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Numerical Analysis of Pressure Propagation Emitted by Collapse of a Single Cavitation Bubble near an Oscillating Wall

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ABSTRACT: This study presents a numerical analysis of the effects of a rigid flat wall with oscillating motion on the pressure wave propagation during a single spherical cavitation bubble collapse at different initial bubble positions. Different nondimensional distances $S = 0.8, 0.9, 1.0, 1.1, 1.2$ and 1.3 were considered to investigate the effects of initial in-phase and out-of-phase oscillations of the flat wall. Numerical simulations of cavitation bubble collapse near an oscillating wall were conducted using a compressible two-phase flow model. This model incorporated the Volume of Fluid (VOF) interface-sharpening technique on a general curvilinear moving grid. The numerical results were consistent with published experimental data. The simulation examined the impact of oscillating walls on bubble behavior and the resulting pressure peaks observed on the wall surface. The numerical results demonstrate the significant impact of wall oscillation conditions on bubble collapse and migration behavior, and consequently, the generation of pressure waves with significantly different propagation and pressure peaks induced by shock impact on the rigid wall. Different behaviors were observed in the trendlines of the pressure peaks and maximum jet velocity under in-phase and out-of-phase oscillating walls, with distinct values. At $S \geq 1.0$, a higher-pressure peak on the wall was observed in the case of the out-of-phase oscillating condition, whereas a higher-pressure peak was found in the case of the in-phase condition at $S < 1.0$. The highest-pressure peak was found at $S = 0.8$ in trend lines of in-phase and $S = 1.1$ in trend lines of out-of-phase oscillation effects.

KEYWORDS: Pressure propagation; numerical analysis; cavitation bubble collapse; oscillating wall effects

1 Introduction

Understanding and controlling the dynamics of cavitation bubbles is vital across a wide range of sectors, including underwater research, hydraulic systems, water treatment processes, the shipping sector, and healthcare. Numerous studies have highlighted the importance of understanding and managing cavitation bubble behavior across various applications. Researchers have explored the dynamics of cavitation bubbles through theoretical, numerical, and experimental approaches, taking into account different environmental conditions, such as it has been studied in a free field and near a rigid wall [1,2] in a narrow tube [3], surrounding a free-moving object [4–7], in an acoustic field [8–12], inside a droplet [13,14], in a soft matter [15], near a fiber [16], near a rigid surface with a hole [17–19], between the free surface and a



rigid wall [20]. The intense pressures and high temperatures produced when bubbles collapse can lead to considerable harm to machinery [21]. The impact of the pressure wave induced by a bubble collapse near a solid structure plays a key role in the damage to the solid structure due to cavitation erosion [22–25].

Recent research has focused on the numerical analysis of pressure propagation resulting from cavitation bubble collapse near solid and oscillating walls. Nguyen et al., 2023 [26] examined shock wave modeling and fluid-material interactions but lacked long-term erosion assessments for industrial applications. Despite these limitations, the collective findings contribute to refining erosion prediction models and enhancing engineering applications. Zhao et al., 2024 [27] highlighted the generation of localized high-pressure zones contributing to material erosion, but their study mainly relied on numerical simulations without experimental validation. Nguyen et al., 2024 [28] examined ambient pressure influences on shock waves and supersonic jets, but did not account for complex material responses to cavitation forces. These studies examine critical factors, including shock waves, jet formation, and impact pressure, with some emphasizing the effects of ambient pressure and fluid-material interactions.

In recent years, various studies have attempted to explore the effects of bubbles on a boundary or the boundary effects on the collapse of laser- and spark-induced bubbles near boundaries, revealing critical insights into fluid–structure interactions and energy focusing mechanisms in various studies, considering a fixed wall a deformable wall, a movable wall, for solid and flexible boundaries [29–38]. Li et al. (2019) [29] and Hu et al. (2021) [30] delve into fluid–structure interactions, presenting models that capture the complex behavior of pulsating bubbles near movable or deformable structures. Andrews et al. (2020) [31] and Trummel et al. (2020) [32] explore cavity and bubble collapse near slot and crevice geometries, revealing how surface features influence jet formation and pressure concentration. Cao et al. (2021) [33] extend this by examining the role of acoustic impedance in shock-induced collapse, highlighting material-dependent energy transfer. Koch et al. (2021) explore the dynamics and examine the behavior of mushroom-shaped bubbles and the fast jet during oscillation of a laser-induced bubble above the flat top of a solid cylinder [34]. Lin et al. (2022) studied single-bubble dynamics with the influence of a mesoscale surface [35], and cavitating flow surrounding a flexible hydrofoil [36]. Zhang et al. (2024) introduce a theoretical model for compressible bubble dynamics that considers phase transition and migration, accounting for the effects of a rigid wall [37]. Sun et al. (2025) investigated the control of cavitation bubble collapse and jet formation on demand using a setup with both free and rigid boundaries [38]. Together, these studies emphasize the significance of boundary conditions, material properties, and structural mobility in shaping collapse dynamics, offering valuable insights for simulations involving oscillatory motion and cavitation phenomena.

In many applications, the cavitation bubble size is small compared to the solid body [39], and the motion of the walls is dominated by the main flow surrounding the solid structure [36,40]. The presence of bubbles near vibrating walls can be observed in pump operations, hydrofoils, control valves, ultrasonic horns, and propellers [39–42]. The examination of oscillatory wall effects on bubble collapse dynamics provides valuable insights for addressing erosion, managing cavitation, and enhancing various cavitation-bubble applications. Scholarly research has demonstrated that the pressure distribution within the surrounding environment significantly influences cavitation behavior [28]. Few experimental observations have revealed that controlled oscillating rigid walls can affect the behavior of bubble collapse [43,44]. In these studies, the dynamics of an air bubble attached to a rigid wall [43] and a single laser cavitation bubble [43,44] have been studied. They found that the laser-induced cavitation bubble collapse near the oscillating wall can be sped up by 15%. Furthermore, a bubble collapse under the effects of a plate approaching was faster than that of a plate-retracting bubble. Numerical simulations of a single cavitation bubble collapse near a controlled oscillating wall were performed with flat [45] and curved shapes [46]. Numerical results indicated that the physical mechanism driving the acceleration or deceleration of bubble motion cycles near oscillating walls

was a consequence of the in-phase or out-of-phase condition. These motions resulted in the compression or expansion of the surrounding fluid, respectively. This phenomenon led to heightened pressure differentials between the bubble's interior and exterior during in-phase movements and reduced pressure differentials during out-of-phase movements. The observed variations in bubble collapse behavior during the study can be attributed to these pressure changes. However, these investigations primarily focused on bubble dynamics under specific motion conditions at a single standoff condition $S = 1.2$. The propagation of pressure during the collapse of cavitation bubbles near a controlled oscillating wall, under different initial standoff conditions, remains unexplored. Thus, it is crucial to accurately model pressure wave propagation during bubble collapse in the presence of an oscillating wall, accounting for various standoff conditions.

