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A New Approach to Degraded Image Processing Based on Two-Dimensional Parameter-Induced Stochastic Resonance

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Abstract: A modified two-dimensional parameter-induced stochastic resonance (2D-PSR) system is proposed. Both theoretical and simulation results indicate that the 2D-PSR system performs a resonant-like behavior when system parameters are properly adjusted. When applied to degraded image processing, 2D-PSR technique is proved to be able to attain higher SNR gain than traditional linear filters. Due to its strong robustness to environmental changes, adaptability, and complementarities with other methods, the proposed 2D-PSR technique turns out to be promising in the field of image processing.

Keywords: Image de-noising, Stochastic resonance, Nonlinear system

1 Introduction

Due to some unexpected disturbances, an image taken by image sensor may suffer from multiple sources of degradations such as noise and low contrast level, which shelter the original information from being recognized. Traditional linear filters have been proved unable to obtain signal-to-noise ratio (SNR) gain greater than one [Reid (1983)]. Thus the image processed by linear filters can only reach a certain level of resolution. However, nonlinear systems may have SNR gain outstrip one in particular cases. Therefore, a natural question arises: can we make use of such profits to design an image processing system that can perform better than traditional filters?

Stochastic Resonance (SR), which was first put forward by Benzi and al [Benzi, Sutera, and vulpiani (1981); Benzi, Parisi, and Vulpiani (1982)] in 1981 to address

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the periodicity of ice ages, is a phenomenon that under certain conditions the disordered noise power may contribute to ordered signal information. The mechanism is described as follow. Consider a heavily damped particle moving in an asymmetric double-well potential which is driven by weak periodic force (see Fig. 1). Generally, the weak force itself cannot cause the particle to shift between the two wells. However, if the system is interfered by noise with certain intensity which is synchronized with the periodic force, the particle will roll between the two wells in accordance with the periodic force [Gammaitoni, Hanggi, Jung, and Marchesoni (1998)]. This synchronization is called the stochastic resonance.

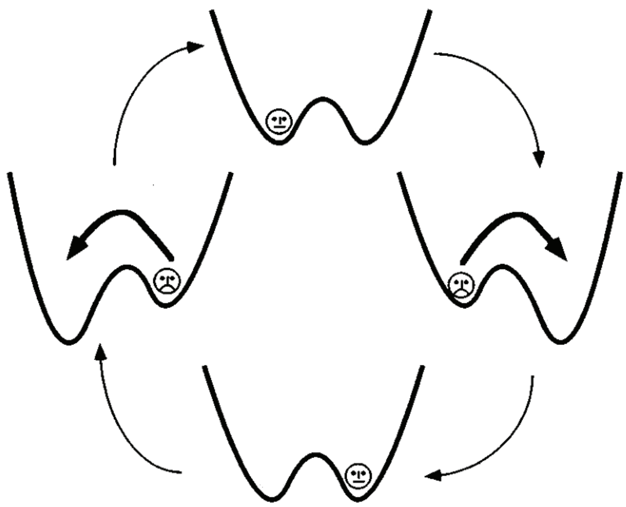

Figure 1: The mechanism of stochastic resonance

Since the concept of SR has been raised and the effect has been experimentally verified, the idea of utilizing noise to enlarge output SNR is widely applied to various scientific fields such as physics, chemistry, biomedical sciences and engineering [Gammaitoni, Hanggi, Jung, and Marchesoni (1998); Harry, Niemi, Priplata, and Collins (2005); Morse and Evans (1996); Zozor and Amblard (2002); Stocks (2001); Chapeau-Blondeau and Rousseau (2004)] over the last few decades, especially in the field of signal processing. The Receiver Operating Characteristic (ROC) curve can be improved when SR occurrs to lower the probability of false alarm under certain detecting level in signal detection [Zozor and Amblard (2002); Jung (1995); Inchiosa and Bulsara (1996); Galdi, Pierro, and Pinto (1998);

Kay (2000); Zozor and Amblard (2003); Bulsara, Seberino, Gammaitoni, Karlsson, Lundqvist, and Robinson (2002); Zhang, Xu, Jiang, and Wu (2008)]. The Bit Error Rate (BER) can be reduced when SR system is applied to conventional encoder [Stocks (2001); Chapeau-Blondeau and Godvier (1996)].

