Computer Modeling in
Engineering & Sciences

Tech Science Press

Doi:10.32604/cmes.2025.064604

ARTICLE

Dombi Power Aggregation-Based Decision Framework for Smart City Initiative Prioritization under t-Arbicular Fuzzy Environment

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Received: 19 February 2025; Accepted: 29 July 2025; Published: 30 October 2025

ABSTRACT: With the rapid growth of urbanization, smart city development has become a strategic priority worldwide, requiring complex and uncertain decision-making processes. In this context, advanced decision-support tools are essential to evaluate and prioritize competing initiatives effectively. To support effective prioritization of smart city initiatives under uncertainty, this study introduces a robust decision-making framework based on the t-arbicular fuzzy (t-AF) set—a recent extension of the t-spherical fuzzy set that incorporates an additional parameter, the radius r, to enhance the representation of uncertainty. Dombi-based operational laws are formulated within this context, leading to the development of four power aggregation operators that integrate a support degree to reflect inter-attribute relationships. The structural and theoretical foundations of the operators are rigorously demonstrated. Further, the proposed operators are embedded into an extended weighted aggregated sum product assessment (WASPAS) method to create a comprehensive multi-criteria decision-making model. The practical utility of the proposed approach is demonstrated through a case study involving the evaluation of seven smart city initiatives against eight critical criteria. Comparative analysis against established models reveals that the proposed approach offers superior ranking consistency, enhanced discrimination power among alternatives, and improved handling of uncertainty—ultimately supporting more reliable and interpretable decision-making outcomes.

KEYWORDS: t-Arbicular fuzzy set; Dombi operations; aggregation operators; decision-making; WASPAS approach

1 Introduction

Multi-criteria decision-making (MCDM) refers to the systematic evaluation and selection of the most suitable alternative from a set of options, each assessed against multiple, and often conflicting, criteria, constraints, and objectives [1–4]. This process is integral to a wide range of domains—from routine personal choices to complex strategic decisions in business and public policy—where it facilitates rational analysis and prioritization. As real-world decision problems frequently involve ambiguous or incomplete information, effective MCDM strategies must account for such uncertainty. To address this, Zadeh [5] introduced the fuzzy set (FS) theory, which allows partial membership values rather than strict binary categorization. Subsequently, Atanassov [6] enhanced this framework by proposing intuitionistic FS (IFS), incorporating degrees of non-membership alongside membership. Later, Yager and Abbasov [7] advanced the theory further through the introduction of Pythagorean FS (PyFS), offering an even more nuanced mechanism for uncertainty representation.

While IFS and PyFS have proven to be effective tools within the broader FS theory, they share a significant shortcoming: their frameworks do not fully accommodate ambiguous or indeterminate opinions that cannot be classified strictly in terms of membership or non-membership. To bridge this conceptual gap, Cuong and Kreinovich [8] proposed the Picture FS (PFS), a more expressive model that permits the representation of human judgments across four distinct dimensions—positive, negative, neutral (abstention), and refusal. This allows decision-makers to express more nuanced evaluations in complex environments. However, like IFS, the PFS model imposes a constraint that the sum of the three primary degrees must not exceed one, limiting its capacity to represent broader uncertainty. To overcome this structural limitation, Mahmood et al. [9] introduced the spherical FS (SFS), a generalization of PFS that defines its membership components on the surface of a unit sphere, thus relaxing the additive constraint and offering a richer modeling framework. Since then, SFS theory has gained traction in decision sciences. Al-Shamiri et al. [10] extended the SFS framework to spherical fuzzy bipolar soft expert sets by introducing certain set-theoretic operations and properties for the proposed model, and addressed several important related problems. Sarwar et al. [11] introduced a quantum spherical fuzzy approach to the technique for order preference by similarity to ideal solution method, incorporating elements from quantum mechanics and fuzzy logic. Further advancements include Ali and Garg's [12] formulation of a norm-based distance metric specific to SFS, and Akram et al.'s [13] exploration of outranking techniques under the same environment. Recently, Ali [14] proposed a symmetry-based ranking approach tailored to sustainable supplier evaluation using SFS. Building upon these developments, Ashraf et al. [15,16] introduced an extended version of the SFS model known as the disc SFS (DSFS), which incorporates an additional radius parameter. This enhancement allows the model to capture variable levels of fuzziness, providing a more flexible and realistic depiction of uncertainty in practical decision-making contexts. Unlike traditional SFS, where the degree values are constrained to lie on the unit sphere, the DSFS introduces a disc-like geometry that adjusts the scale of uncertainty representation. This additional parameter facilitates a more granular differentiation among alternatives and enhances the accuracy of aggregation operations—capabilities that conventional FS models often lack. However, DSFS imposes a rigid constraint that the sum of the squares of the three membership-related components must not exceed unity, i.e., $\mathsf{N}^2 + W^2 + \bar{M}^2 \leq 1$. This condition can be overly restrictive in practical scenarios. For instance, the triple $(0.7, 0.5, 0.7)$ violates this condition, as $0.7^2 + 0.5^2 + 0.7^2 = 1.23 > 1$, thus rendering such information invalid within the DSFS framework. To overcome this limitation, Ali and Popa [17] recently proposed the t-arbicular FS (t-AFS), which generalizes the spherical constraint by allowing the power in the inequality to be any natural number $t \geq 1$. In the previous example, while the triple fails the square-based condition, it satisfies the cubic constraint, i.e., $0.7^2 + 0.5^2 + 0.7^2 = 0.811 < 1$, making it acceptable in the t-arbicular fuzzy (t-AF) context. This flexibility enables the model to accommodate a broader range of data points, which are otherwise excluded under DSFS. To illustrate the practical relevance of this advancement, consider a real-world decision-making scenario involving the selection of a healthcare solution for urban development. Decision-makers may evaluate not only the direct features of a proposed system—such as cost, efficiency, and technological readiness—but also broader contextual factors like long-term scalability, community adaptability, and integration with existing services. While standard models may effectively represent the core technical criteria, capturing these nuanced and often overlapping uncertainties requires a more flexible framework. The t-AF model, by extending the feasible boundary through a customizable power parameter, offers the capability to express such complex interrelations more effectively. It thus ensures that critical alternatives are not prematurely excluded due to rigid structural constraints, leading to more inclusive, accurate, and context-sensitive decision analysis. The existing literature has established the fundamental principles of t-AFS and examined their internal relationships, laying the groundwork for advancing their use in decision-making scenarios. Within this framework, two critical research inquiries arise: (i) What novel operators can be introduced, or how can limitations in current methodologies be resolved? and (ii) What approaches can effectively determine the most suitable option from among various alternatives?

Aggregation operators (AOs) serve as essential tools in real-world decision-making by enabling the synthesis and prioritization of fuzzy information. Over time, researchers have proposed numerous AOs tailored to various fuzzy frameworks. For example, Jana et al. [18] formulated Dombi-based operators within the context of Pythagorean fuzzy sets. Senapati et al. [19] advanced the field by introducing Aczel-Alsina operators for interval-valued intuitionistic fuzzy settings, highlighting their practical relevance in decision analysis. Qiyas et al. [20] explored trigonometric sine-based operators for spherical fuzzy sets to address decision-making challenges, while in a separate study, Qiyas et al. [21] presented Hamacher-type operators for spherical uncertain linguistic environments, emphasizing their applicability in group decision scenarios to unify divergent expert opinions. Similarly, Abdullah et al. [22] conducted an in-depth assessment of decision support systems grounded in 2-tuple spherical fuzzy linguistic aggregation methods.

The Dombi AO, originally formulated by Dombi in 1982 [23], is characterized by its parameter-dependent adaptability, enabling it to function in either a conjunctive or disjunctive mode. This intrinsic flexibility has led to its extensive adaptation within a range of fuzzy set extensions. For instance, Chen and Ye [24] introduced weighted Dombi operators under single-valued neutrosophic sets, establishing a foundational approach for subsequent enhancements. Expanding on this, Liu et al. [25] incorporated the Dombi Bonferroni mean into IFS theory and applied it to MCDM. Similarly, Shi and Ye [26] explored the application of Dombi operators in neutrosophic cubic environments to address MCDM scenarios. More recent advancements include the work of Seikh and Mandal [27], who developed interval-valued Fermatean fuzzy Dombi weighted operators and extended them to interval-valued spherical fuzzy domains, with practical implementation in areas such as plastic waste management [28]. Furthermore, Jana et al. [18] adapted the Dombi operator to picture FSs, thereby broadening its operational landscape. In a recent development, Seikh and Mandal [29] introduced a new class of Dombi averaging and geometric AOs for IFS, applying them to MCDM problems. Despite extensive research on Dombi aggregation operations across various FS extensions, disc spherical fuzzy (DSF) operations have not yet been defined in terms of Dombi operations. Consequently, existing DSF AOs primarily rely on the algebraic product and sum of DSFs, making them less generalized. Moreover, conventional operators often overlook the mutual dependencies among the input values being aggregated, which significantly undermines their suitability for handling intricate decision-making problems. To overcome this shortcoming, and guided by the foundational principles of power aggregation [30–32], it becomes essential to construct a new class of aggregation operators under the DSF environment, wherein the weights are adaptively determined based on the characteristics of the input data. Incorporating Dombi-based operations into this scheme enhances the mutual reinforcement among the aggregated inputs, yielding more coherent and context-sensitive results. Additionally, these advancements call for the application of the weighted aggregated sum product assessment (WASPAS) method in conjunction with the newly proposed operators. A significant benefit of this integrated strategy lies in its capacity to reduce the impact of outliers and unfavorable inputs by leveraging power-based weights within a flexible, hybrid MCDM framework. This methodology combines the advantages of the weighted sum model (WSM) and the weighted product model (WPM), enabling a more stable and discriminative ranking of alternatives. WASPAS produces final scores by synthesizing the outcomes from both models, with its operational mode governed by a threshold parameter. Depending on the selected value, the method can emulate either WSM or WPM behavior, thus drawing on the respective merits of both approaches.

Despite notable progress in fuzzy MCDM, several methodological and practical limitations persist, which serve as the motivation for this study:

i. **Lack of Dombi AOs in DSF environments:** Although Dombi AOs are recognized for their parametric flexibility and generalization capabilities [23], they have not yet been incorporated into DSF frameworks. Existing DSF aggregation models primarily rely on conventional algebraic operations, limiting their adaptability in handling complex uncertainties.
ii. **Static weighting in DSF aggregation:** Current DSF-based AOs utilize fixed weighting schemes, failing to reflect the contextual influence of input data. Inspired by power aggregation mechanisms [30–32], there exists a need to develop DSF-based AOs that dynamically adjust weights in response to varying input arguments, thereby better representing expert judgment and preferences.
iii. **Lack of hybrid aggregation strategies:** Many DSF-based MCDM approaches are restricted to either WSM or WPM, which can limit their robustness in multifactorial decision environments. The hybrid WASPAS method, which balances additive and multiplicative reasoning, has not been adapted to the DSF context.
iv. **Underexplored application in smart city prioritization:** To the best of the authors' knowledge, no prior study has applied DSF-based MCDM frameworks—particularly those involving Dombi operators and hybrid power aggregation—to the domain of smart city initiative prioritization. Given the inherent complexity, uncertainty, and interdependence of smart city components, this application provides an ideal platform to validate the practical value and broader applicability of the proposed model.

Based on the above gaps, this research presents the following contributions:

i. A novel class of DSF AOs based on Dombi functions, enhancing flexibility and generalization in fuzzy environments.
ii. Integration of power aggregation into DSF AOs, allowing context-sensitive weight adjustment that better reflects the influence of input data.
iii. Extension of the WASPAS methodology to the DSF framework using Dombi-based AOs, resulting in improved ranking stability and decision accuracy.
iv. Implementation of the proposed model in a real-world case study on smart city initiative prioritization—an application area not previously addressed in the DSF-MCDM literature—to demonstrate its practical relevance and effectiveness.

The structure of this article is as follows: Section 2 reviews the foundational concepts necessary to comprehend the proposed framework. In Section 3, the formulation and properties of Dombi operations under the DSF environment are explored in detail. Section 4 introduces a set of aggregation operators based on these operations. Section 5 describes the procedural steps of the WASPAS technique, which is then employed in a practical case study in Section 6. Section 7 provides a comparative evaluation to assess the effectiveness of the proposed model. Concluding remarks are offered in Section 8.

2 Prerequisite Knowledge

In this article, the set $\cup$ is consistently regarded as a finite, non-empty collection, defined as the universal set of alternatives. Additionally, the symbol Γ represents the unit interval $[0,1]$.

This section reviews the definitions of DTN, DTCN, and power operators, as well as t-SFS and t-AFS, along with their corresponding theoretical concepts.

Definition 1. *[23] For $\beta_1, \beta_2 \in [0,1]$ with $p \geq 1$, the DTN D and DTCN D^c are defined as follows:*

$$D(\beta_1, \beta_2) = \frac{1}{1 + \left(\left(\frac{1-\beta_1}{\beta_1}\right)^p + \left(\frac{1-\beta_2}{\beta_2}\right)^p\right)^{1/p}}, \tag{1}$$

$$D^c(\beta_1, \beta_2) = 1 - \frac{1}{1 + \left(\left(\frac{\beta_1}{1-\beta_1}\right)^p + \left(\frac{\beta_2}{1-\beta_2}\right)^p\right)^{1/p}}. \tag{2}$$

Definition 2. *[30] Given a sequence of non-negative real numbers $\delta_1, \delta_2, \ldots, \delta_m$, the power average (PA) and the weighted power average (WPA) are defined as follows:*

$$PA(\delta_1, \delta_2, \ldots, \delta_m) = \sum_{l=1}^{m} \frac{(1 + S(\delta_l))}{\sum_{j=1}^{m}(1 + S(\delta_j))} \delta_l, \tag{3}$$

$$WPA(\delta_1, \delta_2, \ldots, \delta_m) = \sum_{l=1}^{m} \frac{\mho_l (1 + S(\delta_l))}{\sum_{j=1}^{m} \mho_j (1 + S(\delta_j))} \delta_l. \tag{4}$$

Here, $\mho_l \in [0, 1]$ denotes a component of the weight vector, and $S(\delta_j)$ is given by $\sum_{i=1}^{m} supp(\delta_j, \delta_i)$, where $supp(\delta_j, \delta_i)$ represents the support measure between δ_j and δ_i. This support measure satisfies the following conditions:

i) $0 \leq supp(\delta_j, \delta_i) \leq 1$, ii) $supp(\delta_j, \delta_i) = supp(\delta_i, \delta_j)$, iii) If $|\delta_j - \delta_i| \leq |\delta_l - \delta_l|$, then $supp(\delta_j, \delta_i) \geq supp(\delta_l, \delta_l)$.

Moreover, the support measure is defined as $supp(\delta_j, \delta_i) = 1 - d(\delta_j, \delta_i)$, where $d(\delta_j, \delta_i)$ represents the distance between δ_j and δ_i.

Definition 3. *[31] Consider a sequence of non-negative real numbers $\delta_1, \delta_2, \ldots, \delta_m$. The power geometric (PG) and weighted power geometric (WPG) operators are defined as follows:*

$$PG(\delta_1, \delta_2, \ldots, \delta_m) = \prod_{l=1}^{m} \delta_l^{\frac{(1+S(\delta_l))}{\sum_{j=1}^{m}(1+S(\delta_j))}}, \tag{5}$$

$$WPG(\delta_1, \delta_2, \ldots, \delta_m) = \prod_{l=1}^{m} \delta_l^{\frac{\mho_l(1+S(\delta_l))}{\sum_{j=1}^{m} \mho_j(1+S(\delta_j))}}. \tag{6}$$

The variables are specified in the same manner as in Definition 2.