This study aims to explore the propagation of pressure waves and their impact on the oscillating rigid wall, considering the effects of standoff conditions by numerical investigation. A numerical investigation of the propagation of pressure waves generated during a single bubble collapse is performed to explore the effects of the standoff condition on pressure propagation during a bubble collapse near an oscillating rigid wall. To investigate the pressure wave propagation during a single bubble collapse near an oscillating wall, a sinusoidal function $Y_{wall} = A \sin(2\pi t/4T_A + \varphi_0)$ is applied to the oscillation of the wall at a large amplitude scale A/R_0 , period time scale T_A/T_R , and Rayleigh's time $T_R = 0.915R_0 \sqrt{\frac{\rho_{water}}{\rho_{water} - \rho_{gas}}}$ [47]. The initial dimensionless distance, S , was chosen as the ratio of the distance L_0 (our initial distance from the bubble center to the rigid wall) to the maximum bubble radius R_0 . In fact, when cavitation occurs near underwater solid structures, the distance between these cavitation bubbles and rigid surfaces varies. However, it remains challenging in literature to study the dynamics of traveling cavitation bubbles and their interactions with surrounding structures. Therefore, the dimensionless initial bubble was considered with three typical positions of the cavitation bubble at the beginning of the interaction process between the oscillating wall and the bubble interface as:

- $S = 0.8$ – 0.9 , cavitation bubble attached to the rigid wall.
- $S = 1.0$, cavitation bubble closed on the rigid wall.
- $S = 1.1$ – 1.3 , cavitation bubbles near the rigid wall with a thin liquid layer.

To the best of the authors' knowledge, this study is the first to examine the effects of oscillating walls on pressure wave propagation during cavitation bubble collapse, considering the influence of different standoff conditions. Fig. 1 portrays a cavitation bubble near an oscillating rigid plate in a flat shape.

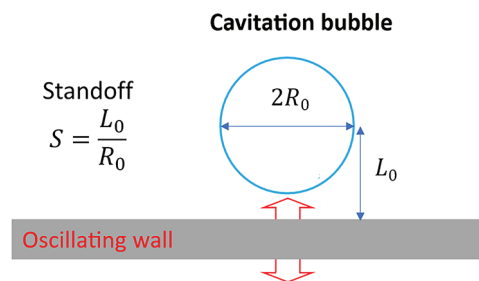


Figure 1: Schematic view of a cavitation bubble near an oscillating surface. Considered standoff values: $S = 0.8, 0.9, 1.0, 1.1, 1.2$ and 1.3

Computational simulations employ a proprietary code for a compressible two-phase flow model that incorporates the VOF interface-sharpening methodology. This model, designed to operate on a generalized curvilinear moving grid, was developed in FORTRAN by researchers at the Computational Fluid Dynamics

Laboratory at the School of Mechanical Engineering, Pusan National University, Busan, Republic of Korea. The accuracy of the numerical model was verified in our previous works [45,46,48,49].

For the reader's convenience, the remaining content of the paper is structured as follows: [Section 2](#) describes the methodology for simulating compressible two-phase flow. [Section 3](#) details the verification of the numerical model against published data. [Section 4](#) presents the numerical analyses and provides an in-depth discussion of the mechanism of pressure wave propagation in the presence of a rigid wall, under both in-phase and out-of-phase conditions. Finally, concluding remarks are provided.

2 Numerical Methodologies

This study used a compressible two-phase flow model to investigate pressure propagation during the collapse of a cavitation bubble near an oscillating wall. The numerical model is based on the Volume of Fluid (VOF) method to accurately capture the sharp bubble interface [48]. A moving-grid algorithm was employed to simulate the effects of wall vibration, and its accuracy has been validated in our previous studies with oscillating flat wall [45] and curved wall [46] and impact problem of free surface and water [49].

The governing equations of liquid-gas flow are given as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (1)$$

$$\frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \tau \quad (2)$$

where t is the physical time; p is the pressure; $\mathbf{u} = (u, v, w)$ is the flow velocity vector; $\tau = \mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T - 2/3 (\nabla \cdot \mathbf{u}) \mathbf{i} \right)$, where τ and $\mathbf{i}$ are the viscous stress tensor and the identity tensor, respectively; ρ and μ are the mixture density and viscosity, respectively, determined by the [Eqs. \(3\)](#) and [\(4\)](#):

$$\rho = \rho_1 \alpha + \rho_2 (1 - \alpha) \quad (3)$$

$$\mu = \mu_1 \alpha + \mu_2 (1 - \alpha) \quad (4)$$

where α is the liquid volume fraction ($\alpha = 1$ for liquid medium; $\alpha = 0$ for gas medium; and $0 < \alpha < 1$ for liquid-gas interface medium), and ρ_1 and ρ_2 are the liquid and gas partial densities, respectively.

The VOF equation of scalar field α for determining the advection of the liquid-gas interface is expressed in Cartesian coordinates as

$$\frac{\partial \alpha}{\partial t} + \mathbf{u} \cdot \nabla \alpha = 0 \quad (5)$$

[Eqs. \(6\)](#) and [\(7\)](#) are expressions of the Tait equations of state for the liquid and gas phases, respectively.

$$\rho_1(p) = \rho_{1,ref} \left[\frac{p - p_{ref}}{B} + 1 \right]^{\frac{1}{\gamma_1}} \quad (6)$$

$$\rho_2(p) = \rho_{2,ref} \left(\frac{p}{p_{ref}} \right)^{\frac{1}{\gamma_2}} \quad (7)$$

The speed of sound c_1 and c_2 are defined for the liquid and gas phases, respectively as

$$c_1^2 = \frac{\partial p}{\partial \rho_1} = B \frac{\gamma_1 \rho_1^{\gamma_1-1}}{\rho_{1,ref}^{\gamma_1}} \quad (8)$$

$$c_2^2 = \frac{\partial p}{\partial \rho_2} = \gamma_2 \frac{p_{ref}}{\rho_{2,ref}} (\rho_2)^{\gamma_2-1} \quad (9)$$

where γ_1 and γ_2 are the specific heat ratio for the liquid and ideal gas, respectively; $B = \frac{\rho_{1,ref} c_1^2}{\gamma_1}$ is the fluid bulk modulus; $\rho_{1,ref}$ and $\rho_{2,ref}$ are reference densities of the liquid and the gas with the reference pressure p_{Ref} and $c_1 = 1500$ m/s. Using $Ma = \frac{U_{ref}}{c_1}$ as an artificial Mach number, the bulk modulus can be calculated as $B = \frac{\rho_{1,ref} U_{ref}^2}{\gamma_1}$ where U_{ref} is the characteristic reference velocity.

In this study, $\gamma_1 = 7.15$ for water and $\gamma_2 = 1.0$ for vapor in standard atmospheric conditions was used [50]. The convective flux derivatives were computed by using a Riemann solver based on the characteristic information of the governing equations with a cell-centered finite-volume procedure. The MUSCL (Monotone Upstream-centered Schemes for Conservation Laws) procedure to extrapolate the variables at the cell interfaces [45,46,51]. The time step, Δt , is adjusted by $\Delta t = \frac{CFL}{\sum_{j=1}^3 \max |\lambda_k|}$ where the Courant–Friedrichs–Lewy (CFL) $CFL < 1$ and λ_k are eigenvalues, which represent the wave propagation of the hyperbolic system.

