 CMES, vol.90, no.1, pp.47-63, 2013

Research on a Triaxial Rate of Penetration (ROP) Model Related to Unloading in Oil & Gas Drilling

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Abstract: In oil & gas drilling engineering, the rock breaking efficiency as well as the trajectory of the Well should be quantitatively and accurately described by an ideal triaxial Rate of Penetration (ROP) model, taking into account the various objective and subjective factors. However, with the existing triaxial ROP models, it is difficult to achieve these goals. The applications of the existing ROP models are limited, especially in under-balanced drilling (air drilling, foam drilling etc.), because the unloading effect (i.e. the effect of the bottom hole differential pressure on the formation force) has been rarely considered. On the basis of a rock/bit interaction model, a new triaxial ROP model including the unloading effect, is developed in this paper. This new model focuses on the effect of the bottom hole differential pressure on the formation-forces. Case studies in this paper also indicate that the inclination and the azimuth forces of the formation will increase with the decrease of bottom hole differential pressure. This explains the reason why the under-balanced drilling has a greater tendency to produce hole deviation, than that in mud drilling. The new model presented in this paper, is applied to predict the well trajectory, and the ROP of the well Longgang-2, in China's Sichuan oilfield. The results of this field application indicate that the new model is quite feasible.

Keywords: Triaxial ROP; Formation force; Differential pressure; Dip angle; Rock anisotropy

1 Introduction

In oil & gas drilling engineering, the rock breaking efficiency as well as the trajectory of the Well should be quantitatively and accurately described by an ideal triaxial rate of penetration (ROP) model, which accounts for the various objective and subjective factors. Although many researchers such as Lubinski (1953), Cheatham

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et al(1981), Ho(1987), Gao et al (1994) and Jian (1990) have established the triaxial ROP models, the effect of the bottom hole differential pressure has rarely been taken into account. This is the primary reason why the effect of the bottom hole differential pressure on the rock breaking efficiency and the well trajectory control could not be described correctly.

The models to describe formation-force, established by Yang (1985), Williamson.et al(1987), Gao et al(1989), and Yan et al (1996), also did not considered the effect of the bottom hole differential pressure. However, the effect of bottom hole differential pressure, on the formation-forces, is dramatic in an under-balanced type of drilling (air drilling, foam drilling etc.). On the basis of a rock/bit interaction model, a new triaxial ROP model, including the unloading effect is developed in this paper, and this model focuses on the effect of the bottom hole differential pressure on the formation-forces. A case study presented in this paper indicates that the formation -force would increase with the decrease of the bottom hole differential pressure. This explains the reason why the problem of the well-trajectory deviation is more serious in an under-balanced drilling than in fluid drilling.

The new triaxial ROP model is then correlated with the various empirical models which were extensively used in engineering practice, and the new triaxial ROP model is found to predict the well trajectory and ROP, both quantitatively and accurately in oil & gas drilling engineering.

2 A New triaxial ROP model Related to Unloading

2.1 Anisotropies of the formation and the Drill-bit

Anisotropy of the formation can be quantified by the formation-anisotropy-index. If an isotropic bit drills a hole in the rectilinearly anisotropy formation, the formation-anisotropy-index [Gao(1989)] can be defined as follows:

$$I_{r1} = D_{dip}/D_n \quad I_{r2} = D_{str}/D_n \tag{1}$$

Where $D_{dip} = R_{dip}/F_{dip}$, $D_{str} = R_{str}/F_{str}$, and $D_n = R_n/F_n$.

Bit anisotropy expressed by the bit-anisotropy-index, which is the ratio of the drillabilities in the lateral and the axial direction of a bit. If an anisotropic bit penetrates in an isotropic formation, anisotropy-index of the bit [Gao (1989)] can be expressed as follow:

$$I_b = D_l/D_a \tag{2}$$

Where $D_a = R_a/F_a$, and $D_l = R_l/F_l$.

2.2 ROP Factors, Which Account for the Bottom Hole Differential Pressure

Laboratory experiments and drilling practice prove that bottom hole differential pressure has a tremendous effect on the ROP. The ROP coefficient can be expressed as follows:

$$\begin{cases} C_1 = e^{-\beta_1 \Delta p} = R_n / R_{n0} \\ C_2 = e^{-\beta_2 \Delta p} = R_{str} / R_{str0} \\ C_3 = e^{-\beta_3 \Delta p} = R_{dip} / R_{dip0} \end{cases} \tag{3}$$

After considering the bottom hole differential pressure, the formation-anisotropy-index can be expressed as follows:

$$\begin{cases} I_{r1} = \frac{D_{dip}}{D_n} = \frac{C_3 R_{dip0}}{F_{dip}} / \frac{C_1 R_{n0}}{F_n} = \frac{C_3}{C_1} I'_{r1} \\ I_{r2} = \frac{D_{str}}{D_n} = \frac{C_2 R_{str0}}{F_{str}} / \frac{C_1 R_{n0}}{F_n} = \frac{C_2}{C_1} I'_{r2} \end{cases} \tag{4}$$