Definition 4. *[9] A t-SFS S on $\Im$ is defined as follows:*

$$S = \left\{\left(\mathsf{N}(u), \mathcal{W}(u), \bar{M}(u)\right) | u \in \Im\right\}, \tag{7}$$

where $\mathsf{N}(u), \mathcal{W}(u), \bar{M}(u) \in \Gamma$ refer to degrees of association, neutrality and non-association, respectively, such that the condition $(\mathsf{N}(u))^t + (\mathcal{W}(u))^t + \left(\bar{M}(u)\right)^t \leq 1$ fulfils for all $u \in \Im$. Further the degree of indeterminacy is expressed by $\pi_s(u) = \sqrt[t]{1 - \left((\mathsf{N}(u))^t + (\mathcal{W}(u))^t + \left(\bar{M}(u)\right)^t\right)}$. The triplet $\left(\mathsf{N}, \mathcal{W}, \bar{M}\right)$ is referred to as t-spherical fuzzy number (t-SFN).

Definition 5. *[9,33] Let $S_1 = \left(\mathsf{N}_1, \mathcal{W}_1, \bar{M}_1\right)$ and $S_2 = \left(\mathsf{N}_2, \mathcal{W}_2, \bar{M}_2\right)$ represent two t-SFNs with $\top > 0$. Then*

1. $S_1 \oplus S_2 = \left(\sqrt[t]{N_1^t + N_2^t - N_1^t N_2^t}, W_1 W_2, \bar{M}_1 \bar{M}_2\right)$;
2. $S_1 \otimes S_2 = \left(N_1 N_2, \sqrt[t]{W_1^t + W_2^t - W_1^t W_2^t}, \sqrt[t]{\bar{M}_1^t + \bar{M}_2^t - \bar{M}_1^t \bar{M}_2^t}\right)$;
3. $\top S_1 = \left(\sqrt[t]{1 - \left(1 - N_1^t\right)^{\top}}, W_1^{\top}, \bar{M}_1^{\top}\right)$;
4. $S_1^{\top} = \left(N_1^{\top}, \sqrt[t]{1 - (1 - W_1^t)^{\top}}, \sqrt[t]{1 - \left(1 - \bar{M}_1^t\right)^{\top}}\right)$.

Definition 6. *[34] Consider* $S_1 = \left(N_1, W_1, \bar{M}_1\right)$ *as a t-SFN. The corresponding score function is defined as:*

$$S(S_1) = \frac{1 + N_1^t - W_1^t - \bar{M}_1^t}{2}. \tag{8}$$

The greater the score function value, the higher the ranking of the corresponding t-SFN.

Definition 7. *[17] A t-AFS on* $\mathfrak{I}$ *is expressed mathematically as:*

$$F = \left\{\left(u, N(u), W(u), \bar{M}(u); r(u)\right) | u \in \mathfrak{I}\right\} = \left\{D_r\left(N(u), W(u), \bar{M}(u)\right) | u \in \mathfrak{I}\right\}, \tag{9}$$

where $N, W, \bar{M} \in \Gamma,\ r \in \left[0, \sqrt[t]{3}\right]$ *such that* $N^t(u) + W^t(u) + \bar{M}^t(u) \le 1$, *and* $D_r\left(N(u), W(u), \bar{M}(u)\right)$ $= \left\{(p, q, s) | p, q, s \in \Gamma \;\&\; \sqrt[t]{(N(u) - p)^t + (W(u) - q)^t + \left(\bar{M}(u) - s\right)^t} \le r(u)\right\} \cap R$;
$R = \left\{(p, q, s) \in \Gamma \;\&\; p^t + q^t + s^t \le 1\right\}$.

The expression $\pi(u) = \left(1 - \left(N^t(u) + W^t(u) + \bar{M}^t(u)\right)\right)^{1/t}$ *represents the degree of refusal of an element* $u \in \mathfrak{I}$*. Further the elements of the t-AFS are termed as the t-AF numbers (t-AFNs), denoted by* $F = \left(N, W, \bar{M}; r\right)$.

Definition 8. *[17] Let* $F_1 = \left(N_1, W_1, \bar{M}_1\right)$ *and* $F_2 = \left(N_2, W_2, \bar{M}_2\right)$ *be two t-AFNs, with* $\top > 0$ *and* $\ddagger \in \{\min, \max\}$*. Then*

1. $F_1 \oplus F_2 = \left(\sqrt[t]{N_1^t + N_2^t - N_1^t N_2^t}, W_1 W_2, \bar{M}_1 \bar{M}_2; \ddagger\{r_1, r_2\}\right)$;
2. $F_1 \otimes F_2 = \left(N_1 N_2, \sqrt[t]{W_1^t + W_2^t - W_1^t W_2^t}, \sqrt[t]{\bar{M}_1^t + \bar{M}_2^t - \bar{M}_1^t \bar{M}_2^t}; \ddagger\{r_1, r_2\}\right)$;
3. $\top F_1 = \left(\sqrt[t]{1 - \left(1 - N_1^t\right)^{\top}}, W_1^{\top}, \bar{M}_1^{\top}; r_1\right)$;
4. $F_1^{\top} = \left(N_1^{\top}, \sqrt[t]{1 - (1 - W_1^t)^{\top}}, \sqrt[t]{1 - \left(1 - \bar{M}_1^t\right)^{\top}}; r_1\right)$;
5. $F_1^c = \left(\bar{M}_1, W_1, N_1; r_1\right)$.

where F_1^c *denotes the complement of* F_1.

Definition 9. *[17] Let* $F_1 = \left(N_1, W_1, \bar{M}_1; r_1\right)$ *be a t-AFN. Then the score function is given by*

$$S(F_1) = \left(\frac{N_1^t - \bar{M}_1^t - \ln(1 + \pi_1^t) + r_1 + 2}{3\sqrt[t]{3}}\right), \tag{10}$$

where $S(F_1) \in [0, 1]$*. The greater value of* $S(F_1)$ *signifies a greater t-AFN* F_1.

3 Dombi Operational Laws

In this section, we establish the Dombi operational laws tailored for t-AFNs, laying the groundwork for subsequent aggregation mechanisms.

Definition 10. *Consider two t-AFNs,* $F_1 = \left(N_1, W_1, \bar{M}_1; r_1\right)$ *and* $F_2 = \left(N_2, W_2, \bar{M}_2; r_2\right)$*, where* $\top > 0$ *and* $\ddagger$ *belongs to the set* $\{\min, \max\}$*. Then,*

1. $$F_1 \oplus F_2 = \left(\begin{array}{c} \sqrt[t]{1 - \frac{1}{1+\left(\left(\frac{N_1^t}{1-N_1^t}\right)^p + \left(\frac{N_2^t}{1-N_2^t}\right)^p\right)^{\frac{1}{p}}}}, \sqrt[t]{\frac{1}{1+\left(\left(\frac{1-\mathcal{W}_1^t}{\mathcal{W}_1^t}\right)^p + \left(\frac{1-\mathcal{W}_2^t}{\mathcal{W}_2^t}\right)^p\right)^{\frac{1}{p}}}}, \\ \sqrt[t]{\frac{1}{1+\left(\left(\frac{1-\bar{M}_1^t}{\bar{M}_1^t}\right)^p + \left(\frac{1-\bar{M}_2^t}{\bar{M}_2^t}\right)^p\right)^{\frac{1}{p}}}}; \ddagger\{r_1, r_2\} \end{array} \right);$$

2. $$F_1 \otimes F_2 = \left(\begin{array}{c} \sqrt[t]{\frac{1}{1+\left(\left(\frac{1-N_1^t}{N_1^t}\right)^p + \left(\frac{1-N_2^t}{N_2^t}\right)^p\right)^{\frac{1}{p}}}}, \sqrt[t]{1 - \frac{1}{1+\left(\left(\frac{\mathcal{W}_1^t}{1-\mathcal{W}_1^t}\right)^p + \left(\frac{\mathcal{W}_2^t}{1-\mathcal{W}_2^t}\right)^p\right)^{\frac{1}{p}}}} \\ \sqrt[t]{1 - \frac{1}{1+\left(\left(\frac{\bar{M}_1^t}{1-\bar{M}_1^t}\right)^p + \left(\frac{\bar{M}_2^t}{1-\bar{M}_2^t}\right)^p\right)^{\frac{1}{p}}}}; \ddagger\{r_1, r_2\} \end{array} \right);$$

3. $$\top F_1 = \left(\sqrt[t]{1 - \frac{1}{1+\left(\top\left(\frac{N_1^t}{1-N_1^t}\right)^p\right)^{\frac{1}{p}}}}, \sqrt[t]{\frac{1}{1+\left(\top\left(\frac{1-\mathcal{W}_1^t}{\mathcal{W}_1^t}\right)^p\right)^{\frac{1}{p}}}}, \sqrt[t]{\frac{1}{1+\left(\top\left(\frac{1-\bar{M}_1^t}{\bar{M}_1^t}\right)^p\right)^{\frac{1}{p}}}}; r_1 \right);$$

4. $$F_1^{\top} = \left(\sqrt[t]{\frac{1}{1+\left(\top\left(\frac{1-N_1^t}{N_1^t}\right)^p\right)^{\frac{1}{p}}}}, \sqrt[t]{1 - \frac{1}{1+\left(\top\left(\frac{\mathcal{W}_1^t}{1-\mathcal{W}_1^t}\right)^p\right)^{\frac{1}{p}}}}, \sqrt[t]{1 - \frac{1}{1+\left(\top\left(\frac{\bar{M}_1^t}{1-\bar{M}_1^t}\right)^p\right)^{\frac{1}{p}}}}; r_1 \right).$$

4 t-AF Dombi Power Operators

Building on the t-AFN operations outlined in Section 3, we introduce a set of Dombi power operators and examine their resulting properties.

4.1 t-AF Dombi Power Average Operator

In the present part, we establish the theoretical foundation of the t-AFDPA operator and explore its characteristics.

Definition 11. *For a given set of t-AFNs $F_l = (N_l, \mathcal{W}_l, \bar{M}_l; r_l)$, where $l = 1, 2, \ldots, m$, the t-AFDPA operator is defined as*

$$t - AFDPA(F_1, F_2, \ldots, F_m) = \overset{m}{\underset{l=1}{\oplus}} \left(\frac{(1 + S(F_l))}{\sum\limits_{1 \le l \le m} (1 + S(F_l))} \right) F_l = \overset{m}{\underset{l=1}{\oplus}} \epsilon_l F_l. \tag{11}$$

Here, ϵ_l represents the power weight of F_l, where

$$S(F_l) = \sum_{1 \le l \le m, l \ne l} supp(F_l, F_l).$$

Theorem 1. *Given a set of t-AFNs $F_l = (N_l, \mathcal{W}_l, \bar{M}_l; r_l)$ for $l = 1, 2, \ldots, m$, we obtain the following result based on Definition 10:*

$$t - AFDPA(F_1, F_2, \ldots, F_m) = \left(\begin{array}{c} \sqrt[t]{1 - \frac{1}{1+\left(\sum\limits_{1 \le l \le m} \epsilon_l \left(\frac{N_l^t}{1-N_l^t}\right)^p\right)^{1/p}}}, \sqrt[t]{\frac{1}{1+\left(\sum\limits_{1 \le l \le m} \epsilon_l \left(\frac{1-\mathcal{W}_l^t}{\mathcal{W}_l^t}\right)^p\right)^{1/p}}}, \\ \sqrt[t]{\frac{1}{1+\left(\sum\limits_{1 \le l \le m} \epsilon_l \left(\frac{1-\bar{M}_l^t}{\bar{M}_l^t}\right)^p\right)^{1/p}}}; \ddagger\{r_1, r_2, \ldots, r_m\} \end{array} \right). \tag{12}$$

Proof. This can be proven using mathematical induction as follows:

For $m = 2$,

$$t-AFDPA\left(F_1,F_2\right)=\text{€}_1F_1\oplus\text{€}_2F_2=$$

$$\left(\sqrt[t]{1-\frac{1}{1+\left(\text{€}_1\left(\frac{N_1^t}{1-N_1^t}\right)^p\right)^{\frac{1}{p}}}},\sqrt[t]{\frac{1}{1+\left(\text{€}_1\left(\frac{1-W_1^t}{W_1^t}\right)^p\right)^{\frac{1}{p}}}},\sqrt[t]{\frac{1}{1+\left(\text{€}_1\left(\frac{1-\bar{M}_1^t}{\bar{M}_1^t}\right)^p\right)^{\frac{1}{p}}}};r_1\right)\oplus$$

$$\left(\sqrt[t]{1-\frac{1}{1+\left(\text{€}_2\left(\frac{N_1^t}{1-N_1^t}\right)^p\right)^{\frac{1}{p}}}},\sqrt[t]{\frac{1}{1+\left(\text{€}_2\left(\frac{1-W_1^t}{W_1^t}\right)^p\right)^{\frac{1}{p}}}},\sqrt[t]{\frac{1}{1+\left(\text{€}_2\left(\frac{1-\bar{M}_1^t}{\bar{M}_1^t}\right)^p\right)^{\frac{1}{p}}}};r_2\right)=$$

$$\left(\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le 2}\text{€}_l\left(\frac{N_l^t}{1-N_l^t}\right)^p\right)^{1/p}}},\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le 2}\text{€}_l\left(\frac{1-W_l^t}{W_l^t}\right)^p\right)^{1/p}}},\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le 2}\text{€}_l\left(\frac{1-\bar{M}_l^t}{\bar{M}_l^t}\right)^p\right)^{1/p}}};\ddagger\{r_1,r_2\}\right).$$

Assume that Eq. (12) holds for $m = x$,

$$t-AFDPA\left(F_1,F_2,\ldots,F_x\right)=$$

$$\left(\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{N_l^t}{1-N_l^t}\right)^p\right)^{1/p}}},\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{1-W_l^t}{W_l^t}\right)^p\right)^{1/p}}},\right.$$
$$\left.\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{1-\bar{M}_l^t}{\bar{M}_l^t}\right)^p\right)^{1/p}}};\ddagger\{r_1,r_2,\ldots,r_x\}\right).$$

Now, to prove it for $m = l + 1$,

$$t-AFDPA\left(F_1,F_2,\ldots,F_x,F_{x+1}\right)=\bigoplus_{l=1}^{x}\text{€}_lF_l\oplus\text{€}_{x+1}F_{x+1}=$$

$$\left(\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{N_l^t}{1-N_l^t}\right)^p\right)^{1/p}}},\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{1-W_l^t}{W_l^t}\right)^p\right)^{1/p}}},\right.$$
$$\left.\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{1-\bar{M}_l^t}{\bar{M}_l^t}\right)^p\right)^{1/p}}};\ddagger\{r_1,r_2,\ldots,r_x\}\right)\oplus$$

$$\left(\sqrt[t]{1-\frac{1}{1+\left(\text{€}_1\left(\frac{N^t_{x+1}}{1-N^t_{x+1}}\right)^p\right)^{\frac{1}{p}}}},\sqrt[t]{\frac{1}{1+\left(\text{€}_{x+1}\left(\frac{1-\mathcal{W}^t_{x+1}}{\mathcal{W}^t_{x+1}}\right)^p\right)^{\frac{1}{p}}}},\sqrt[t]{\frac{1}{1+\left(\text{€}_{x+1}\left(\frac{1-\bar{M}^t_{x+1}}{\bar{M}^t_{x+1}}\right)^p\right)^{\frac{1}{p}}}};r_{x+1}\right)=$$

$$\left(\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le x+1}\text{€}_l\left(\frac{N^t_l}{1-N^t_l}\right)^p\right)^{1/p}}},\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le x+1}\text{€}_l\left(\frac{1-\mathcal{W}^t_l}{\mathcal{W}^t_l}\right)^p\right)^{1/p}}},\right.$$
$$\left.\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le x+1}\text{€}_l\left(\frac{1-\bar{M}^t_l}{\bar{M}^t_l}\right)^p\right)^{1/p}}};\ddagger\{r_1,r_2,\ldots,r_{x+1}\}\right).$$

