Research on Instability Mechanism and Type of Ore Pillar based on the Fold Catastrophe Theory

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Abstract: The stability of ore pillar in mine is essential for the safe and efficient mining. Based on the energy evolvement rule in ore pillar and roadway roof system, the roadway roof and ore pillar are treated as energy release body and energy dissipation body, respectively. Therefore, the double-block mechanical model is established with energy dissipation body and energy release body, and the energy mechanism of ore pillar instability is obtained, based on the fold catastrophe mathematical theory. The research result indicates that the dynamic instability of ore pillar is a physical instability problem caused by the strain softening property of ore mass, and the instability type of ore pillar is determined by the stiffness parameter of double-block mechanical system. When the systematical stiffness parameter is greater than or equal to 1, the equilibrium position of double-block mechanical system passes through shaft K-1 or the origin, from branch 1 of unstable equilibrium state to branch 2 of stable equilibrium state by smooth transition, and the ore pillar shows quasi-static fracture. When the systematical stiffness parameter is less than 1, the equilibrium position of double-block system mutates from branch 1 of unstable equilibrium state to branch 2 of stable equilibrium state by jumping transition, and the ore pillar shows dynamic instability. The research result could provide theoretical guidance for the prevention measures of ore pillar instability.

Keywords: Ore pillar, instability mechanism, double-block mechanical model, systematical stiffness, fold catastrophe theory.

1 Introduction

In metal mining industry and coal mining industry, the long-time and extensive underground mining generate large numbers of cavities, which poses serious threat to the safe mining [Wang (2006); Wang and Li (2010); Ghasemi, Ataei, and Shahriar, K. (2014); Luo, Peng, Su, and Wang (2015); Xin and Ji (2016)]. Ore pillars are key structural columns commonly

employed in underground mining to provide temporary or permanent support for the weight of overburden roof and maintain the stability and safety of mining area [Pan and Wang (2004); Luo, Yang, Tao, and Zeng (2010); Xue, Gao, and Liu (2015); Guo and Xu

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(2016); Jia, Qiao, and Jiang (2016)]. As mining work goes deeper and deeper, dynamic disaster caused by pillar instability becomes more and more frequent and critical due to the remarkable increase in ambient stresses [Guo, Deng, and Zou (2005); He, Li, Zhai, and Tang (2007); Wattimena, Kramadibrata, Sidi, and Azizi (2013); Cao and Zhou (2015); Yi and Cui (2015)]. Therefore, the systematic in-depth research about ore pillar stability is of vital significance to the efficient and safe mining.

Advances in ore pillar stability over the last twenty years have seen various research methods and results [Li, Nan, Zhao, Yang, Tang, Zhang, and Tang (2005); Shabanimashcool and Li (2013); Cao, Zhou, Xu, and Li (2014); Zhang, Stead, and Elmo (2015); Xue, Ranjith, Gao, Zhang, Cheng, Chong, and Hou (2017)]. Based on the mining with pillar supporting in the mined-out area, Wang, Li, and Ma (2010) established a rheological mechanical model of pillars and roof plate system, and analyzed the fracture stages in roof stratum regarding to the time-dependent behavior. Chen, Gu, Zhou, and Su (2012) established the synergistic effect mechanical model of roof-artificial pillar and derived the relationship expression of compression amount of artificial pillar; besides, based on the catastrophe theory, the cusp catastrophe model of artificial pillar destruction was constructed and mechanical judgment expression under the necessary and sufficient condition of instability was derived. In order to design the geometric parameters of artificial pillar for deep mining, Wang, Feng, Yang, Zhao, and Zhao (2012) developed a new calculation formula of artificial pillar reasonable width, which clarifies the bearing mechanism of artificial pillar. Yin, Wu, and Li (2012) adopted the orthogonal polar difference method to evaluate the sensitivity of the factors influencing the ore pillar stability, and put forward the main factors influencing ore pillar stability, by analyzing the ore pillar load，strength，instability potential function，deformation model. Cao, Cao, and Jiang (2014) believed that the stability of section coal pillar is the key to safety of irregular blocks coal mining, and adopted the catastrophe theory to analyze the catastrophe instability mechanism of section coal pillars. Based on the local pillars in Dayingezhuang gold mine, Song, Cao, Fu, Jiang, and Wu (2014) derived the pillar safety factor formulas of different forms, and employed six factors-five levels technology to design the experiment to analyze sensitivity of impact factors. Yang, Xing, Zhang, and Ma (2015) investigated the long-term stability of gypsum pillars in Luneng Taishan Gypsum Mine with laboratory experiments, numerical analysis and in-situ monitoring, and the creep properties of the gypsum is described by the previous modified Burgers model. Wang, Yang, Yan, and Daemen (2015) employed FLAC 3D simulator to investigate factors affecting the allowable width for pillars between two adjacent caverns, and optimized the allowable width of the pillars by discussing the vertical stress, deformation, plastic zone, safety factors, and seepage pressure of pillars between two adjacent salt caverns. Zhao, Yan, Feng, Wang, Zhang, and Zhao (2016) established the simplified mechanical model through analyzing the mechanical properties and failure mode of the artificial pillar, and derived the equation describing total energy of artificial pillar from the energy conservation law.