Recently, the concept of parameter-induced stochastic resonance (PSR) has been proposed to realize the resonant-like phenomenon by optimally tuning system parameters [Xu, Duan, Bao, and Li (2002); Duan and Xu (2003); Xu, Li, and Zheng (2003); Xu, Duan, and Chapeau-Blondeau (2004); Xu, Li, and Zheng (2004); Wu, Jiang, Repperger, and Guo (2006)]. Compared with traditional SR technique, PSR achieves SR effect without adding any noise. It has been proved that the performance of PSR is better than traditional SR technique which is in fact a particular case in PSR region [Xu, Duan, and Chapeau-Blondeau (2004)].

Image processing is another important field to which SR technique can be applied. Based on PSR theory, we propose a modified two-dimensional parameter-induced stochastic resonance (2D-PSR) system with parameters a, b to be adjusted. It will be proved in the paper that the output SNR gain of 2D-PSR system can surpass one when parameters a, b are properly set. The examples we studied in this paper also reveal that the performance of images processed by 2D-PSR technique is much better than by traditional linear filters. Some mainstream nonlinear methods for image processing are also cited to compare with our proposed technique. In addition, an adaptive 2D-PSR system and a combination of 2D-PSR technique with Total Variation method are designed to attain even higher SNR.

The paper is organized as follows. In Section 2, we describe the modified 2D-PSR system and its output probability density function which is derived by solving the corresponding Fokker-Planck Equation [Risken (1989)]. In Section 3, we will introduce how to use the theory discussed in Section 2 to improve SNR gain. Some experimental results of degraded image processing by our proposed technique and the comparison with other methods are presented in Section 4. Finally, in Section 5 we will draw the conclusions of 2D-PSR technique and its prospect.

2 Modified Two-dimensional SR system and corresponding Fokker-Planck Equation

In Ref. [Yang, Jiang, Xu, and Repperger (2009)], we proposed a two-dimensional SR system

$$\frac{\partial^2 w}{\partial x \partial y} = -\gamma\left(\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y}\right) - \frac{\partial \overline{U}}{\partial w} + \overline{\Gamma}(x, y) \tag{1}$$

with $w = w(x,y)$ the state variable (also taken as the system output), $\gamma(\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y})$ $(\gamma > 0)$ the damping term, $\overline{U} = -\frac{1}{2}\overline{a}w^2 + \frac{1}{4}\overline{b}w^4 - \overline{h}w$ the double-well potential where $\overline{h}(x,y)$ is the original signal and $\overline{a}, \overline{b}$ are the system parameters to be adjusted, $\overline{\Gamma}(x,y)$ the Gaussian white noise with

$$\left\langle \overline{\Gamma}(x,y)\overline{\Gamma}(x_1,y_1) \right\rangle = 2\overline{D}\delta(x-x_1, y-y_1). \tag{2}$$

Here $\overline{D}$ is the noise intensity [Zhu (1992)], which is related to noise variance σ^2 by $\sigma^2 = \frac{2\cdot\overline{D}}{\Delta x \cdot \Delta y}$ in the two-dimensional case, where Δx, Δy are the sampling intervals along horizontal and vertical directions.