2.3 A triaxial ROP model, which accounts for unloading

We consider an anisotropic bit penetrating an anisotropic formation. Through a rigorous mathematical derivation, the relationship between the bit penetrating rate and the force can be expressed as follow:

$$\begin{pmatrix} V_{xd} \\ V_{yd} \\ V_{zd} \end{pmatrix} = \frac{D_n}{C_1} [D] [C] \left[IR'\right] [D]^{-1} [F] [IB] [F]^{-1} \begin{pmatrix} F_{xd} \\ F_{yd} \\ F_{zd} \end{pmatrix} \tag{5}$$

Considering the impact of the rotation speed, and the hydraulic factor, on the rate of penetration, the ROP model would be expressed as follow:

$$V_{PC} = K_R N^{\lambda} N_d^a \sqrt{V_{xd}^2 + V_{yd}^2 + V_{zd}^2} \tag{6}$$

3 A model for the formation-force:

A Cartesian system of coordinates is selected, with the Z_d axis opposite to the drilling direction (**Fig.1**). Then the inclination force of the formation, at the bit, can be obtained from the following equation:

$$F_\alpha = \frac{A_1}{A_3 - A_4 \tan \Delta\alpha - A_5 \tan \Delta\psi} WOB \tag{7}$$

Where

$$A_1 = M_1 N_1 + M_2 N_2 + M_3 N_3, \quad A_3 = -M_1 N_3 - M_2 N_5 - M_3 N_6,$$

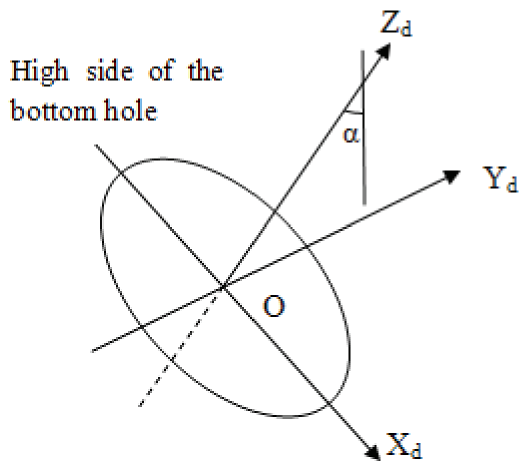

Figure 1: Selected system of coordinates for the calculation of side force at the bit

$$A_4 = M_5N_3 + M_4N_5 + M_1N_6, \quad A_5 = -M_4N_3 - M_6N_5 - M_2N_6,$$

$$M_1 = C_2C_3I'_{r1}I'_{r2}R_1R_2 - C_1C_3I'_{r1}\sin\alpha\cos\alpha\sin^2\Delta\varphi + C_1C_2I'_{r2}R_3R_4,$$

$$M_2 = -C_2C_3I'_{r1}I'_{r2}R_1\sin\beta\sin\Delta\varphi - C_1C_3I'_{r1}\sin\alpha\sin\Delta\varphi\cos\Delta\varphi + C_1C_2I'_{r2}R_4\cos\beta\sin\Delta\varphi,$$

$$M_3 = -C_2C_3I'_{r1}I'_{r2}R_1^2 - C_1C_3I'_{r1}\sin^2\alpha\sin^2\Delta\varphi - C_1C_2I'_{r2}R_4^2,$$

$$M_4 = C_2C_3I'_{r1}I'_{r2}R_2\sin\Delta\varphi\sin\beta - C_1C_3I'_{r1}\cos\alpha\sin\Delta\varphi\cos\Delta\varphi - C_1C_2I'_{r2}R_3\cos\beta\sin\Delta\varphi,$$

$$M_5 = -C_2C_3I'_{r1}I'_{r2}R_2^2 - C_1C_3I'_{r1}\sin^2\Delta\varphi\cos^2\alpha - C_1C_2I'_{r2}R_3^2,$$

$$M_6 = -C_2C_3I'_{r1}I'_{r2}\sin^2\beta\sin^2\Delta\varphi - C_1C_3I'_{r1}\cos^2\Delta\varphi - C_1C_2I'_{r2}\cos^2\beta\sin^2\Delta\varphi,$$

$$R_1 = \cos\alpha\cos\beta - \sin\alpha\sin\beta\cos\Delta\varphi, R_2 = \sin\alpha\cos\beta + \cos\alpha\sin\beta\cos\Delta\varphi,$$

$$R_3 = \sin\alpha\sin\beta - \cos\alpha\cos\beta\cos\Delta\varphi, R_4 = \sin\beta\cos\alpha + \sin\alpha\cos\beta\cos\Delta\varphi,$$

$$N_1 = \theta_x^2(I_b + \theta_y^2) + (\theta_y^2 + 1)^2, N_2 = (I_b - \theta_x^2 - \theta_y^2 - 2)\theta_x\theta_y, N_3 = (I_b - 1)\theta_x,$$

$$N_4 = \theta_y^2(I_b + \theta_x^2) + (\theta_x^2 + 1)^2, N_5 = (I_b - 1)\theta_y, \text{ and } N_6 = I_b + \theta_x^2 + \theta_y^2.$$

F_α is the inclination force of the formation at the bit, in the direction of X_d. It affects only the well-inclination and does not affect the well-azimuth. In a similar way, the azimuth force of formation, F_ϕ, can be calculated by

$$F_\phi = \frac{A_2}{A_3 - A_4\tan\Delta\alpha - A_5\tan\Delta\psi}WOB \tag{8}$$

Where $A_2 = -M_2N_4 - M_1N_2 - M_3N_5$.