In consideration of the significance of ore pillar stability in safe and efficient extraction of underground ores, the instability mechanism and types of ore pillar in mines should be investigated and researched deeply and systematically [Verma, Porathur, Thote, Roy, and Karekal (2014); Xie, Ju, and Li (2005); Gao (2014); Yang, Liu, and Yu (2014)]. In this

paper, the mechanical model of double-block system is established with energy release body (roadway roof) and energy dissipation body (ore pillar), and the energy equilibrium relationship in double-block system is analyzed. Based on the fold catastrophe theory, the instability mechanism of ore pillar is clarified and the instability type of ore pillar is obtained, by deriving the equilibrium equation of double-block mechanical system.

2 Energy mechanism of dynamic instability of ore pillar

2.1 The double-block mechanical model

In the interaction process of ore pillar and roadway roof, the roadway roof is in elastic state in the whole process, which is regarded as the elastic body, therefore, the mechanical model of roadway roof is a spring with the stiffness k_n, whose load-displacement curve is a straight line with the slope $k_n = \tan \alpha$. See Fig. 1.
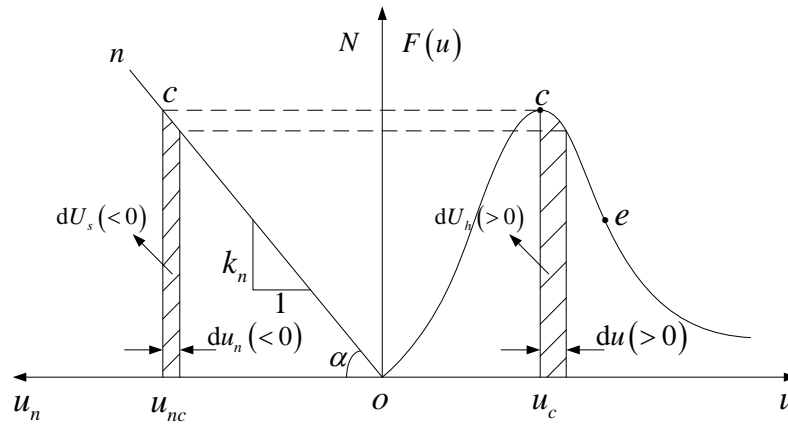

Figure 1: The load-displacement curve of ore pillar and roadway roof

The ore mass exhibits the strain softening property in uniaxial compression test, and the microelement of ore mass satisfies the Weibull distribution rule [Pan and Wang (2004)], so the load-displacement curve of ore mass is shown as a smooth curve with strain softening stage and an inflexion point e, see Fig. 2. According to the statistical damage theory, the load-displacement relationship of ore mass is shown in equation (1),

$$F(u) = \lambda u \exp\left[-\left(\frac{u}{u_c}\right)^m\right]$$ (1)

Where λ refers to the initial stiffness of ore mass, u_c refers to the peak displacement, and m refers to the homogeneous index.

The analytic expressions of each order derivative is

$$\begin{cases} F(u) = \lambda u \exp\left[-\left(\dfrac{u}{u_0}\right)^m \right] \\[2em] F'(u) = \lambda \left[1 - m\left(\dfrac{u}{u_0}\right)^m \right] \exp\left[-\left(\dfrac{u}{u_0}\right)^m \right] \\[2em] F''(u) = \dfrac{\lambda m}{u_0}\left[m\left(\dfrac{u}{u_0}\right)^{2m-1} - (1+m)\left(\dfrac{u}{u_0}\right)^{m-1} \right] \exp\left[-\left(\dfrac{u}{u_0}\right)^m \right] \\[2em] F'''(u) = \dfrac{\lambda m}{u_0{}^2}\left(\dfrac{u}{u_0}\right)^{m-2} \exp\left[-\left(\dfrac{u}{u_0}\right)^m \right]\left\{ -m^2\left(\dfrac{u}{u_0}\right)^{2m} + 3m^2\left(\dfrac{u}{u_0}\right)^m - (m+1)(m-1) \right\} \end{cases} \quad (2)$$

The displacement of inflexion point e in function $F(u)$ is denote as u_e, which could be deduced by $F''(u_e) = 0$, then

$$F''(u_e) = \frac{\lambda m}{u_c}\left[m\left(\frac{u_e}{u_c}\right)^{2m-1} - (1+m)\left(\frac{u_e}{u_c}\right)^{m-1} \right]\exp\left[-\left(\frac{u_e}{u_c}\right)^m \right] = 0 \quad (3)$$

Thus, the relationship of u_e and u_c is obtained in equation (4),

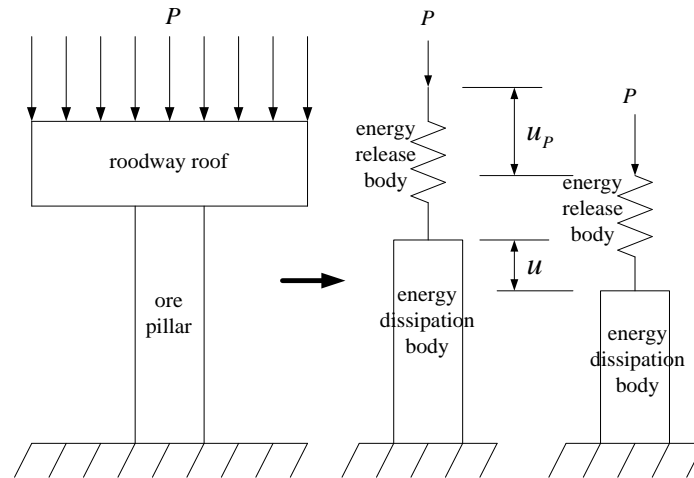$$\frac{u_e}{u_c} = \left(\frac{1+m}{m}\right)^{\frac{1}{m}} \quad (4)$$

Figure 2: The mechanical model of ore pillar and roadway roof system

Based on this basis, the mechanical model of ore pillar and roadway roof system is established in Fig. 2. The force P represents the overburden load applied to the mechanical system of ore pillar and roadway roof, and the displacement of location

applied by force P is donated as u_P; the displacement and internal force of roadway roof is donated as u_n and N, respectively; the displacement and internal force of ore pillar is donated as u and $F(u)$, respectively.

