m} = 0. \tag{35}$$

Next, contact conditions (29) must be satisfied. For this purpose, we represent expressions (33) and (34) in a m-th ($m = -\infty, ..., -2, -1, 0, 1, 2, ..., +\infty$) cylindrical coordinate system. In this case, we use the summation theorem (Watson (1962)) for the $K_n(x)$ function, which can be written as follows for the considered case.

$$r_m \exp i\theta_m = (n-m)R_{12} \exp i\varphi_{mn} + r_n \exp i\theta_n,$$

$$K_\nu(cr_n)\exp i\nu\theta_n =$$

$$\sum_{k=-\infty}^{\infty} (-1)^\nu I_k(cr_m) K_{\nu-k}(c\,|n-m|\,R_{12}) \exp[i(\nu-k)\varphi_{mn}] \exp ik\theta_m\text{'}$$

$$\varphi_{mn} = 0 \text{ for } n > m,\ \varphi_{mn} = \pi \text{ for } m > n,$$

$$c = const. \quad r < R_{12}; \tag{36}$$

Using equations (33)-(36), we obtain from (23), (28) and (29) an infinite system of algebraic equations for the unknown constants in (33) and (34). Introducing the notation

$$C_n^{(1)m} K_n(\xi_1^{(1)}\kappa) = y_{n1}^{(1)m} + iz_{n1}^{(1)m}, \quad A_n^{(1)m} K_n(\xi_2^{(1)}\kappa) = z_{n2}^{(1)m} + iy_{n2}^{(1)m},$$

$$B_n^{(1)m}K_n(\xi_3^{(1)}\kappa) = z_{n3}^{(1)m} + iy_{n3}^{(1)m}, \quad C_n^{(2m)}I_n(\xi_1^{(2)}\kappa) = y_{n1}^{(2)m} + iz_{n1}^{(2)m},$$
$$A_n^{(2m)}I_n(\xi_2^{(2)}\kappa) = z_{n2}^{(2)m} + iy_{n2}^{(2)m}, \quad B_n^{(2m)}I_n(\xi_3^{(2)}\kappa) = z_{n3}^{(2)m} + iy_{n3}^{(2)m},$$
$$Z_n^{(k)q} = \left\| \begin{matrix} z_{n1}^{(k)q} \\ z_{n2}^{(k)q} \\ z_{n3}^{(k)q} \end{matrix} \right\|, Y_n^{(k)q} = \left\| \begin{matrix} y_{n1}^{(k)q} \\ y_{n2}^{(k)q} \\ y_{n3}^{(k)q} \end{matrix} \right\|;$$
$$D_{n\nu}^{(1)q} = \left\| d_{ts}^{(1)q}(n,\nu) \right\|, \quad D_n^{(2)q} = \left\| d_{ts}^{(2)q}(n) \right\|, \quad F_{n\nu}^{(1)q} = \left\| f_{ts}^{(1)q}(n,\nu) \right\|,$$
$$F_n^{(2)q} = \left\| f_{ts}^{(2)q}(n) \right\|, \quad m;q = -\infty,...,-2,-1,0,1,2,...,+\infty;$$
$$i = \sqrt{-1}, \quad t,s = 1,2,3,...,+\infty, \quad \kappa = 2\pi R/\ell \tag{37}$$

we have

$$Z_n^{(1)k} + \sum_{\nu=0}^{\infty}\sum_{q=-\infty}^{+\infty} D_{n\nu}^{(1)q} Z_\nu^{(1)q} + D_n^{(2)k} Z_n^{(2)k} = 0, \tag{38}$$

$$Y_n^{(1)k} + \sum_{\nu=0}^{\infty}\sum_{q=-\infty}^{+\infty} F_{n\nu}^{(1)q} Y_\nu^{(1)q} + F_n^{(2)k} Y_n^{(2)k} = 2\pi\delta_n^3(\sigma_{zz}^{(1),0} - \sigma_{zz}^{(2),0}) \tag{39}$$