Recently, we have found that Eq. 1 will suffer from oscillation when approaching steady state due to the damping term γ, which is unacceptable in image processing. In fact, for large damping term γ we can neglect the second derivative in Eq. 1 [Risken (1989)]. Thus Eq. 1 can be rewritten as

$$-\left(\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y}\right) + \frac{\overline{f}(w)}{\gamma} + \frac{\overline{\Gamma}(x,y)}{\gamma} = 0. \tag{3}$$

Here $\overline{f}(w) = -\frac{\partial \overline{U}}{\partial w} = \overline{a}w - \overline{b}w^3 + \overline{h}$. For convenience, we replace $\frac{\overline{a}}{\gamma}, \frac{\overline{b}}{\gamma}, \frac{\overline{h}}{\gamma}$ by a, b, h, and let

$$\langle \Gamma(x,y)\Gamma(x_1,y_1) \rangle = \frac{2\overline{D}}{\gamma} \cdot \delta(x-x_1, y-y_1) = 2D \cdot \delta(x-x_1, y-y_1). \tag{4}$$

Eq. 3 can be simplified to

$$\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} = aw - bw^3 + h + \Gamma(x,y). \tag{5}$$

Eq. 5 is an over-damped equation, which makes it free of annoying oscillation. The effect is shown in Fig. 2.

According to the characteristic method in Partial Differential Equation (PDE) theory [Courant and Hilbert (1953)], Eq. 5 is equivalent to a set of Ordinary Differential Equations (ODE)

$$\frac{dx}{1} = \frac{dy}{1} = \frac{dw}{aw - bw^3 + h + \Gamma}. \tag{6}$$

The characteristic line is $\frac{dy}{dx} = 1$ or $y = x + C$ with C being the constant. This indicates in any arbitrarily small region, the solution of Eq. 5 is symmetric along the diagonal direction. Thus we can solve Eq. 5 by independently dealing with Eq. 6

$$\begin{cases} \frac{dw}{dx} = aw - bw^3 + h(x) + \Gamma(x) \\ \frac{dw}{dy} = aw - bw^3 + h(y) + \Gamma(y) \end{cases} \tag{7}$$

a b c

Figure 2: (a) Original image (b) Image processed by Eq. 1, due to damping term γ, there exists oscillation when approaching steady state. (c) Image processed by Eq. 5 which is an over-damped equation

The corresponding Fokker-Planck Equation (FPE) [Risken (1989)] for Eq. 7-1 is

$$\frac{\partial \rho(w,x)}{\partial x} = -\frac{\partial}{\partial x}[f(w)\cdot\rho(w,x)] + \frac{D}{\Delta x}\cdot\frac{\partial^2 \rho(w,x)}{\partial w^2}, \tag{8}$$

where $\rho(w,x)$ is the output probability density function (pdf). When $x \to \infty$, we can obtain the static pdf of Eq. 8

$$\rho(w,x\to\infty) = \rho_s(w) = C\cdot\exp[\varphi_0(w)] = C\cdot\exp\left[\int_{-\infty}^{+\infty}\Delta x\cdot\frac{f(w)}{D}dw\right]. \tag{9}$$

Here C is the normalized factor.

We can also deduce the asymptotic dynamic pdf of Eq. 8 as follows [Li and Xu (2006)]

$$\rho(w,x) = \sum_{i=0}^{n-1} C_i\cdot\Phi_i(w)\cdot\exp(-\lambda_i\cdot x) + \left[\rho_s(w) - \sum_{i=0}^{n-1} C_i\cdot\Phi_i(w)\right]\cdot\exp(-\lambda_n\cdot x), \tag{10}$$

where $0=\lambda_0<\lambda_1<...<\lambda_n$ and $\Phi_0=\exp[\varphi_0(w)], \Phi_1,...,\Phi_n$ are the eigenvalues and eigenfunctions of Eq. 7-1, C_i is constant to be determined by orthogonal condition of eigenfunctions [Xu, Li, and Zheng (2003); Pichler and Mang (2000)]. We regard λ_1 which is the dominant factor of system's settling down to steady state as the system response speed.

The static and dynamic pdf of Eq. 7-2 can be derived similarly.