According to the load-displacement curve of mechanical model, as shown in Fig. 1, the area of curved triangle $ou_c c$ in the right side of vertical axis represents the total energy U_h stored in ore pillar when the displacement is u_c, while the area of triangle $ou_n c$ in the left side of vertical axis represents the total energy U_s stored in roadway roof.

When the mechanical system enters the strain softening stage and the ore pillar has the quasi-static displacement increment $du(>0)$, the plastic deformation of ore pillar dissipates the energy $dU_h = F(u)du(>0)$; meanwhile, the corresponding unloading displacement increment of roadway roof is $du_n(<0)$, and the elastic energy released by roadway roof is $dU_s = Ndu_n(<0)$. If the inequality $-du_n < du$ is true, $-Ndu_n < F(u)du$ or $-dU_s < dU_h$ could be obtained, which indicates that the elastic energy released by roadway roof is less than the plastic energy dissipated by ore pillar in strain softening stage. Thus, the compensated work $dW = Pdu_P(>0)$ should be applied by force P, in order to keep the quasi-static movement of ore pillar.

According to the energy evolution process in mechanical system of ore pillar and roadway roof, the roadway roof and ore pillar are regarded as energy release body and energy dissipation body, respectively. The energy release body is the body that releases elastic strain energy; while the energy dissipation body is the body that absorbs elastic strain energy. Thus, the dynamic destabilization of ore pillar is regarded as double-block mechanical problem.

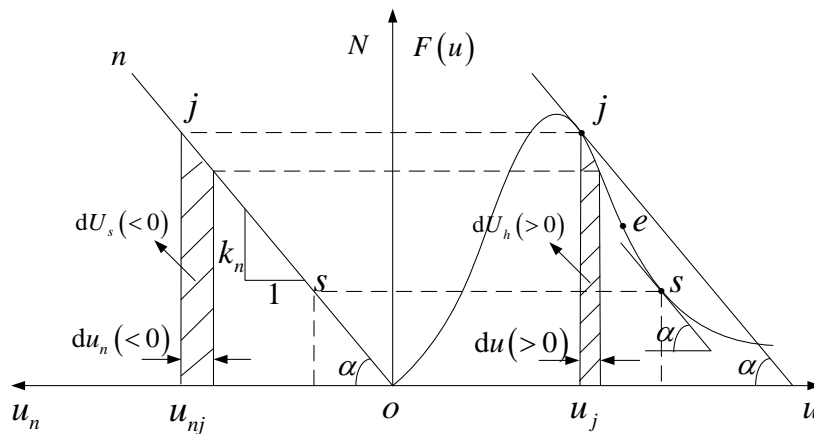### 2.2 The dynamic destabilization mechanism of double-block mechanical model

Figure 3: The energy change relationship of double-block mechanical system

The energy change relationship of double-block mechanical system is shown in Fig. 3, the initial point and terminal point of dynamic destabilization of energy dissipation body is point j and point s, respectively. The energy increment dU_s released by energy release body is larger than the energy increment dU_h dissipated by energy dissipation body from point j to point s, and the excess energy will be transformed into kinetic energy of the mechanical system, leading to the dynamic destabilization of energy dissipation body.
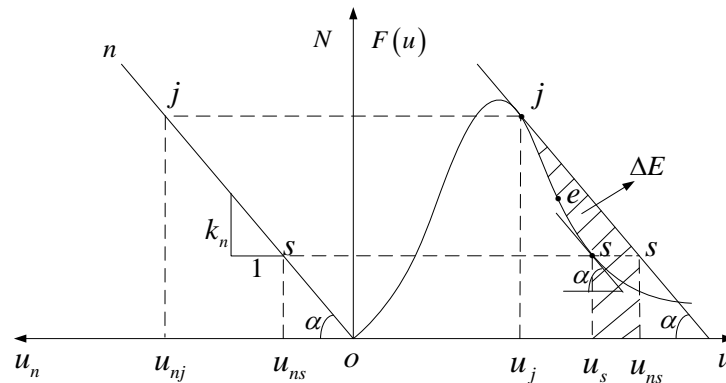

Figure 4: The elastic energy releasing amount of double-block mechanical system

The elastic energy releasing amount of mechanical system is shown in Fig. 4, the trapezoidal area $ju_{nj}u_{ns}sj$ refers to the elastic energy released by energy release body from point j to point s in the left side of vertical axis, while the curved trapezoidal area ju_ju_ssj refers to the plastic energy dissipated by energy dissipation body from point j to point s in the right side of vertical axis. If the trapezoidal ares $ju_{nj}u_{ns}sj$ is shifted to point j to the right side of vertical axis and the curved trapezoidal area ju_ju_ssj is substracted by the trapezoidal ares $ju_{nj}u_{ns}sj$, the shade area in Fig. 4 refers to the excess energy, i.e., the kinetic energy ΔE of the mechanical system.