where $n = 1,2,...,\infty$, $\delta_3^3 = 1$ and $\delta_n^3 = 0$ for $n \neq 3$, $k = -\infty,...,-2,-1,0,1,2,...,+\infty$. Note that under obtaining the equations (37) and (38) the symmetry conditions (8) are used. Moreover, the quantities $D_{n\nu}^{(1)q}$, $F_{n\nu}^{(1)q}$ and $D_n^{(2)k}$, $F_n^{(2)k}$ are obtained from the corresponding formulas mentioned above. Their detailed expressions are rather cumbersome and therefore they are omitted here.

It is seen from equations (38) that $Z_n^{(1)q} = Z_n^{(2)q} = 0$. Moreover, it follows from the periodicity conditions (9) and (10) that the folllowing relations must be satisfied:

for sinphase curving case of the fibers

$$Y_n^{(1)k} = Y_n^{(1)0}, \quad k = -\infty,...,-2,-1,0,1,2,...,+\infty \tag{40}$$

for antiphase curving case of the fibers

$$Y_n^{(1)k} = (-1)^k Y_n^{(1)0}, \quad k = -\infty,...,-2,-1,0,1,2,...,+\infty \tag{41}$$

Taking the relations, (40) and (41) into account from (39) we obtain for sinphase curving case

$$Y_n^{(1)0} + \sum_{\nu=0}^{\infty} Y_\nu^{(1)0}\left(\sum_{q=1}^{+\infty} 2F_{n\nu}^{(1)q}\right) + F_n^{(2)0}Y_n^{(2)0} = \atop 2\pi\delta_n^3(\sigma_{zz}^{(1),0} - \sigma_{zz}^{(2),0}), \tag{42}$$

and for antiphase curving case

$$Y_n^{(1)0}+\sum_{\nu=0}^{\infty}(-1)^{\nu}Y_{\nu}^{(1)0}\left(\sum_{q=1}^{+\infty}F_{n\nu}^{(1)q}\right)+F_n^{(2)0}Y_n^{(2)0}= \\ 2\pi\delta_n^3(\sigma_{zz}^{(1),0}-\sigma_{zz}^{(2),0}) . \tag{43}$$

For numerical investigations the infinite system of algebraic equations (42) and (43) must be approximated by corresponding finite system. To validate of such a replacement, it must be shown that the determinant of the infinite system of equations is of normal type (Kantarovich and Krilov (1962)). Such is the case if the series

$$M=\sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}\left|\sum_{q=1}^{+\infty}F_{n\nu}^{(1)q}\right| \tag{44}$$

converges. For investigating this series, we use the following asymptotic estimates of the functions $I_n(x)$ and $K_n(x)$:

$$I_n(x)<c_1\frac{1}{n!}\left(\frac{|x|}{2}\right)^n,\quad c_1=const.;\quad K_n(x)\approx c_2\,(n-1)!\left(\frac{2}{|x|}\right)^n, \\ c_2=const. \tag{45}$$

These relations hold for large n and fixed x. Let

$$\frac{R}{R_{12}-2L}>\frac{R}{R_{12}},\quad \frac{R_{12}}{R}>2 \tag{46}$$

which means that the fibres do not contact with each other. Then, taking into account equations (45) and (46) and analysing the expressions of $F_{n\nu}^{(1)q}$, we obtain the following estimate for series (44):

$$M<c_3\sum_{n=0}^{\infty}n^{c_4}(\rho-1)^{-n};\quad c_3,c_4=const.,\quad \rho=\frac{R_{12}}{R} \tag{47}$$

As the series on the right hand side converges, so does series (44). Note that such a proof was also performed in Guz (1990), Babaev, Guz and Cherevko (1985), Guz and Chekhov (2007), Guz, Rushchitsky and Guz (2007).