F_ϕ is the azimuth force of the formation at the bit, in the direction of Y_d. It affects only the well-azimuth and not the inclination. The resultant formation force, F_f, can be calculated as follow:

$$F_f = \sqrt{F_\alpha^2 + F_\phi^2} \tag{9}$$

The direction of action for the resultant formation-force is expressed as follows:

$$\theta = \arctan(\frac{F_\alpha}{F_\varphi}) \tag{10}$$

4 The simplified formation-force models

1. In a case when $I_b = 1$, $I'_{r1} = I'_{r2} = I'_r$, $\theta_x = \theta_y = 0$, $\Delta\alpha \neq 0$ and $\Delta\psi \neq 0$, A_1, A_2, A_3, A_4 and A_5 are

$$A_1 = C_1^2 I'^2_r R_1 R_2 - C_1C_3 I'_r \sin\alpha\cos\alpha\sin^2\Delta\varphi + C_1C_3 I'_r R_3R_4,$$

$$A_2 = C_1^2 I'^2_r R_1 \sin\beta\sin\Delta\varphi + C_1C_3 I'_r \sin\alpha\sin\Delta\varphi\cos\Delta\varphi - C_1C_3 I'_r R_4\cos\beta\sin\Delta\varphi,$$

$$A_3 = C_1^2 I'^2_r R_1^2 + C_1C_3 I'_r \sin^2\alpha\sin^2\Delta\varphi + C_1C_3 I'_r R_4^2,$$

$$A_4 = A_1,\ \text{and}\ A_5 = A_2. \tag{11}$$

2. In a case when $I_b = 1$, $I'_{r1} \neq I'_{r2}$, $\theta_x = \theta_y = 0$ and $\Delta\alpha = \Delta\psi = 0$, A_1, A_2 and A_3 are

$$A_1 = C_1C_2 I'_{r1} I'_{r2} R_1R_2 - C_1C_3 I'_{r1}\sin\alpha\cos\alpha\sin^2\Delta\varphi + C_2C_3 I'_{r2} R_3R_4,$$

$$A_2 = C_1C_2 I'_{r1} I'_{r2} R_1\sin\beta\sin\Delta\varphi + C_1C_3 I'_{r1}\sin\alpha\sin\Delta\varphi\cos\Delta\varphi - C_2C_3 I'_{r2} R_4\cos\beta\sin\Delta\varphi,$$

and

$$A_3 = C_1C_2 I'_{r1} I'_{r2} R_1^2 + C_1C_3 I'_{r1}\sin^2\alpha\sin^2\Delta\varphi + C_2C_3 I'_{r2} R_4^2. \tag{12}$$

3. In a case when$I_b \neq 1$, $I'_{r1} = I'_{r2} = 1$, $\theta_x \neq 0$, $\theta_y \neq 0$, $\Delta\alpha \neq 0$ and $\Delta\psi \neq 0$, A_1, A_2, A_3, A_4 and A_5 are

$$A_1 = A_4 = -C_1(I_b - 1)\theta_x,$$

$$A_2 = A_5 = C_1(I_b - 1)\theta_y,$$

$$A_3 = C_1(I_b + \theta_x^2 + \theta_y^2),$$

$$A_4 = A_1, \text{ and } A_5 = A_2. \tag{13}$$

Substituting A_1, A_2,A_3, A_4 and A_5 in Eq. 7 and Eq. 8, by Eq. 13 we obtain

$$F_\alpha = \frac{-(I_b-1)\theta_x}{I_b + \theta_x^2 + \theta_y^2 + (I_b-1)\theta_x \tan\Delta\alpha - (I_b-1)\theta_y \tan\Delta\psi} WOB$$

and

$$F_\phi = \frac{(I_b-1)\theta_y}{I_b + \theta_x^2 + \theta_y^2 + (I_b-1)\theta_x \tan\Delta\alpha - (I_b-1)\theta_y \tan\Delta\psi} WOB \tag{14}$$

From Eq. 14, the following information can be obtained. The bottom hole differential pressure has no effect on the formation-force, when a bit drills in an isotropic formation, regardless of whether the bit itself is anisotropic or not.

5 Effect of the bottom hole differential pressure on the formation-forces

In this section, the influence of bottom hole differential pressure on the formation - force is discussed, assuming that a bit drills in a transversely isotropy formation. In order to better reflect the effect of bottom hole differential pressure on the formation -forces, the incremental multiples of the inclination and azimuth forces are defined as follows:
$n_\alpha = \left|\frac{F_\alpha(\Delta p) - F_\alpha(\Delta p=0)}{F_\alpha(\Delta p=0)}\right|$ and $n_\varphi = \left|\frac{F_\varphi(\Delta p) - F_\varphi(\Delta p=0)}{F_\varphi(\Delta p=0)}\right|$
The formation forces are calculated with the data in Table 1. The results are shown in Fig.2 to Fig.5.