The energy mechnism of dynamic destabilization of energy dissipation body is summarized as follows: the energy dissipation body is under the clamp action of energy release body, and the deformation of energy release body is consistent with that of energy dissipation body to keep the deformation coordination of mechnical system, with elastic deformation energy stored in mechanical system; when the load on energy dissipation body reaches peak stress, then energy dissipation body enters the strain softening stage, meanwhile, the energy release body bounces and unloads caused by the sharp fall of bearing capacity of ore pillar; if the elastic energy released by energy release body is larger than the plastic energy dissipated by energy dissipation body in quasi-static movement, the excess energy will be transformed as kinetic energy of mechanical system, leading to the dynamic destabilization of energy dissipation body. Therefore, the dynamic

destabilization of ore pillar is a physical instability problem caused by the strain softening property of ore mass.

3 Dynamic destabilization of ore pillar based on fold catastrophe model

3.1 Energy equilibrium relationship in double-block mechanical system

The total potential energy of double-block mechanical system consists of elastic deformation energy of ore pillar (energy dissipation body), elastic deformation energy of roadway roof (energy) and potential energy of external force, and the expression is shown in equation (5),

$$\Pi = \frac{1}{2}k_n u_n^2 + \int_0^u F(u)du - \int_0^{u_P} P(u_P)du_P \tag{5}$$

Where u_n and u_P are both the function of ore pillar deformation u; according to the deformation compatibility of mechanical system, the equation $u_P = u_n + u$ is true. Taking the derivative of equation (5) with respect to u, the equilibrium equation of ore pillar deformation is obtained in equation (6),

$$\frac{d\Pi}{du} = k_n u_n \frac{du_n}{du} + F(u) - P(u_P)\frac{du_P}{du} \tag{6}$$

According to the relationship between action and reaction in mechanical system of ore pillar and roadway roof, namely the Newton's third law, the equation (7) can be deduced,

$$P = N = k_n u_n = F(u) \tag{7}$$

Combining the equation (6) with equation (7), the equilibrium equation of ore pillar in quasi-static movement can be obtained,

$$\frac{d\Pi}{du} = k_n u_n \frac{dN}{k_n du} + F(u) - P\frac{du_P}{du} = F(u)\frac{F'(u)}{k_n} + F(u) - P\frac{du_P}{du} = 0 \tag{8}$$

Equation (8) is the work and energy equilibrium relationship of mechanical system when the ore pillar is in strain softening stage, which indicates the basic rule of energy accumulation and evolution. The physical meaning of each term in equation (8) is analyzed as follows:

(1) The first term on the left of equal sign

$$F(u)\frac{F'(u)}{k_n} = k_n u_n \frac{dN}{k_n du} = N\frac{du_n}{du} = \frac{dU_s}{du} \tag{9}$$

$dU_s = Ndu_n$ in equation (9) represents the elastic energy released by the roadway roof in unloading process, when the displacement increment of roadway roof is du_n.

(2) The second term on the left of equal sign

$$F(u) = \frac{F(u)du}{du} = \frac{dU_h}{du} \tag{10}$$

$dU_h = F(u)du$ in equation (10) represents the plastic energy dissipated by the ore pillar, when the displacement increment of ore pillar is du.

(3) The third term on the left of equal sign

$$P(u_P)\frac{du_P}{du} = \frac{dW}{du}$$ (11)

$dW = Pdu_P$ in equation (11) represents the work applied to mechanical system by external force P, when the elastic energy released by roadway roof is less than the plastic energy dissipated by ore pillar in mechanical system to keep the quasi-static movement of ore pillar. If we donate J as $\dfrac{dW}{du}$, which is the energy input by external force P, entitled "the ratio of energy input", the equation (8) could be rewritten as follows:

$$F(u)\frac{F'(u)}{k_n} + F(u) - J = 0$$ (12)

If $J = 0$ in equation (12), ore pillar deformation u increases automatically only by the elastic energy released by roadway roof without the work applied by the external force P, which indicates that the mechanical system is in critical state. Thus, the critical destabilization condition of double-block mechanical system is

$$J = \frac{Pdu_P}{du} = \frac{Pd(u_n + u)}{du} = \frac{P}{k_n}\big[F'(u) + k_n\big] = 0$$ (13)

3. 2Fold catastrophe model of dynamic destabilization of ore pillar

In order to analyze the dynamic destabilization of ore pillar, the Taylor series expansion is performed at the inflexion point e of strain softening curve in equation (12),

$$\left(\frac{u-u_e}{u_e}\right)^2 + 2\left(\frac{u-u_e}{u_e}\right)\frac{(1-K)[F'(u_e)]^2}{u_e F(u_e)F'''(u_e)} + \frac{2(1-K)F'(u_e)}{u_e^2 F'''(u_e)} - \frac{2k_n J}{u_e^2 F(u_e)F'''(u_e)} + o\left((u-u_e)^2\right) = 0$$ (14)

In equation (14), $K = -\dfrac{k_n}{F'(u_e)}$ is the stiffness of mechanical system, which refers to the ratio of roadway roof stiffness k_n to the absolute value of the gradient at the inflexion point on the load-displacement curve of ore pillar.