Consequently, for numerical investigations, the infinite system of algebraic equations (42) and (43) can be replaced with

$$Y_n^{(1)0}+\sum_{\nu=0}^{N_\nu}Y_{\nu}^{(1)0}\left(\sum_{q=1}^{N_q}2F_{n\nu}^{(1)q}\right)+F_n^{(2)0}Y_n^{(2)0}= \\ 2\pi\delta_n^3(\sigma_{zz}^{(1),0}-\sigma_{zz}^{(2),0}) \tag{48}$$

for the sinphase curving case and with

$$Y_n^{(1)0} + \sum_{\nu=0}^{N_\nu} (-1)^\nu Y_\nu^{(1)0} \left(\sum_{q=1}^{N_q} 2F_{n\nu}^{(1)q} \right) + F_n^{(2)0} Y_n^{(2)0} = 2\pi\delta_n^3 (\sigma_{zz}^{(1),0} - \sigma_{zz}^{(2),0}) \quad (49)$$

for the antiphase curving case, where $n = 1,2,...,N_\nu$ in equations (48) and (49). The values of N_ν and N_q in these equations are determined from the convergence requirement of numerical results.

It should be noted that the second and subsecond approximations can be determined in a similar way.

4 Numerical results and discussions

Assume that $\nu^{(1)} = \nu^{(2)} = 0.3$ and introduce the parameter $\in = p/E^{(1)}$ for estimation of the influence of the geometrical non-linearity to the values of the self-balanced normal and shear stresses acting on the interface betweeen the fibers and matrix. Note that these self-balanced stresses arise as a result of the curving of the fibers and the adhesion strength of the material depends mainly on the values of these stresses. In this case taking into account the periodicity of the curving form and the periodicity of the location of the row of fibers, we consider the distribution of these stresses on the surface S_0 only (Fig. 1). We denote the aforementioned normal stress by σ_{nn} and shear stress by $\sigma_{n\tau}$. Note that the stresses σ_{nn} and $\sigma_{n\tau}$ act along the normal $\mathbf{n}$ and tangent $\boldsymbol{\tau}$ vectors to the surface S_0 (Fig. 1). If $\varepsilon = 0$ (i.e. the curving is absent), the stresses σ_{nn} and $\sigma_{n\tau}$ coincide with the stresses σ_{rr} and σ_{rz}, respectively.

Let us also introduce the parameters $\kappa = 2\pi R/\ell$ and $\rho = R_{12}/R$, where R is a radius of the cross-section of the fibers, ℓ is a length of the periodicity of the fibers curving, R_{12} is a distance between two neighboring fibers (Fig. 1). As follows from the present investigations and those carried out in the monograph Akbarov and Guz (2000) that the stress σ_{nn} has a maximum at the point of S_0 determined by equation (12) at $\theta_0 = 0$, $\alpha t_3 = \pi/2$, but the stress $\sigma_{n\tau}$ has a maximum in the vicinity of the point of S_0corresponding to $\theta_0 = 0$, $\alpha t_3 = 0$ ($\alpha = 2\pi/\ell$). These points are denoted by N_1 (for σ_{nn}) and N_2 (for $\sigma_{n\tau}$) in Fig.1. Moreover, the aforementioned investigations show that in the sinphase (antiphase) curving case the shear (normal) stress $\sigma_{n\tau}$ (σ_{nn}) has dominating values. Taking above-stated into account we consider the influence of the problem parameter to the values of $\sigma_{n\tau}$ in the sinphase curving case. But in the antiphase curving case we investigate this influence for the normal stress σ_{nn}. For all numerical investigations presented below it is assumed that $\varepsilon = 0.015$.