Table 1: Formation Forces Calculation Data

WOB	$I_{r'}$	β_1	β_2	$\theta_x(^\circ)$	$\theta_y(^\circ)$	I_b	$\Delta\alpha(^\circ)$	$\Delta\phi(^\circ)$	$\alpha(^\circ)$	$\Delta\Phi(^\circ)$
100kN	0.9	0.1	0.2	0.2	0.1	0.76	5	5	20	60

The following conclusions can be drawn by analyzing Fig.2 to Fig.5:

1. The bottom hole differential pressure has a significant effect on the formation -forces. For different dip angles, the bottom hole differential pressure has a different effect on the formation-forces.

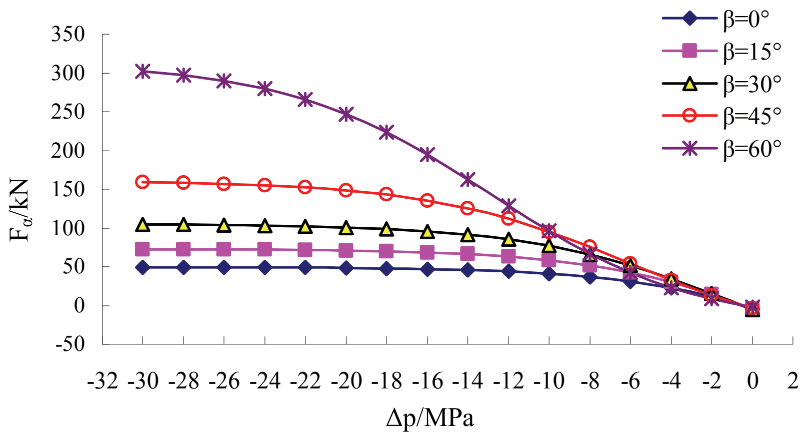

Figure 2: The effect of Δp on inclination side forces of formation

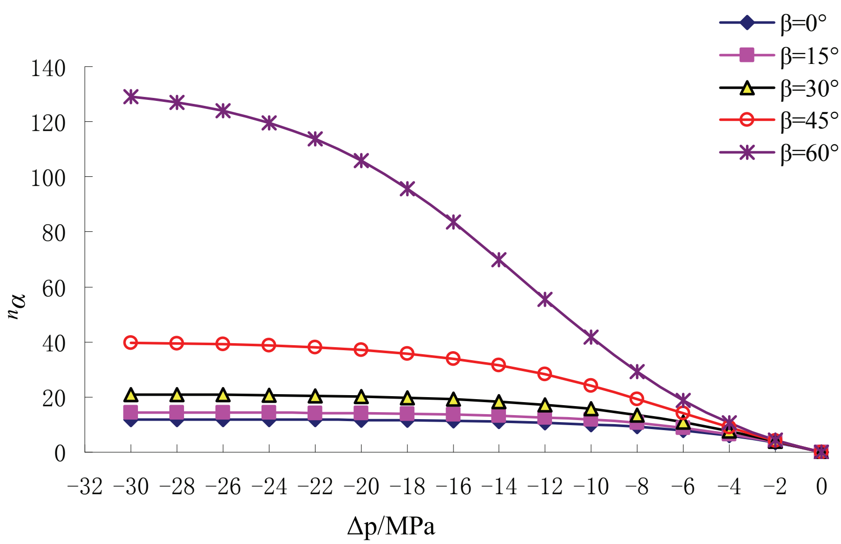

Figure 3: The effect of Δp on the increment multiples of inclination force

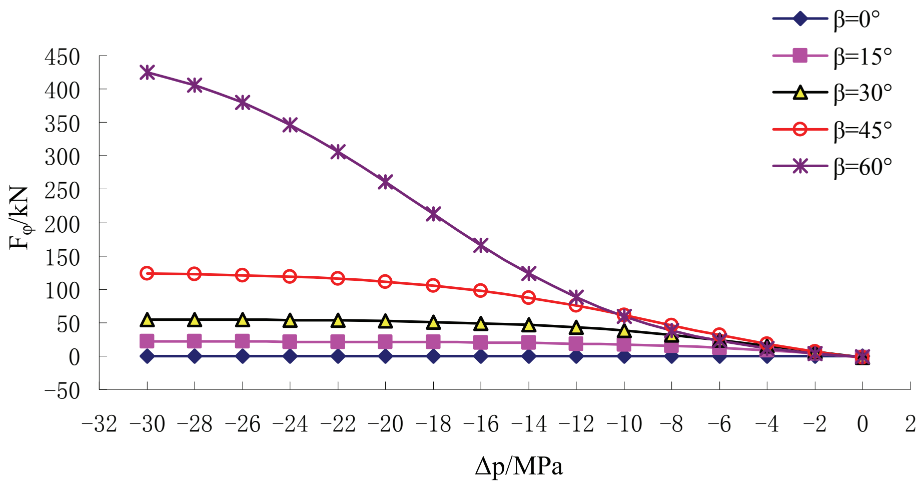

Figure 4: The effect of Δp on azimuth side forces of formation

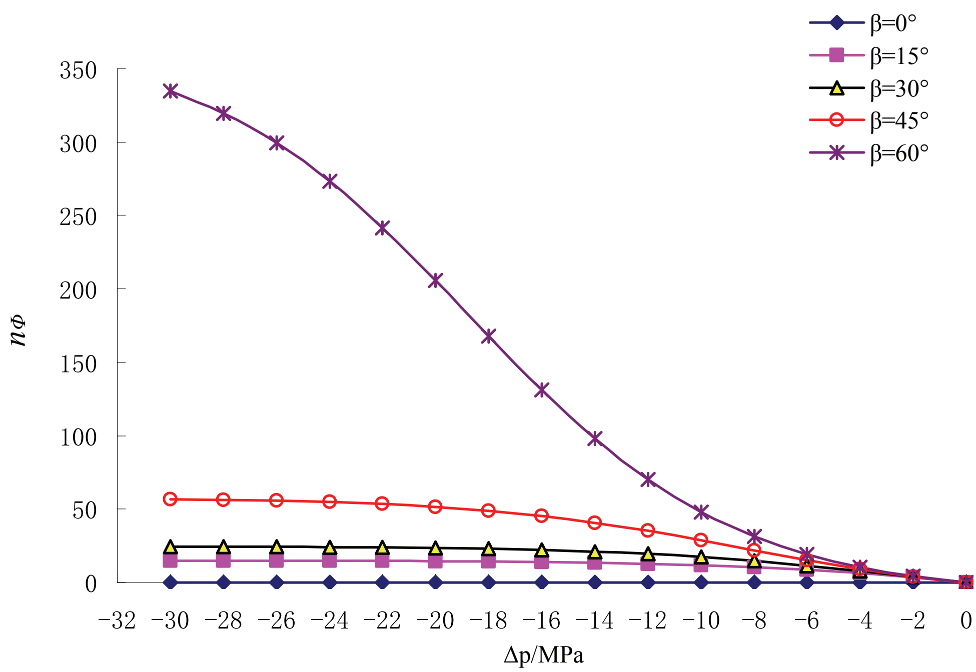

Figure 5: The effect of Δp on the the increment multiples of azimuth force

2. For same dip angle, with the decrease of the bottom hole differential pressure, the inclination and the azimuth forces of the formation will increase, the incremental multiples of the inclination and the azimuth forces will increase as well.