Therefore, Eq. (12) holds for all $x\in\mathbb{N}$. ■

Theorem 2. *For a given set of t-AFNs* $F_l=(N_l,\mathcal{W}_l,\bar{M}_l;r_l)$, *where* $l=1,2,\ldots,m$, *if* $F_l=F$ *for all* l, *then*

$$t-AFDPA(F_1,F_2,\ldots,F_m)=F. \tag{13}$$

Proof. Since $F_l=F$ for all l, it follows from Eq. (12) that

$$t-AFDPA(F_1,F_2,\ldots,F_m)=\left(\begin{array}{c}\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{N^t_l}{1-N^t_l}\right)^p\right)^{1/p}}},\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{1-\mathcal{W}^t_l}{\mathcal{W}^t_l}\right)^p\right)^{1/p}}},\\ \sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{1-\bar{M}^t_l}{\bar{M}^t_l}\right)^p\right)^{1/p}}};\ddagger\{r_1,r_2,\ldots,r_m\}\end{array}\right)$$

$$=\left(\begin{array}{c}\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{N^t}{1-N^t}\right)^p\right)^{1/p}}},\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{1-\mathcal{W}^t}{\mathcal{W}^t}\right)^p\right)^{1/p}}},\\ \sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{1-\bar{M}^t}{\bar{M}^t}\right)^p\right)^{1/p}}};\ddagger\{r,r,\ldots,r\}\end{array}\right)$$

$$=\left(\begin{array}{c}\sqrt[t]{1-\frac{1}{1+\left(\left(\frac{N^t}{1-N^t}\right)^p\right)^{1/p}}},\sqrt[t]{\frac{1}{1+\left(\left(\frac{1-\mathcal{W}^t}{\mathcal{W}^t}\right)^p\right)^{1/p}}},\\ \sqrt[t]{\frac{1}{1+\left(\left(\frac{1-\bar{M}^t}{\bar{M}^t}\right)^p\right)^{1/p}}};\ddagger\{r,r,\ldots,r\}\end{array}\right)=(N,\mathcal{W},\bar{M};r).\quad\blacksquare$$

Theorem 3. *For any two sets of t-AFNs,* $F_l=(N_l,\mathcal{W}_l,\bar{M}_l;r_l)$ *and* $\ddot{F}_l=(\ddot{N}_l,\ddot{\mathcal{W}}_l,\ddot{\bar{M}}_l;\ddot{r}_l)$, *where* $l=1,2,\ldots,m$, *if* $N_l\le\ddot{N}_l$, $\bar{M}_l\ge\ddot{\bar{M}}_l$, *and* $r_l\le\ddot{r}_l$ *for all* l, *then*

$$t-AFDPA(F_1,F_2,\ldots,F_m)\le t-AFDPA(\ddot{F}_1,\ddot{F}_2,\ldots,\ddot{F}_m). \tag{14}$$

Proof. As $\mathsf{N}_{l} \leq \ddot{\mathsf{N}}_{l}$, it follows that

$$\frac{1}{1+\left(\sum\limits_{1\leq l\leq m}\epsilon_{l}\left(\frac{\mathsf{N}_{l}^{t}}{1-\mathsf{N}_{l}^{t}}\right)^{p}\right)^{1/p}} \geq \frac{1}{1+\left(\sum\limits_{1\leq l\leq m}\epsilon_{l}\left(\frac{\ddot{\mathsf{N}}_{l}^{t}}{1-\ddot{\mathsf{N}}_{l}^{t}}\right)^{p}\right)^{1/p}}$$

$$\sqrt[t]{1-\frac{1}{1+\left(\sum\limits_{1\leq l\leq m}\epsilon_{l}\left(\frac{\mathsf{N}_{l}^{t}}{1-\mathsf{N}_{l}^{t}}\right)^{p}\right)^{1/p}}} \leq \sqrt[t]{1-\frac{1}{1+\left(\sum\limits_{1\leq l\leq m}\epsilon_{l}\left(\frac{\ddot{\mathsf{N}}_{l}^{t}}{1-\ddot{\mathsf{N}}_{l}^{t}}\right)^{p}\right)^{1/p}}}.$$

Furthermore, given that $\mathcal{W}_{l} \geq \ddot{N}_{l}$, $\bar{M}_{l} \geq \ddot{\bar{M}}_{l}$, we can express it as

$$\sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\leq l\leq m}\epsilon_{l}\left(\frac{1-\mathcal{W}_{l}^{t}}{\mathcal{W}_{l}^{t}}\right)^{p}\right)^{1/p}}} \geq \sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\leq l\leq m}\epsilon_{l}\left(\frac{1-\ddot{\mathcal{W}}_{l}^{t}}{\ddot{\mathcal{W}}_{l}^{t}}\right)^{p}\right)^{1/p}}},$$

$$\sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\leq l\leq m}\epsilon_{l}\left(\frac{1-\bar{M}_{l}^{t}}{\bar{M}_{l}^{t}}\right)^{p}\right)^{1/p}}} \geq \sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\leq l\leq m}\epsilon_{l}\left(\frac{1-\ddot{\bar{M}}_{l}^{t}}{\ddot{\bar{M}}_{l}^{t}}\right)^{p}\right)^{1/p}}}.$$

Additionally, since $r_{l} \leq \ddot{r}_{l}$, it follows that

$$\ddagger\left\{r_1, r_2, \ldots, r_m\right\} \leq \ddagger\left\{\ddot{r}_1, \ddot{r}_2, \ldots, \ddot{r}_m\right\}.$$

Hence, we obtain

$$t-AFDPA\left(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m\right) \leq t-AFDPA\left(\ddot{\mathcal{F}}_1, \ddot{\mathcal{F}}_2, \ldots, \ddot{\mathcal{F}}_m\right). \quad \blacksquare$$

Theorem 4. *For any sequence of t-AFNs given by* $\mathcal{F}_{l} = \left(\mathsf{N}_{l}, \mathcal{W}_{l}, \bar{M}_{l}; r_{l}\right)$ *for* $l = 1, 2, \ldots, m$*, if* $\mathcal{F}^{-} = \left(\min\limits_{1\leq l\leq m}\mathsf{N}_{l}, \max\limits_{1\leq l\leq m}\mathcal{W}_{l}, \max\limits_{1\leq l\leq m}\bar{M}_{l}; \min\limits_{1\leq l\leq m}r_{l}\right)$ *and* $\mathcal{F}^{+} = \left(\max\limits_{1\leq l\leq m}\mathsf{N}_{l}, \min\limits_{1\leq l\leq m}\mathcal{W}_{l}, \min\limits_{1\leq l\leq m}\bar{M}_{l}; \max\limits_{1\leq l\leq m}r_{l}\right)$*, then*

$$\mathcal{F}^{-} \leq t-AFDPA\left(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m\right) \leq \mathcal{F}^{+}. \tag{15}$$

Proof. Given that $\mathcal{F}^{-} = \left(\min\limits_{1\leq l\leq m}\mathsf{N}_{l}, \max\limits_{1\leq l\leq m}\mathcal{W}_{l}, \max\limits_{1\leq l\leq m}\bar{M}_{l}; \min\limits_{1\leq l\leq m}r_{l}\right) = \left(\bar{\mathsf{N}}, \bar{\mathcal{W}}, \bar{\bar{M}}; \bar{r}\right)$ and $\mathcal{F}^{+} = (\max\limits_{1\leq l\leq m}\mathsf{N}_{l}, \min\limits_{1\leq l\leq m}\mathcal{W}_{l}, \min\limits_{1\leq l\leq m}\bar{M}_{l}; \max\limits_{1\leq l\leq m}r_{l}) = (\mathsf{N}^{+}, \mathcal{W}^{+}, \bar{M}^{+}; r^{+})$. We derive the following inequalities:

$$\sqrt[t]{1-\frac{1}{1+\left(\sum\limits_{1\leq l\leq m}\epsilon_{l}\left(\frac{\bar{\mathsf{N}}^{t}}{1-\bar{\mathsf{N}}^{t}}\right)^{p}\right)^{1/p}}} \leq \sqrt[t]{1-\frac{1}{1+\left(\sum\limits_{1\leq l\leq m}\epsilon_{l}\left(\frac{\mathsf{N}_{l}^{t}}{1-\mathsf{N}_{l}^{t}}\right)^{p}\right)^{1/p}}} \leq \sqrt[t]{1-\frac{1}{1+\left(\sum\limits_{1\leq l\leq m}\epsilon_{l}\left(\frac{\mathsf{N}^{+t}}{1-\mathsf{N}^{+t}}\right)^{p}\right)^{1/p}}}$$

$$\sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\le l\le m}\epsilon_{l}\left(\frac{1-\bar{\mathcal{W}}^{t}}{\bar{\mathcal{W}}^{t}}\right)^{p}\right)^{1/p}}}\le\sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\le l\le m}\epsilon_{l}\left(\frac{1-\mathcal{W}_{l}^{t}}{\mathcal{W}_{l}^{t}}\right)^{p}\right)^{1/p}}}\le\sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\le l\le m}\epsilon_{l}\left(\frac{1-\mathcal{W}^{+t}}{\mathcal{W}^{+t}}\right)^{p}\right)^{1/p}}}$$

$$\sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\le l\le m}\epsilon_{l}\left(\frac{1-\bar{\bar{M}}^{t}}{\bar{\bar{M}}^{t}}\right)^{p}\right)^{1/p}}}\le\sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\le l\le m}\epsilon_{l}\left(\frac{1-\bar{M}_{l}^{t}}{\bar{M}_{l}^{t}}\right)^{p}\right)^{1/p}}}\le\sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\le l\le m}\epsilon_{l}\left(\frac{1-\left(\bar{M}^{+}\right)^{t}}{\left(\bar{M}^{+}\right)^{t}}\right)^{p}\right)^{1/p}}}$$

$$\ddagger\left\{\bar{r},\bar{r},\ldots,\bar{r}\right\}\le\ddagger\left\{r_1,r_2,\ldots,r_m\right\}\le\ddagger\left\{r^{+},r^{+},\ldots,r^{+}\right\}.$$

Therefore, $\mathcal{F}^{-}\le t-AFDPA\left(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m\right)\le\mathcal{F}^{+}$. ■

4.2 t-AF Dombi Power Weighted Average Operator

Definition 12. *For a given set of t-AFNs $\mathcal{F}_{l}=\left(\mathcal{N}_{l},\mathcal{W}_{l},\bar{M}_{l};r_{l}\right)$ where $l=1,2,\ldots,m$, the t-AFDPWA operator is defined as:*

$$t-AFDPWA\left(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m\right)=\overset{m}{\underset{l=1}{\oplus}}\left(\frac{w_{l}\left(1+S\left(\mathcal{F}_{l}\right)\right)}{\sum\limits_{1\le l\le m}w_{l}\left(1+S\left(\mathcal{F}_{l}\right)\right)}\right)\mathcal{F}_{l}=\overset{m}{\underset{l=1}{\oplus}}\mathfrak{m}_{l}\mathcal{F}_{l},\tag{16}$$

here, $\mathfrak{m}_{l}$ represents the power weight of $\mathcal{F}_{l}$, where $w_{l}\in[0,1]$ and the weight vector for the criteria is given by $\sum\limits_{1\le l\le m}w_{l}=1$. Additionally, the function $S\left(\mathcal{F}_{l}\right)$ is defined as $\sum\limits_{1\le l\le m,l\ne l}supp\left(\mathcal{F}_{l},\mathcal{F}_{l}\right)$.

Theorem 5. *For any set of t-AFNs given by $\mathcal{F}_{l}=\left(\mathcal{N}_{l},\mathcal{W}_{l},\bar{M}_{l};r_{l}\right)$ for $l=1,2,\ldots,m$, we obtain the following result based on Definition 10.*

$$t-AFDPWA\left(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m\right)=\left(\begin{array}{c}\sqrt[t]{1-\frac{1}{1+\left(\sum\limits_{1\le l\le m}\mathfrak{m}_{l}\left(\frac{\mathcal{N}_{l}^{t}}{1-\mathcal{N}_{l}^{t}}\right)^{p}\right)^{1/p}}},\sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\le l\le m}\mathfrak{m}_{l}\left(\frac{1-\mathcal{W}_{l}^{t}}{\mathcal{W}_{l}^{t}}\right)^{p}\right)^{1/p}}},\\ \sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\le l\le m}\mathfrak{m}_{l}\left(\frac{1-\bar{M}_{l}^{t}}{\bar{M}_{l}^{t}}\right)^{p}\right)^{1/p}}};\ddagger\left\{r_1,r_2,\ldots,r_m\right\}\end{array}\right).\tag{17}$$

Theorem 6. *For any collection of t-AFNs represented as $\mathcal{F}_{l}=\left(\mathcal{N}_{l},\mathcal{W}_{l},\bar{M}_{l};r_{l}\right)$ for $l=1,2,\ldots,m$, if $\mathcal{F}_{l}=\mathcal{F}$ for all l, then*

$$t-AFDPWA\left(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m\right)=\mathcal{F}.\tag{18}$$

Theorem 7. *For any two sets of t-AFNs, denoted as $\mathcal{F}_{l}=\left(\mathcal{N}_{l},\mathcal{W}_{l},\bar{M}_{l};r_{l}\right)$ and $\ddot{\mathcal{F}}_{l}=\left(\ddot{\mathcal{N}}_{l},\ddot{N}_{l},\ddot{\bar{M}}_{l};\ddot{r}_{l}\right)$ for $l=1,2,\ldots,m$, if the conditions $\mathcal{N}_{l}\le\ddot{\mathcal{N}}_{l}$, $\bar{M}_{l}\ge\ddot{\bar{M}}_{l}$, and $r_{l}\le\ddot{r}_{l}$ hold for all l, then*

$$t-AFDPWA\left(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m\right)\le t-AFDPWA\left(\ddot{\mathcal{F}}_1,\ddot{\mathcal{F}}_2,\ldots,\ddot{\mathcal{F}}_m\right).\tag{19}$$

Theorem 8. *For any set of t-AFNs given by* $\digamma_l = (N_l, W_l, \bar{M}_l; r_l)$ *for* $l = 1, 2, \ldots, m$, *if* $\digamma^- = \left(\min_{1\le l\le m} N_l, \max_{1\le l\le m} W_l, \max_{1\le l\le m} \bar{M}_l; \min_{1\le l\le m} r_l\right)$ *and* $\digamma^+ = \left(\max_{1\le l\le m} N_l, \min_{1\le l\le m} W_l, \min_{1\le l\le m} \bar{M}_l; \max_{1\le l\le m} r_l\right)$, *then*

$$\digamma^- \le t - AFDPWA(\digamma_1, \digamma_2, \ldots, \digamma_m) \le \digamma^+. \tag{20}$$

4.3 t-AF Dombi Power Geometric Operator

In the present section, we establish the theoretical foundation of the t-AFDPG operator and analyze its properties.

Definition 13. *For a given set of t-AFNs* $\digamma_l = (N_l, \bar{M}_l; r_l)$, *where* $l = 1, 2, \ldots, m$, *the t-AFDPG operator is defined as*

$$t - AFDPG(\digamma_1, \digamma_2, \ldots, \digamma_m) = \bigotimes_{l=1}^{m} \digamma_l^{\left(\frac{(1+S(\digamma_l))}{\sum_{1\le l\le m}(1+S(\digamma_l))}\right)} = \bigotimes_{l=1}^{m} \digamma_l^{\epsilon_l}, \tag{21}$$

here, ϵ_l *represents the power weight of* $\digamma_l$, *where the score function is defined as* $S(\digamma_l) = \sum_{1\le l\le m, l\ne l} supp(\digamma_l, \digamma_l)$.