The effect of surface tension is neglected, as the flow during the collapse of small cavitation bubbles is primarily governed by inertia over short timescales. For instant, in cavitation bubble collapse, surface tension has no significant effect on small bubble collapse [52], and an important role only in the growth stage, and for the bubble size $R_{max} \leq 15$ μm , $We^{-1} > \frac{1}{74.9} \approx 0.013$ [53]. On the other hand, nearby boundary effects can also cause jet formation due to pressure gradient, which dominates over gravity if $\lambda \equiv L_0^2 \rho g / (R_{max} \Delta p) < 0.2$ as reported by Obreschkow (2011) [54], where g is the gravity acceleration. For validation of the model and numerical simulation of this study, the collapse of spark-induced cavitation bubble has been considered, which is clarified as a small-scale bubble as reported by (Huang et al., 2015) [52] with $R_{max} = 23$ mm, $We^{-1} = \frac{1}{114,853} \approx 0.897 \times 10^{-6}$, and λ varies from 0.001441 to 0.003805 when investigating the standoff value S increase from 0.8 to 1.3 in the study. Phase transition effect is not considered in this study because it has no significant effect at the first collapse, which is the scope of this study, as observed in our previous work [55,56] and in literature [57]. Furthermore, this research marks the initial effort to explore how pressure propagates when affected by an oscillating wall under different standoff conditions. The impact of phase transition is particularly pronounced during the expansion and collapse stages of cavitation across multiple cycles [58]. Given the complexity of modeling the intricate interactions among the system, the bubble, and the oscillating rigid wall, as well as the rapid bubble collapse, further research on the effects of phase transition is essential for a comprehensive understanding of this phenomenon. Future research is called to investigate the effects of phase transitions to gain a comprehensive understanding of this phenomenon.

3 Verification

A comparison between the present numerical simulations and the published literature of Reuter (2022) [59] for initial cavitation bubble collapse near a rigid wall is conducted to verify the numerical model. The computational domain was axisymmetric with a width of $10R_0$. An axisymmetric boundary condition was applied along the symmetry axis. At the top and right boundaries, the pressure outlet conditions are adopted. A no-slip wall condition is applied to at the bottom surface, corresponding to the rigid wall. The initial conditions of the water and gas bubble are set as follows: $\rho_{water} = 980$ kg/m³, $\rho_{gas,ref}/\rho_{water} = 0.7428 \times 10^{-3}$, $p_{water} = 101300$ Pa, $p_{gas} = 3100$ Pa, and $R_0 = 23$ mm at standoff $S = L_0/R_0 = 1.2$ [60].

The mesh configuration and convergence study were validated with experimental data in our previous work [45] for $S = 1.2$. In this study, the comparison with theoretical results is present to further evaluate the accuracy of the numerical model at various standoff conditions ($S = 0.8$ – 1.3). Fig. 2 shows the validation of the Rayleigh prolongation factor, $k = \frac{T_c}{T_R}$, obtained from the presented results and compared with the data of

Reuter (2022) [59]. The nondimensional time $\frac{t}{T_R}$ refers to the time scale normalized by the bubble collapse time, $T_R \approx 2.1$ ms.

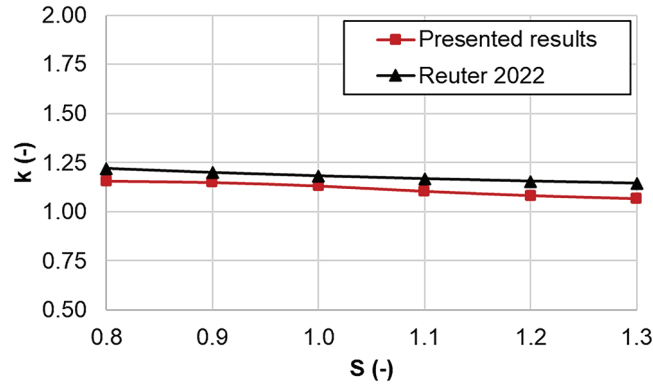


Figure 2: Rayleigh prolongation factor in comparison with the numerical results from the literature [59]

As can be seen in Fig. 2, the results for various standoff distance ($S = 0.8$ – 1.3) are well-represented by a polynomial fit of the data from Reuter (2022) [59]. The maximum deviation in the Rayleigh prolongation factor is approximately 6%, indicating a good agreement with the reference data.

4 Numerical Results and Discussion

This section describes the numerical analysis of bubble dynamics and pressure propagation induced by a single bubble collapse near a rigid wall. Numerical simulations were conducted under various wall oscillation conditions for six standoff ratios ($S = 0.8, 0.9, 1.0, 1.1, 1.2$ and 1.3) to examine their influence on interaction between bubble and the oscillating wall.

At the moment of the maximum bubble size, i.e., rest state, the bubble is assumed to be in a quasi-static state, allowing us to approximate the internal pressure as uniformly distributed. This simplification helps streamline the simulation while still capturing the essential deformation behavior. Therefore, much literature considers only the collapse stage. Thus, in our current model, we did not explicitly consider the initial pressure distribution due to the complexity of bubble conditions and wall motion. Although this, we acknowledge that pressure gradients may arise during the bubble expansion stage of the full-cycle bubble. During the bubble expansion phase, a shock wave can be emitted from the bubble induction position and reflect after impact on the rigid wall. The reflected shock wave can impact the bubble boundary. This leads to non-uniform distributions that could significantly influence the deformation and stress fields. However, due to the movement of the rigid wall and bubble interface, the interaction of the system oscillating wall-liquid-bubble is a strong nonlinear phenomenon and causes much confuse to understand bubble behavior and requires, firstly, a good understanding of the interaction between the oscillating wall and cavitation bubble in the collapse phase, i.e., the remain phase of the full cycle of bubble.

The oscillation parameters are chosen as $A/R_0 = 0.5$, $T_A/T_R = 1.0$ based on observation from our period study [45] of the effects of the oscillating wall on the maximum radius of the cavitation bubble during the collapse process, considering different amplitude scales in the range $A/R_0 = 0.1$ – 1.5 and period time scale in the range $T_A/T_R = 0.125$ – 2.0 , which found that these values are critical values, the cavitation bubble maximum radius is maintained equal to R_0 at $A/R_0 \leq 0.5$ and $T_A/T_R \geq 1.0$. In particular, the considered cavitation bubble has an oscillation frequency of approximately 120 Hz. This frequency matches earlier studies on how a hydrofoil behaves by Lin et al. (2022) [36]. The study showed that vibration frequency is

more noticeable in the higher range, between 90 and 140 Hz. Using these values, the simulation captures realistic dynamic patterns. The initial phase φ_0 condition of the relative motion between the bubble interface and the moving wall, in which the in-phase is ($\varphi_0 = 0^\circ$) and out-of-phase is ($\varphi_0 = 180^\circ$), is investigated. Table 1 presents the controlled parameters of 18 simulation cases conducted in this study to investigate the influence of in-phase and out-of-phase oscillating walls on the collapse of cavitation bubbles and the emitted pressure wave under varying standoff conditions.