Thus, consider the graphs given in Figs. 2, 3 and 4 which show the dependencies between the $\sigma_{n\tau}/|p|$ and the parameter κ for $\rho = 2.1$, 2.5 and 5, respectively, for various suitible values of the parameter $\in$ and for $E^{(2)}/E^{(1)} = 50$. The graphs of the dependencies between $\sigma_{nn}/|p|$ and κ with the same sequencies of the problem parameters are given in Figs. 5, 6 and 7. In these figures the graphs denoted by (a) and (b) correspond to the tension and compression of the consireded body, respectively. Note that under the consideration of compression we assume that $|\in| < |\in_{cr.}|$, where $\in_{cr.}$ is the critical values of the parameter $\in$ obtained for the stability loss problem of the row of fibers in an infinite matrix. The investigations of this stability loss problem and the values of $\in_{cr.}$ are given in Guz (1990), Babaev, Guz and Cherevko (2007).

Thus, the numerical results given in the foregoing figures show that the dependencies among $\sigma_{n\tau}$, σ_{nn} and κ have non-monotonic character, i.e. there is such value of the parameter κ (denote it by κ^*) under which the absolute values of the considered stresses have its absolute maximum. According to the numerical results the values of κ^* decrease with increasing ρ (i.e. with increasing the distance between the two neighboring fibers). Absolute maximum values of $\sigma_{n\tau}$ in the sinphase curving case, and absolute maximum values of σ_{nn} in the antiphase curving case increase with decreasing ρ. In this case, as a result of the geometrical non-linearity the absolute values of $\sigma_{n\tau}$ and σ_{nn} decrease (increase) under tension (compression) with the parameter $\in$. It follows clearly from the foregoing numerical results that the maximum effect of the influence of the geometrical non-linearity arise for the cases where $\kappa = \kappa^*$. Moreover, this effect increases with increasing ρ. Note that the numerical results obtained under compression and tension in the case where $\in = \pm 5.10^{-5}$ coincide with each other and with the corresponding ones obtained in Akbarov, Kosker and Ucan (2004) (for sinphase curving) and in Kosker and Ucan (2006) (for antiphase curving)). Moreover, with increasing ρ and decreasing $\in$ these results approach to the corresponding ones obtained in Akbarov and Guz (1985b) for a single periodically curved fiber and treated in the monograph Akbarov and Guz (2000). This situation also agree well with the mechanical consideration and confirm the trustness of the algorithm and programms used in the present numerical investigations.

Consider Tables 1 and 2 which show the values of $\sigma_{n\tau}/|p|$ (for the sinphase curving case) and $\sigma_{nn}/|p|$ (for the antiphase curving case) respectively obtained for various $\in$ and $E^{(2)}/E^{(1)}$ under $\rho = 2.1$ and $\kappa = \kappa^*$. It follows from these tables that the influence of the geometrical non-linearity to the values of the considered stresses increase with $E^{(2)}/E^{(1)}$.

Note that the foregoing numerical results are obtained in the case where $N_v = 130$ and $N_q = 17$ in the equations (48) and (49). For illustration of the convergence