3 Theory of Two-dimensional Parameter-induced Stochastic Resonance (2D-PSR) for degraded image processing

In previous investigations [Yang, Jiang, Xu, and Repperger (2009)], we set λ_1 to be around 3 so that the error between dynamic pdf $\rho(w,x)$ and static pdf $\rho_s(w)$ is within $e^{-3} \approx 5\%$. However, this restriction will lessen the available choice for parameter a,b. Therefore, we introduce the concept of dynamic signal-to-noise ratio (DSNR)

$$DSNR(a,b) = \frac{E[w]}{\sqrt{Var[w]}}, \tag{11}$$

where $E[\bullet]$ is an expectation operator and $Var[\bullet] = E[\bullet^2] - E^2[\bullet]$.

Without the restriction of $\lambda_1 \approx 3$, the system response speed can either be greater or smaller. Consequently the valid sample points of the output will vary following the value of λ_1. Previously, when processing a noisy image through 2D-PSR system, we just directly pick up the last sample point of each sample period, which is assumed to have the best statistic characteristic in the means of probability density. According to the dynamic solution, when the system response speed grows faster, the output performance will decrease, however, there will be more sample points valid to be considered. If these available sample points are averaged, we might get a better result.

The statistic characteristics of averaged outputs can be calculated by the theory of local average random field [Vanmarcke (1983); Manjuprasad and Manohar (2007)]. Assume $w(x)$ is a random field with expectation m and variance σ^2. $W_X(x)$ is length average of $w(x)$ over a period X. Here $W_X(x)$ is called the local average random field, which has expectation and variance:

$$E\left[W_X(x)\right] = m = \int w \cdot \rho(w,x)dw, \tag{12}$$

$$Var\left[W_X(x)\right] = \Omega(X) \cdot \sigma^2, \tag{13}$$

where $\Omega(X)$ is called the variance function of $W_X(x)$.

Let $\rho(\xi)$ be the normalized correlation function of $w(x)$

$$\rho(\xi) = \frac{Cov(\xi)}{\sigma^2}. \tag{14}$$

The relationship between $\Omega(X)$ and $\rho(\xi)$ can be described as follows

$$\begin{aligned} &\Omega(X) = \tfrac{1}{X^2}\int_0^X \int_0^X \rho(x_1 - x_2)dx_1 dx_2 \\ &= \tfrac{1}{X^2}\int_{-X}^X (X - |\xi|)\rho(\xi)d\xi \\ &= \tfrac{2}{X}\int_0^X \left(1 - \tfrac{\xi}{X}\right)\rho(\xi)d\xi \end{aligned} \tag{15}$$

In order to obtain variance function $\Omega(X)$ from Eq. 15, we have to figure out the second-order statistic characteristics of $w(x)$. To this end, we rewrite Eq. 8 as

$$\frac{\partial\rho(w,x|w_0,x_0)}{\partial x} = -\frac{\partial}{\partial x}\left[f(w)\cdot\rho(w,x|w_0,x_0)\right] + \frac{D}{\Delta x}\cdot\frac{\partial^2\rho(w,x|w_0,x_0)}{\partial w^2}, \tag{16}$$

where $\rho(w,x|w_0,x_0)$ is the conditional pdf which satisfies the following initial condition

$$\rho(w,x_0|w_0,x_0) = \delta(w-w_0), \tag{17}$$

and

$$\lim_{x\to\infty}\rho(w,x|w_0,x_0) = \rho(w,x). \tag{18}$$

Similar to Eq. 8, the first-order approximate solution of Eq. 16 is

$$\rho(w,x|w_0,x_0) = \rho(w,x) + \left[\delta(w-w_0) - \rho(w,x)\right]\cdot\exp\left[-\lambda_1\cdot(x-x_0)\right], \tag{19}$$

with λ_1 the system response speed. Thus the covariance function of $w(x)$ can be written as

$$\begin{aligned} &Cov(\xi) = \int\int w'\cdot w\cdot\rho(w',x+\xi|w,x)\cdot\rho(w,x)dw'dw \\ &-\int w\cdot\rho(w,x)dw\cdot\int w'\cdot\rho(w',x)dw' \\ &= \sigma^2\cdot\exp(-\lambda_1\cdot|\xi|) \end{aligned} \tag{20}$$

where

$$\sigma^2 = \int w^2\cdot\rho(w,x)dw - E^2[w]. \tag{21}$$

Substituting Eq. 14, Eq. 20, Eq. 21 into Eq. 15 we finally come up with the variance function

$$\Omega(X) = \frac{2}{X}\int_0^X\left(1-\frac{\xi}{X}\right)\exp(-\lambda_1\xi)d\xi. \tag{22}$$