Table 1: Parameters of simulation cases

Case No.	Standoff condition $S = \frac{L}{R_0}$	Initial phase condition φ_0	Remark
1	0.8	–	For validation, fixed wall conditions
2	0.9	–	
3	1.0	–	
4	1.1	–	
5	1.2	–	
6	1.3	–	
7	0.8	0°	In-phase oscillating wall conditions
8	0.9	0°	
9	1.0	0°	
10	1.1	0°	
11	1.2	0°	
12	1.3	0°	
13	0.8	180°	Out-of-phase oscillating wall conditions
14	0.9	180°	
15	1.0	180°	
16	1.1	180°	
17	1.2	180°	
18	1.3	180°	

The effects of in-phase and out-of-phase oscillating walls on the propagation of pressure waves and jet formation are discussed in Section 4.1. Section 4.2 provides a discussion of the evolution of pressure load on a rigid wall during the interaction of the bubble collapse process and the oscillating wall. The migration of the bubble and the resulting pressure peak resulting from the shock wave's impact on the rigid wall are discussed in Section 4.3.

4.1 Effects of Initial In-Phase and Out-of-Phase Conditions on Pressure Wave Propagation

Figs. 3 and 4 show the propagation of pressure waves generated during the cavitation bubble collapse with $\varphi_0 = 0^\circ$ and 180° for in-phase and out-of-phase conditions, respectively, with different standoff conditions.

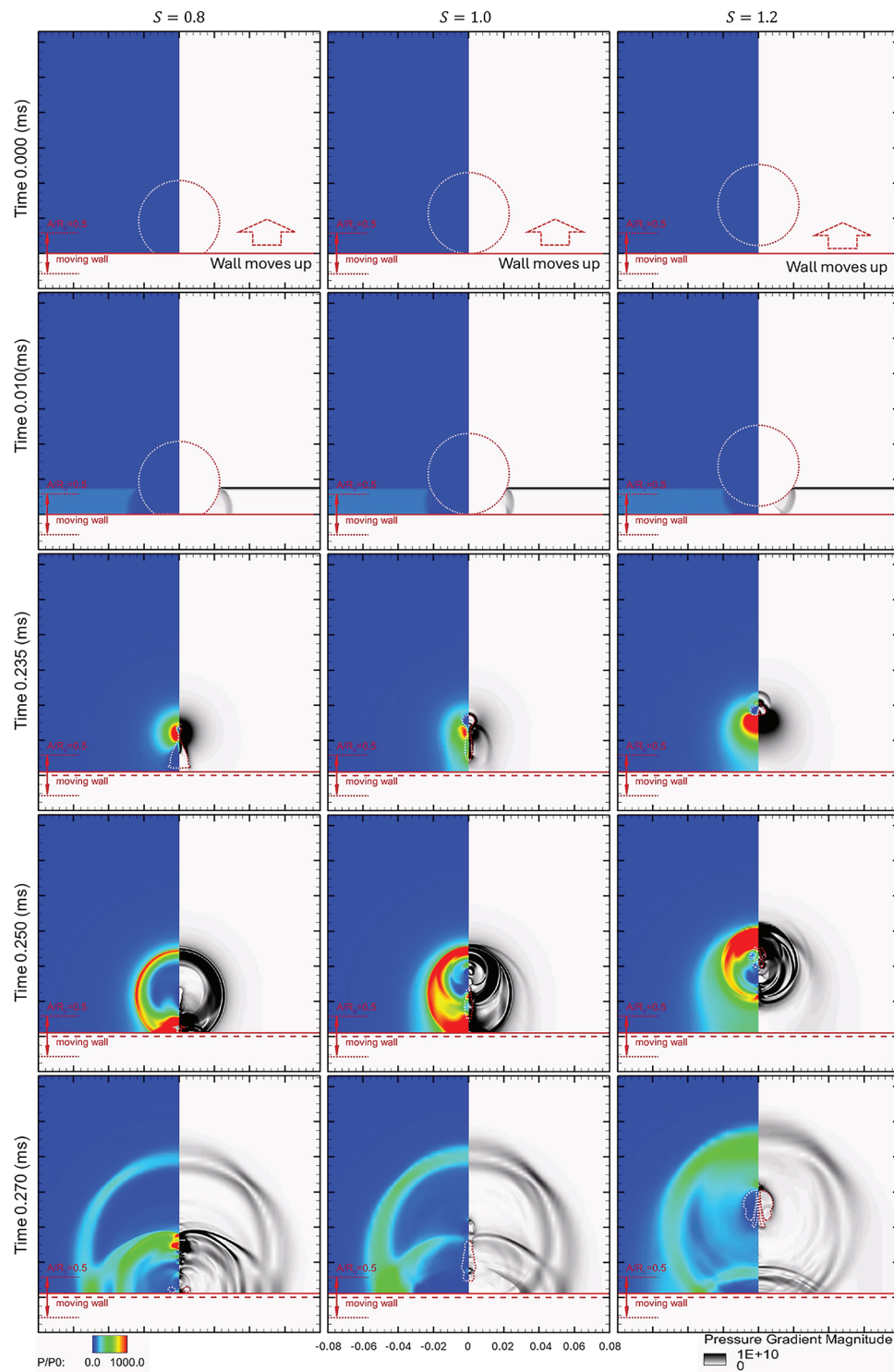


Figure 3: Typical snapshots of Schlieren-type images of pressure distribution at conditions $\varphi_0 = 0^\circ$, $S = 0.8, 1.0$ and 1.2 . Dash curve indicates the bubble interface