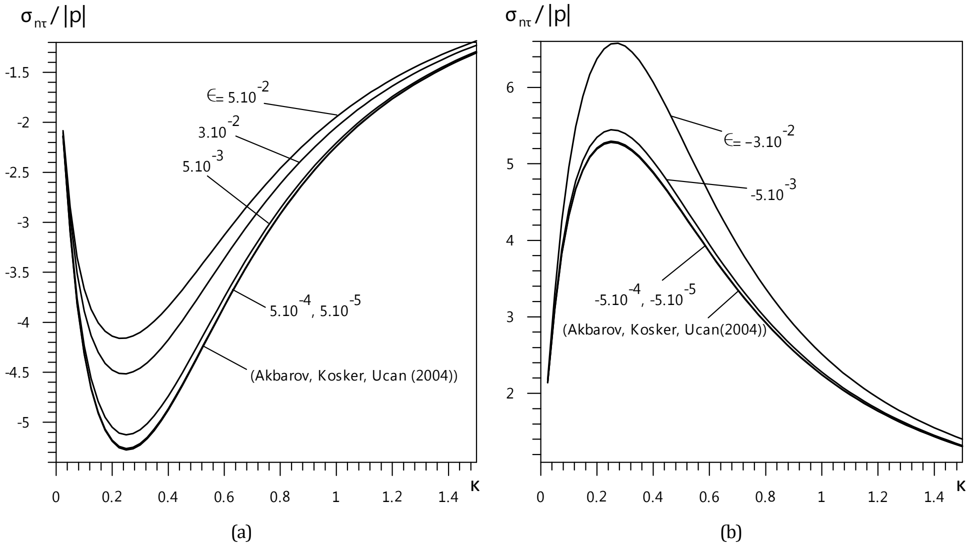

Figure 2: The graphs of the dependencies between $\sigma_{n\tau}/|p|$ and parameter κ for various values of $\in$ under $\rho = 2.1$

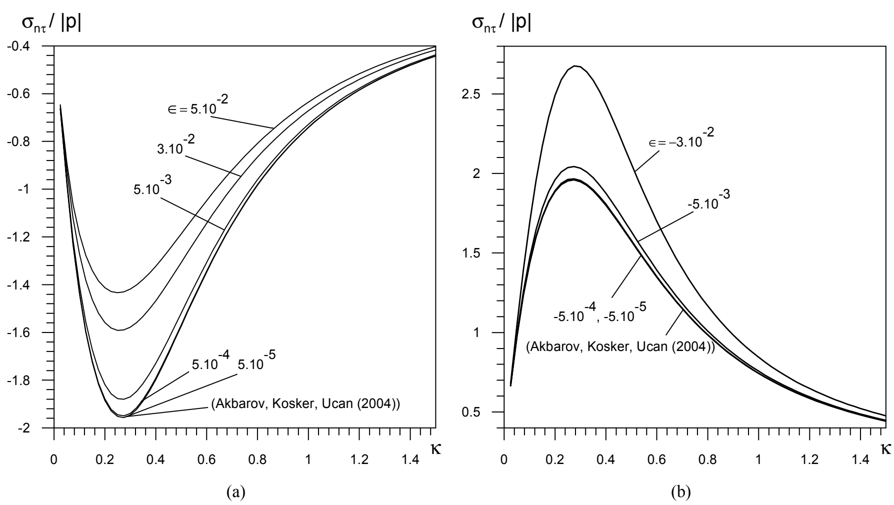

Figure 3: The graphs of the dependencies between $\sigma_{n\tau}/|p|$ and parameter κ for various values of $\in$ under $\rho = 2.5$

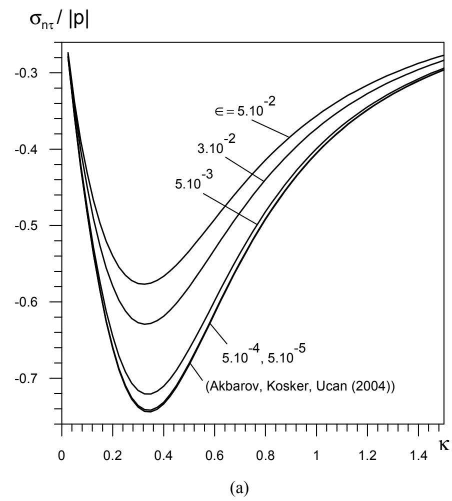

Figure 4: The graphs of the dependencies between $\sigma_{n\tau}/|p|$ and parameter κ for various values of $\in$ under $\rho = 5.0$