3. For the same bottom hole differential pressure, with the increase of the dip angle, the inclination and the azimuth forces of the formation will increase, and the incremental multiples of the inclination and the azimuth forces will increase as well.

6 A model to predict the trajectory of the well

Assuming that the formation consists of transversely isotropic strata, the inclination side force and the azimuth side force can be calculated by a three-dimension analysis of the bore hole assembly. The parameter inversion flow chart is shown as below:

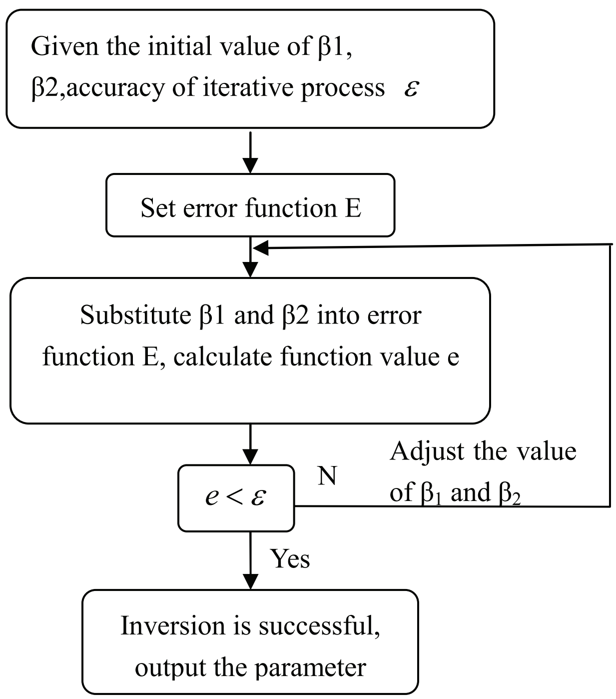

Figure 6: Parameter inversion flow chart

The error function in the inversion calculation is:

$$E = \sum_{i=1}^{n} (F_{\alpha 0}(\vec{X}) - F_{\alpha c}(\vec{X}))^2$$

where n is the number of measuring points; $F_{\alpha 0}$ is the measured inclination force for the i-th measurement point, kN; $F_{\alpha c}$ is the calculated inclination force for the i-th measurement point, kN; $\vec{X}$ is a set of inversion parameters for the i-th measurement point; E represents the difference between the measured and the calculated values of the inclination force.