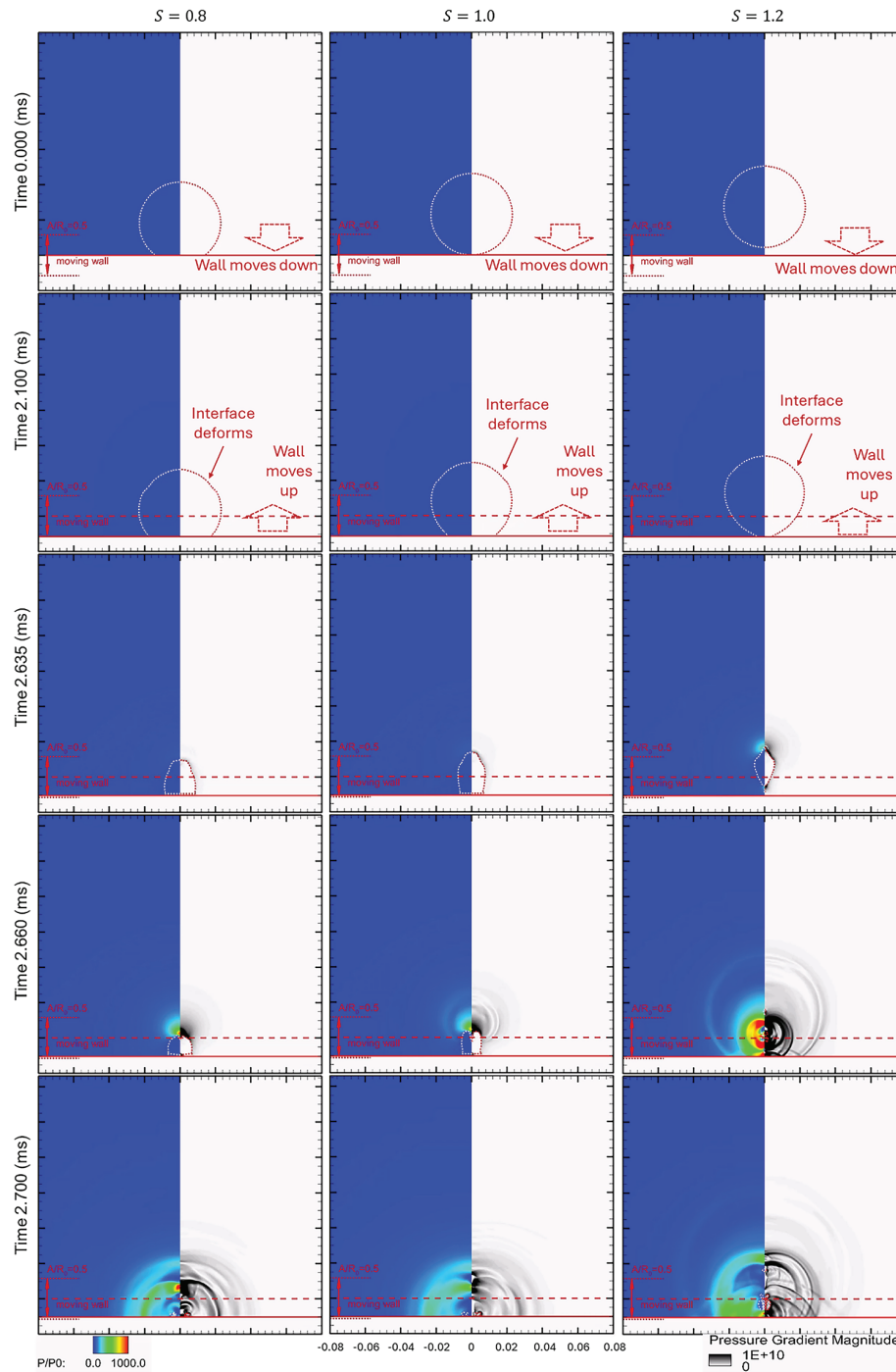


Figure 4: Typical snapshots of Schlieren-type images of pressure distribution at conditions $\varphi_0 = 180^\circ$, $S = 0.8, 1.0$ and 1.2 . Dash curve indicates the bubble interface

In Fig. 3, the in-phase oscillating walls significantly affect the bubble behaviors and the pressure propagation. Under the influence of in-phase oscillating walls, the cavitation bubble collapses rapidly, forming an upward jet at its bottom. During jet development, the jet flow interacts with the topmost point of the bubble interface, generating a pressure wave. Due to the resistance at the bubble interface, there is a

difference in pressure between the upper and lower sides of the pressure wave. The generated pressure wave propagates in the flow field during bubble collapse, with the stronger side moving upward and away from the bubble and the rigid wall, while the weaker side moves toward the rigid wall. At lower standoff conditions, the generated pressure wave is stronger than that at higher standoff conditions.

In Fig. 4, the out-of-phase oscillating wall causes different bubble behavior and pressure propagation compared to the in-phase condition, as shown in Fig. 3. In all standoff conditions, the cavitation bubble deforms slowly as the rigid wall moves far away from the original position until the wall changes its motion direction after reaching maximum oscillation amplitude at time $t = 2.1 \text{ ms} = T_R$. There is no pressure generation in this phase of bubble deformation. When time $t > T_R$, the rigid wall moves up, and the bubble violently collapses during jets' formation and development, varying by different standoff conditions. For $S = 0.8$ and 1.0 , the rigid wall reaches a maximum.

Fig. 5 illustrates the jet formation under the oscillating-wall effects compared with the fixed-wall condition at different S . Under the fixed-wall condition (top row), the bubble collapse produces symmetrical, long, and thick jets directed toward the rigid wall. In contrast, oscillating-wall conditions (middle and bottom rows) generate complex flow structures that are strongly influenced by both the standoff distance and the phase angle.

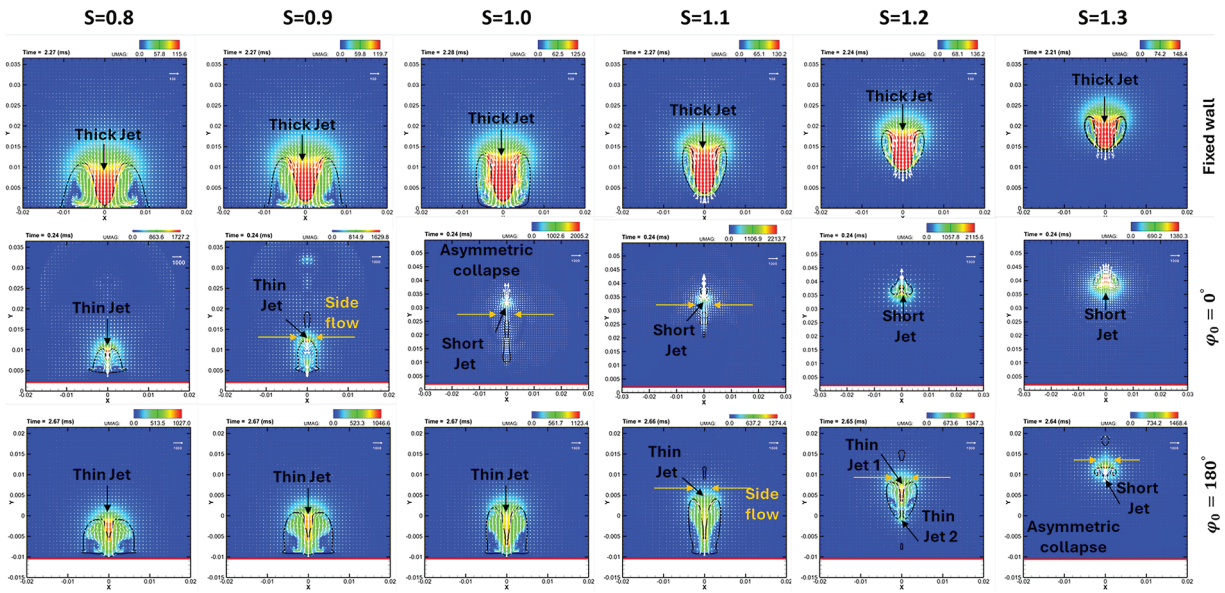


Figure 5: Jet formation under the effects of different standoff conditions of fixed walls and oscillating walls

As S increases from 0.8 to 1.3 , the jets tend to become thinner and shorter under the oscillating-wall effects. The phase angle φ_p also plays a critical role: for $\varphi_0 = 0^\circ$, a reversal of jet direction is observed at $S > 1.0$ whereas for $\varphi_0 = 180^\circ$, two opposite jets are formed at $S = 1.2$. Additionally, side flow formation is observed for $S = 0.9$ – 1.1 in case of $\varphi_0 = 0^\circ$ and for $S > 1.2$ at $\varphi_0 = 180^\circ$. These phenomena arise from the interaction between the bubble collapse process and the oscillating wall motion. At this stage, lateral flows develop on both sides of the bubble, possibly due to a rapid local pressure increase that accelerates the interface velocity and splits the collapsing bubble, leading to the formation of secondary bubbles beneath it.

Overall, the figure illustrates how the wall motion and spacing can significantly affect jet behavior and pressure propagation, providing valuable insights for applications in fluid control, propulsion, and mixing. We currently believe this is the first attempt to study pressure propagation during the collapse of a single cavitation bubble near a controlled oscillating wall. Hence, no experimental study is available to confirm our

results. The numerical results in this study are consistent with similar published experiments [43,44], which indicates that the controlled oscillating wall results in reduced collapse time [44] and a strong deformation on the bubble interface [43].