Theorem 9. *For any collection of t-AFNs represented as* $\digamma_l = (N_l, \bar{M}_l; r_l)$, *where* $l = 1, 2, \ldots, m$, *we derive the following result based on Definition 10.*

$$t - AFDPG(\digamma_1, \digamma_2, \ldots, \digamma_m) = \left(\begin{array}{c} \sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_l \left(\frac{1-N_l^t}{N_l^t}\right)^p\right)^{1/p}}}, \sqrt[t]{1 - \frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_l \left(\frac{W_l^t}{1-W_l^t}\right)^p\right)^{1/p}}}, \\ \sqrt[t]{1 - \frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_l \left(\frac{\bar{M}_l^t}{1-\bar{M}_l^t}\right)^p\right)^{1/p}}}; \ddagger\{r_1, r_2, \ldots, r_m\} \end{array} \right). \tag{22}$$

Proof. This can be proven using mathematical induction as follows:

For the base case $m = 2$,

$$t - AFDPG(\digamma_1, \digamma_2) = \digamma_1^{\epsilon_1} \otimes \digamma_2^{\epsilon_2} =$$

$$\left(\sqrt[t]{\frac{1}{1+\left(\epsilon_1\left(\frac{1-N_1^t}{N_1^t}\right)^p\right)^{\frac{1}{p}}}}, \sqrt[t]{1-\frac{1}{1+\left(\epsilon_1\left(\frac{W_1^t}{1-W_1^t}\right)^p\right)^{\frac{1}{p}}}}, \sqrt[t]{1-\frac{1}{1+\left(\epsilon_1\left(\frac{\bar{M}_1^t}{1-\bar{M}_1^t}\right)^p\right)^{\frac{1}{p}}}}; r_1 \right) \otimes$$

$$\left(\sqrt[t]{\frac{1}{1+\left(\epsilon_2\left(\frac{1-A_1^t}{N_1^t}\right)^p\right)^{\frac{1}{p}}}}, \sqrt[t]{1-\frac{1}{1+\left(\epsilon_2\left(\frac{W_1^t}{1-W_1^t}\right)^p\right)^{\frac{1}{p}}}}, \sqrt[t]{1-\frac{1}{1+\left(\epsilon_2\left(\frac{\bar{M}_1^t}{1-\bar{M}_1^t}\right)^p\right)^{\frac{1}{p}}}}; r_2 \right) =$$

$$\left(\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le 2} \epsilon_l \left(\frac{1-N_l^t}{N_l^t}\right)^p\right)^{1/p}}}, \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le 2} \epsilon_l \left(\frac{W_l^t}{1-W_l^t}\right)^p\right)^{1/p}}}, \right.$$

$$\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le 2}\text{€}_l\left(\frac{\bar{M}_l^t}{1-\bar{M}_l^t}\right)^p\right)^{1/p}}};\ddagger\{r_1,r_2\}\Bigg).$$

Assume that Eq. (22) holds for $m = x$,

$$t-AFDPG\left(F_1,F_2,\ldots,F_x\right)=$$

$$\left(\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{1-\mathrm{N}_l^t}{\mathrm{N}_l^t}\right)^p\right)^{1/p}}},\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{\mathcal{W}_l^t}{1-\mathcal{W}_l^t}\right)^p\right)^{1/p}}},\right.$$

$$\left.\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{\bar{M}_l^t}{1-\bar{M}_l^t}\right)^p\right)^{1/p}}};\ddagger\{r_1,r_2,\ldots,r_x\}\right).$$

Now, to prove it for $m = l + 1$,

$$t-AFDPG\left(F_1,F_2,\ldots,F_x,F_{x+1}\right)=\bigotimes_{l=1}^{x}F_l^{\text{€}_l}\otimes F_{x+1}^{\text{€}_{x+1}}=$$

$$\left(\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{1-\mathrm{N}_l^t}{\mathrm{N}_l^t}\right)^p\right)^{1/p}}},\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{\mathcal{W}_l^t}{1-\mathcal{W}_l^t}\right)^p\right)^{1/p}}},\right.$$

$$\left.\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le x}\text{€}_l\left(\frac{\bar{M}_l^t}{1-\bar{M}_l^t}\right)^p\right)^{1/p}}};\ddagger\{r_1,r_2,\ldots,r_x\}\right)\otimes$$

$$\left(\sqrt[t]{\frac{1}{1+\left(\text{€}_1\left(\frac{1-\mathrm{N}_{x+1}^t}{\mathrm{N}_{x+1}^t}\right)^p\right)^{\frac{1}{p}}}},\sqrt[t]{1-\frac{1}{1+\left(\text{€}_{x+1}\left(\frac{\mathcal{W}_{x+1}^t}{1-\mathcal{W}_{x+1}^t}\right)^p\right)^{\frac{1}{p}}}},\sqrt[t]{1-\frac{1}{1+\left(\text{€}_{x+1}\left(\frac{\bar{M}_{x+1}^t}{1-\bar{M}_{x+1}^t}\right)^p\right)^{\frac{1}{p}}}};r_{x+1}\right)=$$

$$\left(\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le x+1}\text{€}_l\left(\frac{1-\mathrm{N}_l^t}{\mathrm{N}_l^t}\right)^p\right)^{1/p}}},\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le x+1}\text{€}_l\left(\frac{\mathcal{W}_l^t}{1-\mathcal{W}_l^t}\right)^p\right)^{1/p}}},\right.$$

$$\left.\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le x+1}\text{€}_l\left(\frac{\bar{M}_l^t}{1-\bar{M}_l^t}\right)^p\right)^{1/p}}};\ddagger\{r_1,r_2,\ldots,r_{x+1}\}\right).$$

Therefore, Eq. (22) holds for all $x \in \mathbb{N}$. ■

Theorem 10. *For a given set of t-AFNs* $\digamma_{l} = \left(N_{l}, \mathcal{W}_{l}, \bar{M}_{l}; r_{l}\right)$, *where* $l = 1, 2, \ldots, m$, *if* $\digamma_{l} = \digamma$ *for all* l, *then*

$$t - AFDPG\left(\digamma_{1}, \digamma_{2}, \ldots, \digamma_{m}\right) = \digamma. \tag{23}$$

Proof. Given that $\digamma_{l} = \digamma$ for all l, it follows from Eq. (22) that

$$t - AFDPG\left(\digamma_{1}, \digamma_{2}, \ldots, \digamma_{m}\right) = \left(\begin{array}{c} \sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{1-N_{l}^{t}}{N_{l}^{t}}\right)^{p}\right)^{1/p}}}, \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{\mathcal{W}_{l}^{t}}{1-\mathcal{W}_{l}^{t}}\right)^{p}\right)^{1/p}}}, \\ \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{\bar{M}_{l}^{t}}{1-\bar{M}_{l}^{t}}\right)^{p}\right)^{1/p}}}; \ddagger\left\{r_{1}, r_{2}, \ldots, r_{m}\right\} \end{array}\right)$$

$$= \left(\begin{array}{c} \sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{1-N^{t}}{N^{t}}\right)^{p}\right)^{1/p}}}, \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{\mathcal{W}^{t}}{1-\mathcal{W}^{t}}\right)^{p}\right)^{1/p}}}, \\ \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{\bar{M}^{t}}{1-\bar{M}^{t}}\right)^{p}\right)^{1/p}}}; \ddagger\left\{r, r, \ldots, r\right\} \end{array}\right)$$

$$= \left(\begin{array}{c} \sqrt[t]{\frac{1}{1+\left(\left(\frac{1-N^{t}}{N^{t}}\right)^{p}\right)^{1/p}}}, \sqrt[t]{1-\frac{1}{1+\left(\left(\frac{\mathcal{W}^{t}}{1-\mathcal{W}^{t}}\right)^{p}\right)^{1/p}}}, \\ \sqrt[t]{1-\frac{1}{1+\left(\left(\frac{\bar{M}^{t}}{1-\bar{M}^{t}}\right)^{p}\right)^{1/p}}}; \ddagger\left\{r, r, \ldots, r\right\} \end{array}\right) = \left(N, \mathcal{W}, \bar{M}; r\right).$$

Theorem 11. *Consider two sets of t-AFNs, denoted as* $\digamma_{l} = \left(N_{l}, \mathcal{W}_{l}, \bar{M}_{l}; r_{l}\right)$ *and* $\ddot{\digamma}_{l} = \left(\ddot{N}_{l}, \ddot{N}_{l}, \ddot{\bar{M}}_{l}; \ddot{r}_{l}\right)$ *where* $l = 1, 2, \ldots, m$. *If the conditions* $N_{l} \le \ddot{N}_{l}$, $\mathcal{W}_{l} \ge \ddot{N}_{l}$, $\bar{M}_{l} \ge \ddot{\bar{M}}_{l}$, *and* $r_{l} \le \ddot{r}_{l} \forall l$ *hold for all* l, *then*

$$t - AFDPG\left(\digamma_{1}, \digamma_{2}, \ldots, \digamma_{m}\right) \le t - AFDPG\left(\ddot{\digamma}_{1}, \ddot{\digamma}_{2}, \ldots, \ddot{\digamma}_{m}\right). \tag{24}$$

Proof. Given that $N_{l} \le \ddot{N}_{l}$, it follows that

$$\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{1-N_{l}^{t}}{N_{l}^{t}}\right)^{p}\right)^{1/p}} \le \frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{1-\ddot{N}_{l}^{t}}{\ddot{N}_{l}^{t}}\right)^{p}\right)^{1/p}}$$

$$\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{1-N_{l}^{t}}{N_{l}^{t}}\right)^{p}\right)^{1/p}}} \le \sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{1-\ddot{N}_{l}^{t}}{\ddot{N}_{l}^{t}}\right)^{p}\right)^{1/p}}}.$$

Furthermore, since $\mathcal{W}_{l} \ge \ddot{N}_{l}$, $\bar{M}_{l} \ge \ddot{\bar{M}}_{l}$, we can express it as

$$\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{\mathcal{W}_{l}^{t}}{1-\mathcal{W}_{l}^{t}}\right)^{p}\right)^{1/p}}} \ge \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m} \epsilon_{l}\left(\frac{\ddot{\mathcal{W}}_{l}^{t}}{1-\ddot{\mathcal{W}}_{l}^{t}}\right)^{p}\right)^{1/p}}},$$

$$\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{\bar{M}_l^t}{1-\bar{M}_l^t}\right)^p\right)^{1/p}}} \ge \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{\ddot{\bar{M}}_l^t}{1-\ddot{\bar{M}}_l^t}\right)^p\right)^{1/p}}}.$$

Moreover, since $r_l \le \ddot{r}_l$, it follows that $\ddagger\{r_1, r_2, \ldots, r_m\} \le \ddagger\{\ddot{r}_1, \ddot{r}_2, \ldots, \ddot{r}_m\}$. Consequently, we obtain $t-AFDPG(F_1, F_2, \ldots, F_m) \le t-AFDPG(\ddot{F}_1, \ddot{F}_2, \ldots, \ddot{F}_m)$. ■

Theorem 12. *For any set of t-AFNs represented as* $F_l = \left(\mathsf{N}_l, \mathcal{W}_l, \bar{M}_l; r_l\right)$, *where* $l = 1, 2, \ldots, m$, *if*

$$F^- = \left(\min_{1\le l\le m}\mathsf{N}_l, \max_{1\le l\le m}\mathcal{W}_l, \max_{1\le l\le m}\bar{M}_l; \min_{1\le l\le m} r_l\right)$$

and

$$F^+ = \left(\max_{1\le l\le m}\mathsf{N}_l, \min_{1\le l\le m}\mathcal{W}_l, \min_{1\le l\le m}\bar{M}_l; \max_{1\le l\le m} r_l\right),$$

then the following holds.

$$F^- \le t-AFDPG(F_1, F_2, \ldots, F_m) \le F^+. \tag{25}$$

Proof. Given that $F^- = \left(\min_{1\le l\le m}\mathsf{N}_l, \max_{1\le l\le m}\mathcal{W}_l, \max_{1\le l\le m}\bar{M}_l; \min_{1\le l\le m} r_l\right) = \left(\bar{\mathsf{N}}, \bar{\mathcal{W}}, \bar{\bar{M}}; \bar{r}\right)$ and $F^+ = (\max_{1\le l\le m}\mathsf{N}_l, \min_{1\le l\le m}\mathcal{W}_l, \min_{1\le l\le m}\bar{M}_l; \max_{1\le l\le m} r_l) = \left(\mathsf{N}^+, \mathcal{W}^+, \bar{M}^+; r^+\right)$.

We derive the following inequalities.

$$\sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{1-\bar{\mathsf{N}}^t}{\bar{\mathsf{N}}^t}\right)^p\right)^{1/p}}} \le \sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{1-\mathsf{N}_l^t}{\mathsf{N}_l^t}\right)^p\right)^{1/p}}} \le \sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{1-\mathsf{N}^{+t}}{\mathsf{N}^{+t}}\right)^p\right)^{1/p}}}$$

$$\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{\bar{\mathcal{W}}^t}{1-\bar{\mathcal{W}}^t}\right)^p\right)^{1/p}}} \le \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{\mathcal{W}_l^t}{1-\mathcal{W}_l^t}\right)^p\right)^{1/p}}} \le \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{\mathcal{W}^{+t}}{1-\mathcal{W}^{+t}}\right)^p\right)^{1/p}}}$$

$$\sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{\bar{\bar{M}}^t}{1-\bar{\bar{M}}^t}\right)^p\right)^{1/p}}} \le \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{\bar{M}_l^t}{1-\bar{M}_l^t}\right)^p\right)^{1/p}}} \le \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m}\text{€}_l\left(\frac{(\bar{M}^+)^t}{1-(\bar{M}^+)^t}\right)^p\right)^{1/p}}}$$

$$\ddagger\{\bar{r}, \bar{r}, \ldots, \bar{r}\} \le \ddagger\{r_1, r_2, \ldots, r_m\} \le \ddagger\{r^+, r^+, \ldots, r^+\}.$$

Therefore, $F^- \le t-AFDPG(F_1, F_2, \ldots, F_m) \le F^+$. ■

4.4 t-AF Dombi Power Weighted Geometric Operator

Definition 14. *Given an array of t-AFNs defined as* $F_l = \left(N_l, W_l, \bar{M}_l; r_l\right)$ *for* $l = 1, 2, \ldots, m$, *the t-AFDPWG operator is defined as:*

$$t - AFDPWG\left(F_1, F_2, \ldots, F_m\right) = \overset{m}{\underset{l=1}{\otimes}} F_l^{\left(\frac{w_j\left(1+S\left(F_l\right)\right)}{\sum_{1\le l\le m} w_j\left(1+S\left(F_l\right)\right)}\right)} = \overset{m}{\underset{l=1}{\otimes}} F_l^{\mathfrak{m}_l}, \tag{26}$$

here, $\mathfrak{m}_l$ *represents the power weight of* F_l, *where* $w_l \in [0, 1]$ *and* $\sum_{1\le l\le m} w_l = 1$ *denotes the weight vector of the criteria. The term* $S(F_l)$ *is defined as* $S(F_l) = \sum_{1\le l\le m,\, l\ne l} supp(F_l, F_l)$.

Theorem 13. *Given an array of t-AFNs* $F_l = \left(N_l, W_l, \bar{M}_l; r_l\right)$ *for* $l = 1, 2, \ldots, m$, *the result follows from Definition 10 as:*

$$t - AFDPWG\left(F_1, F_2, \ldots, F_m\right) = \left(\begin{array}{c} \sqrt[t]{\frac{1}{1+\left(\sum_{1\le l\le m} \mathfrak{m}_l\left(\frac{1-N_l^t}{N_l^t}\right)^p\right)^{1/p}}}, \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m} \mathfrak{m}_l\left(\frac{W_l^t}{1-W_l^t}\right)^p\right)^{1/p}}}, \\ \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le l\le m} \mathfrak{m}_l\left(\frac{\bar{M}_l^t}{1-\bar{M}_l^t}\right)^p\right)^{1/p}}}; \ddagger\left\{r_1, r_2, \ldots, r_m\right\} \end{array}\right). \tag{27}$$

Theorem 14. *Consider an array of t-AFNs* $F_l = \left(N_l, W_l, \bar{M}_l; r_l\right)$ *for* $l = 1, 2, \ldots, m$. *If all* F_l *are identical, i.e.,* $F_l = F$ *for every* l, *then:*

$$t - AFDPWG\left(F_1, F_2, \ldots, F_m\right) = F. \tag{28}$$

Theorem 15. *Given two arrays of t-AFNs,* $F_l = \left(N_l, W_l, \bar{M}_l; r_l\right)$ *and* $\ddot{F}_l = \left(\ddot{N}_l, \ddot{N}_l, \ddot{\bar{M}}_l; \ddot{r}_l\right)$ *for* $l = 1, 2, \ldots, m$, *if* $N_l \le \ddot{N}_l$, $\bar{M}_l \ge \ddot{\bar{M}}_l$, *and* $r_l \le \ddot{r}_l$ *for all* l, *then:*

$$t - AFDPWG\left(F_1, F_2, \ldots, F_m\right) \le t - AFDPWG\left(\ddot{F}_1, \ddot{F}_2, \ldots, \ddot{F}_m\right). \tag{29}$$

Theorem 16. *Consider an array of t-AFNs* $F_l = \left(N_l, W_l, \bar{M}_l; r_l\right)$ *for* $l = 1, 2, \ldots, m$. *If we define* $F^- = \left(\min_{1\le l\le m} N_l, \max_{1\le l\le m} W_l, \max_{1\le l\le m} \bar{M}_l; \min_{1\le l\le m} r_l\right)$ *and* $F^+ = \left(\max_{1\le l\le m} N_l, \min_{1\le l\le m} W_l, \min_{1\le l\le m} \bar{M}_l; \max_{1\le l\le m} r_l\right)$, *then:*

$$F^- \le t - AFDPWG\left(F_1, F_2, \ldots, F_m\right) \le F^+. \tag{30}$$

5 Proposed WASPAS Method

This section presents a WASPAS-based method that incorporates the developed power AOs. Let $\mathsf{T} = \{\mathsf{T}_j\}$ for $1 \le j \le m$ represent the set of m alternatives, and let $\mathsf{L} = \{\mathsf{L}_i\}$ for $1 \le i \le n$ represent the set of n criteria. Each criteria L_i is associated with a corresponding weight from the weight vector $W = (w_i)^t$, where $w_i > 0$ and $\sum_{i=1}^{n} w_i = 1$.