According to the determined principle in catastrophe theory, the quadratic term of $(u-u_e)$ is the highest term in equation (14), of which coefficient is not null, thus the equation (14) corresponds to the equilibrium equation of fold catastrophe model. Therefore, it can be employed to discuss the stability of ore pillar without considering the terms $(u-u_e)$ of higher than cubic term. Equation (14) can be expressed in equation (15):

$$\left\{\frac{u-u_e}{u_e}+\frac{(1-K)[F'(u_e)]^2}{u_e F(u_e)F'''(u_e)}\right\}^2-\left\{\frac{(1-K)[F'(u_e)]^2}{u_e F(u_e)F'''(u_e)}\right\}^2+\frac{2(1-K)F'(u_e)}{u_e^2 F'''(u_e)}-\frac{2k_n J}{u_e^2 F(u_e)F'''(u_e)}=0 \quad (15)$$

Substituting equation (1) into equation (15), the equilibrium equation of mechanical system, in which the ore mass stratifies Weibull distribution rule, is obtained in equation (16),

$$\left[\frac{u-u_e}{u_e}+\frac{m(1-K)}{(1+m)^2}\right]^2-\left[\frac{m(1-K)}{(1+m)^2}\right]^2-\frac{2(1-K)}{(1+m)^2}-\frac{2KJ}{F(u_e)(1+m)^2}=0 \quad (16)$$

K is the stiffness of mechanical system, which refers to the ratio of roadway roof stiffness to the absolute value of the gradient at the inflexion point on the load-displacement curve of ore pillar. In equation (16), $K=-\dfrac{k_n}{F'(u_e)}=\dfrac{k_n}{\lambda m\exp\left(-\dfrac{1+m}{m}\right)}$.

Substituting the variable as follows:

$$\begin{cases} x=\dfrac{u-u_e}{u_e}+\dfrac{m(1-K)}{(1+m)^2}\\[3mm] a=-\left[\dfrac{m(1-K)}{(1+m)^2}\right]^2-\dfrac{2(1-K)}{(1+m)^2}-\dfrac{2KJ}{F(u_e)(1+m)^2}\end{cases} \quad (17)$$

Equation (16) can be expressed as

$$x^2+a=0 \quad (18)$$

Thus, the state variable x is obtained in equation (19),

$$\begin{cases} x_1=\dfrac{u-u_e}{u_e}+\dfrac{m(1-K)}{(1+m)^2}=-\sqrt{-a}=-\sqrt{\left[\dfrac{m(1-K)}{(1+m)^2}\right]^2+\dfrac{2(1-K)}{(1+m)^2}+\dfrac{2KJ}{F(u_e)(1+m)^2}}\\[4mm] x_2=\dfrac{u-u_e}{u_e}+\dfrac{m(1-K)}{(1+m)^2}=\sqrt{-a}=\sqrt{\left[\dfrac{m(1-K)}{(1+m)^2}\right]^2+\dfrac{2(1-K)}{(1+m)^2}+\dfrac{2KJ}{F(u_e)(1+m)^2}}\end{cases} \quad (19)$$

The symbol u_* is donated as the value of variable u when the equation $x=0$ is true in equation (17), i.e., $\dfrac{u_*-u_e}{u_e}+\dfrac{m(1-K)}{(1+m)^2}=0$, then

$$u_*=\left[1-\frac{m(1-K)}{(1+m)^2}\right]u_e \quad (20)$$

The equation (18) is the regularization form of equilibrium equation of fold catastrophe model, in which x is the state variable, a is the control variable. It is clear that the fold catastrophe satisfies the features of ore pillar instability. The system is in dummy

status if $a > 0$, while the figure of equation (18) is a parabola if $a \le 0$; the straight line $a = 0$ (or the axis $K - 1$) divides the parabola into upper and lower branches, just as Fig. 5 shown. x_1 and x_2 represent the expression of state variable x on branch 1 and branch 2, which corresponds to the upper and lower section of the point u_* in the strain softening stage of the curve $F(u)$, respectively. In branch 1, $x_1 < 0$, which corresponds to $u < u_*$ in double-block mechanical system; while in branch 2, $x_2 > 0$, which corresponds to $u > u_*$ in double-block mechanical system.
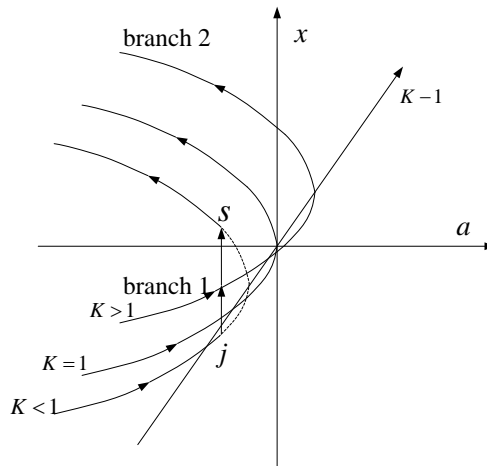

Figure 5: The equilibrium surface of fold catastrophe model

The energy input ratio *J* represents the energy required by the double-block mechanical system, in order to keep the quasi-static movement. Therefore, the equilibrium stability of double-block mechanical system is analyzed by energy input ratio *J* as follows.

Taking the derivative of equation (16) with respect to u,

$$\frac{dJ}{du} = \frac{(1+m)^2 F(u_e)}{Ku_e}\left[\frac{u - u_e}{u_e} + \frac{m(1-K)}{(1+m)^2}\right] = \frac{(1+m)^2 F(u_e)}{Ku_e}x \tag{21}$$

Combining the inequality $x_1 < 0$ and $x_2 > 0$ with the equation (21), in branch 1, $\left.\frac{dJ}{du}\right|_{x=x_1} < 0$, which indicates that the energy input ration *J* diminishes gradually to keep the quasi-static movement, and the equilibrium state of double-block mechanical system is instable; in branch 2, $\left.\frac{dJ}{du}\right|_{x=x_2} > 0$, which indicates that the energy input ration *J* increases gradually to keep the quasi-static movement, and the equilibrium state of double-block mechanical system is stable.