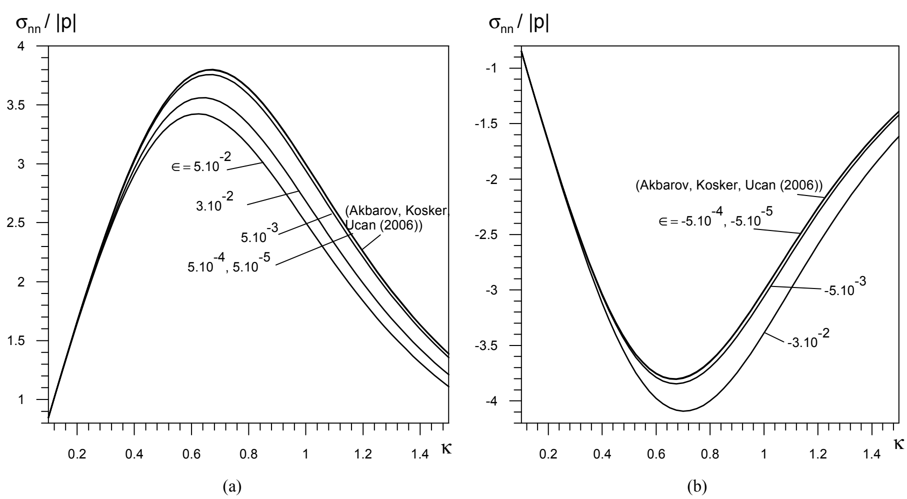

Figure 5: The graphs of the dependencies between $\sigma_{nn}/|p|$ and parameter κ for various values of $\in$ under $\rho = 2.1$

of the numerical results with respect to the number of the N_ν and N_q in Tables 3

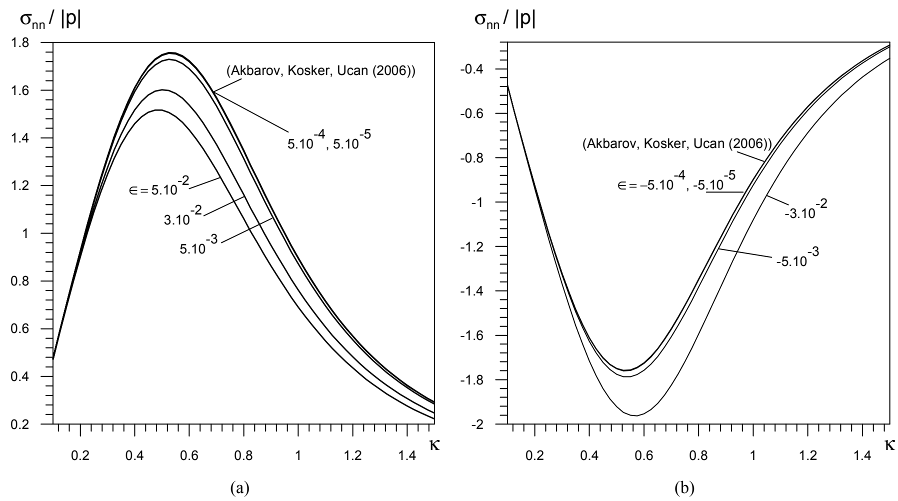

Figure 6: The graphs of the dependencies between $\sigma_{nn}/|p|$ and parameter κ for various values of $\in$ under $\rho = 2.5$