Decision experts assess each criteria L_i for every alternative T_j using t-AFNs of the form $\mathcal{F}_{ji} = \left(\mathsf{N}_{ji}, \mathcal{W}_{ji}, \bar{M}_{ji}; r_{ji}\right)$. These evaluations form the decision matrix $M = \left(\mathcal{F}_{ji}\right)_{m\times n}$, as shown below:

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \mathsf{L}_1 & \cdots & \mathsf{L}_i & \cdots & \mathsf{L}_n \end{array} \\
M = \begin{array}{c} \mathsf{T}_1 \\ \vdots \\ \mathsf{T}_j \\ \vdots \\ \mathsf{T}_m \end{array} &
\begin{pmatrix}
\mathcal{F}_{11} & \cdots & \mathcal{F}_{1i} & \cdots & \mathcal{F}_{1n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\mathcal{F}_{m1} & \cdots & \mathcal{F}_{mi} & \cdots & \mathcal{F}_{mn} \\
\mathcal{F}_{j1} & \cdots & \mathcal{F}_{ji} & \cdots & \mathcal{F}_{jn} \\
\vdots & \ddots & \vdots & \ddots & \vdots
\end{pmatrix}
\end{array}
$$

The following steps describe the procedure of the proposed t-AF-WASPAS method:

Step 1: Normalize the decision matrix $M = \left(\mathcal{F}_{ji}\right)_{m\times n}$ using the expression provided below.

$$
\widetilde{\mathcal{F}}_{ji} = \begin{cases} \left(\mathsf{N}_{ji}, \mathcal{W}_{ji}, \bar{M}_{ji}; r_{ji}\right), & \text{if } \mathsf{L}_i \text{ benefit-type} \\ \left(\bar{M}_{ji}, \mathcal{W}_{ji}, \mathsf{N}_{ji}; r_{ji}\right), & \text{otherwise.} \end{cases} \tag{31}
$$

Step 2: Calculate the support value for each criteria L_i, where $i = 1, 2, \ldots, n$, using Eq. (32):

$$
supp\left(\mathcal{F}_{ji}, \mathcal{F}_{jl}\right) = 1 - d\left(\mathcal{F}_{ji}, \mathcal{F}_{jl}\right); (j = 1, 2, \ldots, m; i, l = 1, 2, \ldots, n; l \neq i), \tag{32}
$$

where $d\left(\mathcal{F}_{ji}, \mathcal{F}_{jl}\right) = \frac{1}{3}\left(\frac{\left|r_{ji} - r_{jl}\right|}{\sqrt[t]{3}} + \left|A^{t}_{ji} - A^{t}_{jl}\right| + \left|\mathcal{W}^{t}_{ji} - \mathcal{W}^{t}_{jl}\right| + \left|\bar{M}^{t}_{ji} - \bar{M}^{t}_{jl}\right|\right)$.

Step 3: Determine the total support for $\mathcal{F}_{ji}$ using Eq. (33):

$$
S\left(\mathcal{F}_{ji}\right) = \sum_{\substack{1\leq l\leq n \\ l\neq i}} supp\left(\mathcal{F}_{ji}, \mathcal{F}_{jl}\right). \tag{33}
$$

Then, compute the power weight corresponding to $\mathcal{F}_{ji}$ as follows:

$$
\mathfrak{m}_{ji} = \frac{w_i\left(1 + S\left(\mathcal{F}_{ji}\right)\right)}{\sum\limits_{1\leq i\leq n} w_i\left(1 + S\left(\mathcal{F}_{ji}\right)\right)}, \tag{34}
$$

here, $\mathfrak{m}_{ji} \in [0, 1]$ and $\sum\limits_{1\leq i\leq n} \mathfrak{m}_{ji} = 1 (j = 1, 2, \ldots, m)$.

Step 4: Apply the t-AFDPWA operator to calculate the WSM $\mathcal{B}^{s}_{j}$ for each alternative $j = 1, 2, \ldots, m$, as given below:

$$
\begin{aligned}
&\mathcal{B}^{s}_{j} = t - AFDPWA\left(\mathcal{F}_{j1}, \mathcal{F}_{j2}, \ldots, \mathcal{F}_{jn}\right) = \\
&\left(\begin{array}{c}
\sqrt[t]{1 - \frac{1}{1+\left(\sum\limits_{1\leq i\leq n} \mathfrak{m}_{ji}\left(\frac{\mathsf{N}^{t}_{ji}}{1-\mathsf{N}^{t}_{ji}}\right)^{p}\right)^{1/p}}}, \sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\leq i\leq n} \mathfrak{m}_{ji}\left(\frac{1-\mathcal{W}^{t}_{ji}}{\mathcal{W}^{t}_{ji}}\right)^{p}\right)^{1/p}}}, \\
\sqrt[t]{\frac{1}{1+\left(\sum\limits_{1\leq i\leq n} \mathfrak{m}_{ji}\left(\frac{1-\bar{M}^{t}_{ji}}{\bar{M}^{t}_{ji}}\right)^{p}\right)^{1/p}}}; \ddagger\left\{r_1, r_2, \ldots, r_n\right\}
\end{array}\right).
\end{aligned} \tag{35}
$$

Step 5: Use the t-AFDPWG operator to calculate the WPM $\mathcal{B}_j^p$ for each alternative, where $j = 1, 2, \ldots, m$, as shown below:

$$\mathcal{B}_j^p = t - AFDPWG\left(F_{j1}, F_{j2}, \ldots, F_{jn}\right) =$$
$$\left(\begin{array}{c}\sqrt[t]{\frac{1}{1+\left(\sum_{1\le i\le n}\mathfrak{m}_{ji}\left(\frac{1-\mathrm{N}_{ji}^{t}}{\mathrm{N}_{ji}^{t}}\right)^{p}\right)^{1/p}}}, \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le i\le n}\mathfrak{m}_{ji}\left(\frac{\mathcal{W}_{ji}^{t}}{1-\mathcal{W}_{ji}^{t}}\right)^{p}\right)^{1/p}}}, \\ \sqrt[t]{1-\frac{1}{1+\left(\sum_{1\le i\le n}\mathfrak{m}_{ji}\left(\frac{\bar{M}_{ji}^{t}}{1-\bar{M}_{ji}^{t}}\right)^{p}\right)^{1/p}}}; \ddagger\left\{r_1, r_2, \ldots, r_n\right\}\end{array}\right). \tag{36}$$

Step 6: Using Eq. (37), combine the WSM and WPM values to obtain the aggregated preference measure $\mathcal{B}_j$ for each alternative, where $j = 1, 2, \ldots, m$:

$$\mathcal{B}_j = £\mathcal{B}_j^s \oplus (1 - £)\mathcal{B}_j^p, \tag{37}$$

here, £ is a decision-precision parameter in the range $0 \le £ \le 1$, used to control the aggregation of $\mathcal{B}_j^s$ and $\mathcal{B}_j^p$. This parameter allows decision-makers to adjust the balance between the two measures, offering greater flexibility in deriving more accurate overall preference values in line with the WASPAS method.

Step 7: Calculate the score of each aggregated preference measure $\mathcal{B}_j$ for $j = 1, 2, \ldots, m$, and rank the alternatives accordingly based on these scores.

The pseudocode representation of the proposed WASPAS approach is provided in Algorithm 1.

Algorithm 1: Pseudocode for the developed WASPAS method

Require: The set of alternatives T and the set of criteria L
Ensure: Prioritized ranking of smart city initiatives
Create t-AF decision-matrix $M = \left(F_{ji}\right)_{m\times n}$; execute normalization if needed
for $i = 1$ to n **do**
  for $j = 1$ to m **do**
    Calculate support value $supp\left(F_{ji}, F_{jl}\right)$ for each criteria using Eq. (32);
  end for
end for
for $i = 1$ to n **do**
  for $j = 1$ to m **do**
    Determine the total support $S(F_{ji})$ according to Eq. (33);
  end for
end for
for $i = 1$ to n **do**
  for $j = 1$ to m **do**

(Continued)

Algorithm 1 (continued)

Determine the power weights $\mathfrak{M}_{ji}$ based on Eq. (34);
end for
end for
for $j = 1$ to m **do**
for $i = 1$ to n **do**
Calculate the WSM $\mathcal{B}_j^s$ using Eq. (35);
end for
end for
for $j = 1$ to m **do**
for $i = 1$ to n **do**
Calculate the WPM $\mathcal{B}_j^p$ using Eq. (36);
end for
end for
for $j = 1$ to m **do**
Determine the aggregated preference measure $\mathcal{B}_j$ with Eq. (37);
end for
for $j = 1$ to m **do**
Compute the score values for aggregated preference measure $\mathcal{B}_j$;
end for
Based on the score values rank the alternatives in decreasing order.

The stepwise procedure of the proposed methodology is depicted in Fig. 1.

6 Case Study: Smart City Initiative Prioritization

In the following, we explore the smart city initiative prioritization problem, and implement the proposed algorithm to showcase its practical applicability and effectiveness.

6.1 Problem Description

The rapid expansion of urban populations worldwide has intensified the challenges faced by cities in managing infrastructure, resources, sustainability, and overall quality of life. In response to these growing concerns, the concept of smart cities has gained significant attention, promoting the integration of digital technologies, data-driven systems, and automation to create more livable, efficient, and sustainable urban environments.

However, the successful development of smart cities demands careful prioritization across diverse domains such as energy, healthcare, governance, transportation, and public safety. Given the limitations in financial resources, technological maturity, and public readiness, it is impractical to implement all smart initiatives simultaneously. Consequently, there is a pressing need for a structured approach to prioritize projects based on their potential impact and feasibility.

Due to the multi-dimensional and often conflicting nature of the factors involved, this decision-making process is complex. Therefore, the application of MCDM problems provides a systematic and rational mechanism to evaluate and rank different initiatives by considering both qualitative and quantitative aspects.

In this study, an MCDM-based framework is proposed to prioritize a selection of seven smart city initiatives evaluated against eight critical criteria, aiming to assist urban planners and decision-makers in adopting strategies that are both effective and sustainable.

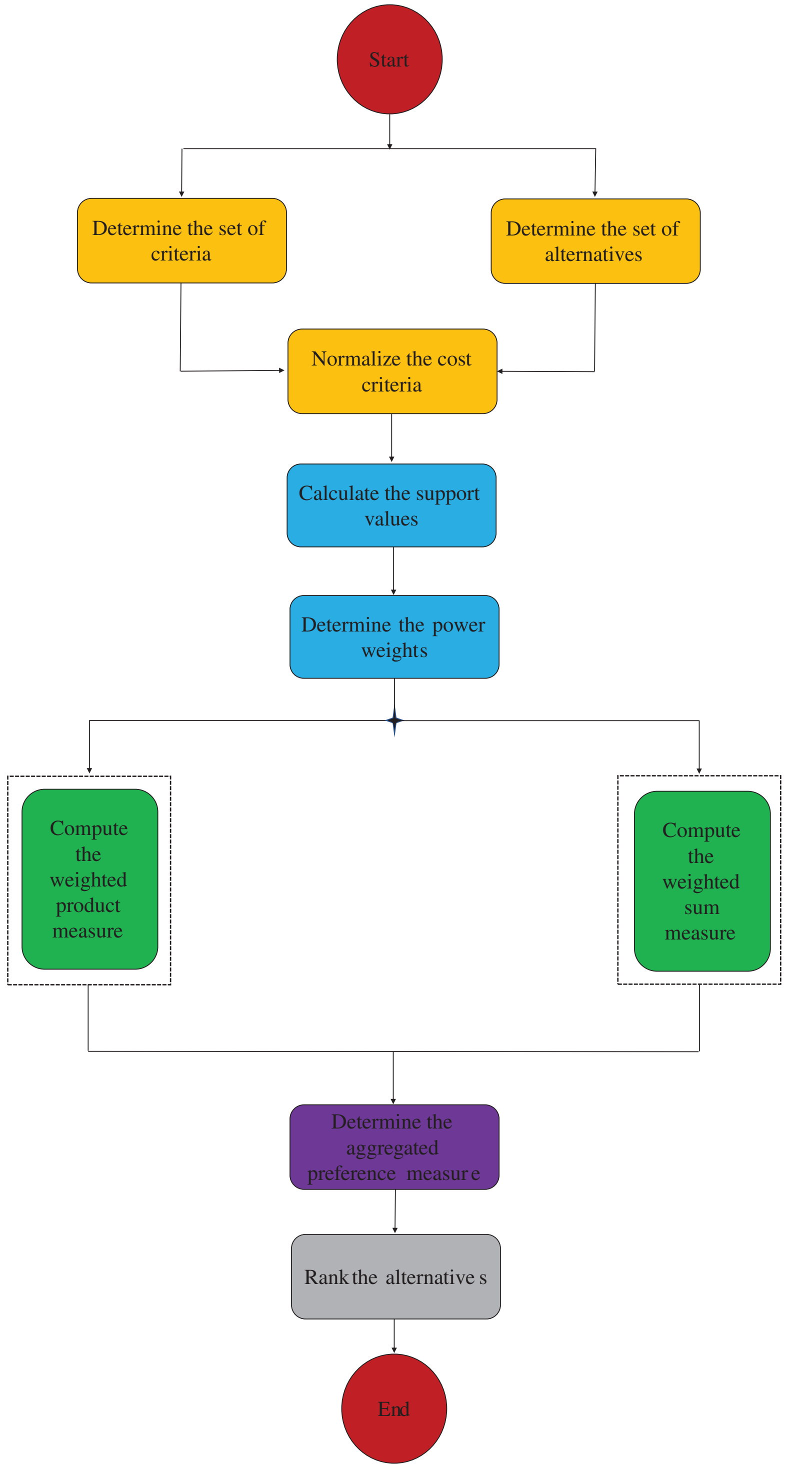

Figure 1: Flowchart of the proposed WASPAS method

Evaluation Criteria: The assessment of smart city initiatives is conducted based on eight carefully selected criteria, as presented in Table 1.

Alternatives: The evaluation considers seven selected smart city initiatives, summarized in Table 2.

Table 1: Evaluation criteria for smart city initiatives

No.	Criterion	Description
L_1	Implementation cost	Estimated total investment, including initial setup and operational expenses.
L_2	Technological feasibility	Practicality of deploying required technologies within existing infrastructure.
L_3	Social acceptance	Likelihood of public support, adoption, and behavioral adaptation.
L_4	Environmental impact	Contribution to environmental preservation and sustainability.
L_5	Data security and privacy	Measures to protect personal and sensitive information.
L_6	Scalability and flexibility	Potential to expand and adapt to future needs and technological advances.
L_7	Operational efficiency	Improvements in service delivery, resource utilization, and administrative performance.
L_8	Return on investment (ROI)	Expected financial and socio-economic returns relative to resources invested.