4.2 Pressure Load Evolution on the Rigid Wall under the Effects of the Oscillating Wall

As discussed above, due to differences in the propagation of pressure waves and jet formation during bubble collapse near oscillating walls, the pressure peaks on the wall are affected by the oscillation condition. Fig. 6 shows the effects of in-phase and out-of-phase conditions on the time evolution of pressure load on the wall at different standoff conditions $S = 0.8$ – 1.3 . The effects of oscillation conditions and pressure peaks on the wall vary with different standoff conditions.

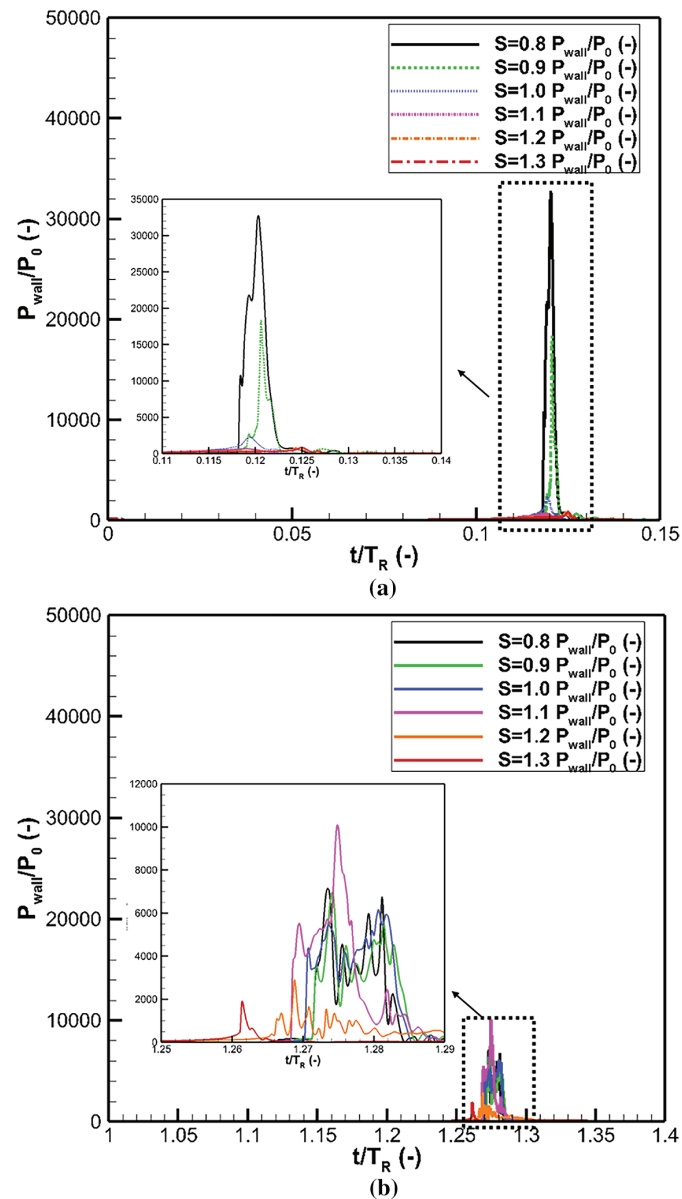


Figure 6: The time evolution of pressure load on the wall during bubble collapse under the effects of an oscillating wall with (a) in-phase ($\varphi_0 = 0^\circ$) and (b) out-of-phase ($\varphi_0 = 180^\circ$) conditions at various standoff values $S = 0.8$ to 1.3

For both in-phase and out-of-phase conditions, the highest-pressure peaks were found in the case of standoff $S = 0.8$ where the initial cavitation bubbles were close to the walls. The lowest pressure peaks were found in cases of $S = 1.1$ where initial cavitation bubbles were near the wall. This is because when a bubble is close to a wall, the jet flow impacts the bubble's bottom side without a liquid layer formed between the rigid wall and bubble interface in the case $S = 0.8$. On the other hand, a thick liquid layer is formed during jet flow development in the case of $S = 1.0$ as shown in the above Fig. 3. Meanwhile, a thicker liquid layer is initial exists near the wall in the case of $S = 1.1$, as shown in the above Figs. 3 and 4. Higher-pressure peaks were observed in the case of out-of-phase conditions at $S > 1.0$ whereas the extremely high-pressure peak was observed in the case of an in-phase condition at $S = 0.8$. This is because the bubble intends to expand and move closer toward the rigid wall as the wall moves down to reach the maximum displacement. This causes the contact of the bubble interface on the rigid wall and thinner liquid layer formation in case of out-of-phase condition at all standoff conditions in comparison with in-phase conditions, as shown in Figs. 3 and 4.

4.3 Bubble Migration with Jet Formation and Pressure Peaks under the Effects of Oscillating Walls

This section discusses the correspondence between the pressure peaks on a rigid wall and bubble migration with jet formation under the oscillating-wall effects, based on quantitative analysis of the numerical results. Fig. 7 presents a comparison of the pressure peaks generated by the impact of pressure waves on the rigid wall under oscillating- and fixed-wall conditions. Both in-phase ($\varphi_0 = 0^\circ$) and out-of-phase ($\varphi_0 = 180^\circ$) oscillation are compared with the fixed-wall case for various standoff values $S = 0.8$ –1.3.

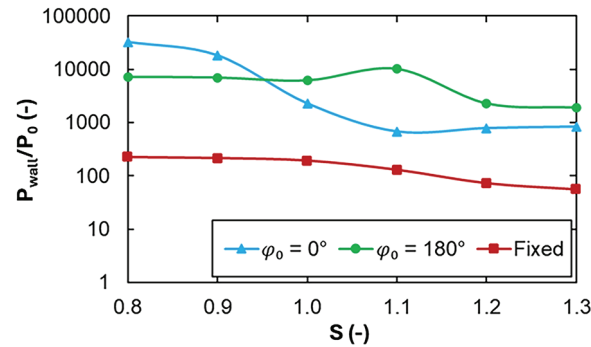


Figure 7: Comparison of the pressure peak due to the impact of pressure on the rigid wall under the effects of an oscillating wall with in-phase ($\varphi_0 = 0^\circ$) and out-of-phase ($\varphi_0 = 180^\circ$) conditions at various standoff values $S = 0.8$ to 1.3

The variations of the pressure peaks show a significant difference between oscillating- and fixed-wall conditions. Generally, the pressure peak decreases as the standoff distance increases. However, under both in-phase ($\varphi_0 = 0^\circ$) and out-of-phase ($\varphi_0 = 180^\circ$) conditions, the pressure peaks are notably higher than those observed with the fixed wall. Under the in-phase condition ($\varphi_0 = 0^\circ$), the pressure peak decreases rapidly as S increases from 0.8 to 1.0, followed by a more gradual decreases for $S = 1.0$ –1.3. Conversely, under the out-of-phase condition ($\varphi_0 = 180^\circ$), the pressure peaks remain relatively unchanged for $S = 0.8$ –1.0 and slightly increase to a local maximum at $S = 1.1$. At higher standoff distances ($S = 1.1$ –1.3), the pressure peak trendlines under out-of-phase conditions gradually decrease as the standoff increases.