Replacing $E[w], Var[w]$ in Eq. 11 with Eq. 12, Eq. 13, Eq. 22, we obtain

$$DSNR(a,b) = \frac{\int w\cdot\rho(w,x)dw}{\frac{2}{X}\int_0^X\left(1-\frac{\xi}{X}\right)\exp(-\lambda_1\xi)d\xi\cdot\left\{\int w^2\cdot\rho(w,x)dw - \left[\int w\cdot\rho(w,x)dw\right]^2\right\}}. \tag{23}$$

Here X is determined as follows: define an allowance error: err. Let

$$\exp(-\lambda_1\cdot\xi) = err \Rightarrow \xi = -\frac{1}{\lambda_1}\ln err. \tag{24}$$

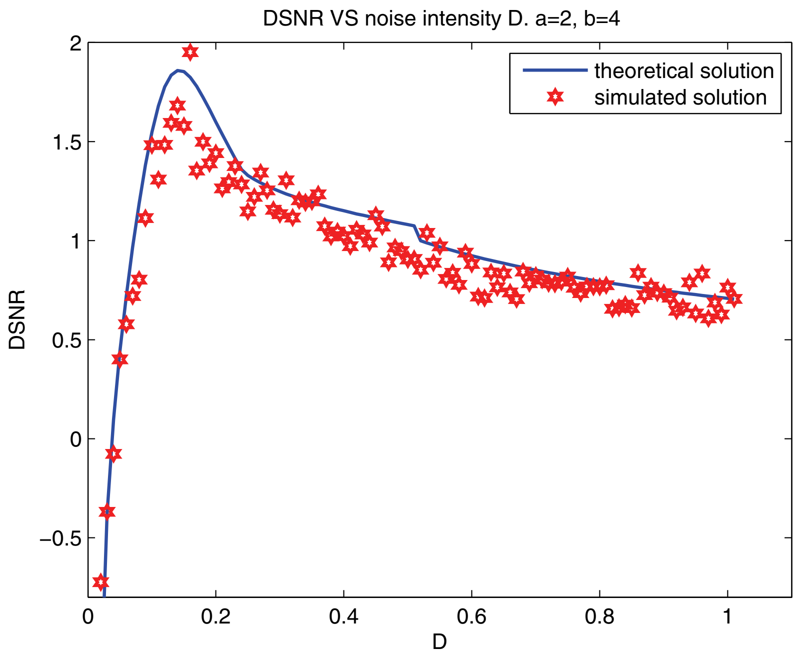

Figure 3: Output DSNR as a function of noise intensity D

If $\xi \geq X_b$ (here X_b means each length period), we directly use the last sample point to calculate the DSNR. If $\xi < X_b$, we take $X = X_b - \xi$. Our goal is to maximize DSNR by optimizing system parameters a, b, which can solved by gradient descent algorithm [Nocedal and Wright (1999)] very efficiently. Fig. 3 shows the dynamic signal-to-noise ratio as a function of noise intensity, with solid line the theoretical solution based on Eq. 23 and hexagon the simulated solution. As the noise intensity grows larger, we can see DNSR reaches a single peak before descending, which means disordered noise can enhance ordered system output. This phenomenon is called Stochastic Resonance.