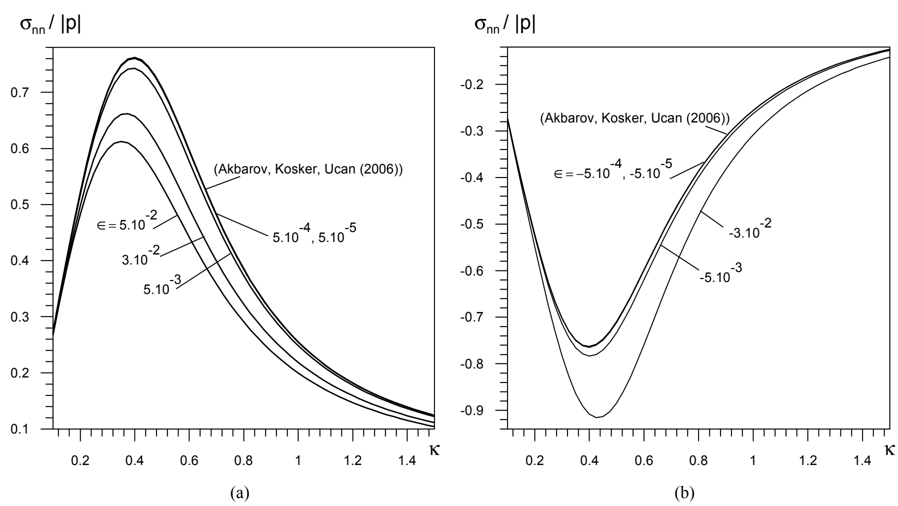

Figure 7: The graphs of the dependencies between $\sigma_{nn}/|p|$ and parameter κ for various values of $\in$ under $\rho = 5.0$

and 4 the values of $\sigma_{n\tau}/|p|$ and $\sigma_{nn}/|p|$ obtained for various N_ν and N_q are given, respectively. These values of $\sigma_{n\tau}/|p|$ and $\sigma_{nn}/|p|$ are calculated in the case where $E^{(2)}/E^{(1)} = 50$, $\rho = 2.1$, $\in = 5.10^{-2}$ for various κ. The convergence of the results obtained for various N_q and N_ν confirm that the used solution method is also highly effective in the convergence sense.

Table 1: The values of $\sigma_{n\tau}/|p|$ obtained in the sinphase curving case for various values of $E^{(2)}/E^{(1)}$ and $\in$ under $\rho = 2.1$

$\in$	$E^{(2)}/E^{(1)}$			
	10 ($\kappa = 0.4$)	20 ($\kappa = 0.4$)	50 ($\kappa = 0.3$)	100 ($\kappa = 0.2$)
5.10^{-5}	-1.1545	-2.2484	-5.2264	-5.1913
5.10^{-4}	-1.1540	-2.2459	-5.2119	-5.1780
5.10^{-3}	-1.1485	-2.2214	-5.0731	-5.0501
3.10^{-2}	-1.1208	-2.1020	-4.4581	-4.4773
5.10^{-2}	-1.1015	-2.0232	-4.0995	-4.1386
-5.10^{-5}	1.1546	2.2490	5.2296	5.1942
-5.10^{-4}	1.1552	2.2515	5.2442	5.2077
-5.10^{-3}	1.1609	2.2774	5.3969	5.3476
-3.10^{-2}	1.2287	2.6183	8.0753	7.7063

Table 2: The values of $\sigma_{nn}/|p|$ obtained in the antiphase curving case for various values of $E^{(2)}/E^{(1)}$ and $\in$ under $\rho = 2.1$

$\in$	$E^{(2)}/E^{(1)}$			
	10 ($\kappa^* = 0.8$)	20 ($\kappa^* = 0.8$)	50 ($\kappa^* = 0.6$)	100 ($\kappa^* = 0.6$)
5.10^{-5}	0.6056	1.4401	3.7514	7.2484
5.10^{-4}	0.6052	1.4389	3.7482	7.2384
5.10^{-3}	0.6015	1.4270	3.7162	7.1398
3.10^{-2}	0.5818	1.3638	3.5467	6.6336
5.10^{-2}	0.5667	1.3164	3.4204	6.2732
-5.10^{-5}	-0.6057	-1.4404	-3.7521	-7.2506
-5.10^{-4}	-0.6060	-1.4416	-3.7554	-7.2606
-5.10^{-3}	-0.6097	-1.4536	-3.7879	-7.3623
-3.10^{-2}	-0.6487	-1.5837	-4.1424	-8.5407

Table 3: The values of $\sigma_{n\tau}/|p|$ obtained in the sinphase curving case for various values of N_q and N_ν in equation (48) in the case where $E^{(2)}/E^{(1)} = 50$, $\rho = 2.1$, $\in = 5.10^{-2}$