Assuming that the current inclination angle at the drill bit is α, and that the current azimuth angle at the drill bit is φ, the new inclination angle and azimuth angle could be predicted as follows:

$$\alpha_1 = \alpha + \arctan\left(\frac{V_z}{V_x}\right) \text{ and } \varphi_1 = \varphi + \arctan\left(\frac{V_y}{V_z \sin\alpha + V_x \cos\alpha}\right) \tag{15}$$

7 Model Validation

7.1 Well trajectory prediction

Longgang-2 and Longgang-9 are located in the Sichuan Yilong County. They are deep vertical wells , the 3545m-4159m sections of these wells are both belong to Xujiahe strata. Longgang-2 uses air drilling while Longgang-9 uses mud drilling. Longgang-2 has a very serious deviation problem in the 3545 m – 4159 m interval, the inclination ranges from 0.74 degrees to 4.68 degrees. This paper would take Longgang-2 as an example to predict the variation of its well trajectory and ROP. Longgang-9 shares the same strata with Longgang-2 in the studied interval, thus the bottom hole differential pressure of Longgang-2 could be calculated from the mud density of Longgang-9. The mud density is 1.45g/cm^3, WOB in the studied section of Longgang-2 is 5 tons. The BHA is:

Φ215.9 mm PDC drill bit + 430×410 back pressure valve + Φ165.1 mm drill collar (two joints) + Φ208 mm centralizer + Φ165.1 mm drill collar + Φ206 mm centralizer + Φ165.1 mm drill collars (12 joints) + Φ165.1 mm drilling jar (1 set) + Φ127 heavy weight drill pipe (15 joints) + Φ127 drill pipe + 411×410 (kelly sub) + 411×410 (kelly cock) + 410×520 (kelly sub) + kelly

By the above inversion model, the differential pressure factors of Longgong-2 can be inversely calculated: β_1=0.109, β_2=0.104.

Fig.6 and Fig.7 compare the predicted values and measured values of the inclination angle and the azimuth angle. These two figures show that the maximum relative error in the inclination angle is 16.85%, the minimum relative error of the inclination angle is 0.07%, the average relative error of the inclination angle is 5.68%; the

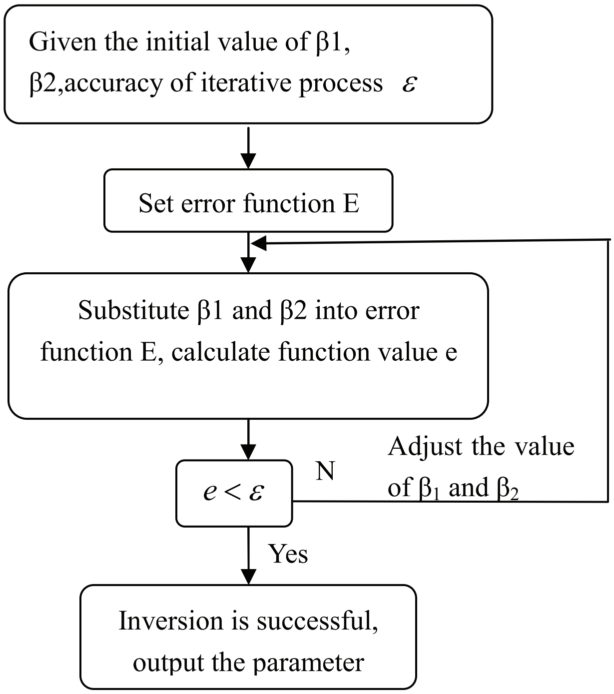

Figure 7: Predicted values vs measured values of inclination angle of Longgang-2

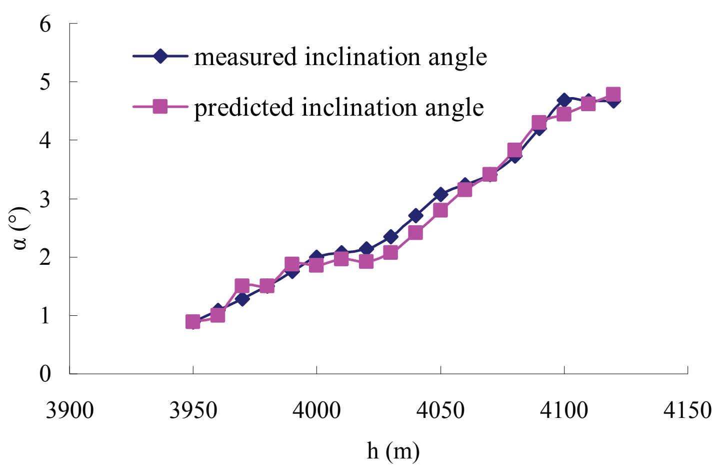

Figure 8: Predicted values vs measured values of the azimuth angle of Longgang-2

maximum relative error of the azimuth angle is 5.74%, the minimum relative error of the azimuth angle is 0.21%, and the average relative error of azimuth angle is 1.93%. The error analysis results show that the calculation results and measured results are in good agreement.

7.2 ROP prediction

The rotation speed of drill bit in the studied interval of well Longgang-2 is 50r/min; the flow rate is 150 L/s; the diameters of all the three nozzles are 33 mm. By a three-dimension analysis of the BHA and using the inversion model, the ROP coefficient, rotation speed index number and fluid power efficiency index number could all be inversely calculated:K_R = 0.86, λ= 0.45, a = 0.17. The predicted values of ROP of Longgang-2 can be seen in Table 2.