It has been proved that the output SNR cannot surpass input SNR under linear systems [Reid (1983)]. Or equivalently, linear systems cannot have SNR gain greater than one. The input SNR in our case can be written as

$$SNR_{input} = \frac{h}{\sqrt{2 \cdot D}}, \tag{25}$$

with h the original image and D the noise intensity. However, in the case of 2D-PSR system, we can obtain an SNR gain greater than one if the parameters a, b are optimized. The SNR gain can be defined as

$$SNR_{gain} = \frac{DSNR(a,b)}{SNR_{input}} = \frac{\sqrt{2\cdot D}\cdot\int w\cdot\rho(w,x)dw}{\frac{2\cdot h}{X}\int_0^X\left(1-\frac{\xi}{X}\right)\exp\left(-\lambda_1\xi\right)d\xi\cdot\left\{\int w^2\cdot\rho(w,x)dw - E^2\left[w\right]\right\}}. \tag{26}$$

Fig. 4 shows the SNR gain as a function of parameter b, where $a = 1.5$ in the former figure and $a = 2$ in the latter one. It indicates when system parameters a, b are properly set, the SNR gain will surpass one, which is impossible for linear filters. In addition, the SNR gain remains high and descending slowly as the parameter b grows larger, which means 2D-PSR technique performs strong robustness to system parameters variations. If the parameters are not best optimized or even biased seriously, we can still obtain high SNR gain through 2D-PSR system.

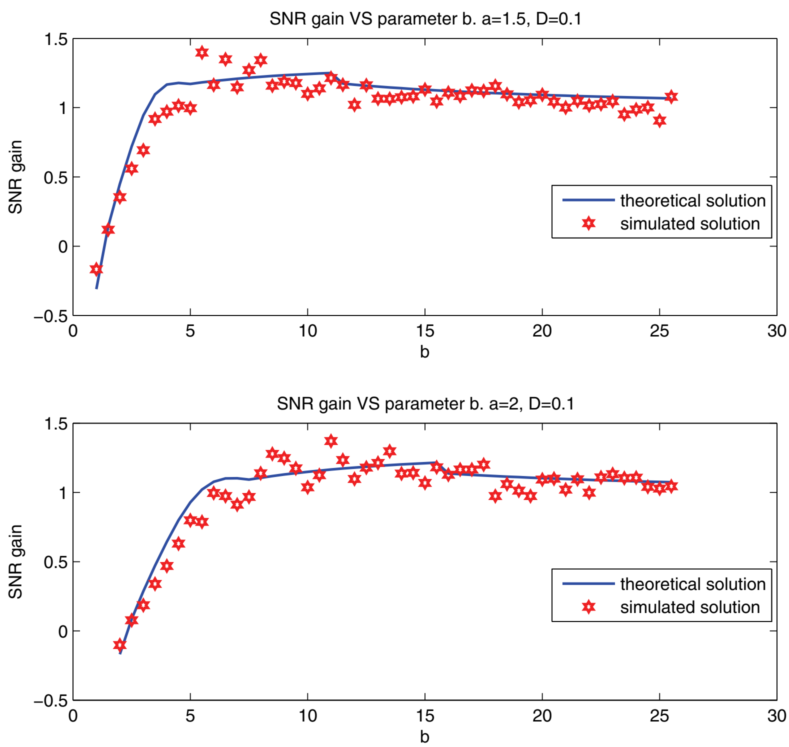

Figure 4: SNR Gain as a function of parameter b

4 Experimental results of 2D-PSR image processing

In practice, the evaluation of output performance equivalent to Eq. 23 can be written as

$$SNR = 10 \cdot \lg \left[\frac{P(\text{img}_{out})}{P(\text{noise}_{out})} \right]. \tag{27}$$

Here $P(\text{img}_{out})$ and $P(\text{noise}_{out})$ are respectively the power of image information and noise under certain filtering system, such as 2D-PSR system and low pass filtering system.