κ	$N_q(N_\upsilon = 130)$							
	10	11	12	13	14	15	16	17
0.1	-3.5276	-3.5673	-3.5971	-3.6196	-3.6366	-3.6496	-3.6595	-3.6671
0.2	-4.1328	-4.1358	-4.1373	-4.1381	-4.1384	-4.1386	-4.1386	-4.1386
0.3	-4.1005	-4.1001	-4.0999	-4.0997	-4.0996	-4.0996	-4.0995	-4.0995
0.4	-3.8438	-3.8436	-3.8436	-3.8435	-3.8435	-3.8435	-3.8435	-3.8435
	$N_\upsilon(N_q = 17)$							
	46	58	70	82	94	106	118	130
0.1	-3.3673	-3.5136	-3.5890	-3.6280	-3.6482	-3.6587	-3.6643	-3.6671
0.2	-3.8010	-3.9656	-4.0505	-4.0944	-4.1173	-4.1291	-4.1354	-4.1386
0.3	-3.7655	-3.9283	-4.0123	-4.0558	-4.0784	-4.0901	-4.0963	-4.0995
0.4	-3.5306	-3.6832	-3.7619	-3.8026	-3.8237	-3.8347	-3.8405	-3.8435

Table 4: The values of $\sigma_{nn}/|p|$ obtained in the antiphase curving case for various values of N_q and N_ν in equation (49) in the case where $E^{(2)}/E^{(1)} = 50$, $\rho = 2.1$, $\in = 5.10^{-2}$

κ	$N_q(N_\upsilon = 136)$							
	13	14	15	16	17	18	19	20
0.1	0.8476	0.8411	0.8463	0.8421	0.8455	0.8427	0.8449	0.8432
0.2	1.6451	1.6423	1.6441	1.6430	1.6437	1.6432	1.6436	1.6433
0.3	2.3492	2.3487	2.3489	2.3488	2.3489	2.3488	2.3489	2.3488
0.4	2.9070	2.9069	2.9069	2.9069	2.9069	2.9069	2.9069	2.9069
	$N_\upsilon(N_q = 20)$							
	52	64	76	88	100	112	124	136
0.1	0.7563	0.7991	0.8212	0.8324	0.8381	0.8410	0.8424	0.8432
0.2	1.4740	1.5574	1.6005	1.6224	1.6335	1.6391	1.6419	1.6433
0.3	2.1066	2.2260	2.2876	2.3189	2.3347	2.3427	2.3468	2.3488
0.4	2.6067	2.7549	2.8312	2.8699	2.8895	2.8994	2.9044	2.9069

5 Conclusions

In the present paper, in the framework of the piecewise homogeneous body model with the use of the three-dimensional geometrically non linear exact equations of the theory of elasticity, the method for determination of the stress-strain state in the infinite body containing periodically located row of periodically curved fibers is developed. It is assumed that the midlines of the fibers are in the same plane and with respect to the location of the fibers according to each other the sinphase and antiphase curving cases are considered.

The numerical results, related to the self-balanced shear (for the sinphase curving case) and normal (for the antiphase curving case) stresses which act on the interface and arises as a result of the fiber curving, are given. In this case, the influence of the geometrical non-linearity to these stresses is analyzed. From the analyses of these results are derived the following conclusions:

1. As a result of the geometrical non-linearity, the absolute values of the considered stresses increase in compression but decrease in tension.

2. The maximum effect of the geometrical non-linearity to the stresses arises under certain values of $\kappa = 2\pi R/\ell$ (where R is a radius of the fiber cross-section, ℓ is a length of the periodicity of the curving form).

3. The effect of the geometrical non-linearity to the considered stresses increases with $E^{(2)}/E^{(1)}$ (where $E^{(2)}$ ($E^{(1)}$) Young's moduli of the fibers (matrix) material).

4. The aforementioned effect of the geometrical non-linearity also increase with increasing of the distance between the neighboring fibers.

5. Obtained numerical results agree well with the well-known mechanical consideration and in the particular cases coincide with the corresponding known results.

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