The differences in the variations of the pressure peaks under in-phase and out-of-phase oscillating-wall conditions can be attributed to the behavior of bubble migration. Figs. 8 and 9 show the time evolution of bubble migration in terms of the normalized displacement of the bubble center DY_c/R_0 , and the normalized bubble size R/R_0 , respectively.

For the in-phase condition, as shown in Fig. 8, the bubble migration behavior can be classified into two types: (Type 1) movement toward the rigid wall for $S = 0.8$ – 1.0 ; and (Type 2) movement away from the rigid wall for $S = 1.0$ – 1.3 . The former (Type 1) results in a stronger pressure-wave impact on the rigid wall compared with the latter (Type 2). This bubble-migration behavior accounts for the observed difference in the pressure peak variation on the rigid wall under in-phase conditions, as shown in Fig. 7.

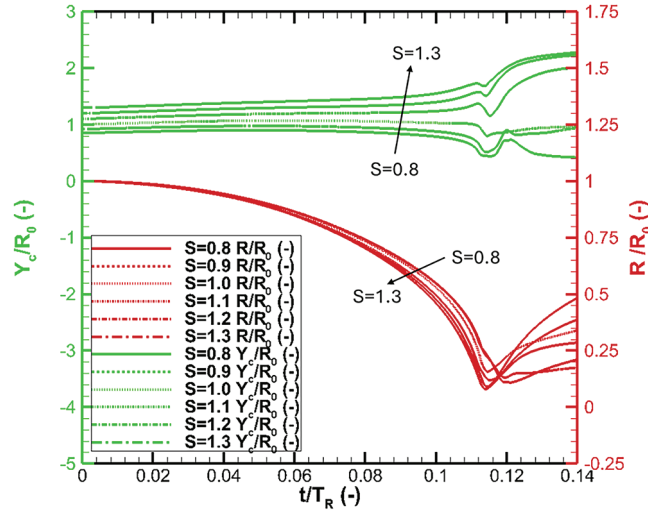


Figure 8: Collapse and migration of cavitation bubble under the effects of in-phase ($\varphi_0 = 0^\circ$) oscillating wall conditions at various standoff values $S = 0.8$ to 1.3

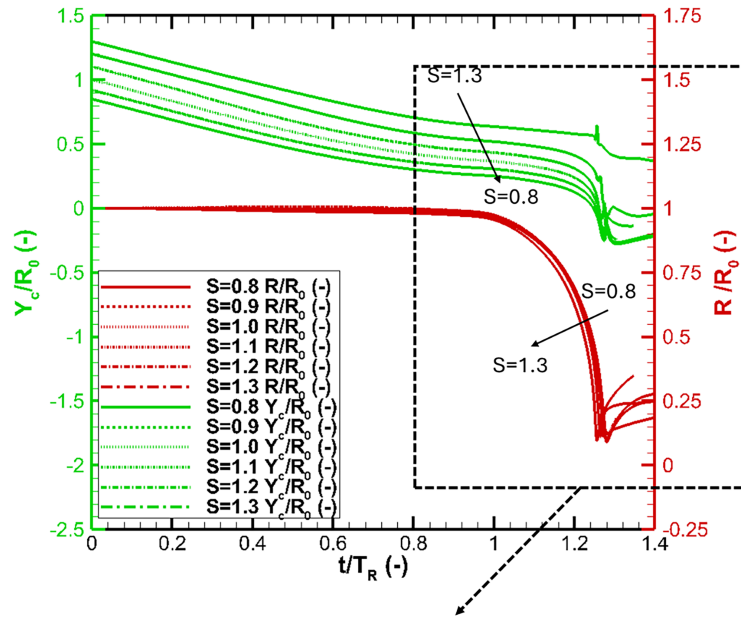


Figure 9: (Continued)

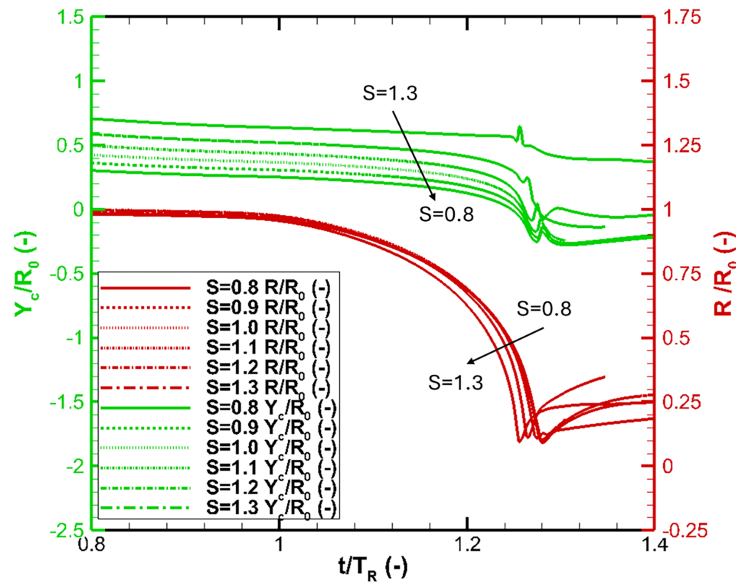


Figure 9: Collapse and migration of cavitation bubble under the effects of out-of-phase ($\phi_0 = 180^\circ$) oscillating wall conditions at various standoff values $S = 0.8$ to 1.3 . Full simulation time (top); zoom in collapse stage (bottom)

For the out-of-phase conditions, we can see a complex migration of bubbles under the oscillating-wall condition at different S (Fig. 9). The bubble migration behaviors remain similar across all standoff conditions for $t/T_R < 1$ (top panel), but differ significantly for $t/T_R > 1$ (bottom panel). This occurs because the wall moves downward during $t < 1.0T_R$ and reverses to an upward motion for $t > 1.0T_R$. During the collapse stage, the bubble migration varies notably with S . At $S = 1.1$, the bubble center approaches the rigid wall most closely, resulting in the highest pressure peaks (see Fig. 8). For $S = 0.8$ – 1.1 , the bubble migration exhibits Type 1 behavior (moving toward the rigid wall), whereas for $S = 1.1$ – 1.3 , it exhibits Type 2 behavior (moving away from the rigid wall). This results in the similar trendline observed in Fig. 8 above.

Fig. 10 presents the trendlines of jet velocity under the effect of an oscillating wall in comparison with the effects of fixed wall conditions that vary with different standoff values. The jet velocity is determined by $U_{jet}^* = \frac{U_{TopMostPoint}}{U_{sound}} (-)$. At some point, the influence of the moving wall on the X-directional flow is stronger than on the Y-directional flow, leading to strong side flows that split the bubbles and form smaller ones. Therefore, to accurately represent the effect of the wall, we use $U_{cMax}^* = \frac{U_{centerline_Max}}{U_{sound}} (-)$ when there is side flow or when the jet current arises from the bottom up instead.