The simulation equation equivalent to Eq. 7 is [Benito, Urena, Gavete, and Alonso (2008)]

$$\begin{cases} w_{m,n} = \left[a w_{m,n-1} - b w_{m,n-1}^3 + h_{m,n-1} + \Gamma_{m,n-1} \right] \cdot \Delta x + w_{m,n-1} \\ w_{m,n} = \left[a w_{m-1,n} - b w_{m-1,n}^3 + h_{m-1,n} + \Gamma_{m-1,n} \right] \cdot \Delta y + w_{m-1,n} \end{cases}, \tag{28}$$

where the subscripts represent the locations of sample points, and $\Delta x, \Delta y$ are the sampling intervals along horizontal and vertical directions.

The intensity value of an image usually distributes in the region of $[0, 255]$. However, the double-well potential of 2D-PSR system is symmetric to zero. Thus we should first subtract the mean value of an image before processing with 2D-PSR technique and add it back later. Fig. 5 (a) shows a computed tomography (CT) image corrupted by additive Gaussian white noise $N(0, 57)$ (Fig. 5 (b)). We sample the degraded image 5×5 times per pixel (five by row and five by column), and then operate it on 2D-PSR system. After optimizing parameters a, b according to Eq. 23 we come up with the recovered image Fig. 5 (c). Fig. 5 (d) shows the result obtained by linear mean filtering, which takes the average value of every 5×5 blocks of Fig. 5 (b). As a comparison, we have further processed Fig. 5 (b) with total variation method [Weickert (1996)], wavelet de-noising [Gonzalez, Woods, and Eddins (2004)] and adaptive Wiener filter [Lim (1990)]. The results are shown respectively in Fig. 5 (e), (f) and (g).

Another advantage of 2D-PSR system, due to nonlinearity, is that it can enhance the contrast of a dim noisy image along with de-noising. Fig. 6 (a) shows an image corrupted by multiplicative Erlang noise [Gonzalez, Woods, and Eddins (2004)] with parameter $A = B = 1$. Fig. 6 (b) shows the result obtained by 2D-PSR technique. Compared with other methods (Fig. 6 (c), (d), (e) and (f)), we can see besides noise reduction, the contrast of the recovered image by 2D-PSR technique is enhanced. Thus some information in the shadow can be recognized such as the cameraman's right hand and details of the jacket. Sometimes such information might be extremely useful.