4 The influence of systematical stiffness on ore pillar stability

The stiffness $K=\dfrac{k_n}{\left|F'(u_e)\right|}=-\dfrac{k_n}{F'(u_e)}>0$ of double-block mechanical system refers to

the ratio of energy release body stiffness k_n to the absolute value $\left|F'(u_e)\right|$ of the gradient at the inflexion point on the load-displacement curve of energy dissipation body. To the fixed value of system stiffness K, when $u<u_*$ and u on the branch 1 augments, $F'(u)$ and J decrease accordingly. Therefore, a and x_1 change from negative to zero, i.e., the equilibrium position (a,x_1) turns rightwards along branch 1. Similarly, when $u>u_*$ and u on the branch 2 augments, $F'(u)$ and J also augment accordingly. Therefore, a augments in negative direction and x_2 augments in positive direction, i.e., the equilibrium position (a,x_2) turns leftwards along branch 2.

In the problem of ore pillar destabilization, the equilibrium position of ore pillar will reach branch 2. To the value of a which is not zero, the system has two equilibrium positions corresponding to a. If the value of K is given and J changes, the equilibrium position transits to branch 2 from branch 1 via origin point or the axis $K-1$, and the ore pillar is destroyed by the progressive failure form; otherwise, the equilibrium position leaps to branch 2 from branch 1, and the ore pillar is destroyed by the dynamic destabilization form.

Since point j and point s are both the critical stabilization state of ore pillar in the equilibrium surface of fold catastrophe model, the ratio of energy input J of the system is zero, i.e., $J(u_j)=J(u_s)=0$, and ore pillar deformation u_j and u_s correspond to x_j in branch 1 and x_s in branch 2, respectively.

In the following part of the paper, we will discuss the instance that u approaches and deviates from u_*, i.e., the equilibrium position approaches and deviates from the axis $K-1$.

4.1 The systematical stiffness is less than 1

When the systematical stiffness is less than 1, i.e., $k_n<-F'(u_e)$, and the equilibrium position turns rightwards along branch 1, it can be proved that there exists a certain point $x_j<0$, or a certain point u_j smaller than u_* in strain softening stage, meets $-F'(u_j)<-F'(u_*)<-F'(u_e)$ and $F'(u_j)+k_n=0$.

When the systematical stiffness is less than 1, the inequality $u_*<u_e$ can be induced in

equation (20), i.e., $u_* = \left[1 - \dfrac{m(1-K)}{(1+m)^2}\right]u_e$. In double-block mechanical system, the

first-order derivative of peak strength point of energy dissipation body is zero, i.e., $F'(u_c) = 0$, therefore, the inequality $F'(u_c) + k_n = k_n > 0$ is true for the peak strength point of energy dissipation body. According to the continuity of function, if the inequality

$F'\left\{\left[1 - \dfrac{m(1-K)}{(1+m)^2}\right]u_e\right\} + k_n < 0$ can   be   proved   to   be   true,   the   equality

$F'(u_j) + k_n = 0$ is met. The inequality $F'\left\{\left[1 - \dfrac{m(1-K)}{(1+m)^2}\right]u_e\right\} + k_n < 0$ is proved

when $m = 1$ as follows:

When $m = 1$, the inequality $F'\left[\left(\dfrac{3+K}{4}\right)u_e\right] + k_n < 0$ need to be proved, and the

equations are obtained in equation (22),

$$\begin{cases} \dfrac{u_e}{u_0} = \left(\dfrac{1+m}{m}\right)^{\frac{1}{m}} = 2 \\[3mm] F'(u) = \lambda\left[1 - m\left(\dfrac{u}{u_0}\right)^m\right]\exp\left[-\left(\dfrac{u}{u_0}\right)^m\right] = \lambda\left[1 - \left(\dfrac{u}{u_0}\right)\right]\exp\left[-\left(\dfrac{u}{u_0}\right)\right] \\[3mm] F'(u_e) = -\lambda m\exp\left(-\dfrac{1+m}{m}\right) = -\lambda\exp(-2) \end{cases} \tag{22}$$

Thus,

$$F'(u) = \lambda\left[1 - \left(\dfrac{u}{u_0}\right)\right]\exp\left[-\left(\dfrac{u}{u_0}\right)\right] = \lambda\left(1 - \dfrac{2u}{u_e}\right)\exp\left(-\dfrac{2u}{u_e}\right) \tag{23}$$

The stiffness of double-block system is

$$K = \dfrac{k_n}{-F'(u_e)} = \dfrac{k_n}{\lambda\exp(-2)} \tag{24}$$

According to equation (23), the equation (25) is induced,

$$F'\left[\left(\dfrac{3+K}{4}\right)u_e\right] = \lambda\left[1 - 2\left(\dfrac{3+K}{4}\right)\right]\exp\left[-2\left(\dfrac{3+K}{4}\right)\right] = \lambda\left(\dfrac{-1-K}{2}\right)\exp\left(\dfrac{-3-K}{2}\right) \tag{25}$$

Since the systematical stiffness satisfies the inequality $0 < K < 1$, $\frac{-3-K}{2} < 0$ can be obtained. According to the basic property of exponential function, the value of exponential function of minus is between 0 and 1, and the inequality $0 < \exp\left(\frac{-3-K}{2}\right) < 1$ is deduced.