Table 2: Alternatives for smart city development

No.	Alternative	Description
T_1	Smart energy grid	Utilization of internet of things (IoT) and AI technologies to optimize energy production, distribution, and consumption.
T_2	E-governance platforms	Digital portals to improve public service delivery, transparency, and citizen engagement.
T_3	Intelligent waste management systems	Smart solutions using sensors and analytics to streamline waste collection and promote recycling.
T_4	Smart water monitoring systems	Real-time tracking and management of water supply and quality to minimize waste.
T_5	Urban mobility solutions	Integrated transport systems with smart traffic management, electric mobility, and shared transportation.
T_6	Smart healthcare systems	Telemedicine, AI-driven diagnostics, and continuous health monitoring solutions.
T_7	Public safety and surveillance systems	Smart surveillance networks and predictive analytics tools for enhanced urban safety.

A panel of experts evaluates seven smart city initiatives $(T_1, T_2, T_3, T_4, T_5, T_6, T_7)$ based on the eight selected criteria, each with its respective weight vector $W = (0.18, 0.18, 0.10, 0.14, 0.18, 0.12, 0.05, 0.05)$. The initiatives range from solar energy systems and wind energy systems to hydroelectric plants and smart grid infrastructure. Each initiative is scored using the proposed MCDM framework to identify the most suitable choice. Matrix M, containing the evaluation data, is used for comparative analysis, assisting experts in selecting initiatives that will contribute to long-term sustainability and the smart city's development.

$$M =$$

$$\begin{array}{cc} & \begin{array}{cccccccc} \mathsf{L}_1 & \mathsf{L}_2 & \mathsf{L}_3 & \mathsf{L}_4 & \mathsf{L}_5 & \mathsf{L}_6 & \mathsf{L}_7 & \mathsf{L}_8 \end{array} \\ \begin{array}{c} \mathsf{T}_1 \\ \mathsf{T}_2 \\ \mathsf{T}_3 \\ \mathsf{T}_4 \\ \mathsf{T}_5 \\ \mathsf{T}_6 \\ \mathsf{T}_7 \\ \mathsf{T}_8 \end{array} & \begin{pmatrix} (0.6,0.2,0.4;0.07) & (0.5,0.3,0.4;0.10) & (0.6,0.3,0.2;0.25) & (0.7,0.4,0.4;0.30) & (0.6,0.3,0.4;0.20) & (0.5,0.4,0.1;0.30) & (0.4,0.3,0.3;0.20) & (0.3,0.2,0.4;0.15) \\ (0.4,0.2,0.4;0.15) & (0.5,0.4,0.1;0.18) & (0.5,0.3,0.5;0.15) & (0.4,0.3,0.4;0.10) & (0.6,0.3,0.5;0.12) & (0.7,0.3,0.2;0.05) & (0.5,0.3,0.4;0.08) & (0.6,0.4,0.3;0.10) \\ (0.4,0.2,0.4;0.15) & (0.5,0.4,0.1;0.18) & (0.5,0.3,0.5;0.15) & (0.4,0.3,0.4;0.10) & (0.6,0.3,0.5;0.12) & (0.7,0.3,0.2;0.05) & (0.5,0.3,0.4;0.08) & (0.6,0.4,0.3;0.10) \\ (0.4,0.2,0.4;0.08) & (0.6,0.5,0.3;0.28) & (0.5,0.2,0.5;0.08) & (0.6,0.4,0.5;0.12) & (0.3,0.3,0.2;0.20) & (0.6,0.5,0.1;0.14) & (0.7,0.4,0.3;0.18) & (0.5,0.2,0.5;0.10) \\ (0.7,0.3,0.3;0.20) & (0.7,0.5,0.4;0.08) & (0.6,0.5,0.2;0.22) & (0.8,0.3,0.3;0.15) & (0.5,0.3,0.4;0.10) & (0.7,0.4,0.3;0.20) & (0.4,0.5,0.4;0.12) & (0.6,0.3,0.2;0.15) \\ (0.5,0.3,0.3;0.16) & (0.6,0.5,0.5;0.14) & (0.5,0.4,0.5;0.20) & (0.7,0.3,0.2;0.08) & (0.4,0.4,0.4;0.05) & (0.6,0.4,0.5;0.10) & (0.5,0.3,0.3;0.12) & (0.6,0.3,0.4;0.08) \\ (0.5,0.2,0.4;0.12) & (0.6,0.4,0.5;0.16) & (0.4,0.3,0.4;0.18) & (0.7,0.3,0.3;0.25) & (0.5,0.3,0.5;0.20) & (0.6,0.4,0.2;0.18) & (0.4,0.4,0.3;0.14) & (0.5,0.3,0.2;0.16) \\ (0.6,0.3,0.3;0.10) & (0.5,0.3,0.2;0.18) & (0.6,0.3,0.4;0.12) & (0.7,0.2,0.3;0.08) & (0.4,0.4,0.4;0.15) & (0.6,0.5,0.3;0.22) & (0.5,0.3,0.4;0.16) & (0.5,0.3,0.5;0.20) \end{pmatrix} \end{array}.$$

6.2 Implementation

In the present section, we employ the proposed WASPAS approach to the data provided in Matrix M.

Step 1: Since the criteria L_1 and L_8 are cost-related, we normalize the matrix M according to Eq. (31) as follows.

$$M =$$

$$\begin{array}{cc} & \begin{array}{cccccccc} \mathsf{L}_1 & \mathsf{L}_2 & \mathsf{L}_3 & \mathsf{L}_4 & \mathsf{L}_5 & \mathsf{L}_6 & \mathsf{L}_7 & \end{array} \\ \begin{array}{c} \mathsf{T}_1 \\ \mathsf{T}_2 \\ \mathsf{T}_3 \\ \mathsf{T}_4 \\ \mathsf{T}_5 \\ \mathsf{T}_6 \\ \mathsf{T}_7 \end{array} & \begin{pmatrix} (0.4,0.2,0.6;0.07) & (0.5,0.3,0.4;0.10) & (0.6,0.3,0.2;0.25) & (0.7,0.4,0.4;0.30) & (0.6,0.3,0.4;0.20) & (0.5,0.4,0.1;0.30) & (0.4,0.3,0.3;0.20) & (0.4,0.2,0.3;0.15) \\ (0.4,0.2,0.4;0.15) & (0.5,0.4,0.1;0.18) & (0.5,0.3,0.5;0.15) & (0.4,0.3,0.4;0.10) & (0.6,0.3,0.5;0.12) & (0.7,0.3,0.2;0.05) & (0.5,0.3,0.4;0.08) & (0.3,0.4,0.6;0.10) \\ (0.4,0.2,0.4;0.08) & (0.6,0.5,0.3;0.28) & (0.5,0.2,0.5;0.08) & (0.6,0.4,0.5;0.12) & (0.3,0.3,0.2;0.20) & (0.6,0.5,0.1;0.14) & (0.7,0.4,0.3;0.18) & (0.5,0.2,0.5;0.10) \\ (0.3,0.3,0.7;0.20) & (0.7,0.5,0.4;0.08) & (0.6,0.5,0.2;0.22) & (0.8,0.3,0.3;0.15) & (0.5,0.3,0.4;0.10) & (0.7,0.4,0.3;0.20) & (0.4,0.5,0.4;0.12) & (0.2,0.3,0.6;0.15) \\ (0.3,0.3,0.5;0.16) & (0.6,0.5,0.5;0.14) & (0.5,0.4,0.5;0.20) & (0.7,0.3,0.2;0.08) & (0.4,0.4,0.4;0.05) & (0.6,0.4,0.5;0.10) & (0.5,0.3,0.3;0.12) & (0.4,0.3,0.6;0.08) \\ (0.4,0.2,0.5;0.12) & (0.6,0.4,0.5;0.16) & (0.4,0.3,0.4;0.18) & (0.7,0.3,0.3;0.25) & (0.5,0.3,0.5;0.20) & (0.6,0.4,0.2;0.18) & (0.4,0.4,0.3;0.14) & (0.2,0.3,0.5;0.16) \\ (0.3,0.3,0.6;0.10) & (0.5,0.3,0.2;0.18) & (0.6,0.3,0.4;0.12) & (0.7,0.2,0.3;0.08) & (0.4,0.4,0.4;0.15) & (0.6,0.5,0.3;0.22) & (0.5,0.3,0.4;0.16) & (0.5,0.3,0.5;0.20) \end{pmatrix} \end{array}.$$

Step 2: Based on Eq. (32), the support values are calculated as given below:

$$supp(\mathcal{D}_{1j},\mathcal{D}_{1l}) = \begin{pmatrix} -- & 0.8809 & 0.7754 & 0.7391 & 0.8250 & 0.7691 & 0.8683 & 0.8946 \\ 0.8809 & -- & 0.8945 & 0.8582 & 0.9441 & 0.8882 & 0.9274 & 0.9204 \\ 0.7754 & 0.8945 & -- & 0.8837 & 0.9504 & 0.9204 & 0.9070 & 0.8808 \\ 0.7391 & 0.8582 & 0.8837 & -- & 0.9141 & 0.8700 & 0.8241 & 0.7978 \\ 0.8250 & 0.9441 & 0.9504 & 0.9141 & -- & 0.8708 & 0.9100 & 0.8837 \\ 0.7691 & 0.8882 & 0.9204 & 0.8700 & 0.8708 & -- & 0.9008 & 0.8745 \\ 0.8683 & 0.9274 & 0.9070 & 0.8241 & 0.9100 & 0.9008 & -- & 0.9737 \\ 0.8946 & 0.9204 & 0.8808 & 0.7978 & 0.8837 & 0.8745 & 0.9737 & -- \end{pmatrix},$$

$$supp(\mathcal{D}_{2j},\mathcal{D}_{2l}) = \begin{pmatrix} -- & 0.8742 & 0.9233 & 0.9737 & 0.8809 & 0.8141 & 0.9399 & 0.8604 \\ 0.8742 & -- & 0.8909 & 0.8813 & 0.8485 & 0.8616 & 0.9074 & 0.8146 \\ 0.9233 & 0.8909 & -- & 0.9304 & 0.9576 & 0.8308 & 0.9565 & 0.8770 \\ 0.9737 & 0.8813 & 0.9304 & -- & 0.8995 & 0.8404 & 0.9662 & 0.8867 \\ 0.8809 & 0.8485 & 0.9576 & 0.8995 & -- & 0.8732 & 0.9256 & 0.8462 \\ 0.8141 & 0.8616 & 0.8308 & 0.8404 & 0.8732 & -- & 0.8742 & 0.7270 \\ 0.9399 & 0.9074 & 0.9565 & 0.9662 & 0.9256 & 0.8742 & -- & 0.8528 \\ 0.8604 & 0.8146 & 0.8770 & 0.8867 & 0.8462 & 0.7270 & 0.8528 & -- \end{pmatrix},$$

$$supp(\mathcal{D}_{3j}, \mathcal{D}_{3l}) = \begin{pmatrix} -- & 0.8015 & 0.9400 & 0.8556 & 0.8969 & 0.8018 & 0.8074 & 0.9362 \\ 0.8015 & -- & 0.8015 & 0.8859 & 0.8246 & 0.9464 & 0.9074 & 0.8054 \\ 0.9400 & 0.8015 & -- & 0.9156 & 0.8369 & 0.8018 & 0.8074 & 0.9962 \\ 0.8556 & 0.8859 & 0.9156 & -- & 0.8013 & 0.8862 & 0.8918 & 0.9195 \\ 0.8969 & 0.8246 & 0.8369 & 0.8013 & -- & 0.8351 & 0.8228 & 0.8408 \\ 0.8018 & 0.9464 & 0.8018 & 0.8862 & 0.8351 & -- & 0.8923 & 0.8056 \\ 0.8074 & 0.9074 & 0.8074 & 0.8918 & 0.8228 & 0.8923 & -- & 0.8113 \\ 0.9362 & 0.8054 & 0.9962 & 0.9195 & 0.8408 & 0.8056 & 0.8113 & -- \end{pmatrix},$$

$$supp(\mathcal{D}_{4j}, \mathcal{D}_{4l}) = \begin{pmatrix} --\,0.6802 & 0.7028 & 0.6737 & 0.8174 & 0.7100 & 0.7979 & 0.9304 & \\ 0.6802 & -- & 0.8897 & 0.8599 & 0.8628 & 0.9236 & 0.8823 & 0.7165 \\ 0.7028 & 0.8897 & -- & 0.8232 & 0.8469 & 0.9062 & 0.8741 & 0.7199 \\ 0.6737 & 0.8599 & 0.8232 & -- & 0.8370 & 0.9170 & 0.7576 & 0.7100 \\ 0.8174 & 0.8628 & 0.8469 & 0.8370 & -- & 0.8541 & 0.9128 & 0.8537 \\ 0.7100 & 0.9236 & 0.9062 & 0.9170 & 0.8541 & -- & 0.8213 & 0.7270 \\ 0.7979 & 0.8823 & 0.8741 & 0.7576 & 0.9128 & 0.8213 & -- & 0.8342 \\ 0.9304 & 0.7165 & 0.7199 & 0.7100 & 0.8537 & 0.7270 & 0.8342 & -- \end{pmatrix},$$

$$supp(\mathcal{D}_{5j}, \mathcal{D}_{5l}) = \begin{pmatrix} -- & 0.8528 & 0.9156 & 0.7813 & 0.9022 & 0.8751 & 0.8856 & 0.9246 \\ 0.8528 & -- & 0.9218 & 0.8218 & 0.8560 & 0.9623 & 0.8528 & 0.8318 \\ 0.9156 & 0.9218 & -- & 0.8036 & 0.9111 & 0.9441 & 0.9079 & 0.8869 \\ 0.7813 & 0.8218 & 0.8036 & -- & 0.8209 & 0.8595 & 0.8956 & 0.7833 \\ 0.9022 & 0.8560 & 0.9111 & 0.8209 & -- & 0.8937 & 0.9099 & 0.9042 \\ 0.8751 & 0.9623 & 0.9441 & 0.8595 & 0.8937 & -- & 0.8828 & 0.8695 \\ 0.8856 & 0.8528 & 0.9079 & 0.8956 & 0.9099 & 0.8828 & -- & 0.8723 \\ 0.9246 & 0.8318 & 0.8869 & 0.7833 & 0.9042 & 0.8695 & 0.8723 & -- \end{pmatrix},$$

$$supp(\mathcal{D}_{6j}, \mathcal{D}_{6l}) = \begin{pmatrix} -- & 0.8856 & 0.9418 & 0.7950 & 0.9379 & 0.8118 & 0.9028 & 0.9356 \\ 0.8856 & -- & 0.8762 & 0.8627 & 0.9323 & 0.9262 & 0.8762 & 0.8700 \\ 0.9418 & 0.8762 & -- & 0.8532 & 0.9362 & 0.8700 & 0.9456 & 0.9262 \\ 0.7950 & 0.8627 & 0.8532 & -- & 0.8570 & 0.9032 & 0.8455 & 0.7793 \\ 0.9379 & 0.9323 & 0.9362 & 0.8570 & -- & 0.8662 & 0.8818 & 0.9223 \\ 0.8118 & 0.9262 & 0.8700 & 0.9032 & 0.8662 & -- & 0.9090 & 0.7962 \\ 0.9028 & 0.8762 & 0.9456 & 0.8455 & 0.8818 & 0.9090 & -- & 0.8795 \\ 0.9356 & 0.8700 & 0.9262 & 0.7793 & 0.9223 & 0.7962 & 0.8795 & -- \end{pmatrix},$$

$$supp(\mathcal{D}_{7j}, \mathcal{D}_{7l}) = \begin{pmatrix} -- & 0.8246 & 0.8395 & 0.7562 & 0.8770 & 0.7436 & 0.8685 & 0.8908 \\ 0.8246 & -- & 0.9118 & 0.8674 & 0.9009 & 0.8856 & 0.9562 & 0.9262 \\ 0.8395 & 0.9118 & -- & 0.9090 & 0.9042 & 0.9041 & 0.9556 & 0.9179 \\ 0.7562 & 0.8674 & 0.9090 & -- & 0.8132 & 0.8597 & 0.8646 & 0.8269 \\ 0.8770 & 0.9009 & 0.9042 & 0.8132 & -- & 0.8665 & 0.9447 & 0.9070 \\ 0.7436 & 0.8856 & 0.9041 & 0.8597 & 0.8665 & -- & 0.8751 & 0.8528 \\ 0.8685 & 0.9562 & 0.9556 & 0.8646 & 0.9447 & 0.8751 & -- & 0.9623 \\ 0.8908 & 0.9262 & 0.9179 & 0.8269 & 0.9070 & 0.8528 & 0.9623 & -- \end{pmatrix}.$$