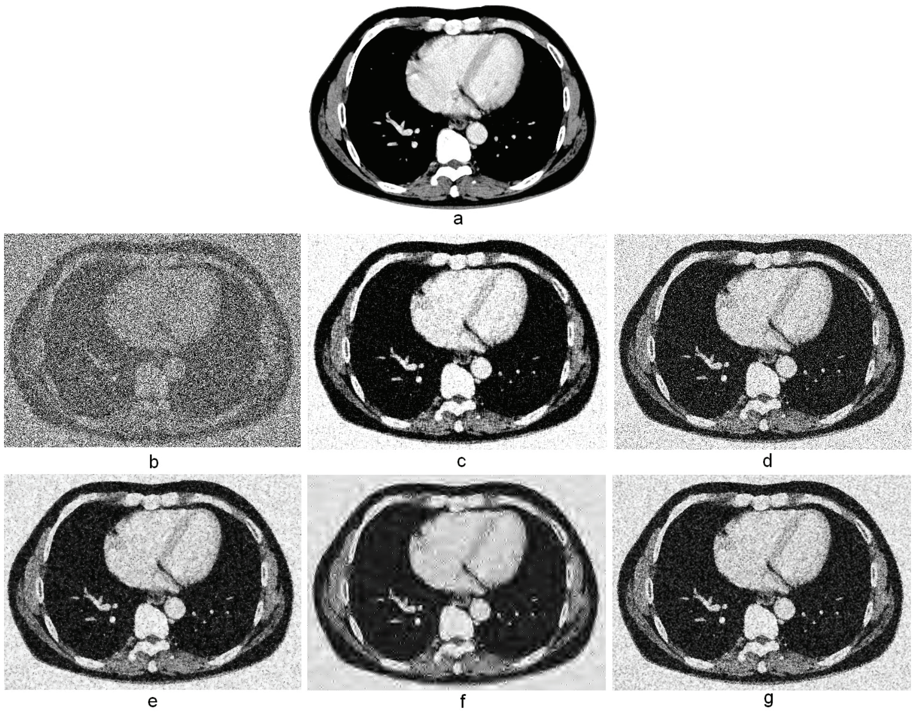

Figure 5: (a) Original CT image. (b) Sampled CT image degraded by additive Gaussian white noise. (c) Image processed by 2D-PSR system, with parameters $a = 0.1, b = 1.1$, the output $SNR = 11.06dB$. (d) Image processed by mean filter, the output $SNR = 5.59dB$. (e) Image processed by total variation method, the output $SNR = 10.45dB$. (f) Image processed by wavelet de-noising, the output $SNR = 10.12dB$. (g) Image processed by adaptive Wiener filter, the output $SNR = 9.72dB$.

In the above discussions, the 2D-PSR system we have presented did not make use of the priori information of the input image. In order to exploit such information, we propose an adaptive 2D-PSR systems. According to Eq. 23 and Eq. 24, once the parameters a, b are optimized, the number of valid sampling points is fixed. However, frequencies in the image may differ from area to area. To address this issue, we define an edge factor

$$K = f(|\nabla g(h + \Gamma)|_{x,y}) \tag{29}$$

as a criterion to conduct down-sampling. $g(\bullet)$ is the Gaussian low-pass filtering operator which is used to remove noise in the input image. $f(\bullet)$ is a mapping function to control the number of sample points. Large K means fast varying in-

Figure 6: (a) Sampled cameraman image degraded by multiplicative Erlang noise. (b) Image processed by 2D-PSR system, with parameters $a = -0.2, b = 0.7$, the output $SNR = 15.32dB$. (c) Image processed by mean filter, the output $SNR = 7.56dB$. (d) Image processed by total variation method, the output $SNR = 14.67dB$. (e) Image processed by wavelet de-noising, the output $SNR = 14.21dB$. (f) Image processed by adaptive Wiener filter, the output $SNR = 13.84dB$. (g) Image processed by adaptive 2D-PSR technique, the output $SNR = 15.57dB$. (h) Image processed by a combination of adaptive 2D-PSR system with Total Variation method, the output $SNR = 17.17dB$.

formation in the original image, thus we take the last few sample points of each pixel to retain more details, while small K means slowly varying in the image, thus we take more valid sample points of each pixel to achieve high static SNR. The performance of adaptive 2D-PSR system is shown in Fig. 6 (g).

Besides, 2D-PSR technique can be combined with other nonlinear methods to further improve the output performance. Fig. 6 (h) shows the example of degraded cameraman image recovered by adaptive 2D-PSR technique combined with Total Variation method. Compared with Fig. 6 (d) and (g), image processed by a combination of the two methods performs better than only by single one. Therefore, 2D-PSR technique and TV method are complementary.

5 Conclusions

In this paper, a modified 2D-PSR system is proposed, which is over-damped thus free of oscillation when approaching steady state. The corresponding static and dynamic pdf of the Fokker-Planck Equation is derived. A new concept of dynamic signal-to-noise ratio (DSNR) is introduced to utilize more valid sample points to upgrade the output performance. According to our theoretical calculation, the output DSNR manifests a resonant-like behavior when system parameters are properly adjusted. The experimental results are in accordance with theoretical solution.

In practice, images processed by 2D-PSR technique are confirmed to achieve higher SNR than by linear mean filtering and other mainstream image processing methods. In order to utilize a priori information of the original image, we recommend an adaptive 2D-PSR system that is able to maintain more details. Moreover, 2D-PSR technique is complementary to other nonlinear methods (such as Total Variation methods) and can be combined with them to obtain even higher SNR gain. We believe the 2D-PSR technique to be promising in image processing.

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