Table 2: Predicted values of ROP in well Longgang-2

Section (m)	Measured ROP (m/h)	Predicted ROP (m/h)	Relative Error (%)
3545∼3590	8.58	10.14	18.2
3590∼3943	13.06	13.14	0.61
3943∼4324	7.28	8.41	15.5

8 Conclusions

1. Using a rock-bit interaction model, and taking into account the effects of drilling circumstances, the bore hole geometry, the bit anisotropy, the formation anisotropy and the bottom hole differential pressure, a new triaxial ROP model related to unloading is developed. This model especially focused on the effect of the bottom hole differential pressure on the formation-forces. The existing models correspond to several simplifications of this model.

2. The bottom hole differential pressure as well as the dip angle, both have significant effects on the formation-forces. The smaller the bottom hole differential pressure is, the greater the deviation tendency of the well will be; the larger the dip angle is, the more severe the deviation problem will be.

3. The predicted results for the well trajectory, and the rate of penetration, of the well Longgang-2 in China's Sichuan oilfield indicate that the new model is quite feasible.

Acknowledgement: The authors gratefully acknowledge the Foundation for Innovative Research Groups of the National Natural Science Foundation of China (Grant No. 51221003), the Natural Science Foundation of China(No.51174220, No.51274220), the Major State Basic Research Development Program of China (973 Program) (No.2010CB226703), National Oil and Gas Major Project (Grant No. 2011ZX05009), the PetroChina Innovation Foundation (2011-5006-0310) and the Science Foundation of China University of Petroleum, Beijing (No. KYJJ2012-02-39).

Nomenclature

D_{a}	axial cutting ability of the bit
D_{l}	lateral cutting ability of the bit
D_{n}	drillability of the formation in its normal direction
D_{dip}	drillability of the formation in its dip direction
D_{str}	drillability of the formation in its strike direction
F_{l}	lateral force of the bit, (kN)
F_a	axial force of the bit , (kN)
F_{dip}	side force of the bit in dip direction of the formation, (kN)
F_{str}	side force of the bit in strike direction of the formation, (kN)
F_{n}	side force of the bit in normal direction of the formation, (kN)
F_α	inclination force of the formation, (kN)
F_φ	azimuth force of the formation, (kN)
F_f	resultant formation force, (kN)
R_{a}	rate of penetration in axial direction of the bit, (m/s)
R_{l}	rate of penetration in lateral direction of the bit, (m/s)
R_{n}	rate of penetration in normal direction of the formation, (m/s)
R_{dip}	rate of penetration in dip direction of the formation, (m/s)
R_{str}	rate of penetration in strike direction of the formation, (m/s)
I_{b}	the bit anisotropy index
I_{r}	rock anisotropy index of the transversely isotropic formation
I_{r1}	rock anisotropy index in its dip direction
I_{r2}	rock anisotropy index in its strike direction
I'_{r}	rock anisotropy index of the transversely isotropic formation when bottom hole differential pressure is zero
I'_{r1}	rock anisotropy index in its dip direction when bottom hole differential pressure is zero
I'_{r2}	rock anisotropy index in its strike direction when bottom hole differential pressure is zero
C_1	rate of penetration coefficient in normal direction of the formation

C_2	rate of penetration coefficient in dip direction of the formation
C_3	rate of penetration coefficient in its strike direction of the formation
β_1	factor in normal direction of the formation
β_2	factor in dip direction of the formation
β_3	factor in its strike direction of the formation
Δp	bottom hole differential pressure, (MPa)
n_α	the increment multiples of inclination force
n_φ	the increment multiples of azimuth force
WOB	weight on bit , (kN)
α	inclination or hole angle, (degree)
β	dip angle of the formation , (degree)
θ	action direction of the resultant formation force , (degree)
θ_x	projection of bit tilt angle on borehole inclination plane , (degree)
θ_y	projection of bit tilt angle on azimuth plane, (degree)
$\Delta\alpha$	change of inclination, (degree)
$\Delta\varphi$	difference between hole azimuth and down dip azimuth of the formation, (degree)
$\Delta\psi$	adjusted change of azimuth angle, (degree)
$\Delta\phi$	increment of the azimuth angle, (degree)
K_R	ROP coefficient
N	rotation speed, (r/min)
N_d	jet power, (kw)
λ	rotation speed index number
a	fluid power efficiency index number

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Appendix: Derivation for Expressions of the Formation Force

Let's consider an anisotropic bit penetrating an anisotropic formation. The vector F is the mechanical resultant force on bit. Coordinates X_d, Y_d and Z_d represent the system with the center of coordinate in the center of the well. Direction Z_d is the outward normal direction of the bottom hole plane, it is opposite to the drilling direction, direction X_d points to downwards and normal to Z_d, direction Y_d is in accordance with the right-hand rule. Coordinates X_f, Y_f and Z_f represent an independent system of coordinates with direction of Z_f positive to the direction of the outward normal direction of the bedding plane and X_f, Y_f indicates upwards direction of dip and direction of formation strike, respectively. The third system of coordinate X_b, Y_b and Z_b represents bit coordinate. Direction Z_b is upward direction of tangent of deformed drill string, direction X_b is downwards and perpendicular to Z_b and direction Y_b is in accordance with the right-hand rule.