Therefore, the equation (26) is obtained, by combining $0 < \exp\left(\frac{-3-K}{2}\right) < 1$ and the equation (25),

$$F'\left[\left(\frac{3+K}{4}\right)u_e\right] < \lambda\left(\frac{-1-K}{2}\right) \tag{26}$$

Then,

$$F'\left[\left(\frac{3+K}{4}\right)u_e\right] + k_n < \lambda\left(\frac{-1-K}{2}\right) + k_n = \lambda\left(\frac{-1-K}{2}\right) + \lambda K \exp(-2) \tag{27}$$

Based on the property of exponential function, the equation $0 < \exp(-2) < 1$ is workable, thus the inequality is obtained as follows:

$$F'\left[\left(\frac{3+K}{4}\right)u_e\right] + k_n < \lambda\left(\frac{-1-K}{2}\right) + \lambda K = -\lambda\left(\frac{1-K}{2}\right) < 0 \tag{28}$$

Therefore, the inequality $F'\left[\left(\frac{3+K}{4}\right)u_e\right] + k_n < 0$ is proved. When $m = 2$, the

inequality $F'\left\{\left[1 - \frac{m(1-K)}{(1+m)^2}\right]u_e\right\} + k_n < 0$ can be proved based on the mathematical induction method. In conclusion, when the equilibrium position turns rightwards along branch 1, there is a point u_j on strain softening curve, which is less than u_*, meets $-F'(u_j) < -F'(u_*) < -F'(u_e)$ and $F'(u_j) + k_n = 0$.

Based on the expression of energy input ratio J in the equation (13), i.e., $J = \frac{P du_P}{du}$, the point j and point s is the initial point and terminal point of dynamic destabilization of ore pillar, respectively, and it can be deduced that the equation $J(u_j) = J(u_s) = 0$ is workable. The double-block mechanical system is in critical state in point j, where $\frac{du_P}{du} = 0$ or $\frac{du}{du_P} \to \infty$. Since du and du_P are the same order

variable, and $\dfrac{du}{du_P} \to \infty$ is true in point j, it can be concluded that u has an abrupt

alteration in Fig. 5, and the equilibrium position will jump from point x_j on branch 1 to

point x_s on branch 2, which indicates that dynamic destabilization of double-block mechanical system.

In summary, the strain softening property of ore pillar and the stiffness parameter of double-block system less than 1, are the necessary conditions of dynamic instability of double-block system, and are the internal cause of dynamic instability of ore pillar. The external load sufficient to make ore pillar into the post-peak deformation zone, is the sufficient condition for the occurrence of dynamic instability, and is the external cause of dynamic instability of ore pillar.

4.2 The systematical stiffness is equal to 1

When the systematical stiffness $K = \dfrac{k_n}{-F'(u_e)} = 1$, $u_* = \left[1 - \dfrac{m(1-K)}{(1+m)^2} \right] u_e = u_e$ is

true. Substituting $K=1$ into equation (19), the expressions of x_1 and x_2 are

$$\begin{cases} x_1 = \dfrac{u - u_e}{u_e} = -\sqrt{-a} = -\sqrt{\dfrac{2KJ}{F(u_e)(1+m)^2}} \\ x_2 = \dfrac{u - u_e}{u_e} = \sqrt{-a} = \sqrt{\dfrac{2KJ}{F(u_e)(1+m)^2}} \end{cases} \tag{29}$$

The equilibrium position (a, x_1) turns rightwards along branch 1, and the energy input ratio J diminishes gradually from positive value. When the deformation of energy dissipation body satisfies $u = u_* = u_e$, the energy input ratio J at the inflection point is zero, $J(u_e)=0$, which indicates that the double-block mechanical system is in critical state. Therefore, the initial destabilization point j and terminal destabilization point s of double-block mechanical system is in coincidence at the inflection point e. Thus, the equilibrium position of double-block mechanical system passes through the origin, from branch 1 of unstable equilibrium state to branch 2 of stable equilibrium state by smooth transition, and the ore pillar is destroyed by quasi-static damage.

4.3 The systematical stiffness is larger than 1

In order to research the change rules of equilibrium position on branch 1 and branch 2 nearby the $K-1$ axis, the state variable x of fold catastrophe model on branch 1 and branch 2 need to be analyze. In consideration of the simplicity of discussion process, the condition of $m=1$ is proved as follows:

Substituting $m=1$ into the expression of state variable x in equation (19) and the

systematical stiffness $K = -\dfrac{k_n}{F'(u_e)} = \dfrac{k_n}{\lambda m \exp\left(-\dfrac{1+m}{m}\right)}$,

$$\begin{cases} x_1 = \dfrac{u - u_e}{u_e} + \dfrac{1-K}{4} = -\sqrt{-a} = -\sqrt{\left(\dfrac{1-K}{4}\right)^2 + \dfrac{1-K}{2} + \dfrac{KJ}{2F(u_e)}} \\[4mm] x_2 = \dfrac{u - u_e}{u_e} + \dfrac{1-K}{4} = \sqrt{-a} = \sqrt{\left(\dfrac{1-K}{4}\right)^2 + \dfrac{1-K}{2} + \dfrac{KJ}{2F(u_e)}} \\[4mm] K = \dfrac{k_n}{-F'(u_e)} = \dfrac{k_n}{\lambda \exp(-2)} \end{cases} \tag{30}$$

According to the expression of energy input ratio $J = \dfrac{F(u)\left[F'(u) + k_n\right]}{k_n}$, when the

ore pillar deformation $u = u_*$, the term $\dfrac{KJ}{2F(u_e)}$ in analytical expression x is

analyzed,

$$\dfrac{K}{2F(u_e)} J\Big|_{u=u_*} = \dfrac{K}{2F(u_e)} \left\{ \dfrac{F(u_*)\left[F'(u_*) + k_n\right]}{k_n} \right\} \tag{31}$$