Step 3: Using Eqs. (33) and (34), the power weights are calculated and presented in the following matrix.

$$\mathfrak{m} = \left(\mathfrak{m}_{ij}\right)_{7\times 8} = \begin{pmatrix} 0.170 & 0.1852 & 0.1015 & 0.1356 & 0.1848 & 0.1197 & 0.0514 & 0.0508 \\ 0.1820 & 0.1773 & 0.1025 & 0.1437 & 0.1811 & 0.1139 & 0.0516 & 0.0478 \\ 0.1808 & 0.1791 & 0.1013 & 0.1430 & 0.1762 & 0.1193 & 0.0495 & 0.0508 \\ 0.1694 & 0.1828 & 0.1008 & 0.1373 & 0.1874 & 0.1227 & 0.0513 & 0.0484 \\ 0.1804 & 0.1794 & 0.1024 & 0.1330 & 0.1819 & 0.1228 & 0.0506 & 0.0496 \\ 0.1806 & 0.1811 & 0.1023 & 0.1343 & 0.1837 & 0.1183 & 0.0504 & 0.0495 \\ 0.1723 & 0.1842 & 0.1033 & 0.1359 & 0.1827 & 0.1180 & 0.0523 & 0.0513 \end{pmatrix}.$$

Step 4: According to Eq. (35), the weighted sum measures $\mathcal{B}_j^s$ for $j = 1, 2, \ldots, 7$ are determined as follows:

$$\mathcal{B}_1^s = (0.6023, 0.2398, 0.1400; 0.3000), \mathcal{B}_2^s = (0.5881, 0.2449, 0.1315; 0.1800),$$
$$\mathcal{B}_3^s = (0.5796, 0.2301, 0.1399; 0.2800), \mathcal{B}_4^s = (0.7110, 0.3105, 0.2674; 0.2200),$$
$$\mathcal{B}_5^s = (0.6004, 0.3180, 0.2630; 0.2000), \mathcal{B}_6^s = (0.6013, 0.2464, 0.2645; 0.2500),$$
$$\mathcal{B}_7^s = (0.5978, 0.2536, 0.2485; 0.2200).$$

Step 5: Using Eq. (36), the weighted product measures $\mathcal{B}_j^p$ for $j = 1, 2, \ldots, 7$ are determined as follows:

$$\mathcal{B}_1^p = (0.4199, 0.3412, 0.4947; 0.3000), \mathcal{B}_2^p = (0.3851, 0.3379, 0.4668; 0.1800),$$
$$\mathcal{B}_3^p = (0.3538, 0.4344, 0.4336; 0.2800), \mathcal{B}_4^p = (0.2949, 0.4397, 0.5940; 0.2200),$$
$$\mathcal{B}_5^p = (0.3516, 0.4202, 0.4905; 0.2000), \mathcal{B}_6^p = (0.3010, 0.3530, 0.4703; 0.2500),$$
$$\mathcal{B}_7^p = (0.3540, 0.3938, 0.4967; 0.2200).$$

Step 6: Applying Eq. (37) with $£ = 0.5$, the aggregated preference measures $\mathcal{B}_j$ for $j = 1, 2, \ldots, 7$ are calculated as follows:

$$\mathcal{B}_1 = (0.5612, 0.2633, 0.1567; 0.3000), \mathcal{B}_2 = (0.5459, 0.2680, 0.1473; 0.1800),$$
$$\mathcal{B}_3 = (0.5367, 0.2559, 0.1566; 0.2800), \mathcal{B}_4 = ([0.6693, 0.3399, 0.2972; 0.2200),$$
$$\mathcal{B}_5 = (0.5570, 0.3454, 0.2920; 0.2000), \mathcal{B}_6 = (0.5572, 0.2706, 0.2934; 0.2500),$$
$$\mathcal{B}_5 = (0.5545, 0.2800, 0.2764; 0.2200).$$

Step 7: The scores of the aggregated preference measures are calculated as follows:

$$S_A(\mathcal{B}_1) = 0.4091, S_A(\mathcal{B}_2) = 0.3813, S_A(\mathcal{B}_3) = 0.3964, S_A(\mathcal{B}_4) = 0.4390,$$
$$S_A(\mathcal{B}_5) = 0.3906, S_A(\mathcal{B}_6) = 0.3949, S_A(\mathcal{B}_7) = 0.3889.$$

Consequently, the ranking of the projects is: $L_4 > L_1 > L_3 > L_6 > L_5 > L_7 > L_2$.

6.3 Theoretical and Policy Implications

Theoretical Implications: This study offers several significant contributions to the field of fuzzy MCDM. Firstly, it enhances the modeling of uncertainty through the use of t-AF sets, incorporating a radius component for a richer and more flexible representation. Secondly, the development of Dombi-based aggregation operators provides an improved mechanism for addressing interdependencies among criteria, a frequently encountered but often underexplored issue. Furthermore, the integration of the WASPAS method within the disc spherical fuzzy environment introduces a robust and efficient mechanism for ranking alternatives, effectively balancing both additive and multiplicative assessment strategies. Collectively, these innovations advance the methodological foundation for researchers tackling high-dimensional and uncertainty-laden decision problems.

Policy Implications: From a practical standpoint, the proposed t-AF-WASPAS method can assist policymakers and planners in making more informed decisions about renewable energy projects. It offers a clear, adaptable structure for evaluating projects based on technical, environmental, and social factors. Importantly, it also accounts for risks related to policy or market changes—an essential feature in today's evolving energy landscape. By aligning with broader sustainability goals and national development targets, this approach supports strategic, long-term planning at various scales—from local initiatives to national programs.

6.4 Analysis of Sensitivity

This section conducts a detailed sensitivity analysis to evaluate the impact of the parameter p on the final ranking outcomes within the proposed t-AF-WASPAS framework. The analysis is based on the numerical example, where the parameter £ is held constant at 0.5 while varying p over a wide range from 1 to 150.

As shown in Table 3, the ranking scores and corresponding preference orderings remain remarkably stable across different values of p. Notably, the alternative T_4 consistently secures the highest rank throughout all cases, while T_2 remains the least preferred option. This consistency demonstrates the robustness and reliability of the proposed method in producing stable decisions even under significant changes to parameter p.

Table 3: Sensitivity results for different values of p

p	Score values							Ranking
	$S(\mathcal{B}_1)$	$S(\mathcal{B}_2)$	$S(\mathcal{B}_3)$	$S(\mathcal{B}_4)$	$S(B_5)$	$S(\mathcal{B}_6)$	$S(\mathcal{B}_7)$	
1	0.3892	0.3591	0.3789	0.3903	0.3625	0.3691	0.3640	$\mathsf{T}_4 > \mathsf{T}_1 > \mathsf{T}_3 > \mathsf{T}_6 > \mathsf{T}_7 > \mathsf{T}_5 > \mathsf{T}_2$
2	0.3992	0.3698	0.3873	0.4181	0.3764	0.3806	0.3759	$\mathsf{T}_4 > \mathsf{T}_1 > \mathsf{T}_3 > \mathsf{T}_6 > \mathsf{T}_7 > \mathsf{T}_5 > \mathsf{T}_2$
3	0.4091	0.3813	0.3964	0.4390	0.3906	0.3943	0.3889	$\mathsf{T}_4 > \mathsf{T}_1 > \mathsf{T}_3 > \mathsf{T}_6 > \mathsf{T}_5 > \mathsf{T}_7 > \mathsf{T}_2$
5	0.4243	0.3986	0.4105	0.4608	0.4082	0.4117	0.4063	$\mathsf{T}_4 > \mathsf{T}_1 > \mathsf{T}_6 > \mathsf{T}_3 > \mathsf{T}_5 > \mathsf{T}_7 > \mathsf{T}_2$
10	0.4415	0.4172	0.4300	0.4808	0.4261	0.4296	0.4242	$\mathsf{T}_4 > \mathsf{T}_1 > \mathsf{T}_3 > \mathsf{T}_6 > \mathsf{T}_5 > \mathsf{T}_7 > \mathsf{T}_2$
15	0.4482	0.4243	0.4389	0.4879	0.4329	0.4365	0.4309	$\mathsf{T}_4 > \mathsf{T}_1 > \mathsf{T}_3 > \mathsf{T}_6 > \mathsf{T}_5 > \mathsf{T}_7 > \mathsf{T}_2$
25	0.4537	0.4301	0.4465	0.4935	0.4385	0.4421	0.4365	$\mathsf{T}_4 > \mathsf{T}_1 > \mathsf{T}_3 > \mathsf{T}_6 > \mathsf{T}_5 > \mathsf{T}_7 > \mathsf{T}_2$
40	0.4568	0.4334	0.4509	0.4966	0.4417	0.4453	0.4396	$\mathsf{T}_4 > \mathsf{T}_1 > \mathsf{T}_3 > \mathsf{T}_6 > \mathsf{T}_5 > \mathsf{T}_7 > \mathsf{T}_2$
60	0.4585	0.4352	0.4533	0.4984	0.4435	0.4471	0.4414	$\mathsf{T}_4 > \mathsf{T}_1 > \mathsf{T}_3 > \mathsf{T}_6 > \mathsf{T}_5 > \mathsf{T}_7 > \mathsf{T}_2$
100	0.4599	0.4367	0.4552	0.4997	0.4449	0.4485	0.4428	$\mathsf{T}_4 > \mathsf{T}_1 > \mathsf{T}_3 > \mathsf{T}_6 > \mathsf{T}_5 > \mathsf{T}_7 > \mathsf{T}_2$
150	0.4606	0.4374	0.4562	0.5004	0.4456	0.4492	0.4435	$\mathsf{T}_4 > \mathsf{T}_1 > \mathsf{T}_3 > \mathsf{T}_6 > \mathsf{T}_5 > \mathsf{T}_7 > \mathsf{T}_2$

Minor shifts are observed in the middle-ranked alternatives (e.g., T_3, T_5, and T_6), which indicates a moderate sensitivity of the model to p in borderline cases. However, these variations do not affect the overall decision reliability, as the most and least favorable options are invariant. This reinforces the method's capability to support decision-making under uncertainty with dependable outcomes, even when the controlling parameter is adjusted over a broad spectrum.

Next, we conduct the sensitivity with respect to the parameter £, while keeping the Dombi parameter fixed at $p = 3$. Since $£ \in [0, 1]$, eleven equally spaced values were tested, ranging from 0.00 to 1.00 in increments of 0.10. The resulting scores and corresponding rankings are presented in Table 4.

Table 4: Sensitivity results for different values of £

£	Score values							Ranking
	$S(\mathcal{B}_1)$	$S(\mathcal{B}_2)$	$S(\mathcal{B}_3)$	$S(\mathcal{B}_4)$	$S(\mathcal{B}_5)$	$S(\mathcal{B}_6)$	$S(\mathcal{B}_7)$	
0.00	0.3563	0.3256	0.3489	0.3159	0.3281	0.3218	0.3294	$T_1 > T_3 > T_7 > T_5 > T_2 > T_6 > T_4$
0.10	0.3850	0.3565	0.3724	0.4005	0.3614	0.3631	0.3605	$T_4 > T_1 > T_3 > T_6 > T_5 > T_7 > T_2$
0.20	0.3939	0.3658	0.3813	0.4166	0.3730	0.3756	0.3715	$T_4 > T_1 > T_3 > T_6 > T_5 > T_7 > T_2$
0.30	0.4001	0.3722	0.3875	0.4264	0.3805	0.3836	0.3788	$T_4 > T_1 > T_3 > T_6 > T_5 > T_7 > T_2$
0.40	0.4051	0.3772	0.3924	0.4334	0.3861	0.3895	0.3844	$T_4 > T_1 > T_3 > T_6 > T_5 > T_7 > T_2$
0.50	0.4091	0.3813	0.3964	0.4390	0.3906	0.3943	0.3889	$T_4 > T_1 > T_3 > T_6 > T_5 > T_7 > T_2$
0.60	0.4126	0.3847	0.3999	0.4435	0.3943	0.3983	0.3927	$T_4 > T_1 > T_3 > T_6 > T_5 > T_7 > T_2$
0.70	0.4157	0.3878	0.4029	0.4474	0.3976	0.4018	0.3960	$T_4 > T_1 > T_3 > T_6 > T_5 > T_7 > T_2$
0.80	0.4184	0.3905	0.4056	0.4507	0.4005	0.4049	0.3990	$T_4 > T_1 > T_3 > T_6 > T_5 > T_7 > T_2$
0.90	0.4208	0.3929	0.4080	0.4537	0.4031	0.4076	0.4016	$T_4 > T_1 > T_3 > T_6 > T_5 > T_7 > T_2$
1.00	0.4231	0.3951	0.4103	0.4564	0.4054	0.4101	0.4040	$T_4 > T_1 > T_3 > T_6 > T_5 > T_7 > T_2$

From the results, it is observed that when £ = 0.00, the WSM component is entirely excluded from the WASPAS formulation. As a result, the ranking significantly deviates from all other cases, placing T_1 at the top and T_4 at the bottom. However, as soon as £ increases to 0.10, the WSM component begins to influence the results, and the ranking stabilizes.

From £ = 0.10 onwards, a consistent ranking order emerges: T_4 consistently secures the top position, followed by T_1, while T_2 remains the least preferred alternative throughout. This notable consistency from £ = 0.10 to £ = 1.00 highlights the robustness of the proposed method under variations in £, particularly when both WSM and WPM components contribute jointly to the final score. In contrast, a slight deviation is observed at £ = 0.00, where the influence of WSM is absent, resulting in a different ranking pattern. However, the stability across the remaining values confirms the reliability of the proposed t-AF-WASPAS framework in maintaining consistent decision outcomes under changes in the aggregation balance.

7 Comparative Analysis

This section presents a comprehensive evaluation of the proposed methodology by comparing its outcomes with various alternative methods previously reported in the literature [16,35–40]. The outcomes derived are presented in Table 5.

(1). From Table 5, it is evident that the operators CSFSWWA and CSFSWWG proposed by Ashraf et al. [16] identify T_1 as the most suitable alternative. In contrast, the proposed methodology ranks T_4 as the top choice, with T_1 appearing as the second-best option. This discrepancy primarily arises from the underlying score formulation and aggregation mechanisms. While Ashraf et al. [16] employ Sugeno-Weber-based operators within a circular spherical fuzzy environment—where all elements are assigned a fixed radius—the proposed approach utilizes Dombi-operation-based aggregation within the WASPAS framework, which enhances ranking reliability and methodological robustness. Moreover, unlike the circular model, the disc spherical fuzzy set adopted in our method allows each element to have a distinct radius, offering greater flexibility and a more generalizable structure for capturing expert preferences more accurately.

(2). From Table 5, it can be observed that the DSF-AAWA and DSF-AAWG operators proposed by Ahmad et al. [35] identify T_1 as the most suitable alternative. Notably, the DSF-AAWG operator ranks T_4 as the least preferred option, whereas both the proposed WASPAS-based approach and the existing CRADIS method [36] consider T_4 to be the most desirable alternative. This discrepancy stems from the distinct treatment of neutral and radius components in the aggregation processes. Specifically, in the averaging operator, these components are handled similarly to the non-membership degree, while in the geometric operator, they are treated analogous to the membership degree. Ahmad et al. [35] not only introduced these AOs but also extended the measurement of alternatives and ranking according to the compromise solution method. However, this method was not employed in the present study due to the nature of the problem, which falls under the MCDM category rather than a multi-criteria group decision-making (MCGDM) context. Their approach also includes a hybrid technique for determining criteria weights—an advantage over the proposed method, which assumes known or pre-defined weights. On the other hand, the CRADIS approach [36] produces results that are largely consistent with those of the proposed model.