In general, bit inclination angles are less than 0.4° [Gao et al (1994)] which can be treated as small quantity. Then there are following relations:

$$\cos\theta_x \approx 1, \quad \sin\theta_x \approx \theta_x, \quad \cos\theta_y \approx 1, \quad \sin\theta_y \approx \theta_y \tag{A-1}$$

Using Eq. A-1 in matrices [F] and [F]$^{-1}$ and neglecting second order of small quantities can lead to

$$[F] \approx \begin{bmatrix} 1 & 0 & \theta_x \\ 0 & 1 & -\theta_y \\ -\theta_x & \theta_y & 1 \end{bmatrix}$$

and

$$[F]^{-1} \approx \begin{bmatrix} 1 & 0 & -\theta_x \\ 0 & 1 & \theta_y \\ \theta_x & -\theta_y & 1 \end{bmatrix} \tag{A-2}$$

Eq. A-2 is the simplification form of matrices [F] and [F]$^{-1}$.

Let's consider a time interval Δt. During this period, the displacement of a bit in X_d, Y_d and Z_d direction in the X_d, Y_d and Z_d system is S_{xd} ,S_{yd} and S_{zd}, respectively. Then S_{xd}, S_{yd} and S_{zd} are

$$S_{xd} = \Delta t \cdot V_{xd}, \quad S_{yd} = \Delta t \cdot V_{yd}, \quad S_{zd} = \Delta t \cdot V_{zd} \tag{A-3}$$

The resultant displacement represents new drilling direction. Then,

$$\tan\Delta\alpha = \frac{S_{xd}}{S_{zd}} = \frac{\Delta t \cdot V_{xd}}{\Delta t \cdot V_{zd}} = \frac{V_{xd}}{V_{zd}} \quad \tan\Delta\psi = \frac{S_{yd}}{S_{zd}} = \frac{\Delta t \cdot V_{yd}}{\Delta t \cdot V_{zd}} = \frac{V_{yd}}{V_{zd}} \tag{A-4}$$

Where $\tan\Delta\psi = \tan\Delta\varphi(\sin\alpha + \tan\Delta\alpha\cos\alpha)$.

Substituting matrices [F] and [F]$^{-1}$ in Eq. 5 by Eq. A-2 yields components of the rate of penetration V_{xd}, V_{yd} and V_{zd}. Using these velocity components in Eq. A-4 and solving out F_{xd} and F_{yd} gives

$$F_{xd} = \frac{-A_1 - A_6\tan\Delta\alpha - A_7\tan\Delta\psi}{A_3 - A_4\tan\Delta\alpha - A_5\tan\Delta\psi} F_{zd},$$

and

$$F_{yd} = \frac{-A_2 - A_8\tan\Delta\alpha - A_9\tan\Delta\psi}{A_3 - A_4\tan\Delta\alpha - A_5\tan\Delta\psi} F_{zd} \tag{A-5}$$

where

$$A_6 = M_5N_1 + M_4N_2 + M_1N_3,$$

$A_7 = -M_4N_1 - M_6N_2 - M_2N_3,$

$A_8 = -M_5N_2 - M_4N_4 - M_1N_5,$

And $A_9 = M_4N_2 + M_6N_4 + M_2N_5.$

It can be observed from the form of Eq. A-5 that the bit force in X_d direction may be divided into three components:

$$F_{xf} = \frac{-A_1}{A_3 - A_4 \tan\Delta\alpha - A_5 \tan\Delta\psi} F_{zd},$$

$$F_{x\alpha} = \frac{-A_6 \tan\Delta\alpha}{A_3 - A_4 \tan\Delta\alpha - A_5 \tan\Delta\psi} F_{zd},$$

And $F_{x\varphi} = \frac{-A_7 \tan\Delta\psi}{A_3 - A_4 \tan\Delta\alpha - A_5 \tan\Delta\psi} F_{zd}$

F_{xf} represents the component of the bit side force in Z_d direction, resulting from the formation anisotropy, bit inclination angle and bit anisotropy. $F_{x\alpha}$ represents the component of the bit side force in Z_d direction that results from the well bore inclination changes. $F_{x\Phi}$ represents the component of the bit side force in Z_d direction that results from the borehole azimuth changes. When the borehole inclination and azimuth are unchanged, $F_{x\alpha}$ and $F_{x\Phi}$ are equal to zero.

Considering the above, the force in the X_d direction to be used for balancing the formation force is F_{xf}, Letting F_{zd}= WOB.

$$F_{\alpha} = \frac{A_1}{A_3 - A_4 \tan\Delta\alpha - A_5 \tan\Delta\psi} WOB \qquad \text{(A-6)}$$

Following the similar procedure, the force in the Y_d direction to be used for balancing the formation force is

$$F_{\phi} = \frac{A_2}{A_3 - A_4 \tan\Delta\alpha - A_5 \tan\Delta\psi} WOB \qquad \text{(A-7)}$$