Substituting $m = 1$ into the equation (2), equation (4) and equation (20),

$$\begin{cases} F(u) = \lambda u \exp\left[-\left(\dfrac{u}{u_0}\right)^m\right] = \lambda u \exp\left(-\dfrac{u}{u_0}\right) \\[4mm] F'(u) = \lambda \left[1 - m\left(\dfrac{u}{u_0}\right)^m\right] \exp\left[-\left(\dfrac{u}{u_0}\right)^m\right] = \lambda \left[1 - \left(\dfrac{u}{u_0}\right)\right] \exp\left[-\left(\dfrac{u}{u_0}\right)\right] \end{cases} \tag{32}$$

And

$$\begin{cases} \dfrac{u_e}{u_c} = \left(\dfrac{1+m}{m}\right)^{\frac{1}{m}} = 2 \\[4mm] u_* = \left[1 - \dfrac{m(1-K)}{(1+m)^2}\right] u_e = \left(\dfrac{K+3}{4}\right) u_e \\[4mm] \dfrac{u_*}{u_c} = \dfrac{K+3}{2} \end{cases} \tag{33}$$

Thus

$$
\begin{cases}
F(u_e) = \lambda u_e \exp\left[-\left(\dfrac{1+m}{m}\right)^{\frac{1}{m}}\right] = \lambda u_e \exp(-2) \\[4mm]
F'(u_e) = -\lambda m \exp\left[-\left(\dfrac{1+m}{m}\right)^{\frac{1}{m}}\right] = -\lambda \exp(-2)
\end{cases}
\tag{34}
$$

Substituting equation (32), equation (33) and equation (34) into equation (31),

$$
\frac{K}{2F(u_e)} J\Big|_{u=u_*} = \frac{K\lambda u_*}{2\lambda u_e k_n \exp(-2)} \exp\left(-\frac{u_*}{u_0}\right)\left\{ \lambda\left[1-\left(\frac{u_*}{u_0}\right)\right]\exp\left[-\left(\frac{u_*}{u_0}\right)\right] + k_n \right\}
\tag{35}
$$

The Taylor series expansion is performed in equation (35), and the higher order term $o\left((K-1)^2\right)$ is omitted,

$$
\frac{K}{2F(u_e)} J\Big|_{u=u_*} = \frac{1}{2}\left(1+\frac{1}{2}\frac{K-1}{2}\right)\left[1-\frac{K-1}{2}+\frac{1}{2}\left(\frac{K-1}{2}\right)^2 + o\left((K-1)^2\right)\right]
$$
$$
\times\left\{ K-\left(1+\frac{K-1}{2}\right)\left[1-\frac{K-1}{2}+\frac{1}{2}\left(\frac{K-1}{2}\right)^2 + o\left((K-1)^2\right)\right]\right\} = \frac{K-1}{2}-\left(\frac{K-1}{4}\right)^2
\tag{36}
$$

Substituting the equation (36) into equation (30),

$$
\begin{cases}
x_1 = \dfrac{u-u_e}{u_e} + \dfrac{1-K}{4} = -\sqrt{-a} = -\sqrt{\left(\dfrac{1-K}{4}\right)^2 + \dfrac{1-K}{2} + \dfrac{KJ}{2F(u_e)}} = 0 \\[5mm]
x_2 = \dfrac{u-u_e}{u_e} + \dfrac{1-K}{4} = \sqrt{-a} = \sqrt{\left(\dfrac{1-K}{4}\right)^2 + \dfrac{1-K}{2} + \dfrac{KJ}{2F(u_e)}} = 0
\end{cases}
\tag{37}
$$

In equation (37), it can be concluded that the variable x and a at each side of equal symbol is zero when $u = u_*$, which indicates that the equilibrium position of double-block mechanical system passes through shaft $K-1$, from branch 1 of unstable equilibrium state to branch 2 of stable equilibrium state by smooth transition, and the ore pillar is destroyed by quasi-static damage.

5   Conclusions

(1) The dynamic destabilization of ore pillar is a physical instability problem caused by the strain softening property of ore mass. When the load on ore mass reaches peak stress, the roadway roof bounces and unloads caused by the sharp fall of bearing capacity of ore

pillar; if the elastic energy released by roadway roof is larger than the plastic energy dissipated by ore pillar in quasi-static movement, the excess energy will be transformed as kinetic energy of the system, leading to the dynamic destabilization of ore pillar.

(2) The ore pillar and the roadway roof are treated as energy dissipation body and energy release body, respectively, and the double-block mechanical model is established with energy dissipation body and energy release body. By analyzing the energy balance relationship of double-block mechanical system, the dynamic instability mechanism of ore pillar can be obtained, based on the fold catastrophe model. The strain softening property of ore pillar and the stiffness parameter of double-block system less than 1, are the necessary condition of dynamic instability of double-block system, and are the internal cause of dynamic instability of ore pillar. The external load sufficient to make ore pillar into the post-peak deformation zone, is the sufficient condition for the occurrence of dynamic instability, and is the external cause of dynamic instability of ore pillar.

(3) When the stiffness parameter of double-block system is greater than or equal to 1, the equilibrium position of double-block system passes through shaft $K-1$ or the origin, from branch 1 of unstable equilibrium state to branch 2 of stable equilibrium state by smooth transition, and the ore pillar shows quasi-static fracture. When the stiffness parameter of double-block system is less than 1, the equilibrium position of double-block system mutates from branch 1 of unstable equilibrium state to branch 2 of stable equilibrium state by jumping transition, and the ore pillar shows dynamic destabilization.

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