However, a notable limitation in both [35] and [36] lies in their use of the score function defined as $S(\digamma) = \frac{1}{4}\left(\mathsf{N} - M - \bar{M} + \sqrt{2r}(2q-1)\right)$, which exhibits certain shortcomings in effectively distinguishing between distinct DSFNs. For instance, consider two DSFNs, $\digamma_1 = (0.5, 0.3; 0.2)$ and $\digamma_2 = (0.4, 0.24; 0.25)$. When applying the aforementioned score function with $q = 2$, the resulting scores are: $S(\digamma_1) = 0.0064, \quad S(\digamma_2) = 0.0064$, indicating that both alternatives receive identical score values despite their evident differences. Similarly, take $\digamma_3 = (0.6, 0.35; 0.2)$ and $\digamma_4 = (0.5, 0.3; 0.25)$. Using the same score function with $q = 1$, we obtain: $S(\digamma_3) = 0.050, \quad S(\digamma_4) = 0.050$, once again resulting in equivalent scores for distinguishable DSFNs. These examples highlight the inadequacy of the existing score formulation in differentiating between alternatives with varying degrees of membership, non-membership, and hesitancy. In contrast, the proposed WASPAS-based model employs a refined score function, as defined in Eq. (10), which addresses these limitations and facilitates a more robust and reliable decision-making process.

(3). In comparison with the approaches proposed by Ashraf et al. [37] and Revathy et al. [38], the radius and neutral components were omitted, respectively, from the dataset in matrix M to maintain compatibility with these existing models. According to the results, Ashraf et al.'s SFDWA operator ranks T_3 as the best alternative, whereas SFDWG identifies T_1 as the top choice. This inconsistency between the two operators within the same study raises concerns regarding the robustness of their proposed models. Furthermore, neither of these rankings aligns with the outcome of the proposed WASPAS-based approach. These discrepancies primarily arise from the loss of essential information caused by the exclusion of the radius component r. This omission diminishes the decision model's ability to capture the full uncertainty and hesitation expressed by experts. For instance, in the case of the SFDWG operator, both T_5 and T_6 receive identical ranking scores, suggesting a reduced capability to distinguish between closely competing alternatives. Similarly, the method introduced by Revathy et al. [38] designates T_1 as the most suitable option, which also diverges from the ranking derived from the proposed approach. The primary reason for these inconsistencies lies in the exclusion of critical information—Ashraf et al. [37] disregard the radius parameter, while Revathy et al. [38] omit neutral grades. Such omissions compromise the comprehensiveness and precision of the final rankings. In contrast, the proposed methodology preserves all relevant components, thereby offering a more accurate and reliable decision-making framework.

(4). From Table 5, it is evident that T_3 and T_4 are identified as the most suitable alternatives by the existing methods proposed in [39] and [40], respectively. Notably, the best alternative obtained by Debnath and Roy [40] aligns with the proposed WASPAS-based approach, thereby supporting the validity and reliability of the developed methodology. Both approaches utilize T-spherical fuzzy data, yet the proposed method provides an enhanced framework by incorporating an additional radius parameter r, enabling better handling of higher levels of uncertainty. In contrast, the method presented in [39] does not account for the radius component r or neutral grades in its input structure. However, it employs a generalized aggregation mechanism capable of capturing interactions between pairs of criteria—an advantage over the proposed method, which assumes criteria independence. This interaction modeling can significantly enhance decision-making in scenarios where interdependencies among criteria are critical. Similarly, the method introduced in [40] offers a unique advantage through the partitioning of criteria, which can be particularly beneficial in problems requiring distinct treatment of criteria groups. Moreover, it effectively handles neutrality through the use of neutral operations, offering further interpretative flexibility.

Table 5: Comparative analysis results

AOs	Final ranking values	Ranking
(R_1)–CSFSWWA [16]	−0.1240, −0.1624, −0.1470, −0.1525, −0.1892, −0.1474, −0.1589	$T_1 > T_3 > T_6 > T_4 > T_7 > T_2 > T_5$
(R_2)–CSFSWWG [16]	0.0999, 0.0644, 0.0815, 0.0554, 0.0335, 0.0765, 0.0661	$T_1 > T_3 > T_6 > T_7 > T_2 > T_4 > T_5$
(R_3)–DSF-AAWA [35]	0.1443, 0.1298, 0.1358, 0.1251, 0.0757, 0.1288, 0.1182	$T_1 > T_3 > T_2 > T_6 > T_4 > T_7 > T_5$
(R_4)–DSF-AAWG [35]	0.0642, 0.0453, 0.0572, 0.0025, 0.0079, 0.0636, 0.0398	$T_1 > T_6 > T_3 > T_2 > T_7 > T_5 > T_4$
(R_5)–DSF-CRADIS [36]	0.2810, 0.2535, 0.2678, 0.3125, 0.1984, 0.2290, 0.1769	$T_4 > T_1 > T_3 > T_2 > T_6 > T_5 > T_7$
(R_6)–SFDWA [37]	0.8971, 0.8992, 0.9066, 0.8475, 0.8412, 0.9025, 0.8978	$T_3 > T_6 > T_2 > T_7 > T_1 > T_4 > T_5$
(R_7)–SFDWG [37]	0.4107, 0.3823, 0.3737, 0.3138, 0.3414, 0.3414, 0.3697	$T_1 > T_2 > T_3 > T_7 > T_5 = T_6 > T_4$
(R_8)–CFFWA [38]	0.1988, 0.1755, 0.1915, 0.1912, 0.1463, 0.1837, 0.1758	$T_1 > T_3 > T_4 > T_6 > T_7 > T_2 > T_5$
(R_9)–CFFWG [38]	0.1660, 0.1388, 0.1573, 0.1399, 0.1194, 0.1583, 0.1439	$T_1 > T_6 > T_3 > T_7 > T_4 > T_2 > T_5$
(R_{10})–AAGWBM [39]	0.2350, 0.2135, 0.2610, 0.1920, 0.1370, 0.1190, 0.1615	$T_3 > T_1 > T_2 > T_4 > T_7 > T_5 > T_6$
(R_{11})–TSFWPPNA [40]	0.3310, 0.2530, 0.3480, 0.3650, 0.2210, 0.3120, 0.2780	$T_4 > T_3 > T_1 > T_6 > T_7 > T_2 > T_5$
(R_{12})–Proposed approach	0.4091, 0.3813, 0.3964, 0.4390, 0.3906, 0.3949, 0.3889	$T_4 > T_1 > T_3 > T_6 > T_5 > T_7 > T_2$

Note: CSFSWWA: Circular spherical fuzzy Sugeno Weber weighted averaging, CSFSWWG: Circular spherical fuzzy Sugeno Weber weighted geometric, DSF-AAWG: DSF-Aczel-Alsina weighted geometric, DSF-AAWA: DSF-Aczel-Alsina weighted average, DSF-CRADIS: Disc spherical fuzzy-CRADIS, SFDWG: Spherical fuzzy Dombi weighted geometric, SFDWA: Spherical fuzzy Dombi weighted averaging, CFFWG: Circular Fermatean fuzzy weighted geometric, CFFWA: Circular Fermatean fuzzy weighted average, AAGWBM: Aczel-Alsina generalized weighted Bonferroni mean, TSFWPPNA: T-spherical fuzzy power partitioned neutral average.

Moreover, Table 6 presents a detailed comparative overview of the proposed approach and the existing methods, highlighting the key characteristics, advantages, and limitations of each.

While the proposed WASPAS-based model offers improved decision-making performance in uncertain environments, several limitations should be acknowledged. (i) The current framework is designed for MCDM problems and may not extend directly to MCGDM settings without further adaptation. (ii) Its reliance on fuzzy data—particularly membership, non-membership, neutrality, and the radius parameter—limits its applicability in crisp decision environments where such inputs are unavailable. (iii) The method treats input criteria as independent and lacks the ability to model interrelationships among them, unlike techniques that use Maclaurin symmetric means or other dependency-aware aggregators. (iv) It also does not support partitioned input structures, which restricts its suitability for hierarchical or thematically grouped criteria. (v) The model's effectiveness depends on predefined weight values, making it sensitive to subjective or imprecise weight assignments. In contrast, hybrid or data-driven weighting strategies offer greater flexibility. (vi) Lastly, although the method performs well for moderate problem sizes, handling very large decision matrices may still pose computational challenges compared to more scalable approaches like CRADIS.

Table 6: Characteristic comparison of employed operators

Operator (with reference)	Ability to consider neutral character	Ability to consider interrelationship of input arguments	Capability to consider radius	Ability to consider partitioned input arguments	Flexibility of decision information	Facility to diminish negative effect
CSFSWWA [16]	✓	✗	✓	✗	✗	✗
CSFSWWG [16]	✓	✗	✓	✗	✗	✗
DSF-AAWA [35]	✓	✗	✓	✗	✗	✗
DSF-AAWG [35]	✓	✗	✓	✗	✗	✗
DSF-CRADIS [36]	✓	✗	✓	✗	✗	✗
SFDWA [37]	✓	✗	✗	✗	✗	✗
SFDWG [37]	✓	✗	✗	✗	✗	✗
CFFWA [38]	✗	✗	✓	✗	✗	✗
CFFWG [38]	✗	✗	✓	✗	✗	✗
AAGWBM [39]	✗	✓	✗	✗	✓	✗
TSFWPPNA [40]	✓	✓	✗	✓	✓	✓
Proposed approach	✓	✗	✓	✗	✓	✓

To overcome the aforementioned limitations, future developments of the framework could focus on incorporating inter-attribute dependency modeling, introducing hybrid weighting mechanisms, enhancing computational scalability for high-dimensional decision problems, and conducting broader real-world validations to improve its generalizability and practical relevance.

In order to statistically validate the robustness of the proposed method in comparison with the existing approach, we apply the Spearman correlation test on the ranking results presented in Table 6. The obtained correlation coefficients between different ranking vectors are graphically illustrated in the heatmap shown in Fig. 2.

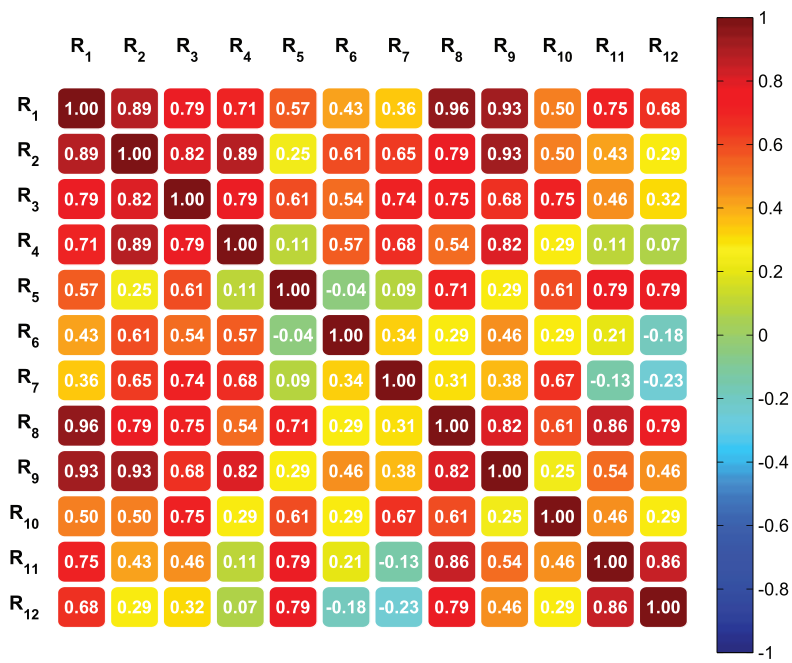

Figure 2: Spearman correlation between rankings

The results, visualized in the heatmap in Fig. 2, reveal that the proposed method (R_{12}) exhibits positive correlation with nine out of eleven existing methods, with coefficients ranging from 0.29 to 0.86, indicating a general agreement in ranking trends. The highest correlations are observed with R_5 (0.79), R_8 (0.79), and R_{11} (0.86), suggesting a strong similarity in ranking results with these methods. However, two methods (R_6 and R_7) show negative correlations (−0.18 and −0.23, respectively), highlighting noticeable differences in the prioritization outcomes. These deviations can be attributed to the exclusion of the radius component r in the traditional methods, which plays a crucial role in uncertainty modeling and has already been discussed in detail in the comparative analysis section. Overall, the predominantly positive correlations reinforce the robustness and compatibility of the proposed method with existing approaches, while the discrepancies further emphasize the added value of incorporating the r parameter in the decision-making process.

8 Conclusions

This study introduced a novel decision-making methodology based on the Dombi operational framework under the t-AF environment. A family of four new aggregation operators—t-AFDPA, t-AFDPWA, t-AFDPG, and t-AFDPWG—was developed by incorporating power weights to effectively capture interrelationships among input arguments while minimizing the influence of extreme values. These operators demonstrate an enhanced capacity to handle complex decision information by integrating support degrees among inputs. A mathematical analysis confirmed the desirable properties of the proposed operators, ensuring their consistency, monotonicity, and idempotency. To operationalize the framework, the WASPAS method was modified to accommodate t-AF data and employed in a realistic decision-making scenario related to smart city initiative prioritization. The results showcased the practicality and effectiveness of the approach, particularly in capturing the nuanced nature of uncertainty and vagueness present in complex environments. A detailed sensitivity analysis, as illustrated in Tables 5 and 6, revealed that the model exhibits strong robustness against variations in both critical parameters—p and £—reinforcing the reliability of the proposed method across different conditions. Additionally, a comparative analysis against several existing MCDM techniques underscored the advantages of the proposed approach. The optimal alternative identified by the model was T_4, which differs from the outcomes of most existing methods. This deviation is primarily attributed to the neglect of certain influencing factors by prior models and the adoption of distinct theoretical underpinnings in their aggregation logic. This further emphasizes the proposed method's capability to deliver more context-sensitive and reliable outcomes in uncertain environments.

Future research could focus on extending the proposed t-AF-based aggregation framework to other advanced decision-making domains, particularly those involving machine learning-driven models, graph theory-based approaches and three-way decision models, where the presence of uncertainty and imprecision is a critical challenge. Additionally, the integration of this framework with hybrid MCDM techniques and confidence-level-based decision strategies could significantly enhance its flexibility and practical applicability in complex, multi-criteria environments.

Acknowledgement: Not applicable.

Funding Statement: The authors received no specific funding for this study.

Author Contributions: Conceptualization, Jawad Ali; methodology, Ioan-Lucian Popa; software, Jawad Ali; validation, Ioan-Lucian Popa; formal analysis, Jawad Ali; writing—original draft preparation, Jawad Ali and Ioan-Lucian Popa; writing—review and editing, Jawad Ali and Ioan-Lucian Popa. All authors reviewed the results and approved the final version of the manuscript.

Availability of Data and Materials: Our manuscript has no associated data.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest to report regarding the present study.

Abbreviations

MCDM	Multi-criteria decision-making
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
PyFS	Pythagorean fuzzy set
PFS	Picture fuzzy set
SFS	Spherical fuzzy set
DSFS	Disc spherical fuzzy set
t-AFS	t-Arbicular fuzzy set
AOs	Aggregation operators
t-AFN	t-Arbicular fuzzy number
WASPAS	Weighted aggregated sum product assessment
PA	Power average
DTN	Dombi t-norm
DTCN	Dombi t-conorm
PG	Power geometric
SFN	Spherical fuzzy number
WPA	Weighted power average
WPG	Weighted power geometric
SFN	Spherical fuzzy number
DSFN	Disc spherical fuzzy number
WSM	Weighted sum measure
WPM	Weighted product measure
DSFDPA	Disc spherical fuzzy Dombi power average
MCGDM	Multi-criteria group decision-making
DSFDPWA	Disc spherical fuzzy Dombi power weighted geometric
DSFDPG	Disc spherical fuzzy Dombi power geometric
DSFDPWG	Disc spherical fuzzy Dombi power weighted average
DSFN	Disc spherical fuzzy number

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