The trend of velocity has correlated with jet flow behavior, as shown in Fig. 5 and the trend line of the pressure peak on the wall, as shown in Fig. 7. The fixed wall condition shows a gradual increase in the trend lines of jet velocity at varying S values. In contrast, the oscillating wall effects result in different trends in jet velocity. This indicates that the higher stand causes higher jet velocities and reduces the impact of the collapse-induced pressure wave on the rigid wall. At the in-phase oscillating wall condition with $\phi_0 = 0^\circ$, the side flow effects give U_{cmax} rise at $S = 0.9$ – 1.1 . This indicates that the side flow reduces the strength of bubble collapse and results in a fast reduce in the trend line of pressure peak as shown in Fig. 7. Meanwhile, the out-of-phase oscillating wall condition with $\phi_0 = 180^\circ$ shows a slight rise in trendlines of velocity with the influence of side flow and resulting in a small peak in the trendline of pressure peak shown in Fig. 7. Overall, we can see the strong linear effects of oscillating wall conditions on jet velocity and pressure peak in the range $0.9 < S < 1.2$, which is relevant for applications involving jet direction control.

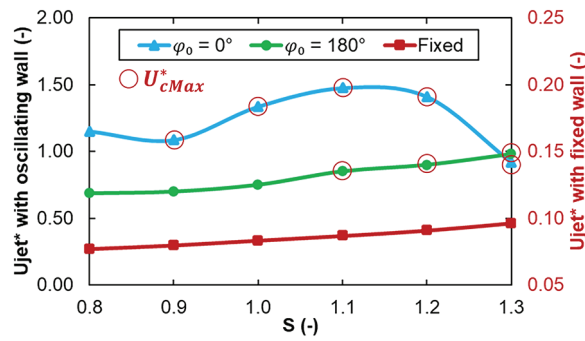


Figure 10: Jet maximum velocity under the effects of out-of-phase ($\phi_0 = 180^\circ$) oscillating wall conditions at various standoff values $S = 0.8$ to 1.3

Overall, the numerical results indicate that wall motion significantly influences the pressure wave generated during cavitation bubble collapse. Under fixed-wall conditions, the pressure response varies with the standoff distance, whereas under oscillating-wall conditions, it is affected by both the standoff distance and the initial phase of wall motion. In practical cases, solid structures are usually not as simple as the rigid flat wall considered in this study. Strong interaction between elastic surfaces and movable particles was reported in the literature [29,61,62]. In particular, the time collapse of bubbles under the effects of oscillating walls shows a good agreement with the experimental observation of Sagar et al., 2024 [44], confirming the validity of the present numerical approach. Further investigation is required to achieve a more comprehensive understanding of these coupled fluid–structure interactions and their implications for practical cavitation control.

5 Conclusion

In this research, we numerically investigated how oscillating wall conditions affect pressure wave propagation during near-wall bubble collapse at different standoff distances. Numerical simulations were conducted using a compressible two-phase flow based on the VOF method and a sharpening technique. The numerical results and experimental data are in good agreement. This demonstrates the numerical model's accuracy. Some conclusions have been drawn as follows:

- The propagation of pressure waves generated during bubble collapse under the effects of in-phase and out-of-phase oscillating wall conditions was observed at different standoff conditions $S = 0.8$ – 1.3 and discussed throughout the paper.
- The numerical analysis showed the significant effects of oscillating wall conditions on the propagation of pressure waves. Extreme pressure wave emissions are observed during bubble collapse under oscillatory wall conditions.
- Quantitative parameters have been analyzed to facilitate an in-depth discussion on bubble migration behaviors and the pressure peaks induced by the impact of pressure waves on a rigid wall. Examinations of pressure peaks on the wall show that the effects of oscillation conditions vary with different standoff conditions. The stronger pressure impact is found in the case of the out-of-phase oscillating walls with $\phi_0 = 180^\circ$ in cases of standoff $S = 1.1$ for near-wall bubble conditions. Whereas with $S = 0.8$ and the bubble close to the rigid wall surface, the extremely strong impact is found in the case of in-phase oscillation conditions with $\phi_0 = 0^\circ$. However, at $S > 1.0$, higher-pressure peaks were found in the case of out-of-phase conditions compared to in-phase conditions. Critical standoff values have been determined. The highest-pressure peak was found at $S = 0.8$ in the trend lines of in-phase and $S = 1.1$ in the trend lines of out-of-phase oscillation effects.

- The paper provides a numerical analysis of pressure wave propagation during bubble collapse near oscillating walls with some limitations, and further research, such as the below, is needed:
 - + The wall motion is assumed to undergo a controlled sinusoidal oscillation; deformation of the solid surface that happens in reality is neglected; the influence of heat transfer and phase change during bubble collapse is not considered; standoff conditions are considered in the range of 0.8–1.3 for typical distance conditions only; rigid surface geometry is assumed as a flat wall only.
 - + Studies on parameters such as characteristics of shock energy and jet flow behavior should be further considered.
 - + In the study, a cavitation bubble is initiated at a maximum size and in a spherical shape. However, an expanding bubble can be significantly deformed from a spherical shock wave emitted at cavitation inception and a reflection wave after impact on the rigid wall. A further study of the expansion stage is required to comprehensively understand the phenomenon.

Overall, this study represents an initial investigation into the pressure propagation during the collapse of a single cavitation bubble near a controlled oscillating wall. Our numerical results align with published experiments, suggesting that the controlled oscillating wall results in shorter collapse time and significant deformation of the bubble interface. The study enhances our understanding of pressure propagation and bubble-structure interaction within the phenomenon. While it is a fundamental study, the findings provide valuable insights into pressure wave propagation and bubble dynamics under oscillating wall conditions. Further research is necessary to examine the relationships between selected oscillation frequencies and bubble sizes, and to assess their application to real-world systems, such as ultrasonic devices and propeller-induced cavitation, to clarify the practical significance of the results.

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Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest to report regarding the present study.

Abbreviations

Term/Symbol	Description	Unit
CFD	Computational Fluid Dynamics	-
VOF	Volume of Fluid	-
CFL	Courant–Friedrichs–Lewy	-
MUSCL	Monotone Upstream-centered Schemes for Conservation Laws	-
L_0	Initial distance between the bubble center and the rigid wall	m
R_0	Initial bubble radius (maximum bubble radius at rest condition)	m

S	Standoff, $S = L_0/R_0$	-
T_R	Rayleigh time	s
T_A	Controlled oscillation time of the rigid wall	s
T_c	Collapse time of the cavitation bubble	s
φ_0	Initial phase parameter of the controlled oscillating wall	-
p	Pressure	Pa
p_{wall}	Pressure on rigid wall at center	Pa
p_0	Ambient pressure	Pa
ρ	Mixture density (partial liquid and gas densities: ρ_1 and ρ_2)	kg/m ³
μ	Mixture viscosity (partial liquid and gas viscosities: μ_1 and μ_2)	Pa·s
t	Physical time	s
$\mathbf{u}$	Flow velocity vector, $\mathbf{u} = (u, v, w)$	m/s
$\mathbf{i}$	Identity tensor	-
τ	Viscous stress tensor, $\tau = \mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T - 2/3 (\nabla \cdot \mathbf{u}) \mathbf{i} \right)$	-
R	Bubble radius	m
α	Liquid volume fraction	-
γ	Specific heat ratio (γ_1 and γ_2 for liquid and ideal gas, respectively)	-
c	Speed of sound (c_1 and c_2 for the liquid and gas, respectively)	m/s
B	Bulk modulus, $B = \frac{\rho c^2}{\gamma}$	Pa
k	Rayleigh prolongation factor, $k = \frac{T_c}{T_R}$	-
Y_c	Equivalent bubble center position	m

Note “-” indicates dimensionless.

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